Three ways to describe Scale-Invariant Turbulence Cosmology: From Navier-Stokes to Burgers Equation to Golden Ratio etc.

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Abstract

In recent years, there is growing interest to describe the Universe we live in from the perspective of scale-invariant turbulence approach. Such an approach is not limited to hydrodynamics Universe model a la Gibson & Schild, but also from Kolmogorov turbulence approach as well as from String theory approach (some researcher began to explore String-Turbulence). In this article, we hope to bring out some correspondence among existing models, so we discuss shortly: the topological vortice approach, Burgers equation in the light of KAM theory and Golden Mean, and the Cantorian Navier-Stokes approach. Of course, this short article is far from being complete. We hope further investigation can be done around this line of approach.

Introduction

From time to time, astronomy and astrophysics discoveries have opened our eyes that the Universe is much more complicated than what it seemed in 100-200 years ago. And despite all pervading popularity of General Relativistic treatment of Cosmology, it seems still worthy to remind us to old concepts of Cosmos, for instance the Hydron theory of Thales ("that water is the essential element in the Cosmos"), and also Heracleitus ("panta rhei"). So we can ask: does it

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mean that the Ultimate theory that we try to find should correspond to hydrodynamics or some kind of turbulence theory?

An indicator of complex turbulence phenomena in Our Universe is the Web like structure. The Cosmic Web is the fundamental spatial organization of matter on scales of a few up to a hundred Megaparsec. Galaxies and intergalactic gas matter exist in a wispy weblike arrangement of dense compact clusters, elongated filaments, and sheetlike walls, amidst large near-empty void regions. The filaments are the transport channels along which matter and galaxies flow into massive high-density cluster located at the nodes of the web. The weblike network is shaped by the tidal force field accompanying the inhomogeneous matter distribution.

May be part of that reason that in recent years, there is growing interest to describe the Universe we live in from the perspective of scale-invariant turbulence approach. Such an approach is not limited to hydrodynamics Universe model a la Gibson & Schild, but also from Kolmogorov turbulence approach as well as from String theory approach (some researcher began to explore String-Turbulence). In this article, we hope to bring out some correspondence among existing models, so we discuss shortly: the topological vortice approach, Burgers equation in the light of KAM theory and Golden Mean, and the Cantorian Navier-Stokes approach. Of course, this short article is far from being complete. We hope further investigation can be done around this line of approach.

A. Topological vortice approach

Two recent papers by Sivaram & Arun, one in *The Open Astronomy Journal* 2012, 5, 7-11 [1], and one in arXiv [2] are found very interesting. They are able to arrive at the observed value of effective cosmological constant by considering background torsion in the teleparallel gravity. According to them, “the background torsion due to a universal spin density not only gives rise to angular momenta of all structures but also provides a background centrifugal term acting as a repulsive gravity accelerating the universe, with spin density acting as effective cosmological constant.”[1] The torsion is given by [1, p.10]:

$$Q = \frac{4\pi G\sigma}{c^3} \approx 10^{-28} cm^{-1}, \qquad (1)$$

And the background curvature [1, p.10] is given by:

$$Q^2 \approx 10^{-56} cm^{-2}. \qquad (2)$$

In the meantime, a recent review of dark energy theories in the literature (including teleparallel gravity) has been given in [4], and present problems in the standard model general relativistic cosmology are discussed by Starkman [5]. These seem to suggest that a torsion model of effective cosmological constant based on teleparallel gravity as suggested by Sivaram and Arun (2012) seems very promising as a description of phenomena related to accelerated expansion of the Universe usually attributed to 'dark energy' (as alternative to cosmological constant explantion).

However, Sivaram & Arun do not make further proposition concerning the connection between quantized vortices (Onsager-Feynman's rule) and the torsion vector. It will be shown here, that such a connection appears possible.

Here we present Bohr-Sommerfeld quantization rules for planetary orbit distances, which results in a good quantitative description of planetary orbit distance in the solar system [6][6b][7]. Then we find an expression which relates the torsion vector and quantized vortices from the viewpoint of Bohr-Sommerfeld quantization rules [3].

Further observation of the proposed quantized vortices of superfluid helium in astrophysical objects is recommended.

