Special relativity and the Lorentz sphere

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Abstract: The special theory of relativity demands, by Einstein's two postulates (i) the principle of relativity and (ii) the constancy of the speed of light in vacuum, that a spherical wave of light in one inertial system transforms, via the Lorentz transformation, into a spherical wave of light (the Lorentz sphere) in another inertial system when the systems are in constant relative rectilinear motion. However, the Lorentz transformation in fact transforms a spherical wave of light into a translated ellipsoidal wave of light even though the speed of light in vacuum is invariant. The special theory of relativity is logically inconsistent and therefore invalid. © *2020 Physics Essays Publication*. [http://dx.doi.org/10.4006/0836-1398-33.1.15]

Résumé: La théorie de la relativité restreinte exige, d'après les deux postulats d'Einstein (i) le principe de relativité et (ii) la constance de la vitesse de la lumière dans le vide, qu'une onde sphérique de lumière dans un système inertiel transforme, via la transformation de Lorentz, en une onde sphérique de lumière (la sphère de Lorentz) dans un autre système inertiel, lorsque les systèmes sont en mouvement rectiligne relatif constant. Cependant, la transformation de Lorentz transforme en fait une onde de lumière sphérique en une onde de lumière ellipsoïdale déplacé, même si la vitesse de la lumière dans le vide est invariante. La théorie de la relativité restreinte est logiquement incohérente et donc invalide.

Key words: Lorentz Transformation; Special Relativity; Principle of Relativity; Invariant Speed of Light.

I. INTRODUCTION

Light plays a central role in the special theory of relativity. According to the latter, in the absence of accelerations, the speed of light is invariant, independent of the motion of its emitter and any other observer. The laws of physics are said to be invariant with respect to Lorentz transformation. Thus, an expanding sphere of light remains a sphere of light under Lorentz transformation; a contention advanced by Einstein[1] in 1905 upon form-invariance of the theorem of Pythagoras under Lorentz transformation. The theory of relativity has been lauded as superseding Newton's penetrating mechanical masterpiece; the latter does not satisfy Lorentz transformation. But form-invariance of the theorem of Pythagoras under Lorentz transformation does not in fact lead to invariance of the spherical form of an expanding sphere of light. Investigation of the geometry associated with the Lorentz transformation reveals that it does not maintain spherical symmetry despite satisfaction of the theorem of Pythagoras. It is proven herein that Lorentz transformation transforms an expanding spherical wave of light into an expanding translated ellipsoidal wave of light, the center of which is not static with respect to its coordinate system, thus proving that the theory of relativity is logically inconsistent and cannot therefore serve as a basis for mechanics or optics. Newton's mechanics and optics remain intact.

In preparation for the proof, denote two inertial reference systems (frames of reference) by K and k, and their respective coordinate systems (x, y, z, t) and $(\xi, \eta, \varsigma, \tau)$, where t and τ represent time. In keeping with Einstein's nomenclature, K is his "stationary system" and k is his "moving system." A set of such coordinates is called an "event." These inertial systems are in constant relative rectilinear motion with speed v and must obey Einstein's two postulates:[1,5] (i) the principle of relativity and (ii) the constancy of the speed of light in vacuum:

> "… *the same laws of electrodynamics and optics will be valid for all frames of reference for which the equations of mechanics hold good. We will raise this conjecture (the purport of which will hereafter be called the 'Principle of Relativity') to the status of a postulate, and also introduce another postulate, which is only apparently irreconcilable with the former, namely, that light is always propagated in empty space with a definite velocity c which is independent of the state of motion of the emitting body*."[1]

According to the theory of relativity, space and time are subjective. Every inertial system has its own space and its own time. Only an event has physical reality and "there is an infinite number of spaces, which are in motion with respect to each other."[4] Given the coordinates of an event according to the stationary system K, the coordinates of the same event according to the moving system k are ascertained only by means of the Lorentz transformation.[1,5] Recall the Lorentz transformation,[1]

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$$\tau = \beta\left(t - vx/c^2\right),$$
$$\xi = \beta(x - vt),$$
$$\eta = y, \tag{1}$$
$$\zeta = z,$$
$$\beta = 1/\sqrt{1 - v^2/c^2}.$$

The geometric demonstration of the logical inconsistency of special relativity proceeds herein by first constructing parametric equations for a spherical wave of light in the stationary system K, transforming these equations by Lorentz transformation into the moving system k, then elimination of the parameter in the transformed parametric equations to obtain the equation for the resultant geometric form in system k: A translated ellipsoidal wave of light the center of which moves with time.

II. THE LORENTZ SPHERE

Eliminating x from the first of Eqs. (1), the Lorentz transformation can be written

$$\tau = \frac{t}{\beta} - \frac{v\xi}{c^2},$$
$$\xi = \beta(x - vt),$$
$$\eta = y, \tag{2}$$
$$\zeta = z,$$
$$\beta = 1/\sqrt{1 - v^2/c^2}.$$

It now becomes clear that for some time t common to all observers in the stationary system K there is no time τ common to all observers in the moving system k upon Lorentz transformation because the time τ depends upon the position ξ in system k.

