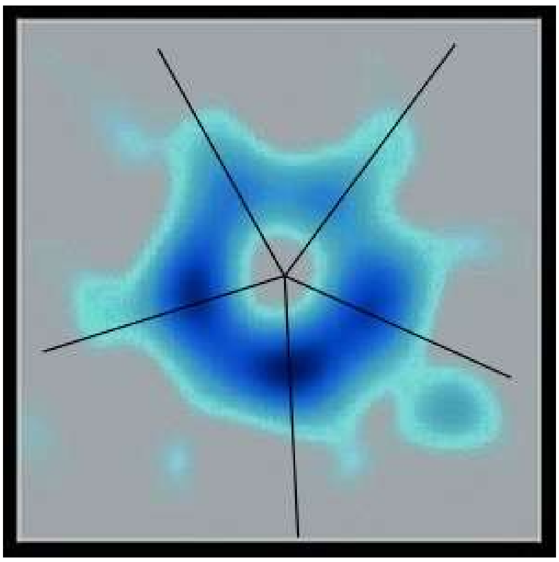
The black hole in M87 (EHT2017) may provide evidence for a Poincare Dodecahedral Space universe

- by Harry K. Hahn -

Germany , 26.5.2019



1 Introduction - The EHT2017 images may provide hints for the universal physical theory

The **EHT-team** has done a great job to provide the first visible evidence of a **Black hole** with the help of the **EH-Telescope !**

One feature in your fiducial image sequence caught my eye and forced me to write this paper here ! It's the apparent existence of **5 knots** in the ring-structure and the nearly symmetrical **pentagonal-shape of the black hole shadow !**

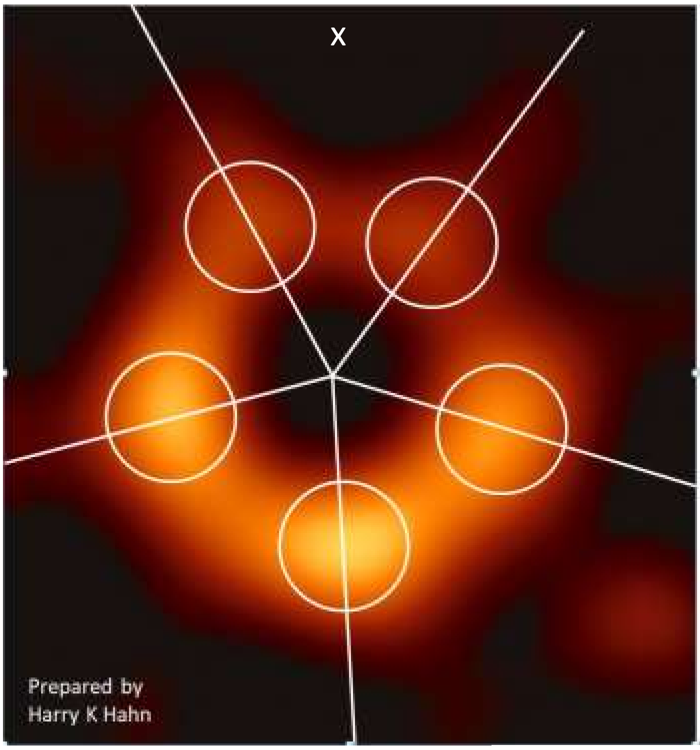
In Paper IV you wrote that your current image reconstructions and visibility-domain analyses are not able to confirm or reject the reality of the "knots" seen in the images, and these features should therefore be interpreted with caution.

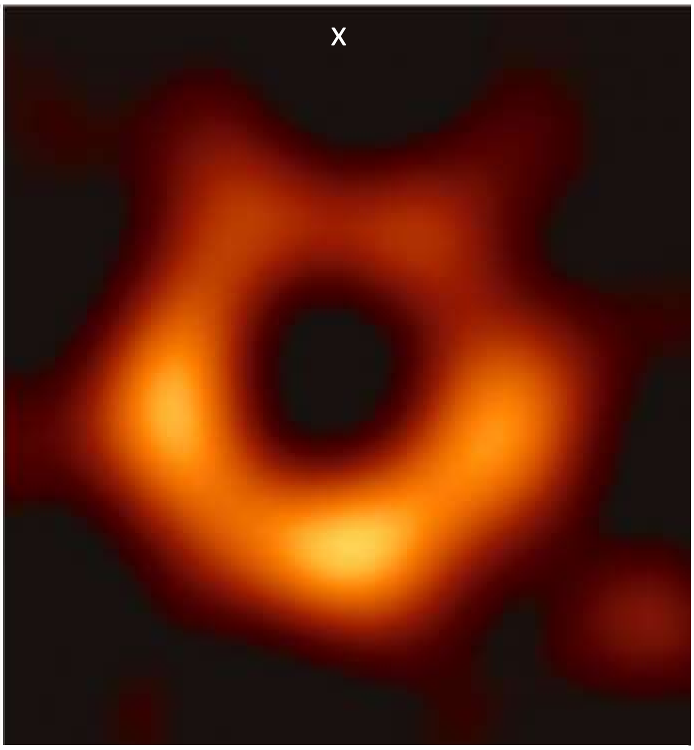
But if these knots turn out to be real and stable, then we may look at a gravitational singularity which indicates a more complex universe then expected ! The pentagonal ring structure may indicate a **Poincare Dodecahedral Space (PDS) structure** of our universe, and a **dodecahedral shaped** gravitational singularity (black hole) in the center of **M87.**
Your images may provide evidence for the **underlying universal theory of our universe** Albert Einstein was searching for.
I.V. Volovich describes the construction of a universal physical theory based on pure numbers as fundamental entities.
And I want to show the **importance of number 5** for the distribution of primes and non-primes as a base for this theory.

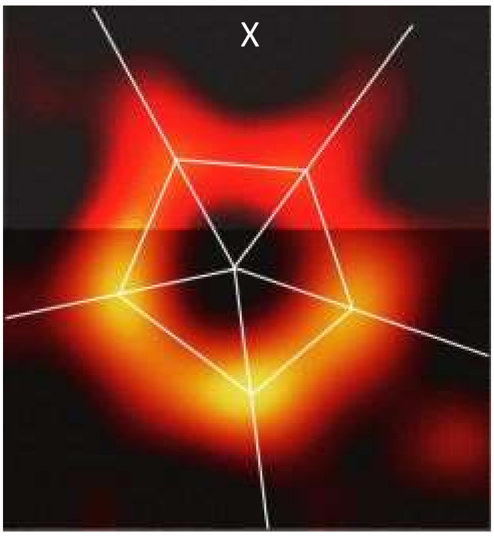
Questions to be answered :
Are the five knots are real and stable in their position ?
Do the knots correspond to a slightly tilted pentagon ?
Is the visible pentagonal structure stable over time ?
Are there structures and extensions around the ring ?
Is the visible anti-clockwise flow over the knots real ?

What should be done ? :
Further image processing & analyses of the sharpest images.

Computer simulations of gravitational singularities in order-4 and order-5 Poincare-Dodecahedral Spaces should be made.
How would the shadow of a comparable black hole look like in an order-4 & -5 PDS-universe ? Then compare it with M87 !







The image on the top (with and without markings) is from the fiducial image sequence of the M87 Black Hole Shadow from the 10th of April from page 18 of Paper IV)
Because the ring is brighter in the south than in the north **I have increased the brightness of the northern section** of the ring, in order to show the ring structure with more homogeneous brightness.

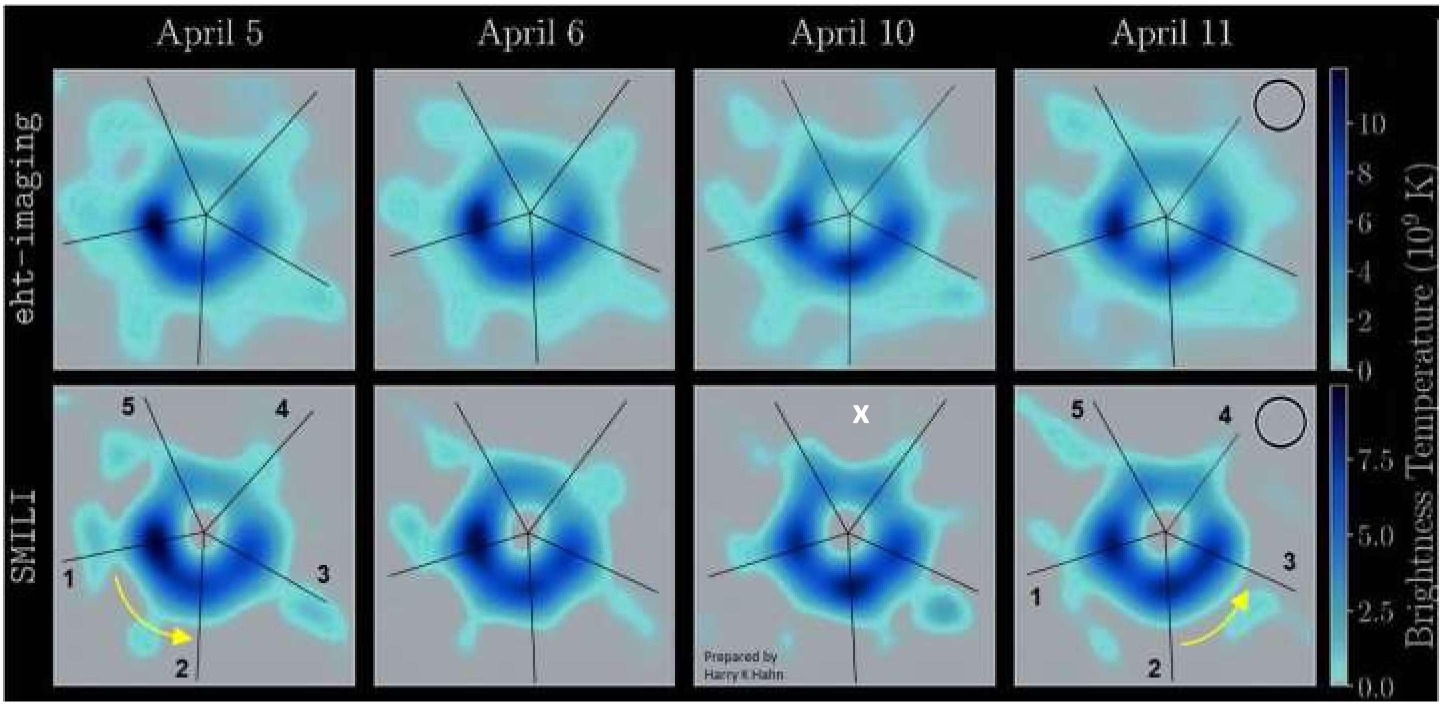
And it becomes clear that the ring has a pentagonal structure !

In Paper I you wrote that the asymmetry in brightness in the ring can be explained in terms of relativistic beaming of the emission from a plasma rotating close to the speed of light around a black hole.

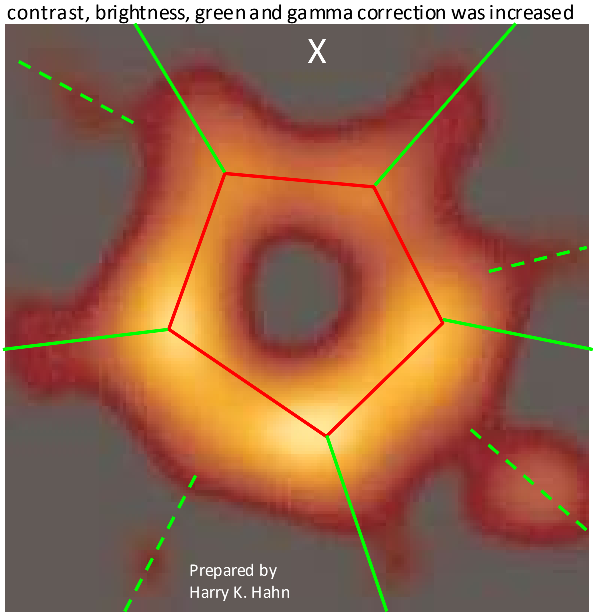
EHT2017 papers :

Paper I: The Shadow of the Supermassive Black Hole
Paper II: Array and Instrumentation
Paper III: Data processing and Calibration
Paper IV: Imaging the Central Supermassive Black Hole
Paper V: Physical Origin of the Asymmetric Ring
Paper VI: The Shadow and Mass of the Central Black Hole

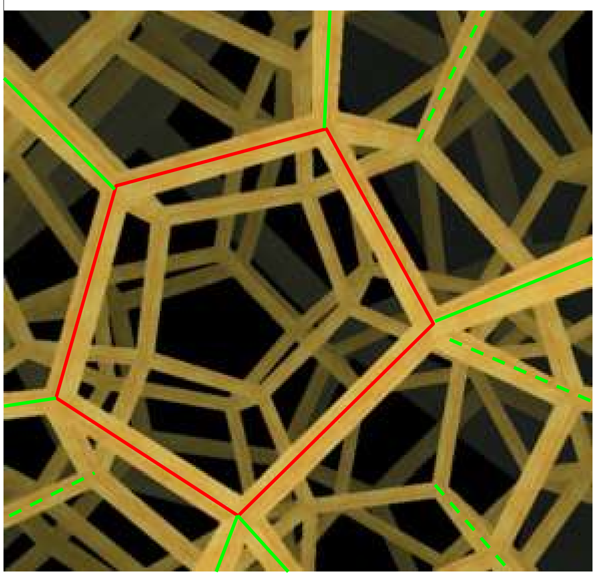
2 The M87 Black Hole Shadow seems to have a pentagonal shape and may have a dodecahedral structure



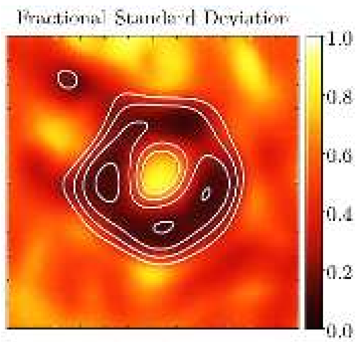
Reprocessed inverted color fiducial image sequence from Paper IV
contrast, brightness, green and gamma correction was increased



The EHT2017 images may indicate a dodecahedral lattice structure of a PDS universe around the black hole



The image above is a **contrast enhanced & inverted color image of the fiducial image sequence of the M87 Black Hole Shadow** shown on page 18 of Paper IV of the EHT2017 observing campaign.
Only the images processed by the **SMILI & eht-imaging algorithm** were used for this inspection.



There was no modification made on the visible structures in the images. All fiducial images calculated by the SMILI & eht-imaging algorithms **clearly indicate a pentagonal ring structure !**
The SMILI-images even show **two flow events** ! On April 5 flow is visible from knot 1 to 2, and on April 11 flow from knot 2 to 3.
This means over 24h the velocity of the flow probably must have been > 50000 km/s.

The above shown contrast-enhanced image sequence also shows lobes extending outward from the corners (knots) of the pentagonal ring structure. If these lobes are real structures, visible in the shadow of the black hole, then these lobes may indicate a Poincare Dodecahedral Space (PDS) structure around the black hole !
We may see a distorted mirror image of a dodecahedral lattice structure around the black hole as shown on the image on the left.

Several studies have proposed that the preferred model of the comoving spatial 3-hypersurface of the Universe may be a Poincare dodecahedral space (PDS) rather than a simply connected flat space

As the pentagon structure in the EHT images isn't perfect symmetric we may see the shadow of a dodecahedral shaped black hole, whose top-tile isn't pointing exactly towards the shadow area & the brightness of the dodecahedral lattice structure seems to fluctuate.

The M87 black hole images indicate a dodecahedral structure of the gravitational singularity

The following images are from the movie (documentation) of the EHT2017-project **„ Black Hole Hunters“**
See weblink (www.welt.de) **:** Black Hole Hunters ; you can also find the movie on :

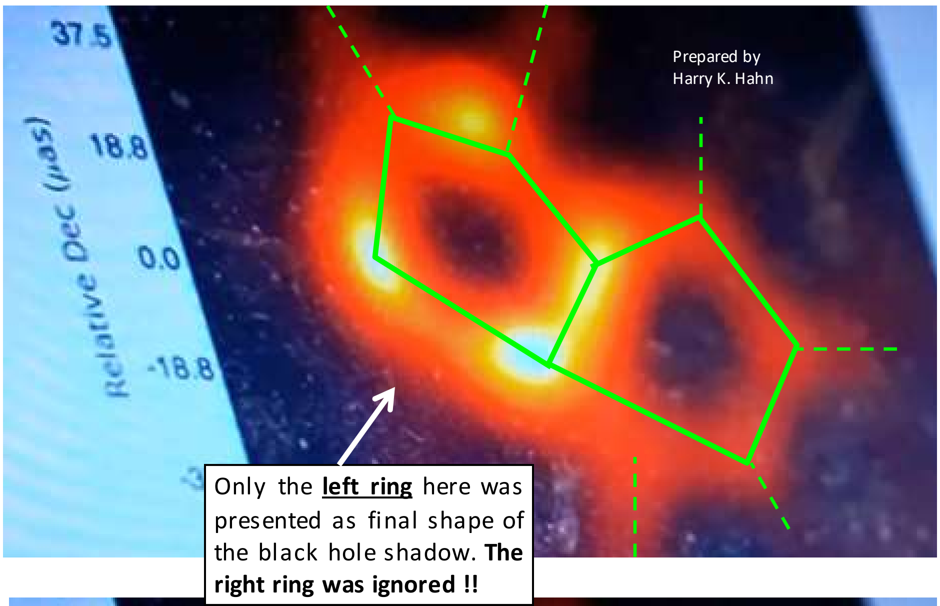
→ **or on YouTube.com : Title : „Black Hole Hunters“ - weblink :** https://www.youtube.com/watch?v=o_F3KVAPMpo

The images are from the section which shows how the algorithms calculate the first images of the M87 black hole
This is the **sequence from around 39:40 to 41:00 minutes.**

In the abstract of Paper I you wrote that when surrounded by a transparent emission region, black holes are expected to reveal a dark shadow caused by gravitational light bending, and photon capture at the event horizon.

When the algorithm started to calculate the first image there were distinct polygonal structures visible !

If you have a close look at this polygonal structure then **two pentagons can be recognized** ! (→ see markings in the image)
And this **pentagon array structure** seems to **extend further into space** as the doted lines indicate !



For comparison **here the original image** of the image calculation in progress.

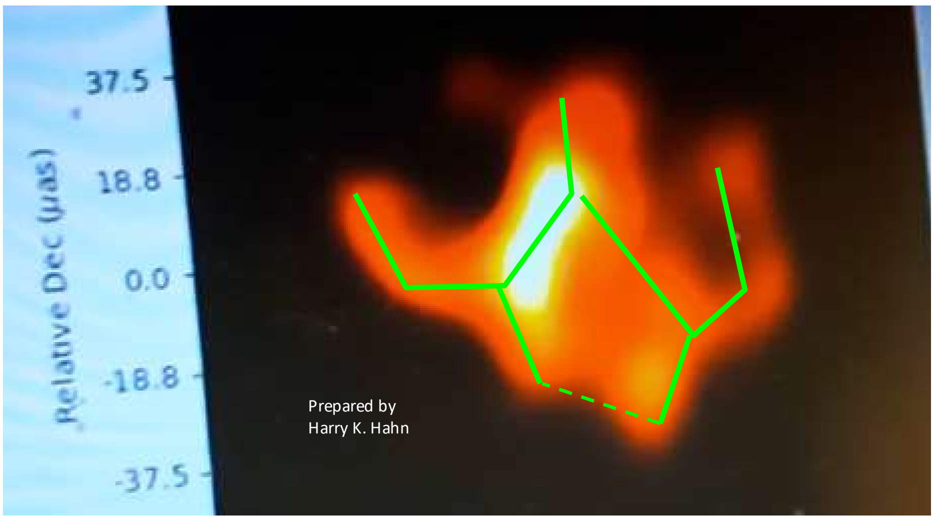
Here an image of the start of the movie sequence, which shows the calculation prozess.

The first impression is that we are looking here at **a complex 3-dimensional structure !**

This structure also indicates that the M87 black hole shadow seems to have a **3D-dodecahedral structure !!**

This image seems to show the dodecahedral structure in a slightly rotated position in regards to above shown images.

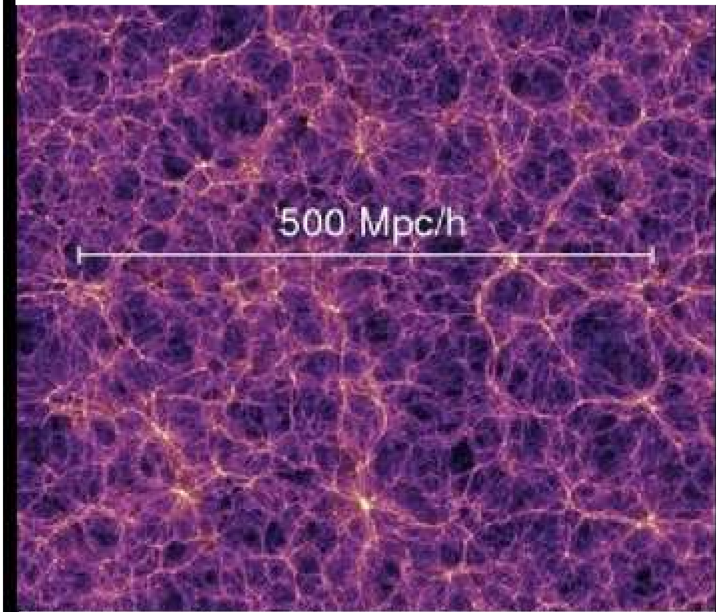
Was this image taken a **considerable time** earlier or later than the above shown image ?