Bohr-Sommerfeld quantization rules and quantized vortices

The quantization of circulation for nonrelativistic superfluid is given by [1][3]:

$$\oint v dr = N \frac{\hbar}{m_s} \tag{3}$$

Where $N, \hbar, m_s$ represents winding number, reduced Planck constant, and superfluid particle's mass, respectively [3]. And the total number of vortices is given by [1]:

$$N = \frac{\omega . 2\pi r^2 m}{\hbar} \tag{4}$$

And based on the above equation (4), Sivaram & Arun [1] are able to give an estimate of the number of galaxies in the universe, along with an estimate of the number stars in a galaxy.

However, they do not give explanation between the quantization of circulation (3) and the quantization of angular momentum. According to Fischer [3], the quantization of angular momentum is a relativistic extension of quantization of circulation, and therefore it yields Bohr-Sommerfeld quantization rules.

Furthermore, it was suggested in [6] and [7] that Bohr-Sommerfeld quantization rules can yield an explanation of planetary orbit distances of the solar system and exoplanets. Here, we begin with Bohr-Sommerfeld's conjecture of quantization of angular momentum. As we know, for the wavefunction to be well defined and unique, the momenta must satisfy Bohr-Sommerfeld's quantization condition:

$$\oint_{\Gamma} p . dx = 2\pi . n\hbar, \tag{5}$$

for any closed classical orbit Γ. For the free particle of unit mass on the unit sphere the left-hand side is:

$$\int_0^T v^2.d\tau = \omega^2.T = 2\pi.\omega, \tag{6}$$

Where $T = \frac{2\pi}{\omega}$ is the period of the orbit. Hence the quantization rule amounts to quantization of the rotation frequency (the angular momentum): $\omega = n\hbar.$ Then we can write the force balance relation of Newton's equation of motion:

$$\frac{GMm}{r^2} = \frac{mv^2}{r}. \tag{7}$$

Using Bohr-Sommerfeld's hypothesis of quantization of angular momentum (6), a new constant g was introduced:

$$mvr = \frac{ng}{2\pi}. \tag{8}$$

Just like in the elementary Bohr theory (just before Schrodinger), this pair of equations yields a known simple solution for the orbit radius for any quantum number of the form:

$$r = \frac{n^2.g^2}{4\pi^2.GMm^2}, \tag{9}$$

or

$$r = \frac{n^2.GM}{v_o^2}, \tag{10}$$

Where r, n, G, M, v_o represents orbit radii (semimajor axes), quantum number (n=1,2,3,...), Newton gravitation constant, and mass of the nucleus of orbit, and specific velocity, respectively. In equation (10), we denote:

$$v_0 = \frac{2\pi}{g} GMm. \tag{11}$$

The value of m and g in equation (11) are adjustable parameters.

Interestingly, we can remark here that equation (10) is exactly the same with what is obtained by Nottale using his Schrodinger-Newton formula [8]. Therefore here we can verify that the result is the same, either one uses Bohr-Sommerfeld quantization rules or Schrodinger-Newton equation. The applicability of equation (10) includes that one can predict new exoplanets (extrasolar planets) with remarkable result.

Therefore, one can find a neat correspondence between Bohr-Sommerfeld quantization rules and motion of quantized vortice in condensed-matter systems, especially in superfluid helium [3]. Here we propose a conjecture that Bohr-Sommerfeld quantization rules also provide a good description for the motion of galaxies, therefore they should be included in the expression of torsion vector. We will discuss an expression of torsion vector of quantized vortices in the next section.

Torsion and quantized vortices

We cite here a rather old paper of Garcia de Andrade & Sivaram, 1998 [9], where they discuss propagation torsion model for quantized vortices. They consider the torsion to be propagating and it can be expressed as derivative of scalar field:

$$Q = \nabla \phi. \tag{12}$$

Therefore $\oint QdS$ can be written as [9]:

$$\oint QdS = \oint \nabla \phi dS = \int \nabla(\nabla \phi) dV \cong \int \nabla^2 \phi dV. \tag{13}$$

Also $\oint QdS$ must have dimensions of length, and thus quantized as [9]:

$$\oint QdS \cong \frac{n\hbar c}{M} \tag{14}$$

Now we invoke a result from the preceding section discussing Bohr-Sommerfeld quantization rules. Assuming that Bohr-Sommerfeld quantization rules also govern the galaxies motion as well as stars motion, then we can insert equation (11) into equation (14), to yield a new expression:

$$\oint QdS \cong \frac{n\hbar c.2\pi Gm}{v_0 g} \tag{15}$$

Therefore, we submit a viewpoint that the torsion vector is also a quantized quantity, and it is a function of Planck constant, speed of light, Newton gravitation constant, vortex particle's mass, a specific velocity and an adjustable parameter, g. It is interesting to find out whether this proposition agrees with observation data or not.