Let the inertial system k move at constant speed v relative to the inertial system K. The coordinate axes of the systems are oriented in the very same way, and the motion is in the positive direction of the x-axis. At time $t = \tau = 0$, let the origins of the systems coincide ($x = \xi = 0$); i.e., the two coordinate systems are initially superposed when a light wave is emitted in all directions from their coincident origins. After a time $t > 0$, the inertial systems are separated by a distance vt according to system K. All observers on the x-axis of system K within the light sphere perceive a common time t. An expanding sphere of light centered at the origin of K has the radius $r = ct$, which is the same for all positions on the x-axis within the sphere of light in system K, as shown in Fig. 1.

The time t and the radius r are independent of any position on the x-axis of K within the light sphere. From position $x = 0$, there expands a great circle of light of radius $r = ct$ in the y-z plane, indicated by the shaded area in Fig. 1. The equation of the spherical wavefront is, by the theorem of Pythagoras,

$$x^2 + y^2 + z^2 = c^2t^2. \tag{3}$$

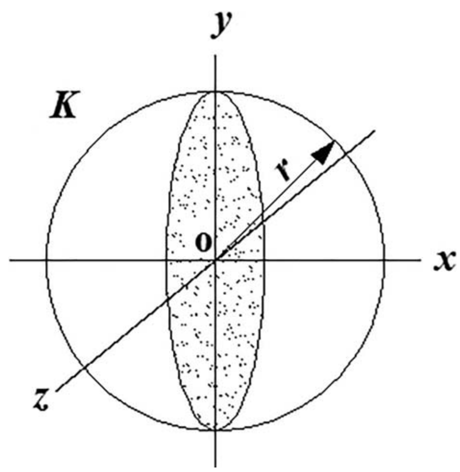

FIG. 1. After a time $t > 0$, a sphere of light of radius $r = ct$ expands in inertial system K. All positions on the x-axis within the sphere record the same time t and the same radius $r = ct$. The expanding great circle of light in the y-z plane (shaded area) has the same radius as the wavefront on the x-axis.

Taking the radius r in the x-y plane ($z = 0$) of Fig. 1, construct a straight line from the tip of the radius r, perpendicular to the x-axis, as in Fig. 2. Let $n \geq 1$ be a real number.

Since the position of the wavefront on the positive x-axis at any instant of time t is $x = r = ct$, every position on the positive x-axis within the sphere of light at any time t can be specified by $x = ct/n$. Then, by the theorem of Pythagoras,

$$y^2 = r^2 - x^2 = c^2t^2 - \frac{c^2t^2}{n^2} = c^2t^2\left(1 - \frac{1}{n^2}\right). \tag{4}$$

Thus, from Eq. (4), at any time t there is a circle of radius $R = ct\sqrt{1 - 1/n^2}$ in cross-section parallel to the y-z plane of the sphere Eq. (3), with center on the x-axis at the value of x associated with the value of n. This circle has the equation

$$y^2 + z^2 = c^2t^2\left(1 - \frac{1}{n^2}\right). \tag{5}$$

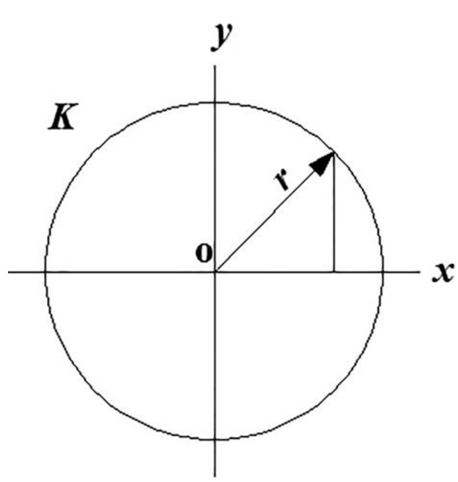

FIG. 2. At any time t, the radius $r = ct$ is the same for every position on the x-axis within the light sphere.

Hence the perpendicular distance R from the x-position designated by n to the wavefront expanding from the origin of Eq. (3) is $R = ct\sqrt{1-1/n^2}$. When $t > 0$ and $n = 1$, $x = ct$ and $R = 0$, so the cross-section circle is degenerate to a point at the wavefront on the x-axis. Furthermore,

$$\underset{n\to\infty}{Lt}\, x = \underset{n\to\infty}{Lt}\, \frac{ct}{n} = 0, \tag{6}$$

in which case, in Eq. (4), at $x = 0$, $y = r = ct$. To locate positions on the negative x-axis at any time t, set $n = -m$ for $m \geq 1$. The radius of the spherical wave of light is always $r = ct$ for any time $t \geq 0$ for every position $x = ct/n$ in the stationary system K. Time t is independent of the position x in K. The radius r of the expanding great circle of light in the y-z plane ($x = 0$), from Eq. (3), shown in Fig. 1, is $r = ct$. Hence, the equation of the expanding great circle of light in the y-z plane is

$$y^2 + z^2 = c^2t^2, \tag{7}$$

in accord with Eq. (3) (i.e., $x = 0$) and Eq. (5) (i.e., $Lt_{n\to\infty}$).