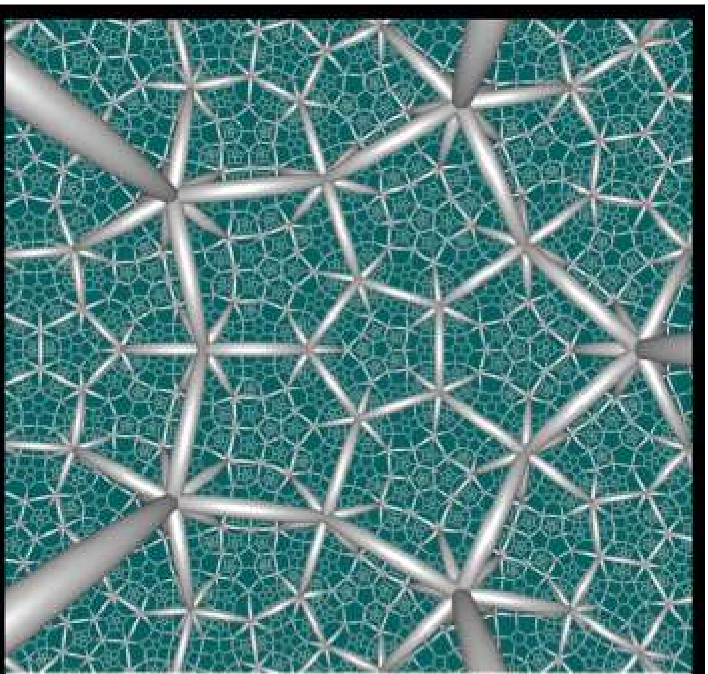
3 The large-scale distribution of matter in our universe is similar to an O-5 dodecahedral honeycomb structure

If we compare the large scale distribution of matter in our universe with the structure of a regular order-5 dodecahedral honeycomb in hyperbolic 3-space, then there are similarities visible ! The order-5 dodecahedral honeycomb has a better „visual" fit to the large-scale filament structure of our universe compared to an order-4 dodecahedral honeycomb.
In our universe matter clearly is distributed along the edges (filaments) of the visible cell structure, if we look at the universe on a large scale. Refering to the „similar-looking" order-5 dodecahedral honeycomb, that means matter (and the photons emitted by it) would be distributed and moving along the edges (filiaments) of the dodecahedral cells.
If we consider a black hole in the order-5 dodecahedral honeycomb then it probably would also have a dodecahedral shape, and matter and photons would move along the edges of the structure. However because our universe does't have a precise cell structure, **the real structure probably is caused by a superposition of all possible honeycomb structures.**

The Millenium Simulation : large scale universe Structure



This is how the universe looks on a large scale; a very large scale. It's not a photo but the outcome of a simulation called The Millennium Simulation. Since the outcome matches the observed distribution of galaxies in the universe very well, we can use it for visualising the biggest structures of the universe, the filaments. The structure of the universe resembles a sponge with the matter in the form of hundreds of billions of galaxies not being uniformly distributed over space but gathering in so-called filaments. In between the filaments there are voids, which are almost empty areas in the universe with diameters of between 40 million and 400 million light years.

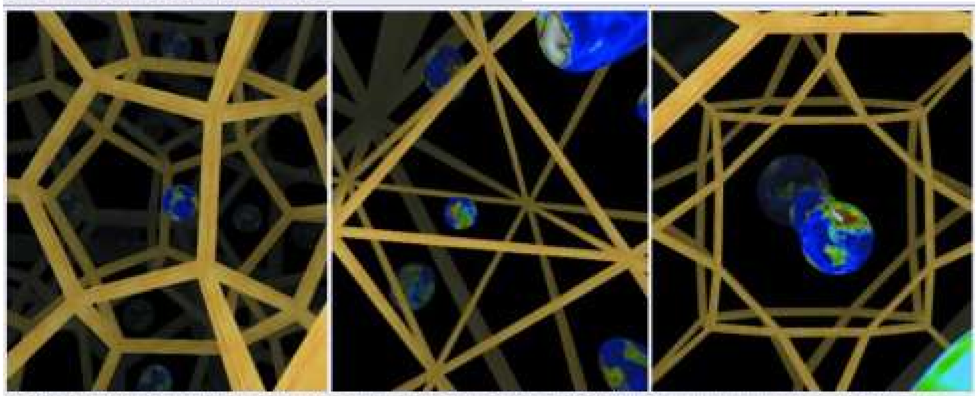


This is the regular {5,3,5} honeycomb in hyperbolic 3-space. The model is cell-centered in the Poincare Ball model, with the viewpoint then placed at the ball origin.

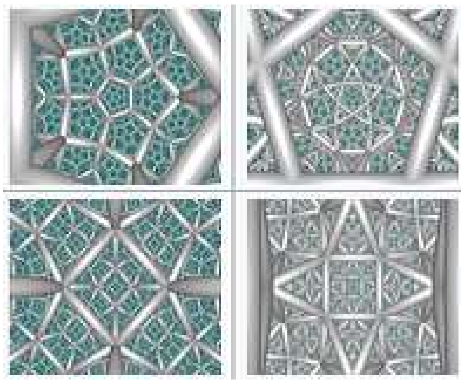
Prepared by
Harry K. Hahn

See : http://www.sun.org/images/structure-of-the-universe-1 and : https://en.wikipedia.org/wiki/Order-5_dodecahedral_honeycomb
and https://en.wikipedia.org/wiki/Galaxy_filament

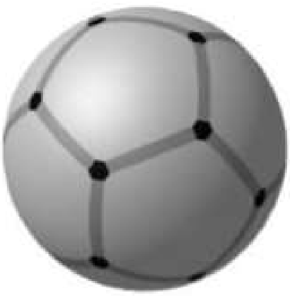
3 The many shapes of the universe → **See article : A cosmic hall of mirrors**



The Poincaré dodecahedral space (left) can be described as the interior of a "sphere" made from 12 slightly curved pentagons. However, there is one big difference between this shape and a football because when one goes out from a pentagonal face, one immediately comes back inside the ball from the opposite face after a 36° rotation. Such a multiply connected space can therefore generate multiple images of the same object, such as a planet or a photon. Other such well-proportioned, spherical spaces that fit the WMAP data are the tetrahedron (middle) and octahedron (right).



Other possible honeycomb structures
See :
→ **Uniform_honeycombs_in_hyperbolic_space**
e.g. Order-4_dodecahedral_honeycomb



Poincare Dodecahedral Space (PDS) Desribed as the interior of a hypersphere tiled with 12 slightly curved pentagons. When one goes out from a pentagonal face, one comes immediately back inside the PDS from the opposite face after a 36° rotation. Such a PDS is finite, although without edges or boundaries so that one can indefinitely travel within it. As a result an observer has the illusion to live in a space 120 times vaster, made of tiled dodecahedra which duplicate like in a mirror hall. As light rays crossing the faces go back from the other side, every cosmic object has multiple images

4 Analyses of the CMB indicate that the universe may have a Poincare dodecahedral space (PDS) structure

Here are the weblinks to two studies which indicate a Poincare dodecahedral space (PDS) structure of the universe:

1.) Dodecahedral space topology as an explanation for weak wide-angle temperature correlations in the cosmic microwave background (CMB)

Jean-Pierre Luminet, Jeffrey R. Weeks, Alain Riazuelo, Roland Lehoucq & Jean-Philippe Uzan

Weblink 1: http://ceadserv1.nku.edu/longa//classes/2004fall/mat115/days/luminet-nat.pdf
Weblink 2: https://luth.obspm.fr/~luminet/physworld.pdf

2.) The optimal phase of the generalised Poincare dodecahedral space hypothesis implied by the spatial cross-correlation function of the WMAP sky maps

Boudewijn F. Roukema, Zbigniew Bulin´ski, Agnieszka Szaniewska, Nicolas E. Gaudin

Weblink 1: https://arxiv.org/abs/0801.0006   Weblink to PDF: https://arxiv.org/pdf/0801.0006.pdf

ABSTRACT

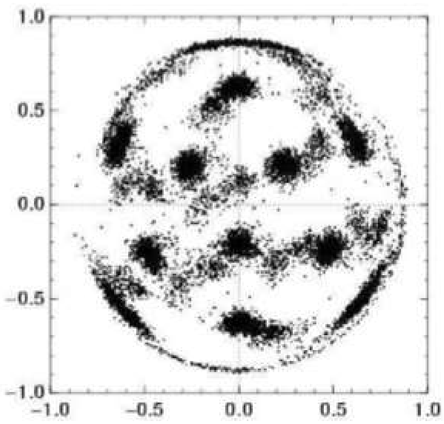
Context. Small universe models predicted a cutoff in large-scale power in the cosmic microwave background (CMB). This was detected by the Wilkinson Microwave Anisotropy Probe (WMAP). Several studies have since proposed that the preferred model of the comoving spatial 3-hypersurface of the Universe may be a Poincaré dodecahedral space (PDS) rather than a simply connected, flat space. Both models assume an FLRW metric and are close to flat with about 30% matter density.

Aims. We study two predictions of the PDS model. (i) For the correct astronomical positioning of the fundamental domain, the spatial two-point cross-correlation function ξ_C of temperature fluctuations in the covering space (where the two points in any pair are on different copies of the surface of last scattering (SLS)) should have a similar order of magnitude to the auto-correlation function ξ_A on a single copy of the SLS. (ii) Consider a "generalised" PDS model for an *arbitrary* "twist" phase $\phi \in [0, 2\pi]$. The optimal orientation and identified circle radius for a generalised PDS model found by maximising ξ_C relative to ξ_A in the WMAP maps should yield one of the two twist angles $\pm 36°$.

Methods. Comparison of ξ_C to ξ_A extends the identified circles method, using a much larger number of data points. We optimise the ratio of these functions at scales $\lesssim 4.0h^{-1}$ Gpc using a Markov chain Monte Carlo (MCMC) method over orientation (l, b, θ), circle size α, and twist ϕ.

Results. Both predictions were satisfied: (i) An optimal generalised PDS solution was found for two different foreground-reduced versions of the WMAP 3-year all-sky map, both with and without the kp2 galactic contamination mask. This solution yields a strong cross-correlation between points which would be distant and only weakly correlated according to the simply connected hypothesis. The face centres are $\{(l,b)\}_{i=1,6} \approx \{(184°, 62°), (305°, 44°), (46°, 49°), (117°, 20°), (176°, -4°), (240°, 13°)\}$ (and their antipodes) to within $\approx 2°$; (ii) This solution has twist $\phi = (+39 \pm 2.5)°$, in agreement with the PDS model. The chance of this occurring in the simply connected model, assuming a uniform distribution $\phi \in [0, 2\pi]$, is about 6-9%.

Conclusions. The PDS model now satisfies several different observational constraints.



lower panel

Fig. 5. Full sky map showing the optimal orientation of dodec ahedral face centres based on 100,000 steps in 10 MCMC chain using the ILC map and either the kp2 mask (upper panel) c no mask (lower panel), showing face centres for which $P > 0.$ (see Eq. (25)). The projection is a Lambert azimuthal equal are projection (Lambert 1772) of the full sky, centred on the Nort Galactic Pole (NGP). The 0° meridian is the positive vertic axis and galactic longitude increases clockwise. These face cer tres are derived from the MCMC chains *without any constrain*

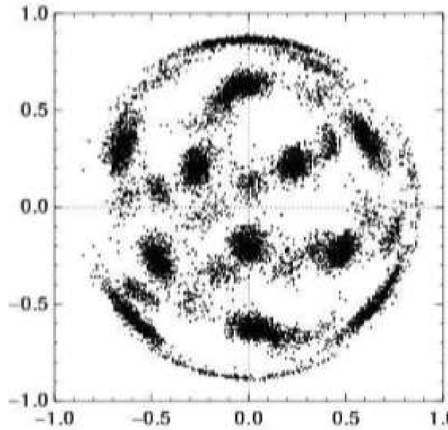


Fig. 10. Full sky map showing the optimal dodecahedral orier tation for the ILC map with the kp2 mask, as for Fig. 5, centre on the North Galactic Pole, from an MCMC chain starting the PDS orientation and circle size suggested in Roukema et a (2004), for an initial twist of $\phi = -\pi/5$. The optimal orient tion is clearly very close to what is found from arbitrary initi positions, shown in the previous figures.

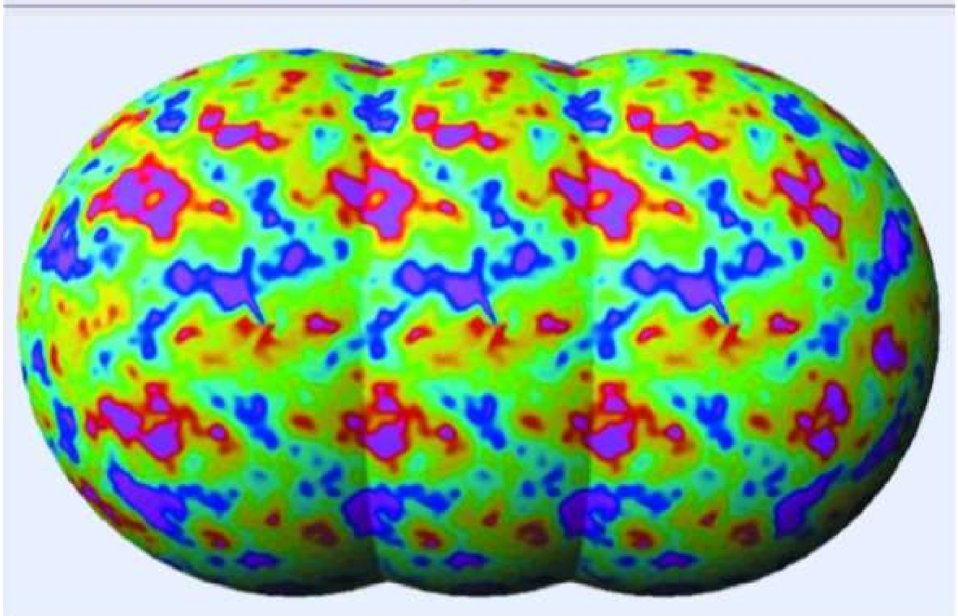
5 How the traces of a Poincare dodecahedral space unsiverse would appear on the CMB map

→ The image below is from the news article : **„A cosmic hall of mirrors“**

Weblink : **A cosmic hall of mirrors**

Extract 1 from article : If physical space is indeed smaller than the observable universe, some points on the map of the cosmic microwave background (**CMB**) will have several copies. As first shown by Neil Cornish of Montana State University and co-workers in 1998, these ghost images would appear as pairs of so-called matched circles in the cosmic microwave background where the temperature fluctuations should be the same (figure 4). This "lensing" effect, which can be precisely calculated, is thus purely attributable to the topology of the universe. Due to its 12-sided regular shape, the **Poincare dodecahedral model** actually predicts six pairs of diametrically opposite matched circles with an angular radius of 10–50°, depending on the precise values of cosmological parameters such as the mass–energy density.

4 Simulated circle matching



The topology of the universe describes how different regions are connected and could therefore leave its imprint on the cosmic microwave background. For example, if our physical space is smaller than the observable universe (as recent data suggest it is) then the horizon sphere wraps around the universe and intersects itself. As a result, duplicated images of the cosmic microwave background (in which the colours represent temperature fluctuations) will intersect along a circle and we would observe this circle on different sides of the sky.

Extract 2 from article :

In June 2004, Boud Roukema and colleagues at the Torun Centre for Astronomy in Poland independently searched for circles in the WMAP data. By only looking for back-to-back circles within a limited range of angular sizes and neglecting all other possible matches, the computer time was reduced drastically. Remarkably, the Polish team found six pairs of matched circles distributed in a dodecahedral pattern and twisted by 36°, each with an angular size of about 11°. This implied that $\Omega = 1.010 \pm 0.001$, which is perfectly consistent with our dodecahedral model, although the result was much less publicized than the earlier negative results.

In fact, the statistical significance of the match still needs to be improved, which means that the validity of the Poincaré dodecahedron model is still open to debate. In the last few months, however, there has been much theoretical progress on well-proportioned spaces in general.

Early this year, for example, Frank Steiner and co-workers at the University of Ulm in Germany went on to prove that the fit between the power spectrum predicted by the Poincaré dodecahedron model and that observed by WMAP was even better than we had previously thought. But the German team also extended its calculations to well-proportioned tetrahedral and octahedral spherical spaces in which $\Omega > 1$ (see figure 3).

These spaces are somewhat easier to understand than a dodecahedral space, but they require higher values of the density: $\Omega > 1.015$ for octahedral spaces and $\Omega > 1.025$ for tetrahedral spaces, compared with $\Omega > 1.009$ for dodecahedral spaces. However, these values are still compatible with the WMAP data. Furthermore, Steiner and co-workers found that the signal for pairs of matched circles could have been missed by current analyses of the cosmic microwave background due to various measurement effects that damage or even destroy the temperature matching.

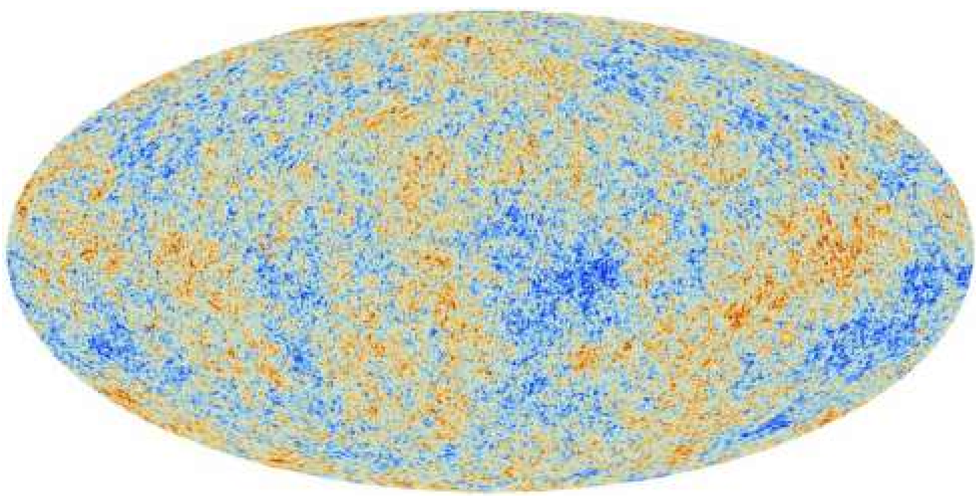
Studies from : Prof. Frank Steiner :

(Institute of Theoretical Physics Ulm University)

Cosmic_microwave_background_alignment_in_multi-connected_universes

CMB_Anisotropy_of_the_Poincare_Dodecahedron

other related studies :
https://www.researchgate.net/profile/Frank_Steiner5



Cosmic microwave background map (All sky map of the **CMB** created from **WMAP-**data)

Cosmic microwave background (CMB) is faint electromagnetic radiation, as remnant, from an early

6 To Albert Einsteins's work on a Unified field theory - Some clues about the final theory he had in mind

One letter to his girlfriend Ilse Rosenthal-Schneider in the year 1945 gives some insight about his thoughts to a TOE

Essentiell he was looking for a theory where the universal constants which appear in the base equations only have units which are reduced to kg , m and s (the units for mass , length and time). Albert Einstein considered the most other physical units and constants equipped with these units as fictitious (manmade) , and said that they can be eliminated.

He went further and said that by multiplying these universal constants with factors, which are formed out of powers of c1, c2, c3 (the universal physical constants), that the new universal constants c4*, c5*, c6* are pure numbers.

Essentiell he was searching for an underlying universal mathematical theory only containing constants like Pi(π) & e

→ Weblink to the german book which contains the original letter from 13th October 1945 in german language :

→ http://docplayer.org/69639849-Ilse-rosenthal-schneider-begegnungen-mit-einstein-von-laue-und-planck.html
see also : - description oft he book contents in english : http://blog.alexander-unzicker.com/?p=27

The letter to Ilse Rosenthal-Schneider (dated 13.10.1945) can be found on the pages 23 bis 27 (pages of online-document 30-34) in Chapter 2 : The universal natural constants (Kapitel 2: Die universellen Naturkonstanten)

→ I have translated this letter. but I can't guarantee a 100% correct tranlation of each expression :

Dear Ms Rosenthal !

I can see from your (last) letter that you haven't understood my hints regarding the universal constants of physics. Therefore I want to try now to make the subject clearer.

1.) …"There are fictitious constants and true constants. The fictitious constants can be eliminated. …the true constants are real numbers…. I believe that these true constants must be of a -„rationell" type-, like **Pi** (π) and **e**

…"The fictious constants just come from the introduction of arbitrary units. Such constants doesn't exist. Their existence is only based on the fact, that we haven't penetrated physics deep enough"
"In contrast to these constants of the „rationell type" there is the rest of the numbers which do not come through a transparent construction out of number 1"
„It lies in the nature of this matter, that such constants of the „rationell type" are not very different to number 1 from their order (of magnitude) → see for example : **Mathematical constants**
- on page 24 (31)
(comment : the expression -„rationell" type- is not exactly clear. It's not clear what Albert Einstein had in his mind when he wrote this expression. In the german language „rationell" can also mean in general : „based on logic" or „based on (logic) reason" / „well reasoned" or he precisely meant the group of irrational and transcendental numbers like Pi (π) and **e** , or more in general a group of irrational numbers comprising also square roots of a selected group of small non quadratic numbers like 2, 3, 5 and 7,….

2.) „There is now a complete theory of physics, were the universal constants c1,c2,…cn appear in the base equations. The Units are reduced somehow to kg (gr), m (cm), s (sek.)…….The choice of these three units is obvious very conventional. Each of the c1,c2,…cn has a dimension in these units. Now we want to chose it in such a way, that c1,c2,c3 have such dimensions, that we can't form a dimensionfree product $c^{\alpha}1\, c^{\beta}2\, c^{\gamma}3$

Then we can multiply c4, c5, etc. in such a way with factors, which are formed out of powers (potenzen) of c1, c2, c3, that these new c4*, c5*, c6* are pure numbers (without units).