The above proposition (15) connects torsion vector with gravitation constant, which seems to give a torsion description of gravitation. There are numerous other models to describe alternative or modified gravitation theories, for instance Wang is able to derive Newton's second law and Schrodinger equation from fluid mechanical dynamics. [10][11]

In the mean time, for discussion of galaxy disk formation, see [12]. And [13] gives alternative vortices argument for dark matter.

B. Golden ratio is directly related to KAM turbulence via Burgers equation

The Cosmic Web is the fundamental spatial organization of matter on scales of a few up to a hundred Megaparsec. Galaxies and intergalactic gas matter exist in a wispy weblike arrangement of dense compact clusters, elongated filaments, and sheetlike walls, amidst large near-empty void regions. The filaments are the transport channels along which matter and galaxies flow into massive high-density cluster located at the nodes of the web. The weblike network is shaped by the tidal force field accompanying the inhomogeneous matter distribution.[1]

Structure in the Universe has risen out of tiny primordial (Gaussian) density and velocity perturbations by means of gravitational instability. The large-scale anisotropic force field induces anisotropic gravitational collapse, resulting in the emergence of elongated or flattened matter configurations. The simplest model that describes the emergence of structure and complex patterns in the Universe is the Zeldovich Approximation (ZA).[1]

It is our hope that the new approach of CA Adhesion model of the Universe can be verified either with lab experiments, computer simulation, or by large-scale astronomy observation data.

From Zeldovich Approximation to Burgers' equation to Cellular Automaton model

In this section, we will outline a route from ZA to Burgers' equation and then to CA model.

The simplest model that describes the emergence of structure and complex patterns in the Universe is the Zeldovich Approximation (ZA). In essence, it describes a ballistic flow, driven by a constant (gravitational) potential. The resulting Eulerian position x(t) at some cosmic epoch t is specified by the expression:[15]

$$x(t) = q + D(t)u_o(q), \tag{16}$$

where q is the initial "Lagrangian" position of a particle, D(t) the time-dependent structure growth factor and

$$u_0 = -\nabla_q \Phi_0 \tag{17}$$

its velocity. The nature of this approximation may be appreciated by the corresponding source-free equation of motion,

$$\frac{\partial u}{\partial D} + (u \cdot \nabla_x)u = 0. \tag{18}$$

The use of ZA is ubiquitous in cosmology. One major application is its key role in setting up initial conditions in cosmological N-body simulations. Of importance here is its nonlinear extension in terms of *Adhesion Model*.[15]

The ZA breaks down as soon as self-gravity of the forming structures becomes important. To 'simulate' the effects of self-gravity, Gurbatov *et al*. included an artificial viscosity. This results in the Burgers' equation as follows:[15]

$$\frac{\partial u}{\partial D} + (u \cdot \nabla_x)u = \nu.\nabla_x^2 u, \tag{19}$$

a well known PDE from fluid mechanics. This equation has an exact analytical solution, which in the limit of $\nu \to 0$, the solution is: [15]

$$\phi(x, D) = \max_q \left[\Phi_0(q) - \frac{(x-q)^2}{2D} \right]. \quad (20)$$

This leads to a geometric interpretation of the Adhesion Model. The solution follows from the evaluation of the convex hull of the velocity potential modified by a quadratic term. We found that the solution can also be found by computing the weighted Voronoi diagram of a mesh weighted with the velocity potential. For more detailed discussion on Adhesion Model of the Universe, see for example [18].

Now, let us consider another routes to solve Burgers equation: (a) by numerical computation with *Mathematica*, see [17]; and (b) by virtue of CA approach. Let us skip route (a), and discuss less known approach of cellular automata.