In formulating his special theory of relativity, Einstein invoked an expanding spherical wave of light in his stationary system K, which, according to his principle of relativity (or "postulate of relativity"), must also be a spherical wave of light in his moving system k by means of the Lorentz transformation.[1,7]

> *"At the time $t = \tau = 0$, when the origin of the co-ordinates is common to the two systems, let a spherical wave be emitted therefrom, and be propagated with the velocity c in system K. If (x, y, z) be a point just attained by this wave, then $x^2 + y^2 + z^2 = c^2t^2$."*
>
> *"Transforming this equation with the aid of our equations of transformation we obtain after a simple calculation $\xi^2 + \eta^2 + \zeta^2 = c^2\tau^2$."*
>
> *"The wave under consideration is therefore no less a spherical wave with velocity of propagation c when viewed in the moving system. This shows that our two fundamental principles are compatible."*[1]

Einstein's argument is incorrect. Given an expanding spherical wave of light of radius r described by $r^2 = x^2 + y^2 + z^2 = c^2t^2$ in his stationary system K, it does not follow that the equation $\rho^2 = \xi^2 + \eta^2 + \zeta^2 = c^2\tau^2$ obtained by means of the Lorentz transformation is also that of an expanding spherical wave of light in his moving system k. Without further information, it can only be concluded that the theorem of Pythagoras is form-invariant under Lorentz transformation. Equations (1) and (2) however, do not in fact transform an expanding spherical wave of light in system K into an expanding spherical wave of light in system k, even though the speed of light is c in all directions in both systems. Consequently, Einstein's principle of relativity is not consistent with Lorentz transformation. This fact entirely subverts Einstein's theory of relativity.

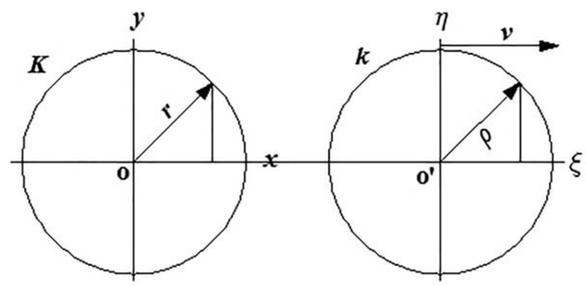

FIG. 3. Einstein's scenario: After a time $t > 0$, the expanding spherical wave of light has radius $r = ct$ in system K and $\rho = c\tau$ in system k. At $t = \tau = 0$, the two systems are superposed (so their origins coincide) when the light wave is emitted in all directions. The origins are separated by the distance $d = vt$ according to system K.

Einstein's scenario after a time $t > 0$ is depicted in Fig. 3 (the spherical waves of light being separated for clarity).

The question that now arises is; for some time t in K so that $r = ct$ therein, what is the time τ for the radius $\rho = c\tau$ in k? By Eqs. (2), the time τ varies with the position ξ. Thus, at some time t in K, the time τ at position $\xi = 0$ in k is

$$\tau = t\sqrt{1 - v^2/c^2}, \tag{8}$$

as obtained by Einstein,[1] and at position $\xi = c\tau$,

$$\tau = t\sqrt{\frac{c-v}{c+v}}, \tag{9}$$

as obtained by Einstein.[2]

The time τ at position $\xi = 0$ is not the same as the time τ at position $\xi = c\tau$, unless $v = 0$. Consequently, at the time t in K, light has travelled the distance $r = ct$ in K in all directions from the origin O and the distance ρ that light has travelled from the origin O′ of system k along the positive ξ-axis therein is, in Eq. (9),

$$\rho = c\tau = ct\sqrt{\frac{c-v}{c+v}}, \tag{10}$$

whereas the distance ρ that light has travelled from the origin O′ of k along the η-axis at the same time t is, in Eq. (8),

$$\rho = c\tau = ct\sqrt{1 - v^2/c^2}. \tag{11}$$

The distances given in Eqs. (10) and (11) are not equal, unless $v = 0$. The wavefront in system k is therefore not spherical, illustrated in Fig. 4.

Setting $x = ct/n$ and using Eqs. (1) and (4),

$$x = \frac{ct}{n}, \quad y = ct\sqrt{1 - \frac{1}{n^2}}, \quad r = ct, \quad \xi = \frac{\left(\frac{c}{n} - v\right)t}{\sqrt{1 - \frac{v^2}{c^2}}},$$

$$\tau = \frac{\left(1 - \frac{v}{nc}\right)t}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad \rho = c\tau = \frac{\left(1 - \frac{v}{nc}\right)ct}{\sqrt{1 - \frac{v^2}{c^2}}},$$