„These (pure numbers) are the real universal constants of the theoretical system (of the universal mathematical theory) which haven't anything to do with conventional units". – on page 26 (33)
(comment : Albert Einstein is talking here about a universal (physical) theory (Theory of everything) which only contains true (pure) mathematical constants as mentioned in 1.) without (manmade) physical units !

3.) „My expectation is now, that these constants c4*, c5*, ...etc. are of the „rationell" type, and that their value is based on the logical base of the complete theory."

„You can also say it so : In a reasonable theory there are no dimensionfree numbers, with a value, which can only be determined empirical."

Of course I can't proof this. But I can't imagine a universal and reasonable theory , which contains a number, which could have been chosen just as good differently by the mood of the creator (god), whereby the world would have turned out qualitatively different in their physical legality."

„You can also say it so : A universal theory, which contains in their base equations a constant which is not of the „rationell" type, would somehow have to be added together out of pieces, which are logically independent from each other. But I have confidence, that this world isn't like that, that we need such an ugly construction to capture the theory of the world."

4.) „Of course, there is no consistent theoretical base for the complete physical science yet. And not at all a theoretical base which would meet the radical demands described above. It is not so difficult to think about possible formulations. Unfortunately these are such relativistic theories , in which it is extremely hard, to progress to proofable conclusions. It is difficult to solve non-linear differential equations so, that there are nowhere Singularities.

But this problems haven't anything to do with the principle question.

With friendly greetings and wishes.

Your Albert Einstein

Some weblinks to studies about natural constants :

- Dimensionless Physical Constant Mysteries
  http://www.rxiv.org/pdf/1205.0050v1.pdf

  Looking_for_Those_Natural_Numbers_Dimensionless_Constants_and_the_Idea_of_Natural_Measurement_1
  https://www.academia.edu/35881283/

- Do we live in an eigenstate of the "fundamental constants" operators?
  https://arxiv.org/pdf/1809.05355.pdf

- Fine structure constant
  https://oeis.org/wiki/Fine-structure_constant

Two quotes from Albert Einstein :

"If you can't explain it in a simple way, then you haven't understood it good enough" (Wenn Du es nicht einfach erklären kannst, hast Du es nicht gut genug verstanden)

"Not everything that counts can be counted, and not everthing that can be counted counts" (Nicht alles was zählt kann gezählt werden, und nicht alles was gezählt werden kann, zählt)

Albert Einstein in his office in Berlin in the early 1920's

7 Number Theory as the Ultimate Physical Theory - by I. V. Volovich / Steklov Mathematical Institute

The mathematician I. V. Vololovich has written a hypothesis on the quantum fluctuation of the number field.

Our picture of space-time at the Plank-scale doesn't seem to be correct. The fundamental entities (base units) of the universe can't be particles, fields or strings. Numbers are considered to be the fundamental entities. Therefore a new quantum mechanics over an arbitrary number field must be developed. And number theory is the base of physics.

→ **Weblink to the study : http://cdsweb.cern.ch/record/179558/files/198708102.pdf**

Extracted text from page 1 of the study :

NUMBER THEORY AS THE ULTIMATE PHYSICAL THEORY

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Weblink :
I.V.Volovich_publication list

A B S T R A C T

At the Planck scale doubt is cast on the usual notion of space-time and one cannot think about elementary particles. Thus, the fundamental entities of which we consider our Universe to be composed cannot be particles, fields or strings. In this paper the numbers are considered as the fundamental entities. We discuss the construction of the corresponding physical theory.

A hypothesis on the quantum fluctuations of the number field is advanced for discussion. If these fluctuations actually take place then instead of the usual quantum mechanics over the complex number field a new quantum mechanics over an arbitrary field must be developed. Moreover, it is tempting to speculate that a principle of invariance of the fundamental physical laws under a change of the number field does hold.

The fluctuations of the number field could appear on the Planck length, in particular in the gravitational collapse or near the cosmological singularity. These fluctuations can lead to the appearance of domains with non-Archimedean p-adic or finite geometry.

We present a short review of the p-adic mathematics necessary, in this context.

Planck units
Elementary particle
Field (physics)
String (physics)
Quantum fluctuations
Quantum mechanics
Field (mathematics)
Non-Archimedean
P-adic geometry
finite geometry
Black hole

Extracted text from pages 14 and 15 of the study :

It is appropriate to recall the famous Einstein programme to reduce all physics to geometry. It is a promising programme but let us ask the question about which geometry we would like to speak of? Why should we pick Riemannian geometry? Are there the reasons in favour of Riemannian geometry, or can one also use non-Archimedean geometry? One can go farther and ask the question why geometry over the field of real numbers but not over an arbitrary field is the proper geometry for physics. We believe that the contemporary version of Einstein's programme should look like a proposal to reduce all physics to geometry over arbitrary number fields. In fact this means the reduction of the physics to number theory. One can agree with the Pythagoreans according to whom we have to understand number, in order to understand the Universe.

If these ideas are true then number theory and the corresponding branches of algebraic geometry are nothing else than the ultimate and unified physical theory.

Number theory

Of course, it is possible to generalize the above general principle and to consider some algebras instead of fields. In superanalysis[4)], we exchange the field of real numbers for superalgebra with a norm. But in this paper we restrict ourselves to the case of the field.

Extracted text from pages 15 and 16 of the study :

We will discuss now an appropriate modification of quantum mechanics. In the usual quantum mechanics, the principle of superposition of probability amplitudes plays the main rôle and it can be written in the form

$$\langle a|c\rangle = \sum_{b} \langle a|b\rangle\langle b|c\rangle$$

where the probability has to be calculated as follows:

$$P_{ab} = |\langle a|b\rangle|^2. \qquad (1)$$

In the construction of the quantum mechanics on an arbitrary field K, one can follow two ways. The first way consists in considering complex-valued functions depending on variables belonging to K, or more generally, functions taking values in a complex Hilbert space. Here the results of representation theory on the local compact fields may be useful [see Ref. 44)] where these constructions are actually considered.

The second way one deals with wave functions taking values in the field K, i.e., $\langle a|b\rangle \in K$. Then in the case of the field K with a norm, Eq. (1) has sense if $|\ |$ means the norm in K. In our opinion it is difficult at present to tell which of these two ways is more favourable, so that it is reasonable to develop both simultaneously. Note that the first way is more closer to traditional quantum mechanics. However, there are important differences. Namely, it seems rather inappropriate to formulate dynamics in this case using the Schrödinger equation. A more appropriate way is to deal with unitary representation of the translation group.

If the above-mentioned hypothesis on the fluctuations of the number field is indeed realized then it is possible to suggest also the following hypothesis. It is common wisdom that in the Big Bang or in the final collapse, time and space do not have their usual meanings. But this is a purely negative answer to the question about the meaning of the time and space co-ordinates in such circumstances. What is a positive answer? Our proposal is as follows. The space and time co-ordinates would be, for example, p-adic. Of course this is an unusual world. For example, p-adic variables are not ordered. In this case, there is no meaning to the words "greater" or "less". But nevertheless we can write differential equations in such variables and we can try to understand processes in the Big Bang in a constructive way.

Then the strongest fluctuations take place in the Big Bang and a newly born Universe can have non-Archimedean or finite or other geometry over non-standard number fields. It may be that the corresponding exotic domain exists at present. An analogous hypothesis can also be considered in the context of the gravitational collapse. By this we mean that in the process of the collapse as a result of quantum effects, matter can collapse into a space with non-Archimedean geometry.

8 - What are the points where we have to look at to find the universal mathematical and physical theory ?

Albert Einstein was on the right track. There is a more profound logic (or theory) which we haven't grasped yet.

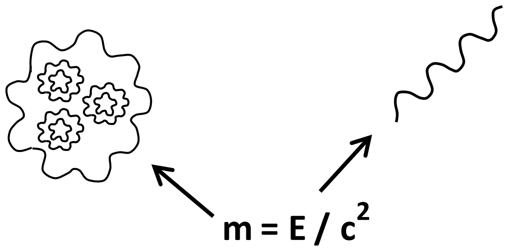
The main reason why he failed to find the fundamental equations for an universal (physical) theory was that he tried to base it on the existing physical theories. He was coming from the physical side.

In his letter from 13.10.1945 Einstein wrote in principle that „too many arbitrary units & -constants" had been introduced into physical theories. Therefore it was a desperate and impossible task to try to eliminate all these arbitrary units and constants, in order to come to a simple and fundamental set of equations.

A better way ist to come from the mathematical side. Especially from the branch of mathematics which deals with the fundation of mathematics : Number Theory !
And we also have to consider all branches of Geometry as important foundation stones to find the universal theory. There are no arbitrary units to eliminate in mathematics ! This ist the big advantage of mathematics.

Regarding the existing physical theory we have to ask the serious question if base units like kg (for mass) and m (for length) not also arbitrary units, which only exist because we may look to „narrow-minded" to the design of the universe.

Maybe we should have a look to the famous equation $\mathbf{E = mc^2}$ in a more visual way to understand what is going on.



If we only consider the massless photon as carrier of energy for the moment, then we can say : mass may be nothing more than energy moving on narrow curved orbits. Linear moving energy (photons) caught in small curved orbits is the cause of magnetic, electric and gravitational fields. The bending of the linear moving „wave-unit" is causing all the effects which we attribute to matter, including gravity. However **it may only be energy where we look at**, which is geometrically deformed and forced in a narrow orbit, which is the real cause of electromagnetism and gravitation.

The truth may be that only energy exists, either moving on open linear tracks, or moving in closed curved orbits.

I want to mention here the following study :

Weblink : https://arxiv.org/pdf/0706.2043.pdf

„Phase spaces in Special Relativity : Towards eliminating gravitational singularities" - by Peter Danenhower

This study uses general phase spaces in special relativity by expanding Minkowski Space to model the physical world. These spaces appear to indicate that graviational singularities can be eliminated !

In this study a simple (invariant) parameter, the **"energy to length" ratio**, which is $\mathbf{c^4/G}$ was used for any spherical region of space-time-matter.

This study may show a way forward to combine General Relativity with Quantum Mechanics.
Together with a more profound analysis of the start section of the square root spiral → see next page

→ On the following pages I want to give an overview of the points we have to look at to find the universal theory :

I fully agree with Mr. Igor.V.Volovich that numbers must be the fundamental entities of a universal physical theory ! That's why we have to find a simple logical explanation of the distribution of prime numbers.

Because the logic of the distribution of prime numbers must be understood, before we can even try to start to work on the final universal mathematical- and physical theory !

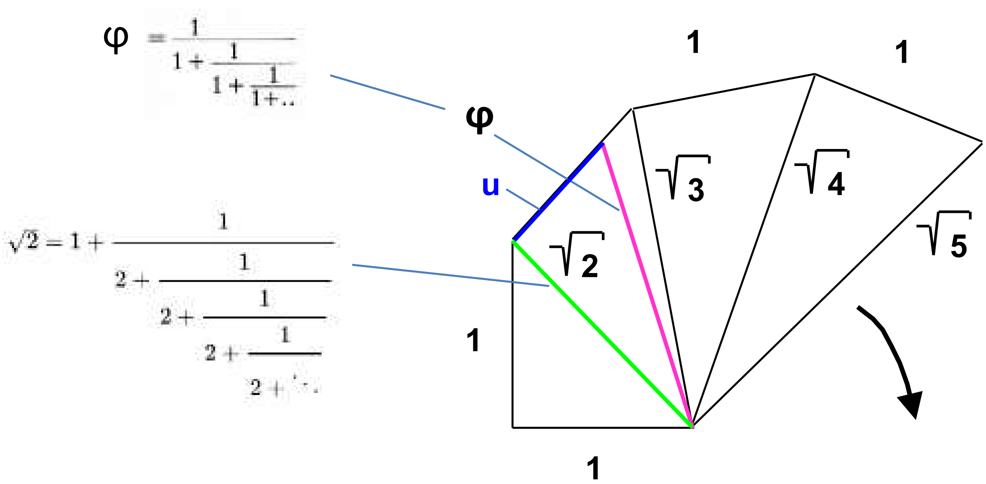
I consider the pure mathematical object Square root spiral (Spiral of Theodorus) where Prime Numbers and Non-Prime Numbers are distributed spatial in a precisely defined godgiven way, defined by the Pythagorean theorem, as an important foundation stone to find the universal mathematical- & physical theory.

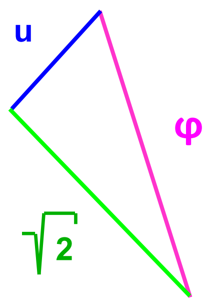
Similar objects are the Ulam spiral (by Stanislaw Ulam) and the Number Spiral (Robert Sachs) which can also be used to understand the spatial behaviour of numbers in similar spiral-shaped number fields. However in these number fields the Pythagorean theorem as link between the numbers is missing.

I consider the first right triangles of the square root spiral which contain the **base unit 1**, and the first irrational numbers defined by it, which are close to 1, like **square root of 2 square root of 3 and square root 5,** including the constant **φ** as important **„space structure constants"**. The constant **u** which defines the cathetus between **square root of 2** and the constant **φ** also plays an import role for finding the final universal theory.

These **„space structure constants"** not only define the complex structure of the square root spiral, but also the Platonic Solids , and they also form the base of number theory and physics as well !

The start of the square root spiral is shown with the constant φ drawn in :





With the help of these constants we will find an important starting point on our way to the universal theory :

From the right triangle φ , square root of 2 & u follows :

$\varphi^2 = (\sqrt{2})^2 + u^2$; application of the Pythagorean theorem

→ $u = \sqrt{\varphi^2 - 2} = 0{,}786151377.....$; we can calculate this value of **u** with the calculator

Research in the internet with Google, found a study where the constant u was expressed with an algebraic term ! With the help of this algebraic term it was possible to find interesting new properties of constant φ

→ See next page !

Here the abstract of the study where an algebraic term was found for constant u :

PHASE SPACES IN SPECIAL RELATIVITY : TOWARDS ELIMINATING GRAVITATIONAL SINGULARITIES

from PETER DANENHOWER → see weblink : **https://arxiv.org/pdf/0706.2043.pdf**

See also document :
→ space-time-matter

Abstract : This paper shows one way to construct phase spaces in special relativity by expanding Minkowski Space. These spaces appear to indicate that we can dispense with gravitational singularities. The key mathematical ideas in the present approach are to include a complex phase factor, such as, eιφ in the Lorentz transformation and to use both the proper time and the proper mass as parameters. To develop the most general case, a complex parameter σ = s + im, is introduced, where s is the proper time, and m is the proper mass, and σ and σ /|σ| are used to parameterize the position of a particle (or reference frame) in space-time-matter phase space. A new reference variable, u = m/r , is needed (in addition to velocity), and assumed to be bounded by 0 and $c^2/G = 1$, in geometrized units. Several results are derived: The equation $E = mc^2$ apparently needs to be modified to $E^2 = (s^2c^{10})/G^2 + m^2c^4$, but a simpler (invariant) parameter is the “energy to length” ratio, which is c^4/G for any spherical region of space-time-matter. The generalized “ momentum vector” becomes completely “masslike” for **u ≈ 0.7861…,** which we think indicates the existence of a maximal gravity field. Thus, gravitational singularities do not occur. Instead, as u → 1 matter is apparently simply crushed into free space. In the last section of this paper we attempt some further generalizations of the phase space ideas developed in this paper.

Extract from page 11 of the study (equation 4.9) :

$$\hat{\mathbf{P}} = \frac{[(\sqrt{1-u^2}-u^2)+i(u\sqrt{1-u^2}+u)]}{\sqrt{1+u^2}}\gamma < 1, v >$$

In this form the real and imaginary part of **P** have a very interesting property, namely, if

(4.10) $u = \frac{\sqrt{2\sqrt{5}-2}}{2}$ **≈ 0.786151377… = u ,** then the real part of **P** is zero, and the imaginary part takes its maximum value (= 1).

I think it makes sense to argue that when the real part of **P** = 0, **P** is entirely “mass like”, which we could understand to be representative of the state of space-time-matter for which the maximal gravity field occurs. In this picture gravity is understood to be the propensity of space-time-matter to become completely mass like. The more mass-like a region of space-time-matter is, then the stronger the external gravity field. Thus, within the discussion of this paper, I think **the only reasonable interpretation of the existence of the special value of u given in equation 4.10 is that there is a maximal gravity field at this value of u.** It is important to observe that the **value of u** considered above, substantially exceeds **the value of u for a typical neutron star (≈ 0.1 – 0.2).**
Thus, I think the maximal gravity field concept can be used to explain all of the experimental evidence for enormous gravity fields.

→ Now we can equate the two algebraic terms which represent the same constant ! :

$$\sqrt{\varphi^2 - 2} = \frac{\sqrt{2\sqrt{5} - 2}}{2}$$; we square both sides

$$\rightarrow \quad 4\varphi^2 - 8 = 2\sqrt{5} - 2$$; and transform

$$\varphi^2 = \frac{\sqrt{5} + 3}{2}$$; (1) we solve for φ^2

$$\sqrt{5} = 2\varphi^2 - 3$$; (2) we solve for $\sqrt{5}$

→ Now we use the following right triangle :

$$(\sqrt{6})^2 = (\sqrt{5})^2 + 1^2$$; Pythagorean theorem

$$6 = (2\varphi^2 - 3)^2 + 1$$; we replace $\sqrt{5}$ by (2)

$$\rightarrow \quad 3 = \frac{\varphi^4 + 1}{\varphi^2} \;(3) \rightarrow \sqrt{3} = \sqrt{\frac{\varphi^4 + 1}{\varphi^2}} \;(4)$$

→ square root 3 expressed by φ and 1 !

With the other right triangles of the square root spiral we can calculate all square roots of the natural numbers expressed only by φ and 1 : (see Appendix of study !)

$$2 = \frac{\varphi^4 - \varphi^2 + 1}{\varphi^2} \;(5) \text{ and } \sqrt{2} = \sqrt{\frac{\varphi^4 - \varphi^2 + 1}{\varphi^2}} \;(6)$$

$$\sqrt{5} = 2\varphi^2 - \frac{\varphi^4 + 1}{\varphi^2} \quad \rightarrow \quad \sqrt{5} = \frac{\varphi^4 - 1}{\varphi^2} \;;\; (7)$$

$$6 = \frac{\varphi^8 - \varphi^4 + 1}{\varphi^4} \;(8) \text{ and } \sqrt{6} = \sqrt{\frac{\varphi^8 - \varphi^4 + 1}{\varphi^4}} \;(9)$$

$$7 = \frac{\varphi^8 + 1}{\varphi^4} \;(10) \rightarrow \sqrt{7} = \sqrt{\frac{\varphi^8 + 1}{\varphi^4}} \;(11)$$

$$8 = \frac{\varphi^8 + \varphi^4 + 1}{\varphi^4} \;(12) \quad \sqrt{8} = \sqrt{\frac{\varphi^8 + \varphi^4 + 1}{\varphi^4}} \;(13)$$

$$10 = \frac{\varphi^8 + 3\varphi^4 + 1}{\varphi^4} \;(14) \quad \sqrt{10} = \sqrt{\frac{\varphi^8 + 3\varphi^4 + 1}{\varphi^4}} \;(15)$$

$$11 = \frac{\varphi^8 + 4\varphi^4 + 1}{\varphi^4} \;(16) \quad \sqrt{11} = \sqrt{\frac{\varphi^8 + 4\varphi^4 + 1}{\varphi^4}} \;(17)$$

$$12 = \frac{\varphi^8 + 5\varphi^4 + 1}{\varphi^4} \;(18) \quad \sqrt{12} = \sqrt{\frac{\varphi^8 + 5\varphi^4 + 1}{\varphi^4}} \;(19)$$

8.1. - The prime number distribution follows a clear logic

→ <u>see study</u> **“About the logic of the prime number distribution“ :** https://arxiv.org/abs/0801.4049

I will focus on the description of this **„Wave Model“ as explanation for the distribution of primes and non-primes :**

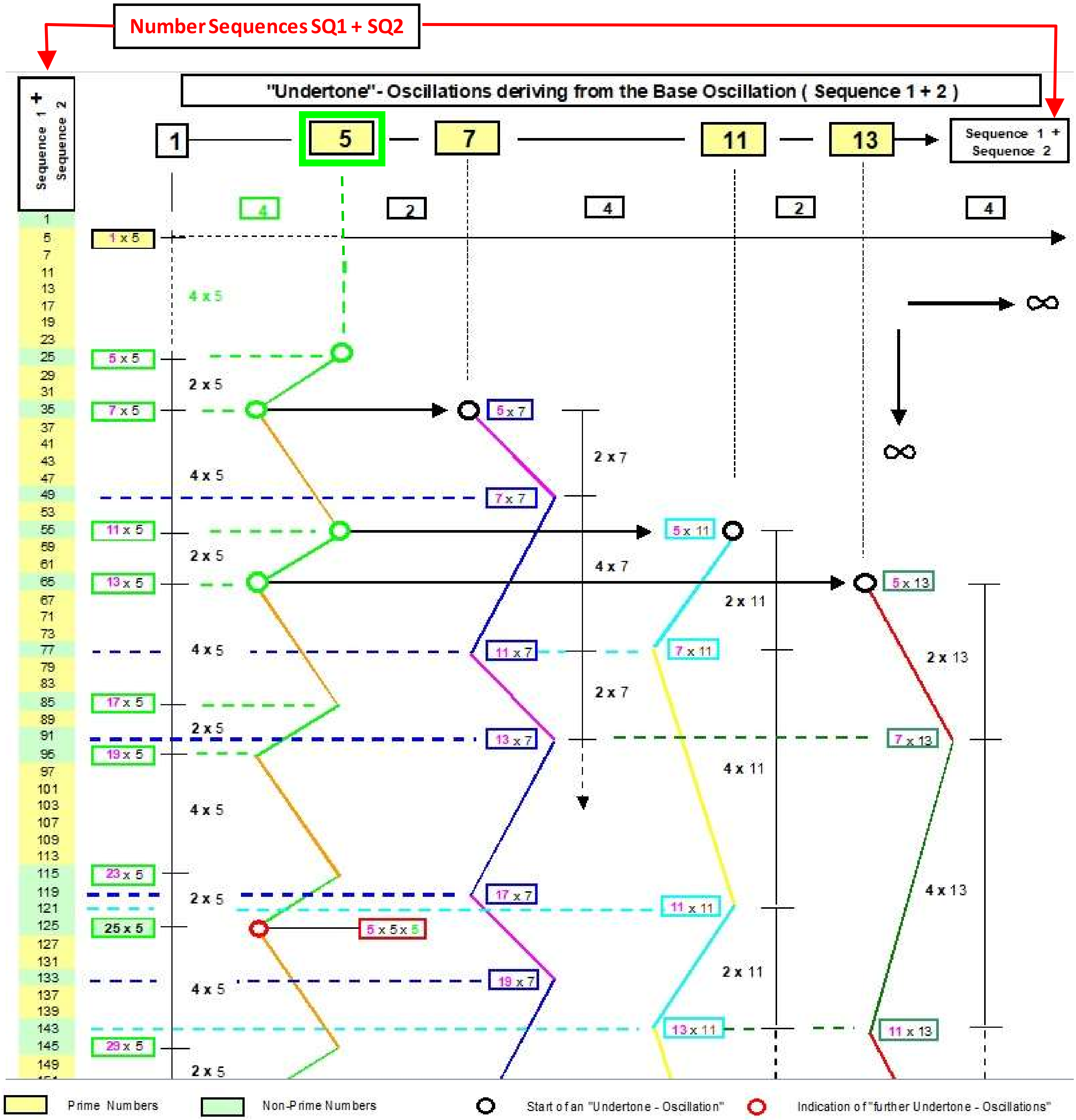
➔ **The distribution of all non-prime numbers in the number sequences SQ1 and SQ2 is defind by a simple logic :**

We consider the following two number sequences which contain all prime numbers :

Sequence 1 (SQ1) : 5, 11, 17, 23, 29, **35**, 41, 47,.... and

Sequence 2 (SQ2) : 1, 7, 13, 19, **25**, 31, 37, 43,....