We start with the Burgers' equation with Gaussian white noise which can be rewritten as follows:[16]

$$\frac{\partial u}{\partial t} + \xi = 2u\frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial x^2} + \eta. \quad (21)$$

By introducing new variables and after straightforward calculations, we have the automata rule:[16]

$$\begin{aligned} \phi_i^{t+1} &= \phi_{i-1}^t + \max[0, \phi_i^t - A, \phi_i^t + \phi_{i+1}^t - B, \Psi_i^t - \phi_{i-1}^t] \\ &- \max[0, \phi_{i-1}^t - A, \phi_{i-1}^t + \phi_i^t - B, \Phi_i^t + \phi_{i-1}^t] \end{aligned} \quad (22)$$

In other words, in this section we give an outline of a plausible route from ZA to Burgers' equation then to CA model, which suggests that it appears possible –at least in theory- to consider a nonlinear cosmology based on CA Adhesion model.

From KAM theory to Golden section

Another possible way to describe the complex structure of Universe, is the Kolmogorov-Arnold-Moser (KAM) theorem states that if the system is subjected to a weak nonlinear perturbation, some of the invariant tori are deformed and survive, while others are destroyed. The ones that survive are those that have "sufficiently irrational frequencies" (the non-resonance condition, so they do not interfere with one another). The golden ratio being the most irrational number is often evident in such systems of oscillators. It is also physically significant in that circles with golden mean frequencies are the last to break up in a perturbed dynamical system, so the motion continues to be quasi-periodic, i.e., recurrent but not strictly periodic or predictable.

An important consequence of the KAM theorem is that for a large set of initial conditions, the motion remains perpetually quasi-periodic, and hence stable. KAM theory has been extended to non-Hamiltonian systems and to systems with fast and slow frequencies.

The KAM theorem become increasingly difficult to satisfy for complex systems with more degrees of freedom; as the number of dimensions of the system increases, the volume occupied by the tori decreases. Those KAM tori that are not destroyed by perturbation become invariant Cantor sets, or "Cantori". The frequencies of the invariant Cantori approximate the golden ratio.

The golden ratio effectively enables multiple oscillators within a complex system to co-exist without blowing up the system. But it also leaves the oscillators within the system free to interact globally (by resonance), as observed in the coherence potentials that turn up frequently when the brain is processing information.

Obviously, this can be tied in to the creation of subatomic particles such as electrons and positrons. At a certain scale of smallness, the media in the local volume becomes isotropic, while larger volumes exhibit occupation by ever-larger turbulence formations and exhibit extremes of **an**isotropy in the media.

The Kolmogorov Limit is 10e -58 m, which is the smallest vortex that can exist in the aether media. Entities smaller than this, down to the SubQuantum infinitesimals (Bhutatmas) (vortex lines) are the primary cause of gravitation (a "sink" model of gravitation caused by superluminal infinitesimals).

[See: LaPlace].

Figure 1 Turbulent flow generated by the tip vortex of the aeroplane wing shown up by red agricultural dye. (after Mae Wan-Ho, [38]).

Shadow gravity is valid in the situation of gravitational interaction between two discrete masses that divert the ambient gravitational flux-density away from each other. This

happens due to absorption (rare), scattering (more common), and refraction (most of the time) of gravitational infinitesimals.
Gravitational flux density is a variable depending on stellar, interstellar, and intergalactic events.

A simplified model of vorticitiy fields in large scale structures of the Universe is depicted below:

