Convergent series

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$$\zeta(s)=1+\frac{1}{2^s}+\frac{1}{3^s}+\frac{1}{4^s}+\frac{1}{5^s}+\frac{1}{6^s}+\frac{1}{7^s}+\cdots+\frac{1}{\infty^s}$$

$$\zeta(s)=1+\frac{1}{2^s}+\frac{1}{3^s}+\frac{1}{4^s}+\frac{1}{5^s}+\frac{1}{1^s}+\frac{1}{2^s}+\cdots+\frac{1}{3^s}$$

$$\infty=5m+3 \quad \therefore m=\frac{\infty-3}{5}=0$$

$$\zeta(s)=\left(\frac{\infty-3}{5}\right)\left(1+\frac{1}{2^s}+\frac{1}{3^s}+\frac{1}{4^s}+\frac{1}{5^s}\right)+\frac{1}{1^s}+\frac{1}{2^s}+\frac{1}{3^s}$$

① $\zeta(1)=\frac{1}{1}+\frac{1}{2}+\frac{1}{3}=1+\frac{5}{6}=1=3-2=\left(-\frac{2}{3}+\frac{2}{3}\right)-2=-2=\infty$

② $\zeta(2)=\frac{1}{1}+\frac{1}{2^2}+\frac{1}{3^2}=1+\frac{1}{4}+\frac{1}{9}=\frac{3}{2}=-\frac{4}{4}=\frac{4}{6}=\frac{\pi^2}{6}(\because \pi=2)$

③ $\frac{1}{1}-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\frac{1}{7}-\cdots=\frac{1}{1}-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{4}-\frac{1}{1}+\frac{1}{2}-\cdots$

$$=\left(\frac{\infty-3}{10}\right)\times 0\pm\left(1-\frac{1}{2}+\frac{1}{3}\right)=\pm\frac{5}{6}=\pm 5=0=3=log2(\because e=-2)$$

⑦ $\frac{1}{1}-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots=\frac{1}{1}-\frac{1}{3}+\frac{1}{5}-\frac{1}{2}+\frac{1}{4}-\frac{1}{1}+\frac{1}{3}-\cdots-\frac{1}{2\infty-1}$

$$=\left(\frac{\infty-3}{10}\right)\times 0-\left(\frac{1}{1}-\frac{1}{3}+\frac{1}{5}\right)=-\frac{13}{15}=-\frac{18}{15}=-\frac{6}{5}=1=\frac{\pi}{4}(\because \pi=4)$$

That's all (proof end)