These two sequences are based on the well known fact, that every prime number is either of the form 6n + 1 or 6n + 5. Or in other words, if a prime number of these two number sequences is divided by 6 the rest of -1 or +1 remains.



Here a short description of the simple rules which define the distribution of the non-prime numbers :

- **Every peak of an Undertone-Oscillation corresponds to a non-prime number in Sequence 1 & 2 (SQ1 & SQ2)**
- **On the contrary "Prime Numbers" represent places in Sequence 1 & 2 which do not correspond with any peak of an Undertone-Oscillation.**

  Prime numbers represent "spots" in the two basic Number-Sequences SQ1 & SQ2 where there is no interference caused by the Undertone Oscillations shown on the righthand side.

- **In every Undertone Oscillation "further Undertone Oscillations" occur, which again are defined by the numbers contained in Sequence 1 & 2.**

  However these "further Undertone Oscillations" are not required to explain the existence of the non prime numbers in Sequence 1 & 2, because the non prime numbers in these sequences are already explained by the undertone oscillations which directly derive from Sequence 1 & 2.

  (→ "further Undertone Oscillations" are marked by red circles on the corresponding peaks of the Undertone Oscillations. Prime factor products of the numbers which belong to these peaks are shown in red and pink boxes)

  Example : The numbers 125, 175, 275 and 325 in the Undertone Oscillation **5** (=1/5f), represent the prime factor products **5**x5x**5**, **5**x5x**7**, **5**x5x**11** and **5**x5x**13.** It is easy to see that these prime factor products form another Undertone Oscillation **5** inside the Undertone Oscillation **5** !!

The following properties are important, because they show the importance of number 5 !! :

- **On every peak of the Undertone Oscillation 5 (=1/5 f) another Undertone Oscillation starts.**
  Undertone Oscillation 5 is the cause (trigger) of all other Undertone Oscillations !
  The green circles on the first few peaks of the Undertone Oscillation **5** mark the starting points of the next 3 Undertone Oscillations **7, 11** and **13** (= 1/**7**f, 1/**11**f and 1/**13**f). More of such Undertone Oscillations will start on every peak of the Undertone Oscillation **5** ad infinitum.

  Note that Undertone Oscillations which are defined by non-prime numbers (e.g. 1/**25**f or 1/**35**f etc.) are not required to explain the non-prime numbers in Sequence 1 & 2 !!

- **If we consider Sequence 1 and 2 (SQ1 & SQ2) simultaneously then it applies that new prime factors at first only occur together with the prime factor 5** !!!

These are very important properties ! It shows that the number 5 oscillation is defining the distribution of all non-prime numbers in the Number Sequences 1 + 2. Mathematicians doesn't seem to know this properties !

For the distribution of the prime numbers the following simple rule applies in the „Wave Model" :

- **Every peak of an Undertone-Oscillation corresponds to a non-prime number in Sequence 1 & 2 (SQ1 & SQ2)**

  Non-Prime Numbers

- **On the contrary "Prime Numbers" represent places in Sequence 1 & 2 which do not correspond with any peak of the clearly (by simple rules) definded Undertone-Oscillation.**

  Prime Numbers

The above described simple rules (properties) clearly define the distribution of all prime numbers !

From these rules the following simple definitions for the groups of primes and non-primes can be derived :

→ **See next page :**

General description of this "Wave Model" and the prime number distribution :

Definition of **Sequence 1 & 2** (Base oscillation with frequency **f**) in mathematical terms :

SQ1 (Sequence 1) : $\mathbf{a_n = 5 + 6n}$ for example $a_0 = 5$; $a_1 = 11$; $a_2 = 17$ etc.

SQ2 (Sequence 2) : $\mathbf{b_n = 1 + 6n}$ for example $b_0 = 1$; $b_1 = 7$; $b_2 = 13$ etc.

with $n \in N = \{0, 1, 2, 3, 4,\dots\}$

Description of the "**Undertone Oscillation 5**" (= 1/**5** f) :

→ undertone oscillation **5** is split into two number sequences $\mathbf{U\text{-}5_1}$ and $\mathbf{U\text{-}5_2}$:

$\mathbf{U\text{-}5_1}$: $\mathbf{a(5)_n = 5(\ 5 + 6n\ )}$ for example $a_0 = 25$; $a_1 = 55$; $a_2 = 85$ etc.

$\mathbf{U\text{-}5_2}$: $\mathbf{b(5)_n = 5(\ 1 + 6n\ )}$ for example $b_1 = 35$; $b_2 = 65$; $b_3 = 95$ etc.

with $n \in N = \{0, 1, 2, 3, 4,\dots\}$ for $\mathbf{U\text{-}5_1}$ and with $n \in N^* = N\backslash\{0\} = \{1, 2, 3, 4,\dots\}$ for $\mathbf{U\text{-}5_2}$

General description of all "**Undertone Oscillations X**" (= 1/**X** f) :

→ every undertone oscillation is split into two number sequences $\mathbf{U\text{-}(x)_1}$ and $\mathbf{U\text{-}(x)_2}$:

$\mathbf{U\text{-}(x)_1}$: $\mathbf{a(x)_n = x(\ 5 + 6n\ )}$ with $n \in N = \{0, 1, 2, 3, 4,\dots\}$

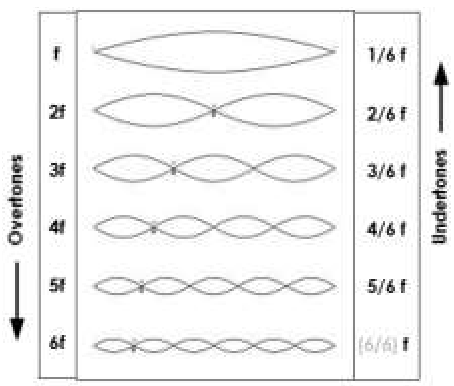
$\mathbf{U\text{-}(x)_2}$: $\mathbf{b(x)_n = x(\ 1 + 6n\ )}$ with $n \in N^* = N\backslash\{0\} = \{1, 2, 3, 4,\dots\}$

and with $X \in (\ SQ1 \cup SQ2\ )\backslash\{1\} = \{5, 7, 11, 13, 17, 19, 23, 25\dots\}$ for both sequences $a(x)_n$ & $b(x)_n$

According to the above described definitions the set of prime numbers (PN) can be defined as follows :

$\mathbf{PN^* = (\ SQ1 \cap SQ2\ )\ \backslash\ (\ U\text{-}(x)_1 \cap U\text{-}(x)_2\ )}$

$\mathbf{PN = \{\ 2, 3\ \} \cap (\ SQ1 \cap SQ2\ )\ \backslash\ (\ U\text{-}(x)_1 \cap U\text{-}(x)_2\ )}$



Overtones (or " harmonics") and Undertones, deriving from a fundamental frequency f

for **PN*** and **PN** the following definition applies :

PN = { 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, ...} ; **set of prime numbers**

and **PN*** = { 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, ...} = **PN \ {** 2, 3 **}**

further the following definitions applies :

$\mathbf{PN^* \subset (\ SQ1 \cap SQ2\ )}$ or $\mathbf{PN^* \subset (\ a_n \cap b_n\ )}$

$\mathbf{PN^* \not\subset (\ U\text{-}(x)_1 \cap U\text{-}(x)_2\ )}$ or $\mathbf{PN^* \not\subset (\ a(x)_n \cap b(x)_n\ )}$ with $\mathbf{x} \in (\ SQ1 \cup SQ2\ )\ \backslash\ \{1\}$

$\mathbf{NPN^* = (\ U\text{-}(x)_1 \cap U\text{-}(x)_2\ )}$ or $\mathbf{NPN^* = (a(x)_n \cap b(x)_n\ )}$; **NPN* =** non-prime-numbers divisible by 2 or 3

8.2. - There are two more fundamental (→ alltogether three) number oscillations :

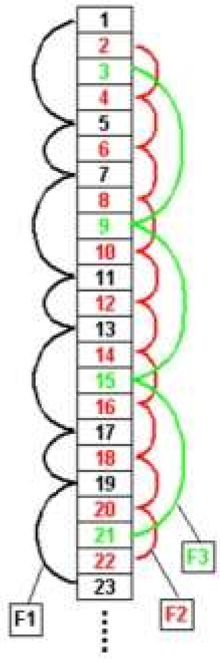


FIG. 7 : There are three fundamental oscillations

The "**Wave Model**" for the logic of the prime number distribution described on the previous pages is **based on all natural numbers not divisible by 2 or 3.**
But there is the rest of the natural numbers which are divisible by 2 or (and) 3, which also have to be taken in consideration !

These numbers represent two more fundamental oscillations which exist parallel to the "base oscillation SQ1 + SQ2" described in Table 2. And the same physical principle of the creation of "Undertones" would of course also apply to **these two additional fundamental oscillations**, whose highest mode frequencies **could be named f2 and f3.**

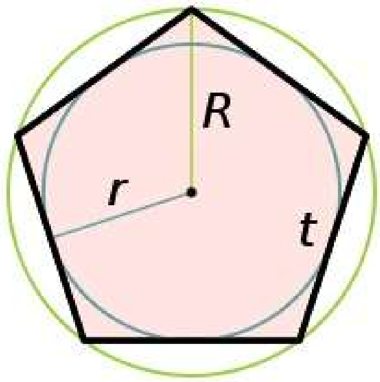
Accordingly the highest mode frequency of **the fundamental oscillation SQ1 + SQ2 would then be named f1**.

The image on the righthand side (FIG. 7) shall give an idea of the coexistence of the described „Three fundamental number oscillations".

8.3. Geometrical objects closely conected to prime number 5 (& sqrt 5) are the Pentagon & Dodecahedron

To the Pentagon :

A *regular pentagon* has five lines of reflectional symmetry, and rotational symmetry of order 5 (through 72°, 144°, 216° and 288°). The diagonals of a convex regular pentagon are in the golden ratio to its sides. Its height (distance from one side to the opposite vertex) and width (distance between two farthest separated points, which equals the diagonal length) are given by :



Side (t), circumcircle radius (R), inscribed circle radius (r), height ($R + r$), width/diagonal (φt)

golden ratio constant **φ** ↓

$$\text{Height} = \frac{\sqrt{5+2\sqrt{5}}}{2} \cdot \text{Side}$$

$$\text{Width} = \text{Diagonal} = \frac{1+\sqrt{5}}{2} \cdot \text{Side} \quad \approx 1.618 \cdot \text{Side}$$

$$\text{Diagonal} = R\sqrt{\frac{5+\sqrt{5}}{2}}$$

where R is the radius of the circumcircle.

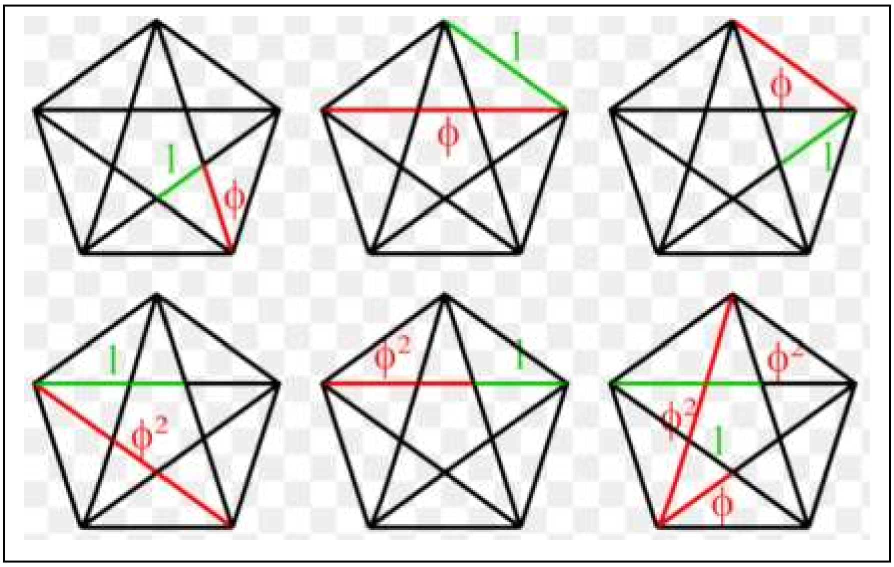
its edge length t is given by the expression

$$t = R\sqrt{\frac{5-\sqrt{5}}{2}}$$

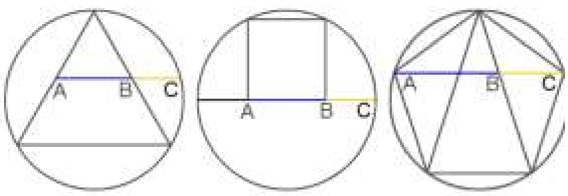
and its area is

$$A = \frac{5R^2}{4}\sqrt{\frac{5+\sqrt{5}}{2}}$$

Interesting is the multiple appearance of the golden ratio (golden mean) constant **φ** in the pentagon :



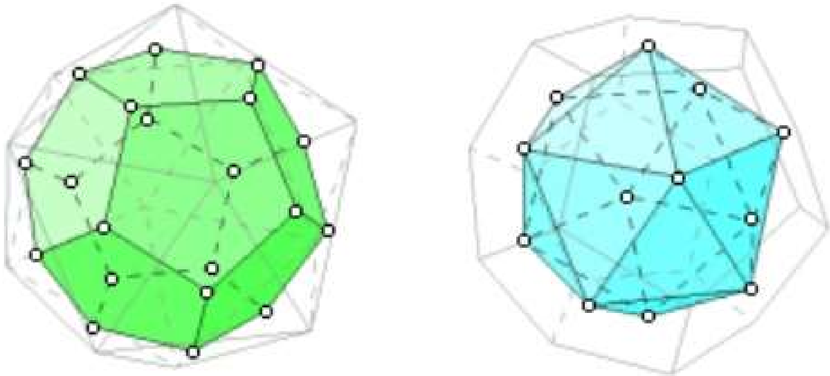
The golden ratio also appears if an equal angle triangle, a square and a pentagon is inscribed in a circle as shown :



8.4. To the Dodecahedron :

A **regular** **Dodecahedron_** or **pentagonal dodecahedron** is a dodecahedron that is regular, which is composed of twelve regular pentagonal faces, three meeting at each vertex.

It is one of the five Platonic Solids. And it is obviously the most important one ! (H.K.Hahn), as the M87 black hole indicates ! It has 12 faces, 20 vertices, 30 edges, and 160 diagonals (60 face diagonals, 100 space diagonals). It is represented by the Schläfli symbol {5,3}.



The dodecahedron is the **dual** of the icosahedron. Connecting the centers of adjacent faces of the dodecahedron results in an icosahedron, and connecting the centers of the icosahedron faces results in a dodecahedron.

If the edge length of a regular dodecahedron is ***a*** , the radius of a circumscribed sphere **ru** (one that touches the regular dodecahedron at all vertices) is :

$$r_u = a\frac{\sqrt{3}}{4}\left(1+\sqrt{5}\right) \rightarrow$$

and the radius of an inscribed sphere **ri** (tangent to each of the regular dodecahedron's faces) is :

$$r_i = a\frac{1}{2}\sqrt{\frac{5}{2}+\frac{11}{10}\sqrt{5}} \rightarrow$$

while the midradius **rm**, which touches the middle of each edge, is :

$$r_m = a\frac{1}{4}\left(3+\sqrt{5}\right) \rightarrow$$

These quantities can also be expressed as :

$$r_u = a\,\frac{\sqrt{3}}{2}\phi$$

$$r_i = a\,\frac{\phi^2}{2\sqrt{3-\phi}}$$

$$r_m = a\,\frac{\phi^2}{2}$$

where ϕ is the golden ratio.

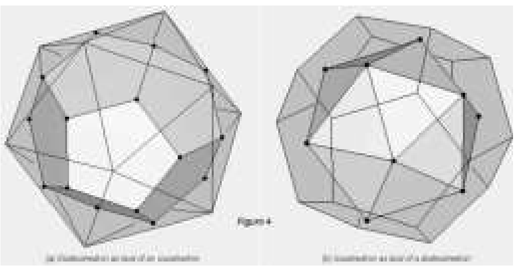
Please also see the following weblink : → **Phi_sacred_Solids** →
→The golden ratio Phi (φ) in Platonic Solids

Weblink to more formulas which describe the dodecahedron

see : polyhedra_dodecahedron

The Dodecahedron has geometric relations to all five Platonic Solids
(see also image above) :

By connecting select vertices of the dodecahedron, it is possible to form a **Tetrahedron** or a **Cube.** By connecting midpoints of certain edges, it is possible to form an **Octahedron**.



The small stellated Dodecahedron contains three powers of Φ (φ)

A,(0,	φ²,	φ,)
B,(0,	φ²,	-φ,)
C,(0,	-φ²,	-φ,)
D,(0,	-φ²,	φ,)
E,(φ²,	φ	0)
F,(φ²,	-φ	0)
G,(-φ²,	-φ	0)
H,(-φ²,	φ	0)
I,(φ,	0,	φ²)
J,(-φ,	0,	φ²)
K,(-φ,	0,	-φ²)
L,(φ,	0,	-φ²)

Table 4: The cartesian coordinates of the small stellated dodecahedron. The vertices of the original dodecahedron (Table 3), though visible, are not vertices of the stellation.

←

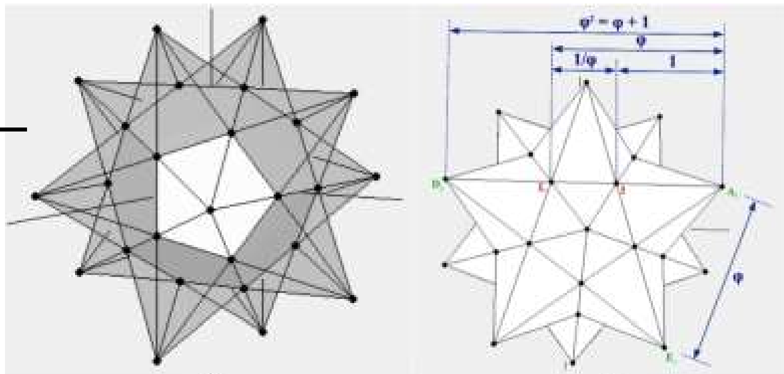


Figure 6: The small stellated dodecahedron contains three powers of the Golden Ratio. This can be clearly appreciated in its pentagram faces. Its outer vertices define an icosahedron whose edge length (A₁E₁) is in a proportion 1:φ² with the edge of the original dodecahedron (IL).