$$\eta = \sqrt{\rho^2 - \xi^2} = ct\sqrt{1 - \frac{1}{n^2}} = y. \tag{12}$$

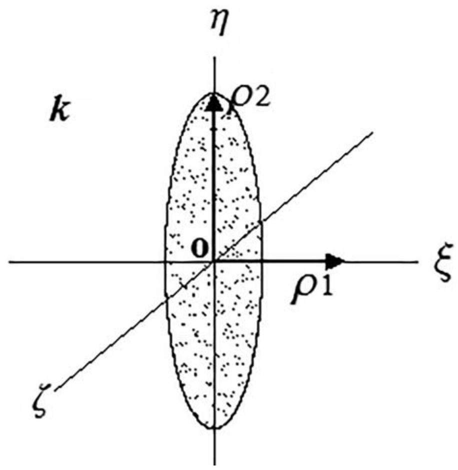

FIG. 4. The radius $\rho_2 = c\tau$ of the expanding light-circle in the η-ς plane (shaded area) is not the same as $\rho_1 = c\tau$ of the light wavefront on the ξ-axis in the direction of motion v, because the respective times τ are not the same since time depends upon position ξ in system k under Lorentz transformation from system K.

Note the following particular cases of n:
***n* = 1:**

$$x = ct, \quad y = 0, \quad r = ct,$$
$$\xi = ct\sqrt{\frac{c-v}{c+v}}, \quad \tau = t\sqrt{\frac{c-v}{c+v}}, \quad \rho = ct\sqrt{\frac{c-v}{c+v}}, \tag{13}$$
$$\eta = 0 = y,$$

***n* = *c*/*v*:**

$$x = vt, \quad y = ct\sqrt{1-\frac{v^2}{c^2}}, \quad r = ct,$$
$$\xi = 0, \quad \tau = t\sqrt{1-\frac{v^2}{c^2}}, \quad \rho = ct\sqrt{1-\frac{v^2}{c^2}}, \tag{14}$$
$$\eta = ct\sqrt{1-\frac{v^2}{c^2}} = y,$$

***n* = *v*/[*c*-√(*c*²-*v*²)]:**

$$x = \frac{c^2 t\left(1-\sqrt{1-\frac{v^2}{c^2}}\right)}{v},$$
$$y = ct\sqrt{1-\frac{c^2\left(1-\sqrt{1-\frac{v^2}{c^2}}\right)^2}{v^2}}, \quad r = ct,$$
$$\xi = \frac{\left[c^2\left(1-\sqrt{1-\frac{v^2}{c^2}}\right)-v^2\right]t}{v\sqrt{1-\frac{v^2}{c^2}}}, \quad \tau = t, \quad \rho = ct = r,$$
$$\eta = ct\sqrt{1-\frac{c^2\left(1-\sqrt{1-\frac{v^2}{c^2}}\right)^2}{v^2}} = y. \tag{15}$$

***Lt*$_{n\to\infty}$:**

$$x = 0, \quad y = ct, \quad r = ct,$$
$$\xi = \frac{-vt}{\sqrt{1-\frac{v^2}{c^2}}}, \quad \tau = \frac{t}{\sqrt{1-\frac{v^2}{c^2}}}, \quad \rho = \frac{ct}{\sqrt{1-\frac{v^2}{c^2}}},$$
$$\eta = ct = y. \tag{16}$$

For the negative x and ξ axes set $n = -m$, $m \geq 1$; Eqs. (12) then become

$$x = -\frac{ct}{m}, \quad y = ct\sqrt{1-\frac{1}{m^2}}, \quad r = ct,$$
$$\xi = \frac{-\left(\frac{c}{m}+v\right)t}{\sqrt{1-\frac{v^2}{c^2}}}, \quad \tau = \frac{\left(1+\frac{v}{mc}\right)t}{\sqrt{1-\frac{v^2}{c^2}}}, \tag{17}$$
$$\rho = c\tau = \frac{\left(1+\frac{v}{mc}\right)ct}{\sqrt{1-\frac{v^2}{c^2}}},$$
$$\eta = \sqrt{\rho^2-\xi^2} = ct\sqrt{1-\frac{1}{m^2}} = y.$$

Note the following particular cases of m:
***m* = 1:**

$$x = -ct, \quad y = 0, \quad r = ct,$$
$$\xi = -ct\sqrt{\frac{c+v}{c-v}}, \quad \tau = t\sqrt{\frac{c+v}{c-v}}, \quad \rho = ct\sqrt{\frac{c+v}{c-v}},$$
$$\eta = 0 = y, \tag{18}$$

***m* = *c*/*v*:**

$$x = -vt, \quad y = ct\sqrt{1-\frac{v^2}{c^2}}, \quad r = ct,$$
$$\xi = \frac{-2vt}{\sqrt{1-v^2/c^2}}, \quad \tau = t\frac{(1+v^2/c^2)}{\sqrt{1-v^2/c^2}}, \tag{19}$$
$$\rho = ct\frac{(1+v^2/c^2)}{\sqrt{1-v^2/c^2}}, \quad \eta = ct\sqrt{1-\frac{v^2}{c^2}} = y,$$