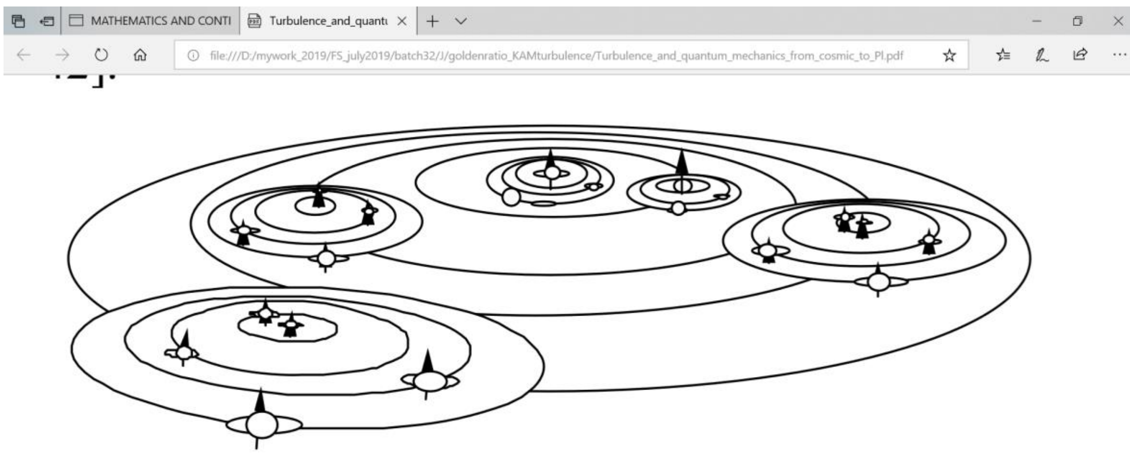

Fig.2 Description of internal (iso-spin) versus external vorticity fields in cosmology [41].

Figure 2. Vorticity fields in cosmology (after {34]])

What is more interesting here, is that it can be shown that there is correspondence between Golden section and in coupled oscilators and KAM Theorem, but also between Golden section and Burgers equation. [35]
Meanwhile, Négadi has shown that there is Fibonacci series (related to Golden mean) which can explain genetic code pattern. [36][37]
For more discussion, on Golden Mean and its ramifications, see for instance [39[40][41].

Table 1. Standard Genetic Code pattern (after [37])

UUU F	UUC F	UCU S	UCC S	CUU L	CUC L	CCU P	CCC P
UUA L	UUG L	UCA S	UCG S	CUA L	CUG L	CCA P	CCG P
UAU Y	UAC Y	UGU C	UGC C	CAU H	CAC H	CGU R	CGC R
UAA stop	UAG stop	UGA stop	UGG W	CAA Q	CAG Q	CGA R	CGG R
AUU I	AUC I	ACU T	ACC T	GUU V	GUC V	GCU A	GCC A
AUA I	AUG M	ACA T	ACG T	GUA V	GUG V	GCA A	GCG A
AAU N	AAC N	AGU S	AGC S	GAU D	GAC D	GGU G	GGC G
AAA K	AAG K	AGA R	AGG R	GAA E	GAG E	GGA G	GGG G

C. Cantorian Navier-Stokes approach

Vorticity as the driver of Accelerated Expansion

According to Ildus Nurgaliev [26], velocity vector V_α of the material point is projected onto coordinate space by the tensor of the second rank $H_{\alpha\beta}$:

$$V_\alpha = H_{\alpha\beta} R^\beta \tag{23}$$

Where the Hubble matrix can be defined as follows for a homogeneous and isotropic universe:

$$H_{\alpha\beta} = \begin{pmatrix} H & \pm\omega & \pm\omega \\ \mp\omega & H & \pm\omega \\ \mp\omega & \mp\omega & H \end{pmatrix} \tag{24}$$

Where the global average vorticity may be zero, though not necessarily [7]. Here the Hubble law is extended to 3x3 matrix.

Now we will use Newtonian equations to emphasize that cosmological singularity is consequence of the too simple model of the flow, and has nothing to do with special or general relativity as a cause [26]. Standard equations of Newtonian hydrodynamics in standard notations read:

$$\frac{d\vec{\upsilon}}{dt} = \frac{\partial\vec{\upsilon}}{\partial t} + \vec{\upsilon}\nabla\vec{\upsilon} = -\nabla\varphi + \frac{1}{\rho}\nabla\rho + \frac{\mu}{\rho}\Delta\vec{\upsilon} + \ldots, \qquad (25)$$

$$\frac{\partial\rho}{\partial t} + \nabla\rho\vec{\upsilon} = 0, \qquad (26)$$

$$\Delta\varphi = 4\pi G\rho \qquad (27)$$

Procedure of separating of diagonal H, trace-free symmetrical σ, and anti-symmetrical ω elements of velocity gradient was used by Indian theoretician Amal Kumar Raychaudhury (1923-2005). The equation for expansion θ, sum of the diagonal elements of [7]

$$\dot{\theta} + \frac{1}{3}\theta^2 + \sigma^2 - \omega^2 = -4\pi G\rho + div(\frac{1}{\rho}\sum f) \qquad (28)$$

is most instrumental in the analysis of singularity and bears the name of its author. [26]