8.5. The constant φ (Φ) is a mathematical constant which appears everywhere in nature

The asymptotic ratio of successive Fibonacci numbers leads to the Golden Ratio constant φ (or Φ)

The Fibonacci Sequence describes morphological patterns in a wide range of living organisms. It is one of the most remarkable organizing principles mathematically describing natural and manmade phenomena

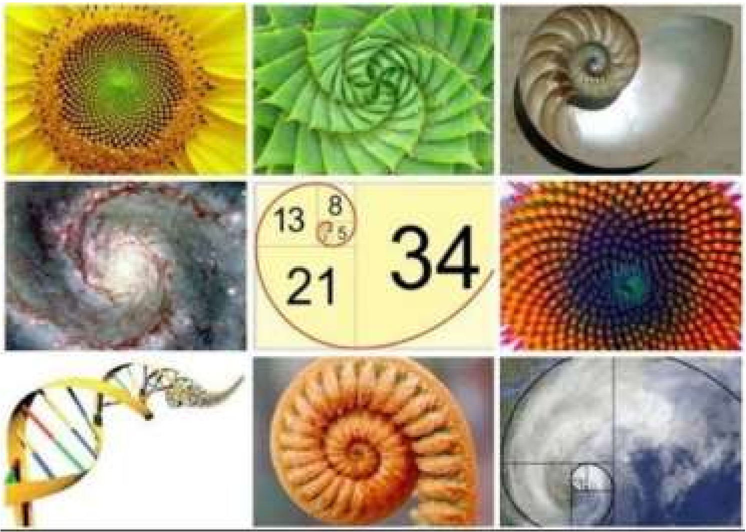
$$\varphi = x = \frac{1+\sqrt{(5)}}{2} = 1.618034...$$

The constant φ is the positive solution of the following quadratic equation :

$$x + 1 = x^2$$

The constant φ can be written in terms of itself :

$$\frac{1.618 + 1}{1.618} = 1.618$$



The **Fibonacci numbers** defined by φ :

1/1 = 1
2/1 = 2
3/2 = 1.5
5/3 = 1.667
8/5 = 1.6
13/8 = 1.625
21/13 = 1.615
34/21 = 1.619
55/34 = 1.618 φ

The following most simple periodic continued fraction describes φ :

Substitute φ = 1 + 1/φ for φ in the denominator

$$\varphi = \frac{\varphi + 1}{\varphi} \rightarrow \varphi = 1 + \frac{1}{\varphi} \rightarrow \varphi = 1 + \frac{1}{1 + \frac{1}{\varphi}} ...$$

periodic continued fraction of φ

Phyllotactic spirals form a distinctive class of patterns in nature. The basic arrangement of leaves and seeds is an opposite alternate (spiral) pattern. The current theory says that a certain hormone (auxin) is responsible for the pattern. But because phyllotactic patterns defined by the Fibonacci sequence also appear outside biology there must be a more profound physical law which is causing these extremely precise patterns. **The following extracts from 6 selected studies clearly indicate that there is a more fundamental universal law at work , responsible for these Fibonacci patterns in nature !**

→ Please also have a read through the study : **„An infinite Fibonacci Number Sequence Table“**

Study 1 : http://surface.iphy.ac.cn/sf03/articles/2006-2007/2007APL-chirality.pdf

➔ This study shows how phyllotactic patterns appear outside biology on spherical or nearly spherical surfaces :

Stressed Fibonacci spiral patterns of definite chirality

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(Received 21 August 2006; accepted 22 February 2007; published online 18 April 2007)

Fibonacci spirals are ubiquitous in nature, but the spontaneous assembly of such patterns has rarely been realized in laboratory. By manipulating the stress on Ag core/SiO_2 shell microstructures, the authors obtained a series of Fibonacci spirals (3×5 to 13×21) of definite chirality as a least elastic energy configuration. The Fibonacci spirals occur uniquely on conical supports-spherical receptacles result in triangular tessellations, and slanted receptacles introduce irregularities. These results demonstrate an effective path for the mass fabrication of patterned structures on curved surfaces; they may also provide a complementary mechanism for the formation of phyllotactic patterns.

 [DOI: 10.1063/1.2728578]

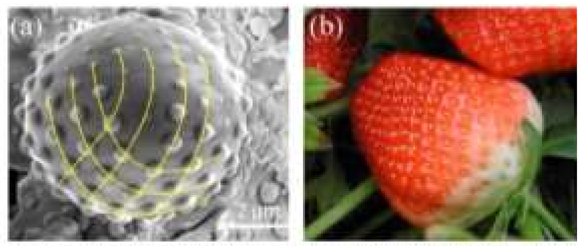

FIG. 4. Parastichous spirals on frustrating surfaces. (a) Stressed pattern on slanted Ag core/SiO_2 shell microstructure, and (b) the "X pattern" of achenes in a strawberry.

In summary, we demonstrated that the Fibonacci spiral patterns of definite chirality can be reproduced through stress manipulation on the Ag core/SiO_2 shell microstructures. These results will be very helpful for the design and fabrication of patterned structures on curved surfaces that can find useful applications in photonics and foldable electronics. Furthermore, these results obtained in a purely inorganic material system hint at the role of stress in influencing the plant patterns. We speculate that the prerequisite for the occurrence of Fibonacci spiral patterns as stressed buckling modes be the availability of a conical support. The robust adherence of the stressed patterns to the geometry of the supports sheds some light on the mechanical rationale underlying the formation of particular plant patterns. Of course, a comprehensive model for the formation of plant patterns should incorporate as well the biochemical and genetic processes that alter growth at deeper levels.

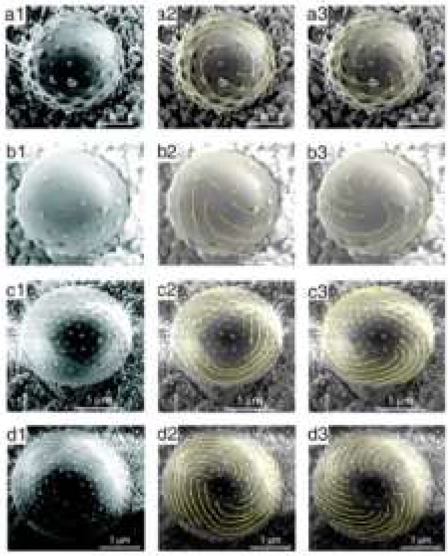

FIG. 1.
Fibonacci spiral patterns in the sinister form grown **on Ag-core/SiO2 shell microstructures: (a) 3x5, (b) 5x8, (c) 8x13 and (d) 13x21.**
Each individual pattern is presented in triad, one original and two with plotted ounterclockwise and clockwise spirals to guide the eyes.

Study 2 : https://hal.archives-ouvertes.fr/jpa-00212565/document

→ Crystalline phases with the cubic symmetry close to the icosahedral one cause **Fibonacci Crystals in Al-Mn/-Si alloys** :

Cubic approximants in quasicrystal structures

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Abstract. — The regular deviations from the exact icosahedral symmetry, usually observed at the diffraction patterns of quasicrystal alloys, are analyzed. It is shown that shifting, splitting and asymmetric broadening of reflections can be attributed to crystalline phases with the cubic symmetry very close to the icosahedral one (such pseudo-icosahedral cubic approximants may be called the Fibonacci crystals). The Fibonacci crystal is labelled as $\langle F_{n+1}/F_n \rangle$, if in this crystal the most intense vertex reflections have the Miller indices $\{0, F_n, F_{n+1}\}$ where F_i are the Fibonacci numbers (F_i = 1, 1, 2, 3, 5, 8, 13, 21, 34...). The deviations of x-ray and electron reflections from their icosahedral positions are calculated. The comparison with available experimental data shows that at least four different Fibonacci crystals have been observed in Al-Mn and Al-Mn-Si alloys ⟨2/1⟩ (MnSi structure), ⟨5/3⟩ (α-Al-Mn-Si), ⟨13/8⟩, and ⟨34/21⟩ with the lattice constants 4.6 Å, 12.6 Å, 33.1 Å, 86.6 Å respectively. It is interesting to note that there are no experimental evidences for the intermediate approximants ⟨3/2⟩, ⟨8/5⟩ and ⟨21/13⟩. The possible space groups of the Fibonacci crystals and their relationships with quasicrystallographic space groups are discussed.

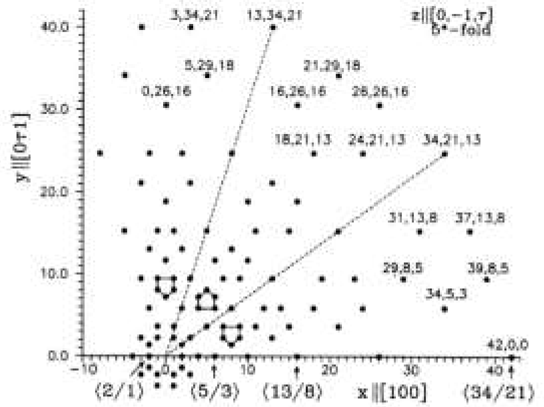
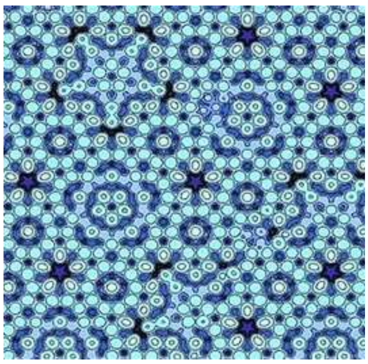


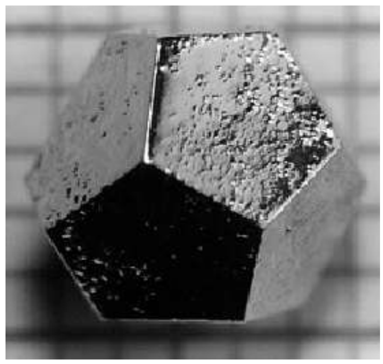
Fig. 3. — Universal pseudo-fivefold electron-diffraction pattern of the Fibonacci crystals (incident beam falls along − z direction). All x-coordinates are integer : x_{hkl} = h. Only about one fourth of the pattern is shown ; the complete picture can be obtained by the mirror reflections : x → − x and (or) y → − y. The scale of the pattern are determined by the positions of the most intense edge reflections ($Q \approx 3$ Å^{-1}) ; for the different Fibonacci crystals those positions are marked by arrows. The Miller indices are shown for outer reflections only ; the indices of inner reflections can be obtained from the outer ones after multiplication by factor τ^{-i} (i = 1, 2, 3, ...) and round-up to the closest integer. The innermost ten reflections are of the type {200} and {211}. The approximate τ-inflation is clearly seen in the figure (pentagons with strongest distortions are marked with solid lines). Pseudo-twofold axes are shown by dashed lines.

Quasi-Crystals :

A quasiperiodic crystal, or **Quasicrystal** , is a structure that is ordered but not periodic. A quasicrystalline pattern can continuously fill all available space, but it lacks translational symmetry. While crystals, according to the classical crystallographic restriction theorem, can possess only two, three, four, and six-fold rotational symmetries, the Bragg diffraction pattern of quasicrystals shows sharp peaks with other symmetry orders, for instance **five-fold.**



Atomic model of a Al-Pd-Mn Quasicrystal



Ho-Mg-Zn Quasicrystal with a dodecahedral shape

Aperiodic tilings were discovered by mathematicians in the early 1960s, and, some twenty years later, they were found to apply to the study of natural quasicrystals. The discovery of these aperiodic forms in nature has produced a paradigm shift in the fields of crystallography. Roughly, an ordering is non-periodic if it lacks translational symmetry, which means that a shifted copy will never match exactly with its original. The more precise mathematical definition is that there is never translational symmetry in more than *n* – 1 linearly independent directions, where *n* is the dimension of the space filled, e.g., the three-dimensional tiling displayed in a quasicrystal may have translational symmetry in two directions. Symmetrical diffraction patterns result from the existence of an indefinitely large number of elements with a regular spacing, a property loosely described as long-range order. Experimentally, the aperiodicity is revealed in the unusual symmetry of the diffraction pattern, that is, symmetry of orders other than two, three, four, or six.

Study 3 : 252736598_Fibonacci_order_in_the_period-doubling_cascade_to_chaos

→ The **Fibonacci number sequence (φ)** appears **in the Feigenbaum scaling of the period-doublin cascade** to chaos.

Fibonacci order in the period-doubling cascade to chaos

G. Linage [a], Fernando Montoya [a], A. Sarmiento [b], K. Showalter [c], P. Parmananda [a,*]

Abstract

In this contribution, we describe how the Fibonacci sequence appears within the Feigenbaum scaling of the period-doubling cascade to chaos. An important consequence of this discovery is that the ratio of successive Fibonacci numbers converges to the golden mean in every period-doubling sequence and therefore the convergence to ϕ, the most irrational number, occurs in concert with the onset of deterministic chaos.

Two of the most remarkable organizing principles mathematically describing natural and man-made phenomena are the Fibonacci number sequence and the Feigenbaum scaling of the period-doubling cascade to chaos. The Fibonacci sequence describes morphological patterns in a wide variety of living organisms [1], and the asymptotic ratio of successive Fibonacci numbers yields the golden mean. The Feigenbaum scaling [2] for the period-doubling cascade to chaos has been observed in a wide range of dynamical systems, from turbulence to cell biology to chemical oscillators [3,4]. Here we describe how the Feigenbaum scaling and the Fibonacci sequence are intimately intertwined.

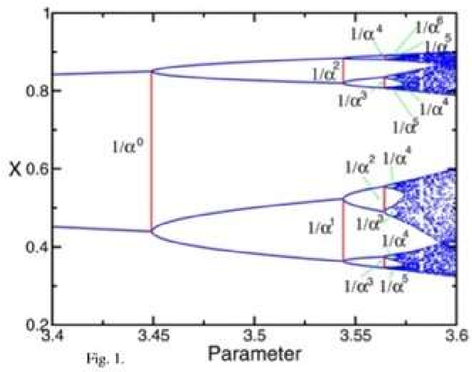


Fig. 1.

Table 1 reveals how the Fibonacci series develops in the period-doubling cascade. We see that after the first bifurcation gives rise to the period-2 oscillation, there is one segment of width $1/\alpha^0$. An examination of the successive bifurcations shows how the Fibonacci sequence expands with each bifurcation. We also see that there is a pattern of branch widths within each period. Every bifurcation contains a $1/\alpha^n$ branch, which is the first component of a binomial expansion of branch widths according to $(1/\alpha + 1/\alpha^2)^n$ as well as the remaining components of the expansion. The exponent n corresponds to the bifurcation, with $n = 0$ for period-2, $n = 1$ for period-4, and so on.

We emphasize that the Fibonacci sequence will be found in all dynamical systems exhibiting the period-doubling route to chaos, as it is directly linked to the Feigenbaum scaling constant α. However, since the scaling of α holds only in the asymptotic limit, the reported relation also holds only for asymptotic values. It follows that the ratio of successive Fibonacci numbers converges to the golden mean [7], ϕ, in every period-doubling cascade. Thus, the convergence to ϕ, the "most irrational number" [8], occurs in concert with the onset of deterministic chaos.

A typical period-doubling bifurcation diagram for the logistic map is shown in Fig. 1. We normalize the width of the period-2 branch to unity ($1/\alpha^0$), allowing the widths of the branches corresponding to higher periods to be written as inverse powers of α. With this normalization, we notice that the number of branches corresponding to the various powers of $1/\alpha$ follows the sequence:

$1/\alpha^0$, $1/\alpha^1$, $2/\alpha^2$, $3/\alpha^3$, $5/\alpha^4$, $8/\alpha^5$, $13/\alpha^6$, $21/\alpha^7$, $34/\alpha^8$, $55/\alpha^9$, ...

where the coefficients 1, 1, 2, 3, 5, 8, ... correspond to the number of branches with widths $1/\alpha^0$, $1/\alpha^1$, $1/\alpha^2$, $1/\alpha^3$, $1/\alpha^4$, $1/\alpha^5$, ..., respectively. These coefficients are the beginning of the Fibonacci sequence.

Table 1
Distribution of powers of $1/\alpha$ in successive period-doubling bifurcations

	P2 $n=0$	P4 $n=1$	P8 $n=2$	P16 $n=3$	P32 $n=4$	P64 $n=5$		F.N.
$1/\alpha^0$	1							1
$1/\alpha^1$		1						1
$1/\alpha^2$		1	1					2
$1/\alpha^3$			2	1				3
$1/\alpha^4$			1	3	1			5
$1/\alpha^5$				3	4	1		8
$1/\alpha^6$				1	6	5	...	13
$1/\alpha^7$					4	10	...	21
$1/\alpha^8$					1	10	...	34

Study 4 : http://www.itp.uni-bremen.de/prichter/download/DoppelpendelIWF.pdf ; **Movie : https://av.tib.eu/media/14902**

→ The pendulum (P1) orbit with the winding number W= g^2=0.3820 (g= φ -1) can resist longest to the chaos

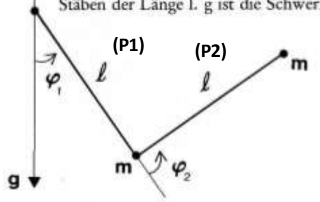
The Planar Double Pendulum   Das ebene Doppelpendel

Verfasser der Publikation: PETER H. RICHTER und HANS-JOACHIM SCHOLZ

Summary of the Film:

The Planar Double Pendulum. Computer experiments made it possible to describe the complex dynamics of this classical example in mechanics. To begin with, the various types of motion of the double pendulum are presented. With the help of the method of Poincaré sections, a qualitative survey of the complex dynamics follows, with special emphasis on irrational winding numbers (golden ratio).

Abb. 1b. Eine mathematische Idealisierung Die Massen m hängen als Punkte an starren Stäben der Länge l. g ist die Schwerkraft



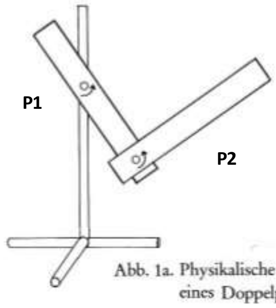


Abb. 1a. Physikalische Realisierung eines Doppelpendels

„Dreihundert Jahre nach Newton sollten wir eigentlich wissen, was seine Gleichungen uns über das qualitative Verhalten konservativer Systeme mit zwei Freiheitsgraden lehren" – so MICHAEL V. BERRY in einem vielbeachteten Übersichtsartikel ([2]), der das aktuelle Interesse von Physikern und Mathematikern an der Klassischen Mechanik reflektiert. Tatsache aber ist, daß wir erst jetzt zu sehen beginnen, was alles in diesen scheinbar so einfachen Gleichungen steckt. Denn erst seit kurzem haben wir Zugang zu ihren Lösungen. Es bedurfte der Entwicklung moderner Computer, den Reichtum an Komplexität aufzudecken, der schon den einfachsten Systemen innewohnt. Gegenwärtig erleben wir eine Phase, in der ausgiebiges mathematisches Experimentieren zunächst die Phänomene zutage fördert, während die theoretische Analyse dann immer wiederkehrende Szenarien zu verstehen sucht.

Figure 5 shows Poincare-Sections for the Double Pendulum at different values of the total energy of the pendulum. Different colors mean different orbits.

X-axis = angle φ1 of Pend. 1 (P1)

y-axis = angular momentum P1

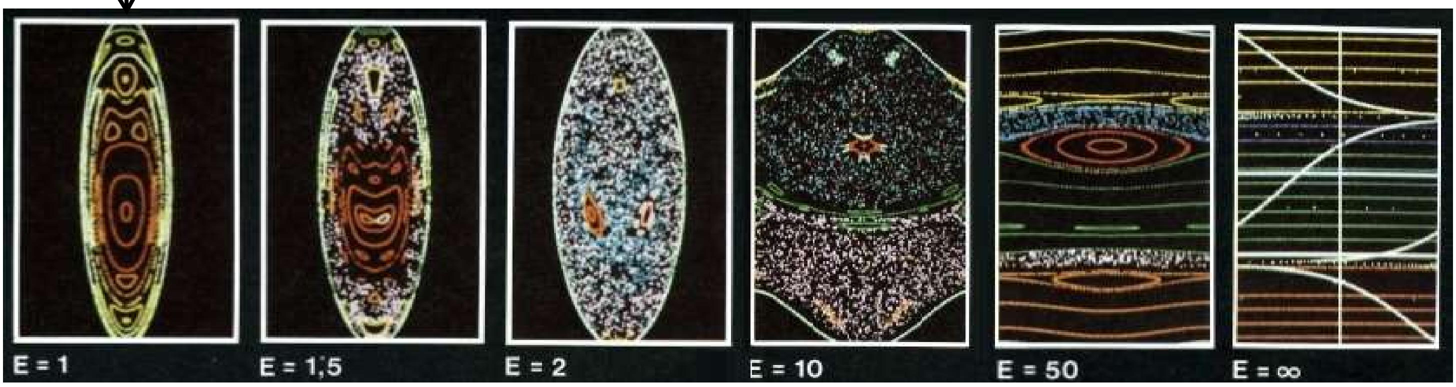


Abb. 5. Poincaré-Schnitte für das ebene Doppelpendel bei verschiedenen Werten der Gesamtenergie E (in Einheiten von mgl). Auf der Abszisse ist der Winkel φ_1 des inneren Pendels, auf der Ordinate der entsprechende Drehimpuls L_1 aufgetragen. Die Bewegung wird immer dann aufgezeigt, wenn $\varphi_2 = 0$ und $\dot{\varphi}_2 > 0$ sind. Jedes Bild wurde aus etwa 10 bis 20 Anfangsbedingungen und 300 Folgepunkten generiert. Mit Hilfe der Farbe können verschiedene Orbits unterschieden werden. Das letzte Bild zeigt den integrierbaren Fall E = ∞ oder g = 0, wo der Drehimpuls konstant bleib

ARNOLD ([1]) und MOSER ([9]) haben unabhängig voneinander mit mathematischer Strenge gezeigt, daß i n v a r i a n t e L i n i e n mit Windungszahlen W a u c h u n t e r S t ö r u n g e n noch existieren, wenn diese hinreichend klein sind und wenn W eine sog. diophantische Bedingung erfüllt: Für jede rationale Approximation p/q muß eine Abschätzung der Art

$$\left| W - \frac{p}{q} \right| > \frac{c}{q^{\tau}} \qquad (4)$$

→ W = winding number of pendulum L1 per winding of pendulum L2 ; (p, q → two numbers not simply divisible)

möglich sein, mit festem c und τ. Es läßt sich zeigen, daß die meisten irrationalen Zahlen eine solche Bedingung erfüllen. Sie läßt sich dazu verwenden, den Grad der Irrationalität einer Zahl W zu definieren. Denn der nach (4) geforderte Abstand zu den rationalen Zahlen wird größer, wenn τ kleiner und c größer werden. Wir nennen eine Zahl W_1 i r r a t i o n a l e r als eine Zahl W_2, wenn das zu W_1 gehörige minimale τ kleiner und das maximale c größer sind als die entsprechenden Werte für W_2.

In diesem Sinne nun erweist sich das Verhältnis des Goldenen Schnitts als die irrationalste Zahl. Erinnern wir uns an die Definition: der Wert g teilt die Strecke 1 im goldenen Verhältnis, wenn der kleinere Teil 1 – g sich zu g verhält wie g zu 1.