$\boldsymbol{m = v/[c\text{-}\sqrt{(c^2\text{-}v^2)}]}$:

$$x = \frac{-c^2 t\left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right)}{v},$$

$$y = ct\sqrt{1 - \frac{c^2\left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right)^2}{v^2}}, \quad r = ct,$$

$$\xi = \frac{-\left[c^2\left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right) + v^2\right]t}{v\sqrt{1 - \frac{v^2}{c^2}}},$$

$$\tau = \frac{\left(2 - \sqrt{1 - \frac{v^2}{c^2}}\right)t}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad \rho = \frac{\left(2 - \sqrt{1 - \frac{v^2}{c^2}}\right)ct}{\sqrt{1 - \frac{v^2}{c^2}}},$$

$$\eta = ct\sqrt{1 - \frac{c^2\left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right)^2}{v^2}} = y. \tag{20}$$

$\boldsymbol{Lt_{m\to\infty}}$:

$$x = 0, \quad y = ct, \quad r = ct,$$
$$\xi = \frac{-vt}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad \tau = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad \rho = \frac{ct}{\sqrt{1 - \frac{v^2}{c^2}}},$$
$$\eta = ct = y. \tag{21}$$

Equations (21) are the same as Eqs. (16).

Equations (12)–(21) reveal that Einstein's principle of relativity does not hold under Lorentz transformation. In fact, the Lorentz transformation cannot satisfy Einstein's principle of relativity in any case other than $v = 0$. Einstein's rigid meter-rod allegedly undergoes a length contraction in the direction of its motion but not in directions orthogonal to the direction of its motion.[1,7] The length of Einstein's moving rigid meter-rod does not depend upon time or position in his moving system k, only upon the relative speed v. (Although it is not the rod which contracts, it is the "moving space" in which the rod is "at rest" that contracts, and imparts its contraction to the rod in the direction of motion of the space containing the rod.) In the case of light, however, the distance light travels in any direction in the moving system k depends upon time τ which, by the Lorentz transformation, depends upon the associated position ξ in system k and time t of system K.

Eliminating the parameter n in Eqs. (12) for the moving system k gives

$$\frac{\left(\xi + \frac{vt}{\sqrt{1 - v^2/c^2}}\right)^2}{\left[\frac{c^2t^2}{(1 - v^2/c^2)}\right]} + \frac{\eta^2}{c^2t^2} = 1. \tag{22}$$

Thus, the wave of light in the x-y plane of stationary system K is circular but by Lorentz transformation is an elliptical wave of light in the ξ-η plane of moving system k, centered at $\left(-vt/\sqrt{1 - v^2/c^2}, 0\right)$ therein, with the length of the semimajor axis a and semiminor axis b given by

$$a = \frac{ct}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad b = ct. \tag{23}$$

Note that ξ seems shortened in the positive ξ-axis because the center of the ellipse is actually translated in the direction of the negative ξ-axis of moving system k. By Eq. (22), at $\xi = 0$,

$$\eta = \pm ct\sqrt{1 - \frac{v^2}{c^2}}. \tag{24}$$

At $\eta = 0$,

$$\xi = \frac{(\pm c - v)\,t}{\sqrt{1 - \frac{v^2}{c^2}}}, \tag{25}$$

that is,

$$\xi = ct\sqrt{\frac{c - v}{c + v}}, \quad \text{and} \quad \xi = -ct\sqrt{\frac{c + v}{c - v}}. \tag{26}$$

The focal length f of the ellipse is

$$f = \sqrt{a^2 - b^2} = \frac{vt}{\sqrt{1 - \frac{v^2}{c^2}}}. \tag{27}$$

The eccentricity e of the ellipse is

$$e = \frac{f}{a} = \frac{v}{c}. \tag{28}$$

As $v \to 0$, the ellipse of system k closes in on the circle of system K with the origin of system k moving toward the origin of K as the center of the ellipse approaches the origin of k and hence also of K. As v increases the origins of k and K separate, the ellipse in k increases its eccentricity and is translated further from the origin of k. The center of the ellipse is not at the origin of coordinates of system k and is not fixed, as it moves with time.

Solving Eqs. (22) for η,

$$\eta = \pm\sqrt{c^2t^2 - \left(1 - \frac{v^2}{c^2}\right)\left(\xi + \frac{vt}{\sqrt{1 - v^2/c^2}}\right)^2},$$
$$-ct\sqrt{\frac{c + v}{c - v}} \le \xi \le ct\sqrt{\frac{c - v}{c + v}}. \tag{29}$$

To simplify the graphical representation of Eq. (29), set $c = 1$ so that $0 \le v < 1$; and set time $t = 1$ unit. The figures are separated for clarity. The origins are actually separated by the distance $d = vt$ according to system K (Figs. 5–7).

Owing to symmetry of the η and ς axes, the ellipsoid corresponding to Eq. (22) is

$$\frac{\left(\xi + \frac{vt}{\sqrt{1 - v^2/c^2}}\right)^2}{\left[\frac{c^2t^2}{(1 - v^2/c^2)}\right]} + \frac{\eta^2}{c^2t^2} + \frac{\zeta^2}{c^2t^2} = 1. \quad (30)$$

The intercepts on the coordinate axes of system k for this ellipsoid are: $(ct\sqrt{(c - v)/(c + v)}, 0, 0)$, $(-ct\sqrt{(c + v)/(c - v)}, 0, 0)$, $\left(0, \pm ct\sqrt{1 - v^2/c^2}, 0\right)$, and $\left(0, 0, \pm ct\sqrt{1 - v^2/c^2}\right)$.

