Division by Zero Calculus and Pompe's Theorem

*Tsutomu Matsuura, **Hiroshi Okumura, and ***Saburou Saitoh

*Division of Mechanical Science and Technology,
Faculty of Science and Technology, Gunma University
29-1 Honcho, Ota, Gunma 373-0057, Japan
matsuura@gunma-u.ac.jp
**Maebashi Gunma 371-0123, Japan
e-mail: hokmr@yandex.com
***Institute of Reproducing Kernels,
Kawauchi-cho, 5-1648-16, Kiryu 376-0041, Japan
saburou.saitoh@gmail.com

Abstract. In this paper, we will introduce the application of the division by zero calculus to geometry and it will show the power of the new calculus.

Keywords. Division by zero calculus, $0/0 = 1/0 = z/0 = 0$, Laurent expansion, Pompe's example.

Mathematics Subject Classification (2010). 00A05, 00A09.

1. Division by zero calculus

We will give the definition of the division by zero calculus. For any Laurent expansion around $z = a$,

$$f(z) = \sum_{n=-\infty}^{-1} C_n(z-a)^n + C_0 + \sum_{n=1}^{\infty} C_n(z-a)^n,$$

we **define** the identity, by the division by zero

$$f(a) = C_0.$$

In addition, we will refer to the naturality of the division by zero calculus.

Recall the Cauchy integral formula for an analytic function $f(z)$; for an analytic function $f(z)$ around $z = a$ and for a smooth simple Jordan closed curve γ enclosing one time the point a, we have

$$f(a) = \frac{1}{2\pi i}\int_\gamma \frac{f(z)}{z-a}dz.$$

Even when the function $f(z)$ has any singularity at the point a, we assume that this formula is valid as the division by zero calculus. We define the value of the function $f(z)$ at the singular point $z = a$ with the Cauchy integral.

The division by zero calculus opens a new world since Aristotele-Euclid. See, in particular, [1] and also the references for recent related results.

On February 16, 2019 H. Okumura introduced the surprising news in Research Gate to Saitoh:

Jose Manuel Rodriguez Caballero
Added an answer

In the proof assistant Isabelle/HOL we have $x/0 = 0$ for each number x. This is advantageous in order to simplify the proofs. You can download this proof assistant here: **https://isabelle.in.tum.de/**.

J.M.R. Caballero kindly showed surprisingly several examples by the system that

$$\tan\frac{\pi}{2} = 0,$$

$$\log 0 = 0,$$

$$\exp\frac{1}{x}(x = 0) = 1,$$

and others to Saitoh. Furthermore, for the presentation at the annual meeting of the Japanese Mathematical Society at the Tokyo Institute of Technology:

March 17, 2019; 9:45-10:00 in Complex Analysis Session, *Horn torus models for the Riemann sphere from the viewpoint of division by zero* with [1],

he kindly sent the message:

It is nice to know that you will present your result at the Tokyo Institute of Technology. Please remember to mention Isabelle/HOL, which is a software in which x/0 = 0. This software is the result of many years of research and a millions of dollars were invested in it. If x/0 = 0 was false, all these money was for nothing. Right now, there is a team of mathematicians formalizing all the mathematics in Isabelle/HOL, where x/0 = 0 for all x, so this mathematical relation is the future of mathematics. https://www.cl.cam.ac.uk/ lp15/Grants/Alexandria/

Surprisingly enough, he sent his e-mail at 2019.3.30.18:42 as follows:

Nevertheless, you can use that $x/0 = 0$, following the rules from Isabelle/HOL and you will obtain no contradiction. Indeed, you can check this fact just downloading Isabelle/HOL: https://isabelle.in.tum.de/

and copying the following code

theory DivByZeroSatoih imports Complex Main

begin

theorem T: ?x/0 + 2000 = 2000? for x :: complex by simp

end

In this paper, from an example of Pompe ([16]), we will see the power of division by zero and division by zero calculus clearly.