Also (1 – g) : g = g : 1. Daraus folgt sofort

$$g^2 + g - 1 = 0, \; g = \frac{\sqrt{5}-1}{2} = 0.618034 \qquad (5)$$

Gelegentlich werden auch die Zahlen

$$G = 1/g = 1 + g = 1.618034$$

oder $\quad g^2 = 1 - g = 0.381966$

als Goldener Schnitt bezeichnet. Ihnen allen ist gemeinsam, daß sie den größtmöglichen Abstand von den rationalen Zahlen haben, denn wenn W = g, G oder g^2 und p, q beliebig, so gilt

$$\left| W - \frac{p}{q} \right| \geq \frac{g^2}{q^2} \qquad (6)$$

Keine Zahl erlaubt eine noch schärfere Abschätzung. Für eine genauere Diskussion ver weisen wir auf Bücher über Zahlentheorie, etwa SCHRÖDER ([11]). Wir begnügen un hier mit dem Hinweis auf die Kettenbruchentwicklung der Zahl g, die sich aus (5) unmit telbar ergibt:

$$g = \frac{1}{1+g} \longrightarrow g = \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + ..}}} \qquad (7$$

Brechen wir die Folge der Brüche nach dem n-ten Schritt ab, so erhalten wir die n-te Ket tenbruchapproximation g_n für g. Es gilt

$$\{ g_1, g_2, \ldots, g_n \ldots \} = \{ 1, \tfrac{1}{2}, \tfrac{2}{3}, \tfrac{3}{5}, \tfrac{5}{8} \ldots \frac{F_n}{F_{n+1}}, \ldots \} \qquad (8$$

wobei die F_n die bekannten Fibonacci-Zahlen sind: $F_1 = F_2 = 1$, $F_{n+1} = F_n + F_{n-1}$. Für G ist die n-te Kettenbruchapproximation $G_n = F_n/F_{n-1}$, für g^2 finden wir $(g^2)_n = F_n/F_{n+2}$. Di Bedeutung dieser Kettenbruchapproximationen liegt darin, daß sie für gegebene (ode kleinere) Nenner jeweils die beste rationale Annäherung an die irrationale Zahl W darstel len. Die langsame Konvergenz der Folgen im Sinne von (6) rührt daher, daß die Entwick lung (7) nur Einsen enhält, also die kleinstmöglichen ganzen Zahlen.

Was aber hat das mit der Dynamik des Doppelpendels zu tun? Abb. 7 gibt darauf die Ant wort. Wenn wir, vom integrablen Grenzfall E = ∞ ausgehend, die Energie erniedrige und dabei anhand der Poincaré-Schnitte die Ausbreitung des Chaos verfolgen, dan sehen wir zunächst noch viele invariante Linien. Die einzelnen Chaosbänder sind relati schmal, die Bewegung hat vorwiegend regelmäßigen Charakter. Allmählich aber ver schwinden mehr und mehr dieser Linien; Chaosbänder verschmelzen zu größeren, un schließlich – etwa bei E = 10 – gibt es nur noch eine letzte solche „K A M - L i n i e" (s genannt nach den Mathematikern KOLMOGOROFF, ARNOLD und MOSER). Diese letzt Form regelmäßiger Bewegung, ehe e i n Chaos die ganze Energieschale überschwemm hat als Windungszahl das goldene Verhältnis:

$$W = g^2 = 0.381\,966 \qquad (9$$

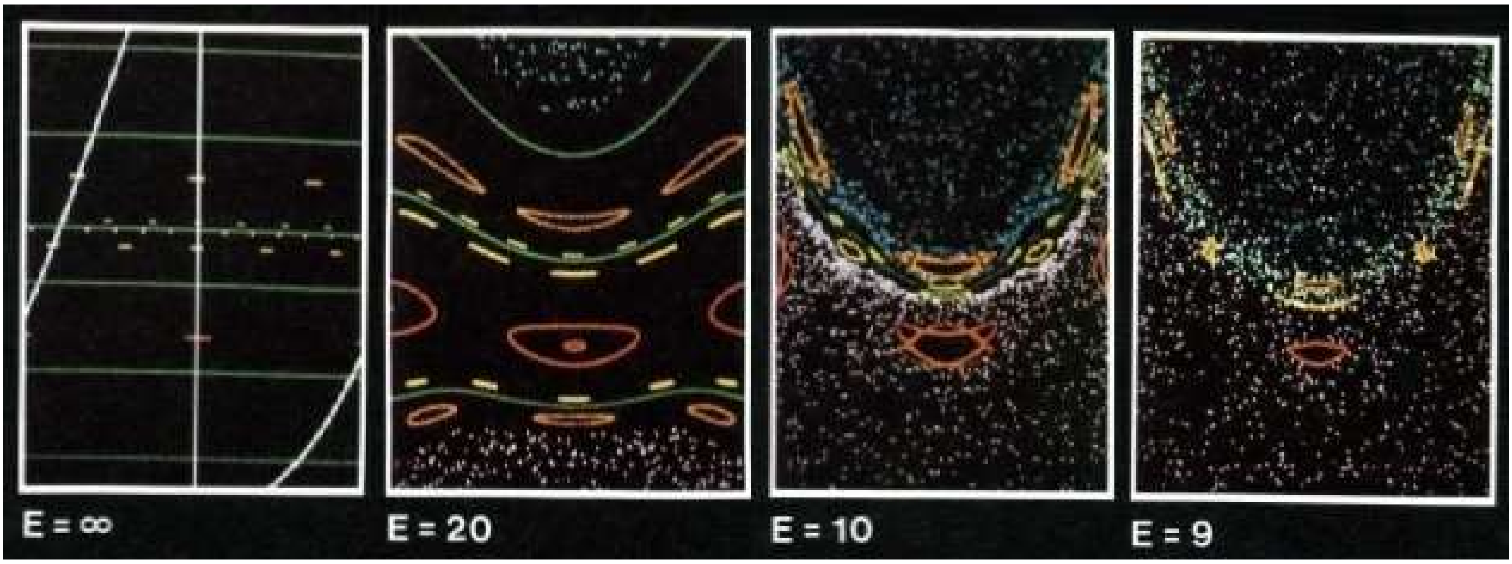


Figure 7 shows details from Images of Figure 5 →

Abb. 7. Diese Serie zeigt jeweils Ausschnitte der entsprechenden Bilder von Abbildung 5. Sie demonstriert das Schicksal einiger periodischer (W = 1/2, 1/3, 2/5, 3/8) und einiger quasiperiodischer Orbits unter Störung. Der Orbit mit der goldenen Windungszahl $W = g^2 = 0{,}3820$ hält dem Einbruch des Chaos am längsten stand (Bild c). Bei E = 9 ist er auch zerfallen

Figure 7 (Abb.7) shows Poincare-Sections for the double pendulum at different values of the energy motion equation. It shows the fate of some periodic (W=1/2, 1/3, 2/5, 3/8) and of some quasiperiodic orbits under disturbance.

If W = g, G(φ) or g^2 (golden ratio numbers, see above), and p, q (any numbers), then the equation (6) is valid.

→ The golden ratio numbers g, G or g^2 (most irrational numbers) have the biggest possible distance to the rational numbers.

→ Note : The orbit with the winding number W= g^2=0.3820 (g = φ -1) can resist longest to the chaos !

→ This behaviour indicates that **φ** somehow must be connected to gravitation ! (H.K. Hahn)

Study 5 : → Ratios of orbital periods clearly show a preference for Fibonacci-Number ratios → Another indication for a link between the constant φ and gravitation !

→ weblink : https://arxiv.org/pdf/1803.02828

Orbital Period Ratios and Fibonacci Numbers in Solar Planetary and Satellite Systems and in Exoplanetary Systems

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6. Conclusions

It was shown that orbital period ratios of successive secondaries in the Solar planetary and giant satellite systems and in exoplanetary systems are preferentially and significantly closer t irreducible fractions formed with the second to the sixth Fibonacci numbers (between 1 and 8 than to other fractions, in a ratio of approximately 60% vs 40%, although there are less irreducibl fractions formed with Fibonacci integers between 1 and 8 than other fractions.
Furthermore, if sets of minor planets are chosen with gradually smaller inclinations an eccentricities, one observes that the proximity to Fibonacci fractions of their period ratios wit Jupiter or Mars' period tends to increase for more "regular" sets with minor planets on les eccentric and less inclined orbits. Therefore, orbital period ratios closer to Fibonacci fraction could indicate a greater regularity in the system.

Abstract
It is shown that orbital period ratios of successive secondaries in the Solar planetary and giant satellite systems and in exoplanetary systems are preferentially closer to irreducible fractions formed with Fibonacci numbers between 1 and 8 than to other fractions, in a ratio of approximately 60% vs 40%. Furthermore, if sets of minor planets are chosen with gradually smaller inclinations and eccentricities, the proximity to Fibonacci fractions of their period ratios with Jupiter or Mars' period tends to increase. Finally, a simple model explains why the resonance of the form $\frac{P_1}{P_2} = \frac{p}{p+q}$, with P_1 and P_2 orbital periods of successive secondaries and p and q small integers, are stronger and more commonly observed.

1 Introduction

The discovery of the Trappist-1 system of seven planets (Gillon et al., 2017; Luger et al., 2017) with five out of six orbital period ratios being close to ratios of Fibonacci integers (Pletser and Basano, 2017) has prompted a search among other planetary and satellite systems of the Solar System and of exo-planetary systems to assess whether Fibonacci numbers intervene more often in integer fractions close to ratios of orbital periods. It is found that ratios of Fibonacci numbers outnumber significantly ratios formed with other integers, when limited to the most significant ratios of small integers between 1 and 8.

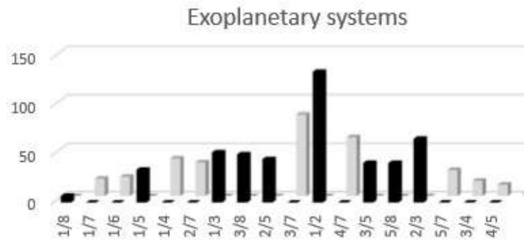


Figure 5: Histogram of the numbers of ratios of orbital periods of all adjacent planets in all exoplanetary systems, close to irreducible fractions of two Fibonacci numbers (black) and other fractions (grey).

It is seen that the highest peak is observed for 1/2 with 135 cases, followed by 3/7 with 84 cases and 2/3 with 66 cases, and in total, there are 473 out of 791 ratios (59.80%) close to Fibonacci fractions, disregarding 115 cases of ratios smaller than 0.1181.

Ratios of orbital periods in exoplanetary systems :

Fibonacci Number ratios :

1/2 (135 cases), 2/3 (66 cases), 3/5 (~35 cases), 3/8 (~40 cases), 5/8 (~35 cases), 3/8 (~35 cases),

Lucas Number ratios :

1/3 (~45 cases), 3/7 (84 cases) , 4/7 (~60 cases)

Study 6 : - Extracts from a study produced by Dr. Iliya Iv. Vakarelov, University of Forestry, Bulgaria (1982-1994)

Title : **"Changes in phyllotactic pattern structure (Fibonacci Sequences) in Pinus mugo due to changes in altitude "**

from the book „Symmetry in Plants" by Roger V. Jean and Denis Barabe, Universities of Quebec and Montreal, Canada (Part I. – Chapter 9 , pages 213 – 229)

Research Site and methods :

Pinus mugo grows in high mountainous parts at altitudes up to 2500m forming vast communities. The vertical profile of the research sites for *Pinus mugo* was situated along the northern slopes of the eastern part of the Ria mountain, and test specimens were collected from the following altitudes : 1900, 2200 and 2500 m. Test specimens were also collected from the city of Sofia (at 550 m) where *Pinus mugo* is grown as decorative plant.
The research was carried out over a period of 12 years (except of altitude 550m where research was carried out only around 6 years). The initation of leaf primordia in the bud (→ **Meristem**) occurs at the end of the growing period. The apical meristem of *Pinus mugo* starts this process around the beginning of mid of August and ends in autumn when the air temperature goes below a certain point.

Fig : *Pinus mugo*

The interesting results of the study :

(3) With the increase of altitude from 1900m to 2500m the phyllotactic pattern structure of "Pinus mugo" twigs changes considerably, the number of patterns (different Fibonacci Sequences) grows from 3 to 12, and the relative frequency of the main sequence decreases from 88 % to 38 %.

At the upper boundary of Pinus mugo natural distribution – at about 2500m, the variation of phyllotactic twig pattern structure (entropy) becomes cyclic, with six year duration of the cycles.

(5) The changes in temperature during the period of phyllotactic pattern formation of Pinus mugo twigs determine about 48 % of the changes in pattern structure, the latter lagging behind with one or two years.

It is obvious that when the altitude increases, the number of phyllotactic patterns (Fibonacci-Sequences) of the vegetative organs of *Pinus mugo* also increases above a given altitude.

	Sequence No.	FIBONACCI–Sequences present in given altitude	Altitute in (m)									
			550		1900		2200		2500		Total	
			Frequency	Relative Frequency	Frequency	Relative Frequency	Frequency	Relative Frequency	Frequency	Relative Frequency	Frequency	Relative Frequency
	F1	⟨1,2,3,5,8,13,...⟩	231	0.902	431	0.885	619	F1 0.812	246	F1 0.381	1527	0.710
	F3	2⟨1,2,3,5,8,13,...⟩	16	0.063	34	0.070	35	F3 0.046	111	F3 0.172	196	0.092
	F2	⟨1,3,4,7,11,18,...⟩	3	0.012	22	0.045	49	F2 0.064	86	F2 0.133	160	0.074
	F4	3⟨1,2,3,5,13,...⟩	6	0.023	–	–	29	F4 0.038	98	F4 0.152	133	0.062
	F8	⟨2,5,7,12,19,31,...⟩	–	–	–	–	10	0.013	50	0.077	60	0.028
	F11	⟨3,7,10,17,27,44,...⟩	–	–	–	–	5	0.007	18	0.028	23	0.011
	F6	⟨1,4,5,9,14,23,...⟩	–	–	–	–	1	0.001	8	0.012	9	0.004
	F9	2⟨1,3,4,7,11,18,...⟩	–	–	–	–	4	0.005	7	0.011	11	0.005
(?)	F6	⟨1,7,8,15,23,38,...⟩	–			–	2	0.003	7	0.011	9	0.004
	F5	4⟨1,2,3,5,8,13,...⟩	–			–	8	0.011	9	0.013	17	0.008
(?)	F13	⟨1,6,7,13,20,33,...⟩	–			–	–	–	3	0.005	3	0.001
	F10	⟨2,7,9,16,25,41,...⟩	–	–	–	–	–	–	3	0.005	3	0.001

Note : The number of Fibonacci-Sequences is **increasing** with altitude !

Table 1 : Data on the frequency and relative frequency of the different phyllotactic patterns for ***Pinus mugo*** twigs at different altitudes. Specimen formed during the period 1982-1994 have been tested for all sites except for the one at 550 m where the period covers the years 1989 – 1993.

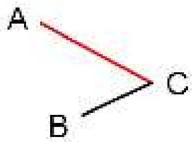
From the Fibonacci-Sequences shown by *Pinus mugo* at 2500m an infinite Fibonacci-Table was developed :

There are clear spatial interdependencies noticable between the different Fibonacci -Sequences, which are connected by the golden ratio **φ** . There is a complex network visible between the numbers of all Sequences. This table of Fibonacci-Number Sequences can be extended towards infinity and all natural numbers are contained in the lower half only once!

For 3 numbers A, B and C in the below shown arrangement, which belong to the same 3 (or 2) different Fibonacci-Sequences, the following rule is true :

The ratio of the difference (C-A) indicated by a "red line", to the difference (B-C) indiated by a "black line" is approaching the golden ratio **φ** for the further progressing Fibonacci-Number Sequences towards infinity (downwards in the table).

„Main Bow-Structures“ are also linked by the „golden ratio“ **φ** !



$$\longrightarrow \quad \lim \frac{C - A}{B - C} = \varphi \quad \text{for} \quad A, B, C \rightarrow \infty$$

FIBONACCI – Number Sequences No. 1 to 14 (F1 - F14) → see <u>extended table</u> in the Appendix !

Row No.	F1 Fibonacci-Base-Sequence	F2 Lucas-Sequence	F3 Fibonacci-Sequence (x 2)	F4 Fibonacci-Sequence (x 3)	F5 Fibonacci-Sequence (x 4)	F6	F7	F8	F9 Lucas-Sequence (x 2)	F10	F11	F12	F13	F14
1	1	1				1	1							
2	2		2					2	2	2				
3	3	3		3							3	3	3	
4		4	4		4	4								4
5	5					5	5	5						
6			6	6			6		6					
7		7						7		7	7			
8	8				8				8			8		
9				9		9				9			9	9
10			10								10			
11		11					11					11		
12					12			12					12	
13	13													13
14						14			14					
15				15										
16			16							16				
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55	55													
56														
57														57
58									58					
59														
60						60								

3 5 8 13 ∞

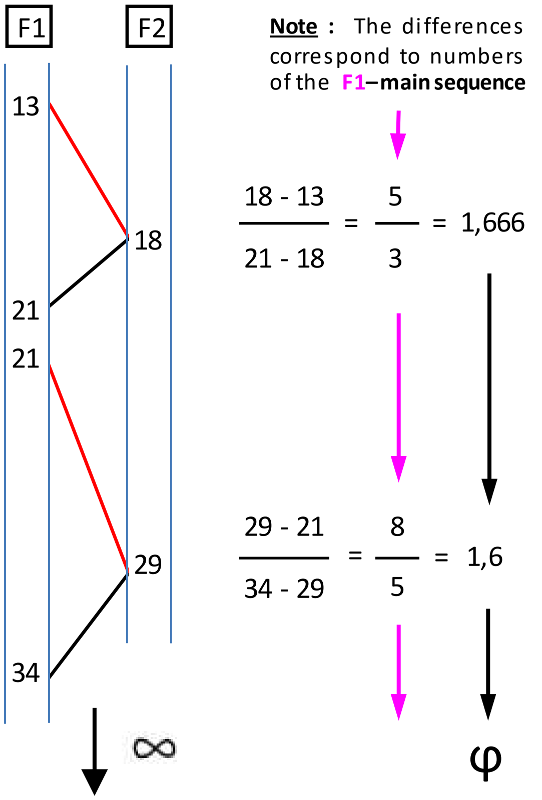
<u>Note</u> : Below this line all natural numbers are contained in the Fibonacci Sequences just <u>once</u> !

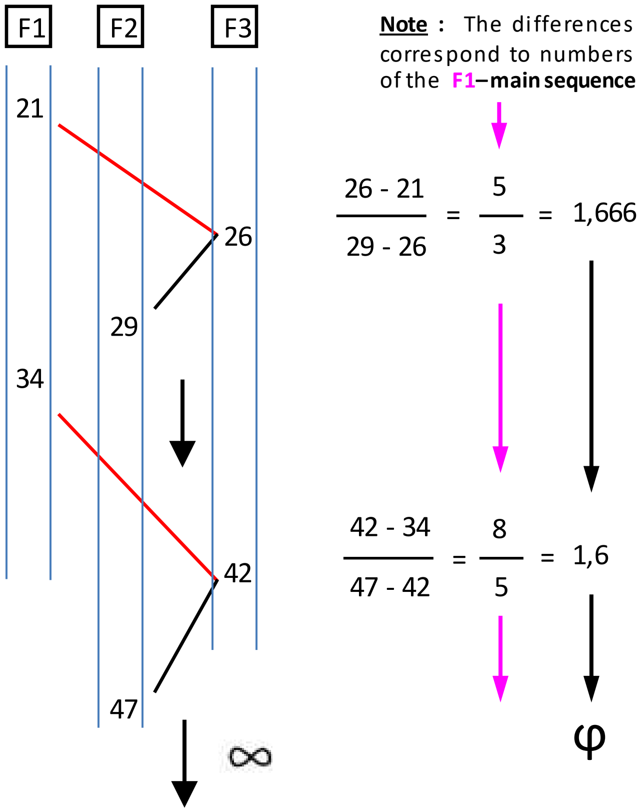
A general rule is visible which connects numbers of different Fibonacci-Sequences by the golden ratio φ

➔ **The following two examples explain the rule which was described in general on the previous page :**

The examples show how the quotient of the differences between the numbers of designated Fibonacci -Sequences (indicated by red- and black-lines in the table), is approaching the golden ratio for the number sequences progressing towards infinity.

For the examples we look at the Fibonacci Sequences **F1, F2** and **F3** (→ F2 is the Lucas-Sequence, F3 = F1 x 2)





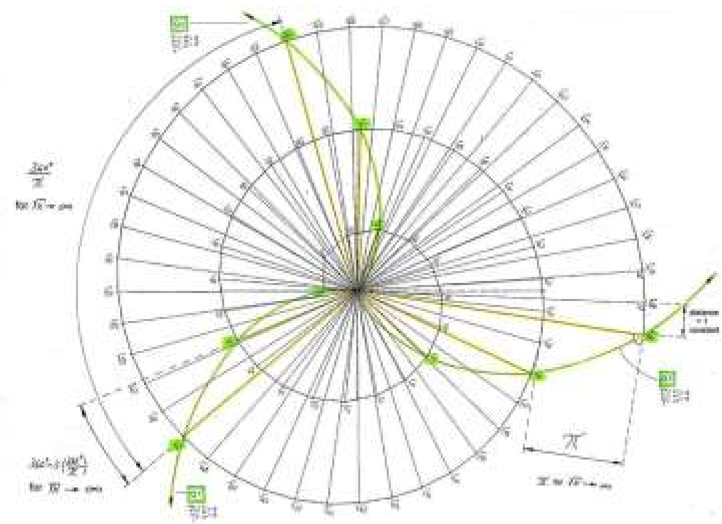
➔ Interesting properties of the Fibonacci-F1 Sequence (and other Fibonacci-Sequences) :

- The numbers of the **Fibonacci F1** – Number Sequence seem to contain all prime numbers as prime factors !
- This is not the case for all other Fibonacci-Sequences where certain prime factors are missing ! (**see Appendix**)
- And all prime factors appear periodic in defined "number-distances" in the sequence (see left side of table)
- This is the case for all Fibonacci-Sequences ! (→ These mentioned properties must be analysed in more detail !)