Each trace parallel to the η-ς plane is either a circle or a single point. For $-ct\sqrt{(c + v)/(c - v)} < \xi_o < ct\sqrt{(c - v)/(c + v)}$, Eq. (30) reduces to

$$\eta^2 + \varsigma^2 = c^2t^2 - \left(\xi_0\sqrt{1 - v^2/c^2} + vt\right)^2, \quad (31)$$

which is the equation of a circle of radius

$$R = \sqrt{c^2t^2 - \left(\xi_0\sqrt{1 - v^2/c^2} + vt\right)^2}. \quad (32)$$

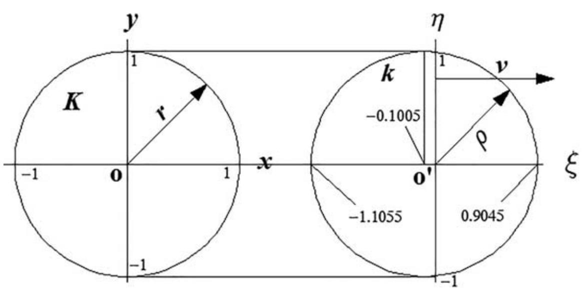

FIG. 5. $v = 0.1$.

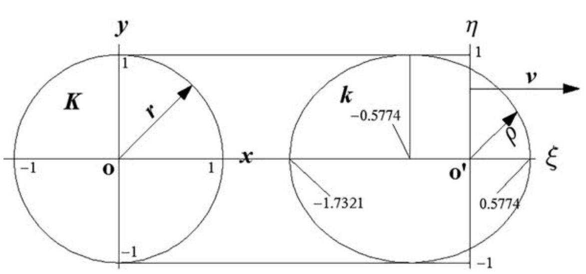

FIG. 6. $v = 0.5$.

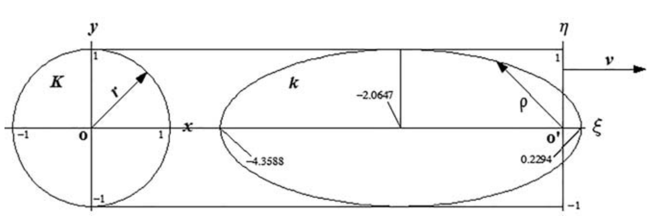

FIG. 7. $v = 0.9$.

At $\xi_0 = ct\sqrt{(c - v)/(c + v)}$ and at $\xi_0 = -ct\sqrt{(c + v)/(c - v)}$, $R = 0$ and Eq. (31) reduces to

$$\eta^2 + \varsigma^2 = 0, \quad (33)$$

which is a point on the ξ-axis.

Denoting by τ_c the τ-time at the center of the ellipsoid in the moving system k, Eq. (30) can be written as

$$\frac{(\xi + v\tau_c)^2}{c^2\tau_c^2} + \frac{\eta^2}{(c^2 - v^2)\,\tau_c^2} + \frac{\zeta^2}{(c^2 - v^2)\,\tau_c^2} = 1, \quad \tau_c \geq 0. \quad (34)$$

The intercepts on the coordinate axes of system k for ellipsoid Eq. (34) are

$$((c - v)\,\tau_c,\ 0,\ 0),\ (-(c + v)\,\tau_c,\ 0,\ 0),$$
$$\left(0,\ \pm\tau_c\sqrt{c^2 - v^2},\ 0\right),\ \left(0,\ 0,\ \pm\tau_c\sqrt{c^2 - v^2}\right).$$

Each trace parallel to the η-ς plane is either a circle or a single point. For $-(c + v)\tau_c < \xi_0 < (c - v)\tau_c$, Eq. (34) reduces to

$$\eta^2 + \varsigma^2 = (c^2 - v^2)\,\tau_c^2 - \frac{(c^2 - v^2)\,(\xi_0 + v\tau_c)^2}{c^2}, \quad (35)$$

which is the equation of a circle of radius

$$R = \sqrt{(c^2 - v^2)\,\tau_c^2 - \frac{(c^2 - v^2)\,(\xi_0 + v\tau_c)^2}{c^2}}. \quad (36)$$

At $\xi_0 = (c - v)\,\tau_c$ and at $\xi_0 = -(c + v)\,\tau_c$, $R = 0$ and Eq. (35) reduces to

$$\eta^2 + \varsigma^2 = 0, \quad (37)$$

which is a point on the ξ-axis.