System of (25)-(27) gets simplified up to two equations [26]:

$$\dot{\theta} + \frac{1}{3}\theta^2 - \omega^2 = 0, \qquad (29)$$

$$\dot{\omega} + \frac{2}{3}\theta\omega = 0. \qquad (30)$$

Recalling $\theta = 3H$, the integral of (30) takes the form [26]

$$H^2 = H_\infty^2 - \frac{3\omega_0^2 R_0^4}{R^4}. \qquad (31)$$

<u>How to write down Navier-Stokes equations on Cantor Sets</u>

Now we can extend further the Navier-Stokes equations to Cantor Sets, by keeping in mind their possible applications in cosmology.

By defining some operators as follows:

1. In Cantor coordinates [28]:

$$\nabla^\alpha \cdot u = div^\alpha u = \frac{\partial^\alpha u_1}{\partial x_1^\alpha} + \frac{\partial^\alpha u_2}{\partial x_2^\alpha} + \frac{\partial^\alpha u_3}{\partial x_3^\alpha}, \tag{32}$$

$$\nabla^\alpha \times u = curl^\alpha u = \left(\frac{\partial^\alpha u_3}{\partial x_2^\alpha} - \frac{\partial^\alpha u_2}{\partial x_3^\alpha}\right)e_1^\alpha + \left(\frac{\partial^\alpha u_1}{\partial x_3^\alpha} - \frac{\partial^\alpha u_3}{\partial x_1^\alpha}\right)e_2^\alpha + \left(\frac{\partial^\alpha u_2}{\partial x_1^\alpha} - \frac{\partial^\alpha u_1}{\partial x_2^\alpha}\right)e_3^\alpha . \tag{33}$$

2. In Cantor-type cylindrical coordinates [29, p.4]:

$$\nabla^\alpha \cdot r = \frac{\partial^\alpha r_R}{\partial R^\alpha} + \frac{1}{R^\alpha}\frac{\partial^\alpha r_\theta}{\partial \theta^\alpha} + \frac{r_R}{R^\alpha} + \frac{\partial^\alpha r_z}{\partial z^\alpha}, \tag{34}$$

$$\nabla^\alpha \times r = \left(\frac{1}{R^\alpha}\frac{\partial^\alpha r_\theta}{\partial \theta^\alpha} - \frac{\partial^\alpha r_\theta}{\partial z^\alpha}\right)e_R^\alpha + \left(\frac{\partial^\alpha r_R}{\partial z^\alpha} - \frac{\partial^\alpha r_z}{\partial R^\alpha}\right)e_\theta^\alpha + \left(\frac{\partial^\alpha r_\theta}{\partial R^\alpha} + \frac{r_R}{R^\alpha} - \frac{1}{R^\alpha}\frac{\partial^\alpha r_R}{\partial \theta^\alpha}\right)e_z^\alpha . \tag{35}$$

Then Yang, Baleanu and Machado are able to obtain a general form of the Navier-Stokes equations on Cantor Sets as follows [28, p.6]:

$$\rho \frac{D^\alpha \upsilon}{Dt^\alpha} = -\nabla^\alpha \cdot (pI) + \nabla^\alpha\left[2\mu\left(\nabla^\alpha \cdot \upsilon + \upsilon \cdot \nabla^\alpha\right) - \frac{2}{3}\mu\left(\nabla^\alpha \cdot \upsilon\right)I\right] + \rho b \tag{36}$$

The next task is how to find observational cosmology and astrophysical implications. This will be the subject of future research.

Concluding remarks

In this article, we hope to bring out some correspondence among existing Turbulence Cosmology models, so we discuss shortly: the topological vortice approach, Burgers equation in the light of KAM theory and Golden Mean, and the Cantorian Navier-Stokes approach. Of course, this short article is far from being complete. We hope further investigation can be done around this line of approach.

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