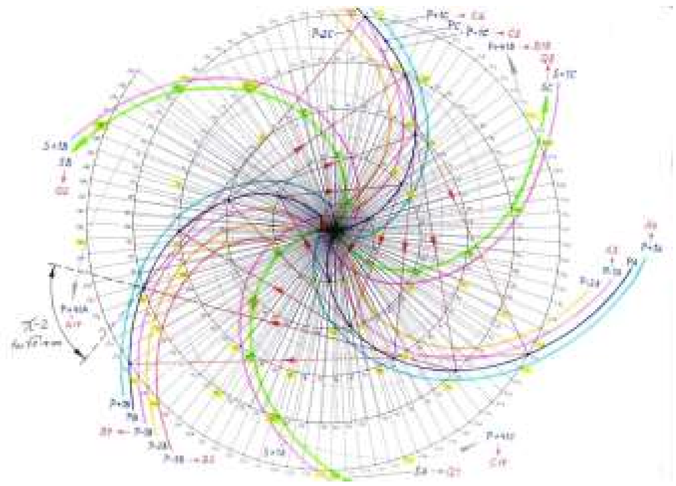
Table 2 : Periodicity of the prime factors of the **Fibonacci F1** - Number Sequence :

some prime factors shown in table form													in prime factors factorized Fibonacci-Numbers		sum of digits	Fibonacci-Sequence F1			
41	37	31	29	23	19	17	13	11	7	5	3	2	repeating products	new products		F	F'	F''	**Nr.**
															1	1			**1**
															1	1			**2**
															2	2	1		**3**
															3	3	1		**4**
															5	5	2	1	**5**
												2^3		2x2x2	8	8	3	1	**6**
															4	13	5	2	**7**
									7		3			3x7	3	21	8	3	**8**
						17						2		2x17	7	34	13	5	**9**
								11		5				5x11	10	55	21	8	**10**
															17	89	34	13	**11**
											3^2	2^4	2x2x2x	3x3	9	144	55	21	**12**
															8	233	89	34	**13**
			29				13							13x29	17	377	144	55	**14**
										5		2		2x5x61	7	610	233	89	**15**
									7		3		3x7x	47	24	987	377	144	**16**
															22	1597	610	233	**17**
					19	17						2^3	2x17x	2x2x19	19	2584	987	377	**18**
	37													37x113	14	4181	1597	610	**19**
41								11		5	3		5x11x	3x41	24	6765	2584	987	**20**

8.9. The square root spiral may represent (partly) a two-dimensional projection of the universal theory



Study : The Ordered Distribution of Natural Numbers on the Square Root Spiral
http://front.math.ucdavis.edu/0712.2184 **PDF :** http://arxiv.org/pdf/0712.2184



The Distribution of Prime Numbers on the Square Root Spiral
http://front.math.ucdavis.edu/0801.1441
PDF : http://arxiv.org/pdf/0801.1441

The complex **square root spiral develops out of a simple right triangle** with the **cathetus lengths of 1** and the **hypotenuse lenght square root of 2**, by the application of the Pythagorean theorem. And it is obvious to see in the square root spiral that **the constant Pi (π), which defines the distance between the winds of the square root spiral, is developing out of this base triangle and the continued application of the Phytagoran Theorem**. It is not the opposite way, that the square root of 2 is developing out of π ! (By the way a perfect circle does't exist. This is just an illusion !)

Pi (π) is developing out of square root of 2 ! Therefore **square root of 2** must be considered as a more fundamental constant than **π** ! Note that in the square root spiral **all other irrational numbers develop out of this base triangle !**

The importance (simplicity) of **square root of 2** is also indicated by a comparison of the Continued fraction of **square root of 2** and **π** : see also weblink to : „Periodic Continued Fraction“ , from : mathworld.wolfram. com

$$\sqrt{2} = 1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \ddots}}}}.$$

The simple continued fraction for π does not exhibit any obvious pattern
But mathematicians have discovered several generalized continued fractions that do, such as :

$$\pi = \cfrac{4}{1 + \cfrac{1^2}{2 + \cfrac{3^2}{2 + \cfrac{5^2}{2 + \cfrac{7^2}{2 + \cfrac{9^2}{2 + \ddots}}}}}} = 3 + \cfrac{1^2}{6 + \cfrac{3^2}{6 + \cfrac{5^2}{6 + \cfrac{7^2}{6 + \cfrac{9^2}{6 + \ddots}}}}}$$

and by **Viete's formula from 1593 :**

$$\pi = \frac{2}{\sqrt{2}} \frac{2}{\sqrt{2+\sqrt{2}}} \frac{2}{\sqrt{2+\sqrt{2+\sqrt{2}}}} \cdots$$

It is also possible to derive from Viète's formula a related formula for π that still involves nested square roots of two, but uses only one multiplication :

$$\pi = \lim_{k\to\infty} 2^k \underbrace{\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\cdots+\sqrt{2}}}}}}}_{k \text{ square roots}}$$

However the most important mathematical constant is Phi (**φ**)
This is already indicated by the continued fraction of **φ** :

$$\varphi = \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + ..}}}$$

Note : **φ** Is the most irrational number and therefore logically the most important constant !

8.10. The constant Pi (π) can also be expressed only by using the constant φ and 1 !

Again to **Viete's formula from 1593 :**

→ It is also possible to derive from Viète's formula a related formula for π that still involves nested square roots of two, but uses only one multiplication :

$$\pi = \frac{2}{\sqrt{2}} \frac{2}{\sqrt{2+\sqrt{2}}} \frac{2}{\sqrt{2+\sqrt{2+\sqrt{2}}}} \cdots$$

$$\pi = \lim_{k\to\infty} 2^k \underbrace{\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\cdots+\sqrt{2}}}}}}}_{k \text{ square roots}}$$

If we replace the number 2 in the above shown formulas by the found equation (5) where number 2 can be expressed by constant φ and 1, then we can express the constant Pi (π) also by only using the constant φ and 1 !

Replace Number 2 in the above shown formulas with this term.

$$\rightarrow \quad 2 = \frac{\varphi^4 + 1}{\varphi^2} - 1 \quad \rightarrow \quad 2 = \frac{\varphi^4 - \varphi^2 + 1}{\varphi^2} \quad (5) \quad \text{and} \quad \sqrt{2} = \sqrt{\frac{\varphi^4 - \varphi^2 + 1}{\varphi^2}} \quad (6)$$

It becomes clear that the irrationality of Pi (π) is also only based on the constant φ and 1, in the same way as the irrationality of all irrational square roots, is only based on constant φ & 1 ! Numbers don't exist ! Only φ & 1 exist !

Constant Pi (π) can now be expressed in this way, by only using constant φ and 1 :

$$\pi = \lim_{k\to\infty} \left[\frac{\varphi^4-\varphi^2+1}{\varphi^2}\right]^k \underbrace{\sqrt{\frac{\varphi^4-\varphi^2+1}{\varphi^2} - \sqrt{\frac{\varphi^4-\varphi^2+1}{\varphi^2} + \sqrt{\frac{\varphi^4-\varphi^2+1}{\varphi^2} + \cdots + \sqrt{\frac{\varphi^4-\varphi^2+1}{\varphi^2}}}}}}_{k \text{ square roots}}$$

It becomes clear that the irrationality of Pi (π) is also only based on the constant φ and 1, in the same way as the irrationality of all irrational square roots, is only based on constant φ & 1 !

Numbers don't seem to exist ! Natural Numbers, their square roots and irrational transcendental constants like Pi (π) can be expressed by only using constant φ and 1 !!

This is an interesting discovery because it allows to describe most (maybe all) geometrical objects only with φ & 1 !

The result of this discovery may lead to a new base of number theory. Not numbers like 1, 2, 3,….. and constants like Pi (π) etc. are the base of number theory ! Only the constant φ and the base unit 1 (which shouldn't be considered as a number) form the base of mathematics and geometry. This will certainly also have an impact on physics !

And constant φ and the base unit 1 must be considered as the fundamental „space structure constants" of the real physical world ! With constant φ and 1 all geometrical objects including the Platonic Solids can be expressed !

There probably isn't something like a base unit if we consider a „wave model" as the base of physics and if we see the universe as one oscillating unit. In the universe everyting is connected with everything. see : Quantum Entanglement

References : → **Regarding the M87 black hole (EHT2017) and a possible Poincare Dodecahedral Space universe**

Webside of the Event Horizon Telescope Organization
https://eventhorizontelescope.org/

The Event Horizon Telescope
https://en.wikipedia.org/wiki/Event_Horizon_Telescope

Scientific papers to the M87-black hole observation (EHT2017)-Project :

Paper I: The Shadow of the Supermassive Black Hole
Paper II: Array and Instrumentation
Paper III: Data processing and Calibration
Paper IV: Imaging the Central Supermassive Black Hole
Paper V: Physical Origin of the Asymmetric Ring
Paper VI: The Shadow and Mass of the Central Black Hole

Movie about the EHT-Project (in german language)
→ **Image Calculation process_live** in action in the movie **at around 39:40 to 41:00 minutes in the movie.**
See weblink : Black Hole Hunters If this doesn't work anymore , then alternatively you can also find the movie on
YouTube.com : → **Title „Black Hole Hunters" :** https://www.youtube.com/watch?v=o_F3KVAPMpo

Dodecahedral space topology as an explanation for weak wide-angle temperature correlations in the cosmic microwave background (CMB)

Jean-Pierre Luminet, Jeffrey R. Weeks, Alain Riazuelo, Roland Lehoucq & Jean-Philippe Uzan
Weblink 1 : http://ceadserv1.nku.edu/longa//classes/2004fall/mat115/days/luminet-nat.pdf
Weblink 2 : https://luth.obspm.fr/~luminet/physworld.pdf

The optimal phase of the generalised Poincare dodecahedral space hypothesis implied by the spatial cross-correlation function of the WMAP sky maps

Boudewijn F. Roukema, Zbigniew Bulin´ski, Agnieszka Szaniewska, Nicolas E. Gaudin
Weblink 1 : https://arxiv.org/abs/0801.0006 Weblink to PDF : https://arxiv.org/pdf/0801.0006.pdf

Studies to a Dodecahedral Space Universe and multi-connected universes - from Prof. Frank Steiner :
Cosmic_microwave_background_alignment_in_multi-connected_universes
CMB_Anisotropy_of_the_Poincare_Dodecahedron
other related studies from Prof. Frank Steiner : https://www.researchgate.net/profile/Frank_Steiner5

The large-scale structure of our universe : http://www.sun.org/images/structure-of-the-universe-1
The Millenium Simulation : https://wwwmpa.mpa-garching.mpg.de/galform/millennium/
see also : https://en.wikipedia.org/wiki/Galaxy_filament

Order-5 Dodecahedral Honeycomb Structure in hyperbolic space
https://en.wikipedia.org/wiki/Order-5_dodecahedral_honeycomb

A cosmic hall of mirrors - Jean-Pierre Luminet - https://arxiv.org/ftp/physics/papers/0509/0509171.pdf
News article : https://www.slideshare.net/Convergent_Technology/the-dodecahedron-universe
https://physicsworld.com/a/a-cosmic-hall-of-mirrors/

Cosmic microwave background map : https://en.wikipedia.org/wiki/Cosmic_microwave_background

Wilkinson Microwave Anisotropy Probe : https://en.wikipedia.org/wiki/Wilkinson_Microwave_Anisotropy_Probe

other References : → Regarding a **universal physical theory** supporting a Poincare Dodecahedral Space universe

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PHASE SPACES IN SPECIAL RELATIVITY : TOWARDS ELIMINATING GRAVITATIONAL SINGULARITIES
from PETER DANENHOWER → see weblink : https://arxiv.org/pdf/0706.2043.pdf

---

A Unified Field Theory : https://en.wikipedia.org/wiki/Unified_field_theory

Number Theory as the Ultimate Physical Theory
by **I. V. Volovich** / Steklov Mathematical Institute - **Study** : http://cdsweb.cern.ch/record/179558/files/198708102.pdf

Space-Time-Matter – by Gerald E. Marsh : https://arxiv.org/ftp/arxiv/papers/1304/1304.7766.pdf

Letters of Albert Einstein, including his letter to natural constants from 13th October 1945 in german language :
http://docplayer.org/69639849-Ilse-rosenthal-schneider-begegnungen-mit-einstein-von-laue-und-planck.html
see also : - description of the book contents in english : http://blog.alexander-unzicker.com/?p=27

Dimensionless Physical Constant Mysteries : www.rxiv.org/pdf/1205.0050v1.pdf

Looking_for_Those_Natural_Numbers_Dimensionless_Constants_&_the_Idea_of_Natural_Measurement_1
https://www.academia.edu/35881283/

Do we live in an eigenstate of the "fundamental constants" operators? - https://arxiv.org/pdf/1809.05355.pdf

About the logic of the prime number distribution - by harry K. Hahn **:** https://arxiv.org/abs/0801.4049

The golden ratio Phi (φ) in Platonic Solids : http://www.sacred-geometry.es/?q=en/content/phi-sacred-solids

Stressed Fibonacci spiral patterns of definite chirality - by Chaorong Li ; Ailing Ji & Zexian Cao
http://surface.iphy.ac.cn/sf03/articles/2006-2007/2007APL-chirality.pdf

Cubic approximants in quasicrystal structures – by V. E. Dimitrienko
https://hal.archives-ouvertes.fr/jpa-00212565/document

Fibonacci order in the period-doubling cascade to chaos - by G.Linage, Fernando Montoya, A. Samiento…
https://www.researchgate.net/publication/252736598_Fibonacci_order_in_the_period-doubling_cascade_to_chaos

The Planar Double Pendulum - by Peter H. Richter & Hans-Joachim-Scholz
http://www.itp.uni-bremen.de/prichter/download/DoppelpendelIWF.pdf ; → **Movie :** https://av.tib.eu/media/14902

Orbital Period Ratios and Fibonacci Numbers in Solar Planetary and Satellite Systems and in Exoplanetary Systems
– by Vladimir Pletser - weblink : https://arxiv.org/pdf/1803.02828

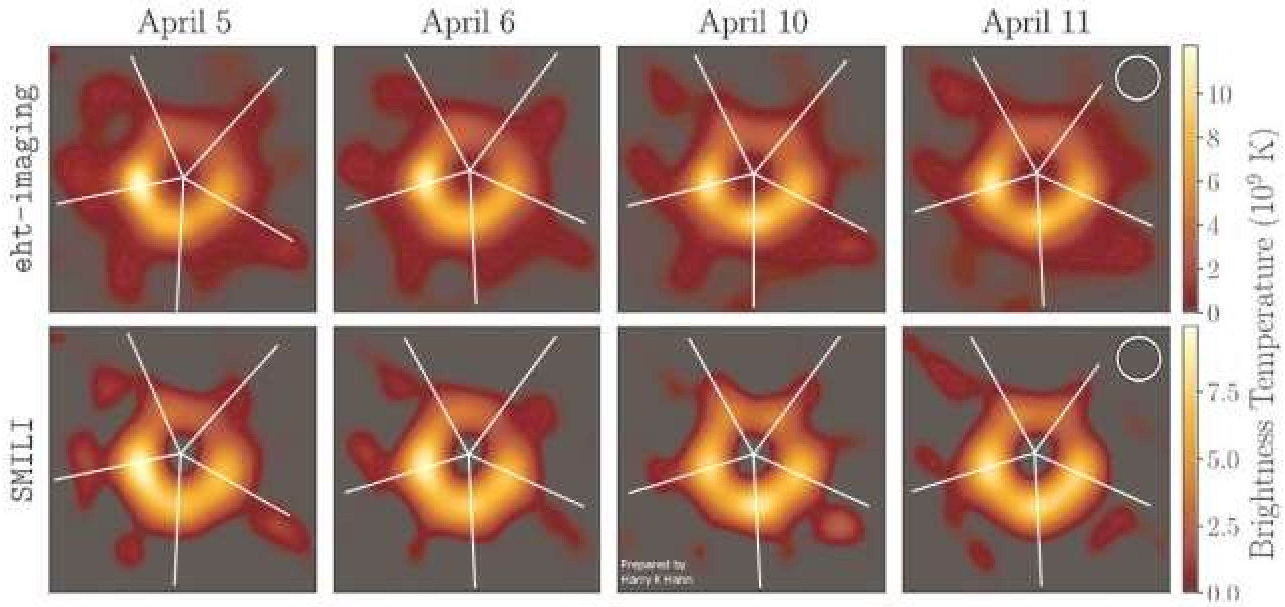
Changes in phyllotactic pattern structure (Fibonacci Sequences) in Pinus mugo due to changes in altitude
Longterm botanical research study by **Dr. Iliya Iv. Vakarelov**, University of Forestry, Bulgaria (1982-1994)
From the book **„Symmetry in Plants"** by Roger V. Jean and Denis Barabe, Universities of Quebec and Montreal, Canada (Part I. – Chapter 9 , pages 213 – 229) – 1998 by World Scientific Publishing , **ISBN : 981-02-2621-7**

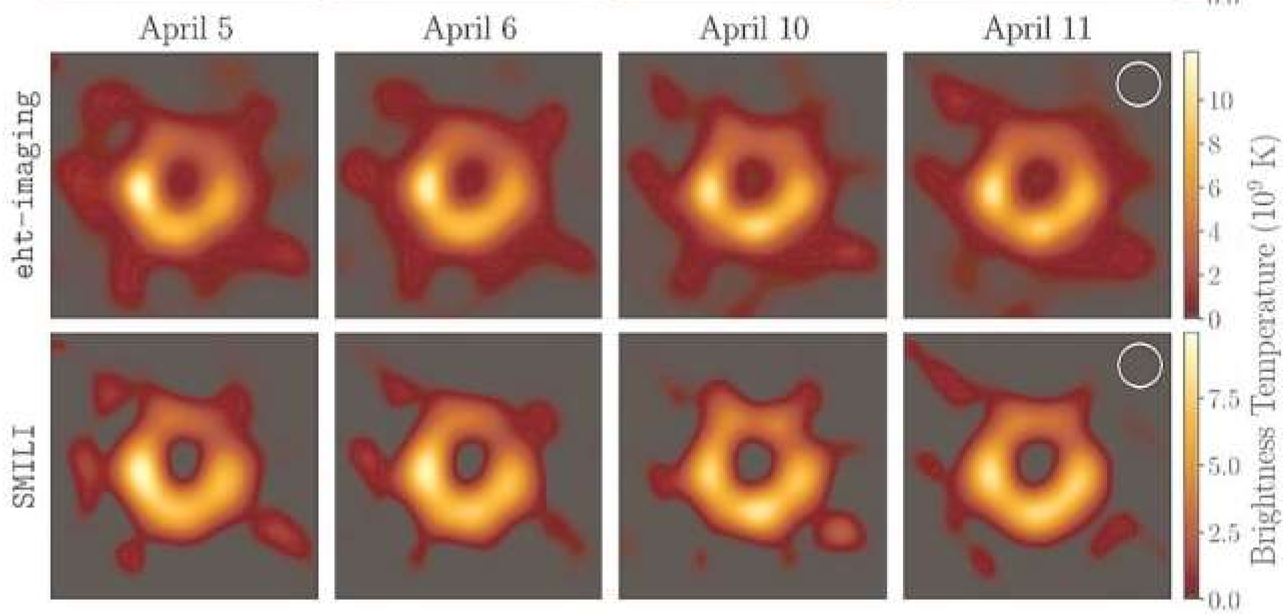
The Ordered Distribution of Natural Numbers on the Square Root Spiral - by Harry K. Hahn
http://front.math.ucdavis.edu/0712.2184 **PDF :** http://arxiv.org/pdf/0712.2184

The Distribution of Prime Numbers on the Square Root Spiral – by Harry K. Hahn
http://front.math.ucdavis.edu/0801.1441 **PDF :** http://arxiv.org/pdf/0801.1441

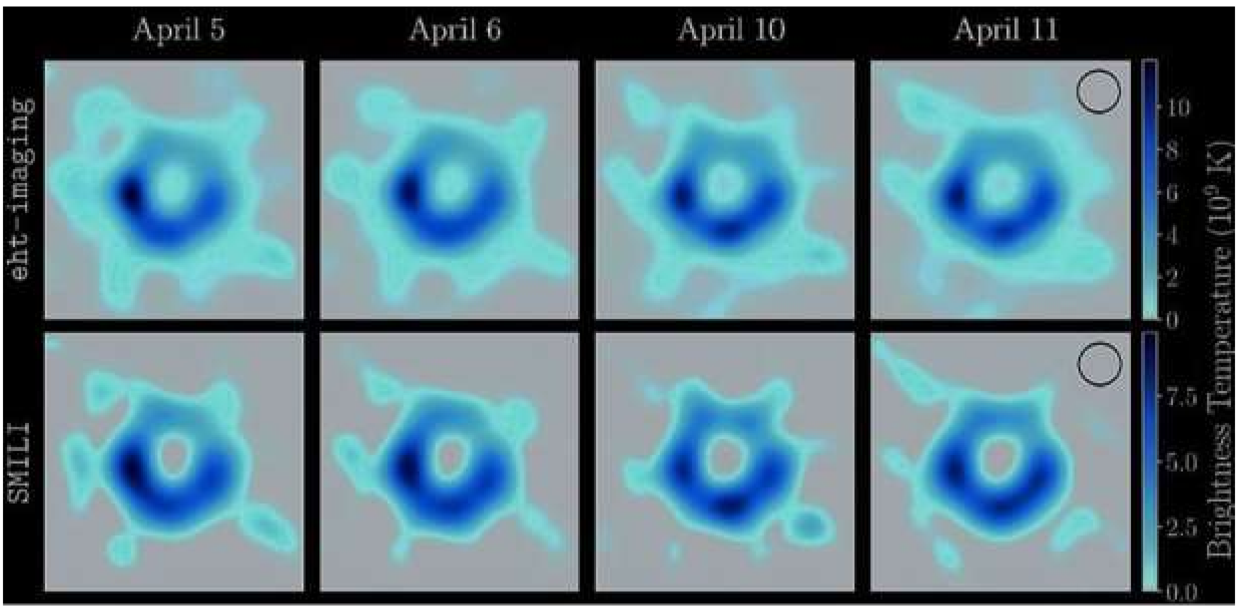
Appendix 1 : Here are the re-processed images of the EHT2017 fiducial images :