The semimajor axis a and the semiminor axis b are then

$$a = c\tau_c; \quad b = \tau_c\sqrt{c^2 - v^2}, \quad (38)$$

and the focal length f and eccentricity e are

$$f = v\tau_c; \quad e = \frac{v}{c}. \quad (39)$$

For the ellipsoid Eq. (30),

$$\xi^2 + \eta^2 + \varsigma^2 = c^2\tau^2 \quad (40)$$

always holds: By Eqs. (2), (5), and (12),

$$\xi^2 + \eta^2 + \varsigma^2 = \frac{\left(\frac{c}{n} - v\right)^2 t^2}{\left(1 - \frac{v^2}{c^2}\right)} + c^2t^2\left(1 - \frac{1}{n^2}\right)$$
$$= \frac{c^2\left(1 - \frac{v}{nc}\right)^2 t^2}{\left(1 - \frac{v^2}{c^2}\right)} = c^2\tau^2, \quad (41)$$

and by Eqs. (2), (5), and (17),

$$\xi^2+\eta^2+\varsigma^2=\frac{\left(\frac{c}{m}+v\right)^2 t^2}{\left(1-\frac{v^2}{c^2}\right)}+c^2t^2\left(1-\frac{1}{m^2}\right)$$
$$=\frac{c^2\left(1+\frac{v}{mc}\right)^2 t^2}{\left(1-\frac{v^2}{c^2}\right)}=c^2\tau^2. \tag{42}$$

From Eqs. (12) and (34),

$$\xi=\left(\frac{c}{n}-v\right)\tau_c,\quad \tau=\left(1-\frac{v}{nc}\right)\tau_c,$$
$$\rho=c\tau=\left(1-\frac{v}{nc}\right)c\tau_c, \tag{43}$$

and from Eqs. (2) and (5),

$$\eta^2+\varsigma^2=\left(1-\frac{1}{n^2}\right)(c^2-v^2)\,\tau_c^2. \tag{44}$$

Similarly from Eqs. (17) and (34),

$$\xi=-\left(\frac{c}{m}+v\right)\tau_c,\quad \tau=\left(1+\frac{v}{mc}\right)\tau_c,$$
$$\rho=c\tau=\left(1+\frac{v}{mc}\right)c\tau_c, \tag{45}$$

and from Eqs. (2) and (5),

$$\eta^2+\varsigma^2=\left(1-\frac{1}{m^2}\right)(c^2-v^2)\,\tau_c^2. \tag{46}$$

Then for the ellipsoid Eq. (34),

$$\xi^2+\eta^2+\varsigma^2=c^2\tau^2 \tag{47}$$

always holds: By Eqs. (43) and (44),

$$\xi^2+\eta^2+\varsigma^2=\left(\frac{c}{n}-v\right)^2\tau_c^2+\left(1-\frac{1}{n^2}\right)(c^2-v^2)\,\tau_c^2$$
$$=c^2\tau^2, \tag{48}$$

and by Eqs. (45) and (46),

$$\xi^2+\eta^2+\varsigma^2=\left(\frac{c}{m}+v\right)^2\tau_c^2+\left(1-\frac{1}{m^2}\right)(c^2-v^2)\,\tau_c^2$$
$$=c^2\tau^2. \tag{49}$$

Ellipsoids Eqs. (30) and (34) are exactly the same.

By the Lorentz transformation Eqs. (2), the relation between any position ξ and its time τ is fixed. Consequently, the ellipsoid Eq. (34), and hence Eq. (30), can be written in terms of any time τ. For example, let τ_o denote the time at $\xi=0$, i.e., the origin of coordinates for system k. Then from Eqs. (2), the ellipsoid in system k due to Lorentz transformation of Eq. (3) becomes

$$\frac{\left[\xi+\frac{v\tau_0}{(1-v^2/c^2)}\right]^2}{\left(\frac{c^2\tau_o^2}{(1-v^2/c^2)^2}\right)}+\frac{\eta^2}{\left(\frac{c^2-v^2}{(1-v^2/c^2)^2}\right)\tau_o^2}$$
$$+\frac{\varsigma^2}{\left(\frac{c^2-v^2}{(1-v^2/c^2)^2}\right)\tau_o^2}=1,\quad \tau_o\geq 0. \tag{50}$$

All such recasting of the equation of the ellipsoid in system k describes exactly the same ellipsoid. In all cases, ρ is the distance to the wavefront from the origin of coordinates for system k.

Equation (3) is the equation of a sphere and also the equation of a hypotenuse. This is not the case with Eqs. (30) and (34). The ellipsoids described in Eqs. (30) and (34) have the associated hypotenuse given in Eq. (40): The distance from the origin of the coordinate system k to the wavefront in k. It is Eq. (40) that is produced directly from Eq. (3) by the Lorentz transformation: Form-invariance of the theorem of Pythagoras. Nevertheless, the Lorentz transformation produces the ellipsoidal wavefront of Eqs. (30) and (34) from the spherical wavefront Eq. (3). The dual character of Eq. (3) (i.e., hypotenuse and sphere) is incorrectly attributed to Eq. (40) by the special theory of relativity. Moreover, Eqs. (1) and (2) actually pertain to only one observer in particular, (privileged) in system K, an observer Einstein incorrectly permitted to speak for all observers in system K, owing to his tacit assumption of the existence of systems of clock-synchronized stationary observers consistent with Lorentz transformation. However, systems of clock-synchronized stationary observers consistent with Lorentz transformation do not exist.[8,9] For the same reason, Minkowski's four-dimensional spacetime continuum violates the theorem of Pythagoras.[10]