2. Pompe's theorem

Generalizing a sangaku problem, W. Pompe gave the following theorem (see Figure 1):

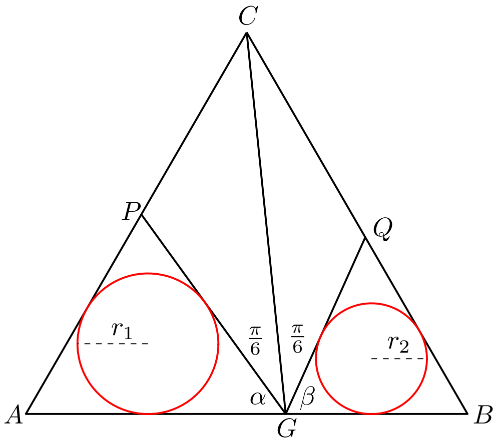

Figure 1.

Theorem 1 ([16]). *Let ABC be an equilateral triangle and let G be a point on the side AB. Points P and Q lie on the sides AC and BC, respectively, and satisfy $\angle PGC = \angle QGC = \pi/6$. Let $\alpha = \angle AGP$ and $\beta = \angle BGQ$. Denote by r_1 and r_2 the inradii of the triangles AGP and BGQ, respectively. Then*

$$\frac{r_1}{r_2} = \frac{\sin 2\alpha}{\sin 2\beta}. \tag{1}$$

We now concern with the case $\beta = \pi/2$ in the sense of division by zero and division by zero calculus. In this case the point G coincides with B, then the triangle BQG degenerates to the point B, i.e., $r_2 = 0$ (see Figure 2). In this case the left side of (1) equals $r_1/0 = 0$. Also the right side equals $\sin 2\alpha / \sin 2\pi = \sin 2\alpha/0 = 0$. Therefore (1) holds.

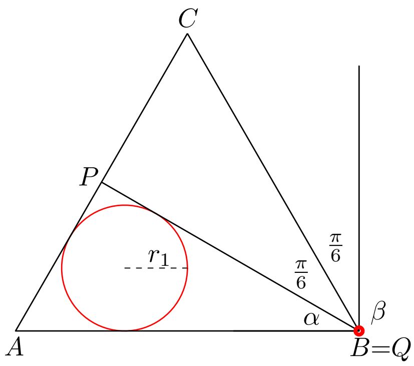

Figure 2.

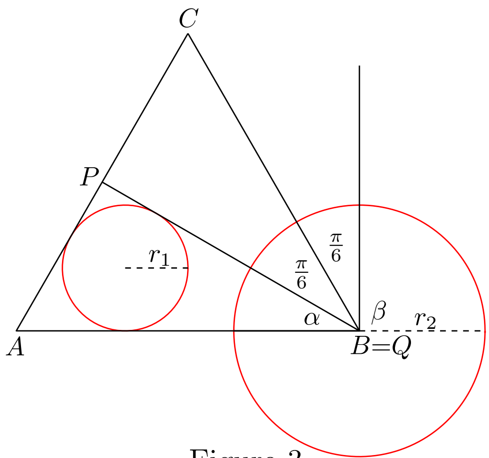

Figure 3.

On the other hand the right side of (1) is a function of β; $\sin 2(2\pi/3 - \beta)/\sin 2\beta$ and

$$\frac{\sin 2(2\pi/3 - x)}{\sin 2x} = -\frac{\sqrt{3}}{4x} + \frac{1}{2} + \frac{x}{\sqrt{3}} + \cdots .$$

This implies that

$$\frac{r_1}{r_2} = \frac{\sin 2\alpha}{\sin 2\beta} = \frac{1}{2}$$

in the case $\beta = 0$ by division by zero calculus. The large circle in Figure 3 has radius $r_2 = 2r_1$ and center $B = Q$. It is orthogonal to the lines AB, BC and the perpendicular to AB at B. Therefore the circle still touches the three lines, since $\tan \pi/2 = 0$, i.e., *it is the circle of radius $2r_1$ touching the lines AB, BC and the perpendicular to AB at B.*

Note that for many cases, we can calculate the division by zero calculus by MATHEMATICA, because it is just a coefficient of Laurent expansion.

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