The image below was in steps contrast enhanced, brightness was slightly increased, a gamma correction was carried out and the color green was slightly increased





The image below is just the inverted color image of the above shown image :



Appendix 2 : With the algebraic term of constant u we can calculate all square roots of all natural numbers expressed only by constant φ and 1 :

$$\sqrt{\varphi^2 - 2} = \frac{\sqrt{2\sqrt{5} - 2}}{2}$$; we equate the two algebraic terms which represent the same constant !

$$\rightarrow \quad 4\varphi^2 - 8 = 2\sqrt{5} - 2$$; we square both sides and transform

$$\varphi^2 = \frac{\sqrt{5} + 3}{2}$$; (1) we solve for φ^2

$$\sqrt{5} = 2\varphi^2 - 3$$; (2) we solve for $\sqrt{5}$

Now we go back to the square root spiral and use the following right triangle :

$$(\sqrt{6})^2 = (\sqrt{5})^2 + 1^2$$; application of the Pythagorean theorem

$$6 = (2\varphi^2 - 3)^2 + 1$$; we replace $\sqrt{5}$ by equation (2) and transform

$$\rightarrow \quad 3 = \frac{\varphi^4 + 1}{\varphi^2} \quad (3) \quad \rightarrow \quad \sqrt{3} = \sqrt{\frac{\varphi^4 + 1}{\varphi^2}} \quad (4)$$; square root 3 expressed by φ and 1

Now we use the following right triangle :

$$(\sqrt{3})^2 = (\sqrt{2})^2 + 1^2$$; application of the Pythagorean theorem & inserting equation (3)

$$\rightarrow \quad 2 = \frac{\varphi^4 + 1}{\varphi^2} - 1 \quad \rightarrow \quad 2 = \frac{\varphi^4 - \varphi^2 + 1}{\varphi^2} \quad (5) \quad \text{and} \quad \sqrt{2} = \sqrt{\frac{\varphi^4 - \varphi^2 + 1}{\varphi^2}} \quad (6)$$

Now we insert equation (3) in equation (2) :

$$\rightarrow \quad \sqrt{5} = 2\varphi^2 - \frac{\varphi^4 + 1}{\varphi^2} \quad \rightarrow \quad \sqrt{5} = \frac{\varphi^4 - 1}{\varphi^2}$$; (7) ; square root 5 expressed by φ and 1

Now we use the following right triangle :

$$(\sqrt{6})^2 = (\sqrt{5})^2 + 1^2$$; application of the Pythagorean theorem & inserting equation (7)

$$\rightarrow \quad 6 = \left(\frac{\varphi^4 - 1}{\varphi^2}\right)^2 + 1 \quad \rightarrow \quad 6 = \frac{\varphi^8 - \varphi^4 + 1}{\varphi^4} \quad (8) \quad \text{and} \quad \sqrt{6} = \sqrt{\frac{\varphi^8 - \varphi^4 + 1}{\varphi^4}} \quad (9)$$

We can now continue and use the following right triangles oft he square root spiral :

$$(\sqrt{7})^2 = (\sqrt{6})^2 + 1^2$$; application of the Pythagorean theorem & inserting equation (8)

$$\rightarrow \quad 7 = \frac{\varphi^8 + 1}{\varphi^4} \quad (10) \qquad \rightarrow \quad \sqrt{7} = \sqrt{\frac{\varphi^8 + 1}{\varphi^4}} \quad (11)$$

In the same way we can now calculate all square roots of all natural numbers with the next right triangles :

$$\rightarrow \quad 8 = \frac{\varphi^8 + \varphi^4 + 1}{\varphi^4} \quad (12) \text{ and } \sqrt{8} = \sqrt{\frac{\varphi^8 + \varphi^4 + 1}{\varphi^4}} \quad (13)$$

$$\rightarrow \quad 10 = \frac{\varphi^8 + 3\varphi^4 + 1}{\varphi^4} \quad (14) \text{ and } \sqrt{10} = \sqrt{\frac{\varphi^8 + 3\varphi^4 + 1}{\varphi^4}} \quad (15)$$

$$\rightarrow \quad 11 = \frac{\varphi^8 + 4\varphi^4 + 1}{\varphi^4} \quad (16) \text{ and } \sqrt{11} = \sqrt{\frac{\varphi^8 + 4\varphi^4 + 1}{\varphi^4}} \quad (17)$$

$$\rightarrow \quad 12 = \frac{\varphi^8 + 5\varphi^4 + 1}{\varphi^4} \quad (18) \text{ and } \sqrt{12} = \sqrt{\frac{\varphi^8 + 5\varphi^4 + 1}{\varphi^4}} \quad (19)$$

From the above shown formulas (equations) we can read a general rule for all natural numbers > 10 :

Note : → The expression (3+n) in the rule can be replaced by products or sums of the equations (3) to (13)

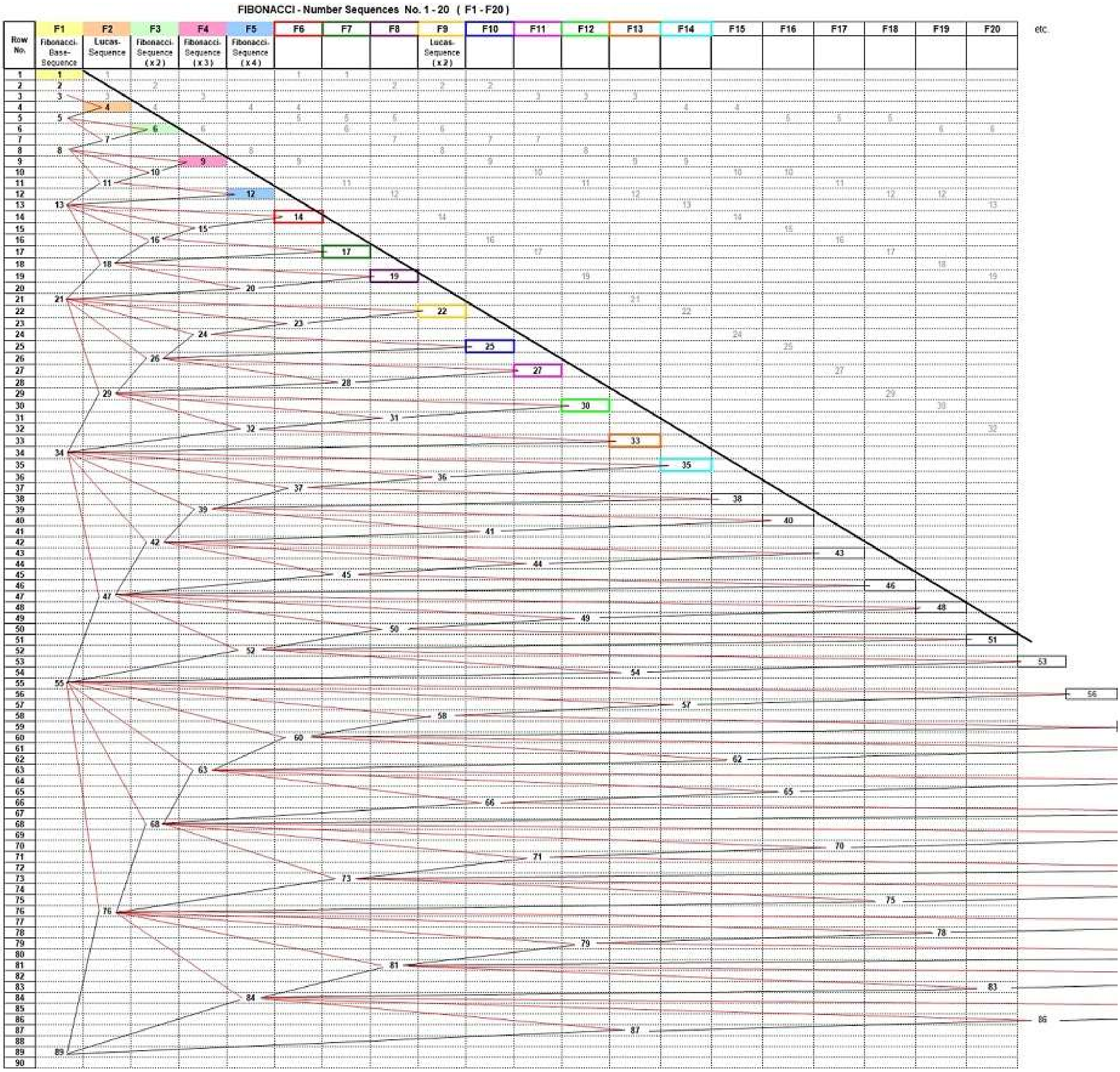
$$\rightarrow \quad (10 + n) = \frac{\varphi^8 + (3+n)\varphi^4 + 1}{\varphi^4} \quad (20) \text{ and } \sqrt{(10 + n)} = \sqrt{\frac{\varphi^8 + (3+n)\varphi^4 + 1}{\varphi^4}} \quad (30)$$

For n → ∞

With this formulas we can express all natural numbers and their square roots only with φ and 1 ! This is an interesting discovery, because it also allows to describe probably most (if not all) geometrical objects only with φ and 1 !

<u>Appendix 3</u> : Additional information to the described study 6 in the main document (carried out by myself)

➔ Extended version of the Fibonacci-Number Sequence Table up to sequence F20 :



Abstract of my study : "An infinite Fibonacci-Number Sequence Table" - by Harry K. Hahn

A Fibonacci-Number-Sequences-Table was developed, which contains infinite Fibonacci-Sequences. This was achieved with the help of research results from an extensive botanical study. This study examined the phyllotactic patterns (Fibonacci-Sequences) which appear in the three species „Pinus mugo" at different altitudes (from 550m up to 2500m) With the increase of altitude above around 2000m the phyllotactic patterns change considerably, the number of patterns (different Fibonacci Sequences) grows from 3 to 12, and the relative frequency of the main Fibonacci Sequence decreases from 88 % to 38 %. The appearance of more Fibonacci-Sequences in the plant clearly is linked to environmental (physical) factors changing with altitude. Temperature (in a wider sense) must be the main factor which defines which Fibonnacci-Patterns appear in which frequency. The developed (natural) Fibonacci-Sequence-Table shows interesting spatial dependencies between the numbers of different Fibonacci-Sequences, which are connected to each other, defined by the golden ratio. Interesting periodic recurrences of the prime factors of factorized Fibbonacci-Numbers of the same sequence were found (see Appendix).
With the help of another study with title: Phase spaces in Special Relativity : Towards eliminating gravitational singularities - a way was found to express (calculate) all natural numbers and their square roots only by using the constant Phi (φ) and number 1. An algebraic term found by Mr Peter Danenhower in this study made this possibe.
With the formulas which I have found it's possible to express all natural numbers and their square roots only with φ & 1 ! This is an interesting discovery because it also allows to describe most (maybe all) geometrical objects only with φ & 1.

Appendix 3 : Additional information to the described study 6 in the main document

➔ To the periodicity of the prime factors of the Fibonacci-Numbers of selected Sequences

Note: The numbers of the Fibonacci F1 – Number Sequence seem to contain all prime numbers as prime factors !
And all prime factors appear periodic in defined "number-distances" in the sequence (see left side of table)

Table 2 : Periodicity of some of the prime factors of the numbers of the **Fibonacci F1** - Number Sequence :

some prime factors shown in table form													in prime factors factorized Fibonacci-Numbers		sum of digits	Fibonacci-Sequence F1			
41	37	31	29	23	19	17	13	11	7	5	3	2	repeating products	new products		F	F'	F"	Nr.
															1	1			1
															1	1			2
															2	2	1		3
															3	3	1		4
															5	5	2	1	5
												2^3		2x2x2	8	8	3	1	6
															4	13	5	2	7
									7		3			3x7	3	21	8	3	8
						17						2		2x17	7	34	13	5	9
								11		5				5x11	10	55	21	8	10
															17	89	34	13	11
											3^2	2^4	2x2x2x	3x3	9	144	55	21	12
															8	233	89	34	13
			29				13							13x29	17	377	144	55	14
										5		2		2x5x61	7	610	233	89	15
									7		3		3x7x	47	24	987	377	144	16
															22	1597	610	233	17
					19	17						2^3	2x17x	2x2x19	19	2584	987	377	18
	37													37x113	14	4181	1597	610	19
41								11		5	3		5x11x	3x41	24	6765	2584	987	20
							13					2		2x13x421	20	10946	4181	1597	21
														89x199	17	17711	6765	2584	22
															28	28657	10946	4181	23
				23					7		3^2	2^5	2x2x2x3x3x	2x2x7x23	27	46368	17711	6765	24
										5^2				5x5x3001	19	75025	28657	10946	25
														233x521	19	121393	46368	17711	26
						17						2		2x17x53x109	29	196418	75025	28657	27
			29				13				3		13x29x	3x281	21	317811	121393	46368	28
															23	514229	196418	75025	29
		31						11		5		2^3	2x5x61x	2x2x11x31	17	832040	317811	121393	30
														557x2417	31	1346269	514229	196418	31
									7		3		3x7x47x	2207	30	2178309	832040	317811	32
												2		2x89x19801	34	3524578	1346269	514229	33
														1597x3571	37	5702887	2178309	832040	34
							13			5				5x13x141961	35	9227465	3524578	1346269	35
					19	17					3^3	2^4	2x2x2x17x19x	2x3x3x3x107	27	14930352	5702887	2178309	36
														73x149x2221	35	24157817	9227465	3524578	37
	37												37x113x	9349	44	39088169	14930352	5702887	38
												2		2x233x135721	43	63245986	24157817	9227465	39
41								11	7	5	3		3x5x11x41x	7x2161	24	102334155	39088169	14930352	40
														2789x59369	31	165580141	63245986	24157817	41
			29				13					2^3	2x13x421x	2x2x29x211	46	267914296	102334155	39088169	42
															41	433494437	165580141	63245986	43
											3		89x199x	3x43x307	33	701408733	267914296	102334155	44
						17				5		2		2x5x17x61x109441	29	1134903170	433494437	165580141	45
														139x461x28657	35	1836311903	701408733	267914296	46
															37	2971215073	1134903170	433494437	47
				23					7		3^2	2^6	2x2x2x2x2x3x3x7x23x	2x47x1103	54	4807526976	1836311903	701408733	48

Note : all prime numbers are marked in yellow and all numbers not divisible by 2, 3 or 5 are marked in orange

Table 3 : Periodicity of some of the prime factors of the numbers of the **Fibonacci F2 (Lucas)** - Number Sequence :

some prime factors shown in table form													in prime factors factorized Fibonacci-Numbers		sum of digits	**Fibonacci-Sequence F2 (Lucas-Sequence)**			
41	37	31	29	23	19	17	13	11	7	5	3	2	repeating products	new products		**L**	**L'**	**L''**	**No.**
															1	1			**1**
															3	3			**2**
												2^2		2x2	4	4	1		**3**
															7	7	3		**4**
															4	11	4	1	**5**
											3^2	2		2x3x3	9	18	7	3	**6**
															11	29	11	4	**7**
															11	47	18	7	**8**
					19							2^2	2x2x	19	13	76	29	11	**9**
41											3			3x41	6	123	47	18	**10**
															19	199	76	29	**11**
				23					7			2		2x7x23	7	322	123	47	**12**
															8	521	199	76	**13**
											3			3x281	15	843	322	123	**14**
		31						11				2^2		2x2x11x31	14	1364	521	199	**15**
															11	2207	843	322	**16**
															16	3571	1364	521	**17**
											3^3	2	2x3x3x	3x107	27	5778	2207	843	**18**
															25	9349	3571	1364	**19**
									7					7x2161	16	15127	5778	2207	**20**
			29									2^2		2x2x29x211	23	24476	9349	3571	**21**
											3			3x43x307	21	39603	15127	5778	**22**
														139x461	26	64079	24476	9349	**23**
												2		2x47x1103	20	103682	39603	15127	**24**
	Prime factor is missing					Prime factor is missing	Prime factor is missing	11		Prime factor is missing				11x101x151	28	167761	64079	24476	**25**
											3			3x90481	21	271443	103682	39603	**26**
					19							2^2	2x2x19x	5779	22	439204	167761	64079	**27**
									7					7x7x14503	25	710647	271443	103682	**28**
														59x19489	29	1149851	439204	167761	**29**
41											3^2	2	3x41x	2x3x2521	36	1860498	710647	271443	**30**
															20	3010349	1149851	439204	**31**
														1087x4481	38	4870847	1860498	710647	**32**
												2^2		2x2x199x9901	40	7881196	3010349	1149851	**33**
											3			3x67x63443	24	12752043	4870847	1860498	**34**
			29					11						11x29x71x911	28	20633239	7881196	3010349	**35**
				23					7			2	2x7x23x	103681	34	33385282	12752043	4870847	**36**
															26	54018521	20633239	7881196	**37**
											3			3x29134601	33	87403803	33385282	12752043	**38**
												2^2		2x2x79x521x859	23	141422324	54018521	20633239	**39**
														47x1601x3041	38	228826127	87403803	33385282	**40**
															34	370248451	141422324	54018521	**41**
											3^2	2	3x281x	2x3x83x1427	54	599074578	228826127	87403803	**42**
														6709x144481	43	969323029	370248451	141422324	**43**
									7					7x263x881x967	52	1568397607	599074578	228826127	**44**
		31			19			11				2^2	2x2x11x31x	19x181x541	41	2537720636	969323029	370248451	**45**
											3			3x4969x275449	30	4106118243	1568397607	599074578	**46**
															62	6643838879	2537720636	969323029	**47**
												2		2x769x2207x3167	47	10749957122	4106118243	1568397607	**48**
			29											29x599786069	46	17393796001	6643838879	2537720636	**49**
41											3			3x41x401x570601	39	28143753123	10749957122	4106118243	**50**

Note : all prime numbers are marked in yellow and all numbers not divisible by 2, 3 or 5 are marked in orange

Table 4 : Periodicity of some of the prime factors of the numbers of the **Fibonacci F6** - Number Sequence :

Periodicity of the prime factors **2 - 41** shown in table form													in prime factors factorized Fibonacci-(F6)-Numbers	sum of digits	**Fibonacci-F6 Sequence**			
41	37	31	29	23	19	17	13	11	7	5	3	2			**F6**	F6'	F6"	**Nr.**
															1			**1**
												2^2	2x2		4			**2**
															5	1		**3**
											3^2		3x3		9	4		**4**
									7			2	2x7		14	5	1	**5**
															23	9	4	**6**
															37	14	5	**7**
										5	3	2^2	2x2x3x5		60	23	9	**8**
															97	37	14	**9**
															157	60	23	**10**
												2	2x127		254	97	37	**11**
											3		3x137		411	157	60	**12**
					19				7	5			5x7x19		665	254	97	**13**
												2^2	2x2x269		1076	411	157	**14**
															1741	665	254	**15**
											3^2		3x3x313		2817	1076	411	**16**
												2	2x43x53		4558	1741	665	**17**
										5^3			5x5x5x59		7375	2817	1076	**18**
															11933	4558	1741	**19**
											3	2^2	2x2x3x1609		19308	7375	2817	**20**
									7				7x4463		31241	11933	4558	**21**
															50549	19308	7375	**22**
										5		2	2x5x8179		81790	31241	11933	**23**
		31									3		3x31x1423		132339	50549	19308	**24**
															214129	81790	31241	**25**
	37											2^2	2x2x37x2341		346468	132339	50549	**26**
															560597	214129	81790	**27**
???			prime factor is missing			prime factor is missing	prime factor is missing	prime factor is missing		5	3^2		3x3x5x6719		907065	346468	132339	**28**
									7			2	2x7x79x1327		1467662	560597	214129	**29**
				23									23x223x463		2374727	907065	346468	**30**
					19								19x202231		3842389	1467662	560597	**31**
											3	2^2	2x2x3x379x1367		6217116	2374727	907065	**32**
										5			5x227x8863		10059505	3842389	1467662	**33**
															16276621	6217116	2374727	**34**
												2	2x641x20543		26336126	10059505	3842389	**35**
											3		3x1637x8677		42612747	16276621	6217116	**36**
									7				7x181x54419		68948873	26336126	10059505	**37**
										5		2^2	2x2x5x5578081		111561620	42612747	16276621	**38**
															180510493	68948873	26336126	**39**
											3^2		3x3x32452457		292072113	111561620	42612747	**40**
												2	2x1109x213067		472582606	180510493	68948873	**41**
													67x2083x5479		764654719	292072113	111561620	**42**
										5^2			5x5x49489493		1237237325	472582606	180510493	**43**
											3	2^2	2x2x3x53x3147629		2001892044	764654719	292072113	**44**
	37								7^2				7x7x37x1788613		3239129369	1237237325	472582606	**45**
													71x3613x20431		5241021413	2001892044	764654719	**46**
												2	2x167x3607x7039		8480150782	3239129369	1237237325	**47**
										5	3		3x5x914744813		13721172195	5241021413	2001892044	**48**
					19								19x83x14078201		22201322977	8480150782	3239129369	**49**
												2^2	2x2x337x2664083		35922495172	13721172195	5241021413	**50**
													129631x448379		58123818149	22201322977	8480150782	**51**
											3^2		3x3x2671x3912239		94046313321	35922495172	13721172195	**52**
									7	5		2	2x5x7x2173859021		152170131470	58123818149	22201322977	**53**
		31		23									23x31x345324607		246216444791	94046313321	35922495172	**54**
															398386576261	152170131470	58123818149	**55**

Note : all prime numbers are marked in yellow and all numbers not divisible by 2, 3 or 5 are marked in orange