The ellipsoidal wavefront generated from a spherical wavefront by the Inverse Lorentz transformation is obtained from Eq. (30) by interchange of the coordinates of systems K and k and changing v to $-v$

$$\frac{\left(x-\frac{v\tau}{\sqrt{1-v^2/c^2}}\right)^2}{\left[\frac{c^2\tau^2}{(1-v^2/c^2)}\right]}+\frac{y^2}{c^2\tau^2}+\frac{z^2}{c^2\tau^2}=1. \tag{51}$$

III. ANGULAR RELATIONS

Let θ be the angle between the positive x-axis and the radius $r=ct$ in the x-y plane, and let θ_ρ be the angle between the positive ξ-axis and the radius $\rho=c\tau$ in the ξ-η plane. Set $1/n=\cos\,\theta$, for $0\leq\theta\leq 90^\circ$. Set $-1/m=\cos\,\theta$, for $0\leq\theta\leq 90^\circ$ or equivalently, $1/m=\cos\,\theta$ for $90^\circ\leq\theta\leq 180^\circ$. Then taking symmetry into account, Eqs. (12) and (17) can be combined and written as

$$x = ct\cos\theta, \quad y = ct\sin\theta, \quad r = ct,$$

$$\xi = \frac{(c\cos\theta - v)\,t}{\sqrt{1-\frac{v^2}{c^2}}}, \quad \tau = \frac{\left(1-\frac{v\cos\theta}{c}\right)t}{\sqrt{1-\frac{v^2}{c^2}}},$$

$$\rho = c\tau = \frac{\left(1-\frac{v\cos\theta}{c}\right)ct}{\sqrt{1-\frac{v^2}{c^2}}},$$

$$\eta = \sqrt{\rho^2 - \xi^2} = ct\sin\theta = y, \quad 0 \le \theta \le 360^\circ. \tag{52}$$

From Eqs. (52), $\cos\theta_\rho = \xi/\rho$, hence,

$$\cos\theta_\rho = \frac{c\cos\theta - v}{c - v\cos\theta}. \tag{53}$$

If $v = 0$ then $\theta_\rho = \theta$ and $\rho = r$. When $v > 0$, the condition $\rho = r$ occurs, from Eqs. (15), only for,

$$\cos\theta = \frac{c - \sqrt{c^2 - v^2}}{v}. \tag{54}$$

Substituting Eq. (54) into Eq. (53) gives the angle θ_ρ for which $\rho = r$

$$\cos\theta_\rho = \frac{\sqrt{c^2 - v^2} - c}{v}, \tag{55}$$

which depends only upon v.

By Eq. (53), for $\theta = 0$, $\theta_\rho = 0$, for $\theta = 180^\circ$, $\theta_\rho = 180^\circ$, and for $\theta = 90^\circ$ and $\theta = 270^\circ$,

$$\theta_\rho = \arccos\left(-\frac{v}{c}\right) = \arccos(-e), \tag{56}$$

where $e = v/c$ is the eccentricity of the ellipsoid. For $\cos\theta = v/c$,

$$\theta_\rho = \arccos(0) = 90^\circ \text{ and } 270^\circ. \tag{57}$$

Solving Eq. (53) for $\cos\theta$,

$$\cos\theta = \frac{c\cos\theta_\rho + v}{c + v\cos\theta_\rho}. \tag{58}$$

Putting Eq. (58) into Eqs. (52) for ρ gives

$$\rho = \frac{(c^2 - v^2)\ t}{(c + v\cos\theta_\rho)\sqrt{1 - v^2/c^2}}. \tag{59}$$

But $t/\sqrt{1 - v^2/c^2} = \tau_c$. Therefore,

$$\rho = \rho(\theta_\rho, \tau_c) = \frac{(c^2 - v^2)\,\tau_c}{(c + v\cos\theta_\rho)}, \quad \tau_c \ge 0. \tag{60}$$

Putting Eq. (55) into Eq. (60) gives

$$\rho = ct = r. \tag{61}$$

Once again, by the Lorentz transformation Eqs. (2), the ellipsoid described in Eq. (60) can be recast in terms of any time τ. All positions ξ within the expanding wavefront describe the very same ellipsoid. In all cases, ρ is the distance to the wavefront from the origin of coordinates for system k. For example, if τ_0 denotes the time at $\xi = 0$, then by the Lorentz transformation Eqs. (2), Eq. (60) becomes

$$\rho = \rho(\theta_\rho, \tau_0) = \frac{(c^2 - v^2)\,\tau_0}{(c + v\cos\theta_\rho)\,(1 - v^2/c^2)}, \quad \tau_0 \ge 0. \tag{62}$$

IV. CONCLUSIONS

The postulates of the theory of relativity are incompatible. A spherical wave of light is not transformed into a spherical wave of light by the Lorentz transformation but into a translated ellipsoidal wave of light with a moving center, even though the speed of light in vacuum is invariant. Consequently, the theory of relativity is logically inconsistent. It is therefore invalid.

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