Spherical Quantum Spacetime

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Understanding gravity at Planck scale is biggest goal of quantum gravity theory. In this paper i will present idea of quantum spacetime that can be thought as gravity in quantum scale, this spacetime is fixed it means this idea is not background independent it lives on specific modified spherical spacetime. That spherical spactime does not break at inside of black hole and works in low energy very close to general relativity- it changes mostly after passing event horizon. Key idea is to use Planck energy units of energy and momentum as measure of curvature of spacetime. Model predicts that if energy goes to Planck energy time stops-all light cones are frozen and it happens from point of view of observer falling into black hole. From field equation there is calculated wave function vector that represents state of quantum system and thus leads to it's gravity effects.

Field Equation

Energy tensor is extension of Einstein energy momentum relation (*1*) to sixteen parts where only ten of them are independent. Tensor itself has four indexes but i use contraction of one index to match metric tensor. I can write energy tensor components as, where indexes i, j, k go from one to three:

$$T^0_{000} = E^2$$
$$T^0_{0ij} = -p_i p_j$$
$$T^k_{kij} = -p^0_{0i} p^0_{0j} + p^k_{ki} p^k_{kj}$$
$$T^k_{k0j} = T^k_{kj0} = -E p^0_{0j} + E p^k_{kj}$$

Energy and momentum are dived by Planck units, it means energy is equal to energy divided by Planck energy and same with momentum- it is so maximum value of energy and momentum can be one. Field equation with wave function vector ψ_μ energy tensor and metric tensor is equal to :

$$\begin{cases} \Delta\psi_\mu - T^\gamma_{\gamma\mu\nu}\psi^\nu = 0 \\ T^\gamma_{\gamma\mu\nu}\psi^\nu - g_{\mu\nu}\psi^\nu = 0 \end{cases}$$

Where Δ is Laplace operator, metric tensor then is equal to, where primed is ν part:

$$g_{\mu\nu} = s(\mu,\nu)\left[\left(1 - \sin(\phi_1)\sin(\phi_1')\right) - \frac{r^2}{dx^\mu dx^\nu}\left[(d\phi_3 d\phi_3' + \sin(\phi_3)\sin(\phi_3')\left(d\phi_2 d\phi_2' + \sin(\phi_2)\sin(\phi_2')d\phi_1 d\phi_1'\right)\right]\right]$$

Function $s(\mu,\nu)$ is a sign function, for zero part is has plus sign $s(0,0) = 1$ for rest part it has minus sign $s(\mu,\nu) = -1$. Radius is equal to: $r^2 = \frac{dx^\mu dx^\nu}{T^0_{0\mu\nu}}$, and ϕ_1 is equal to: $\phi_1 = \arcsin\left((T^0_{0\mu\nu})^{\frac{1}{2}}\right)$. Differential of angle is defined by $d\theta \to \frac{\theta_1 - \theta_0}{2\pi}$, that means that for angle 2π its equal to one, where θ_0 means begin angle and θ_1 final angle. When i want to calculate relative change in length in one frame of reference to another it's equal to:

$$dx^{\mu'} = dx^\mu \frac{\csc((T^0_{0\mu\nu})^{\frac{1}{2}}\pi)}{\csc'((T^0_{0\mu\nu})^{\frac{1}{2}}\pi)}$$

Simplest solutions

Simplest solutions to this equations are plane waves. First i write down energy tensor, where zero component is changing with radius, i can write component $T^0_{000} = \frac{Ml_P}{m_P R}$, M means mass, R is radius and l_p, m_P is Planck length and mass. Rest of the components follow same rule just with momentum, so i can write energy tensor solutions as:

$$
\begin{aligned}
T_{00} &= \left(\frac{Ml_P}{m_P R}\right)^2 - \left(\frac{p^0_{01} l_P}{p_P R}\right)^2 - \left(\frac{p^0_{02} l_P}{p_P R}\right)^2 - \left(\frac{p^0_{03} l_P}{p_P R}\right)^2 \\
T_{11} &= -\left(\frac{p^1_{10} l_P}{p_P R}\right)^2 + \left(\frac{p^1_{11} l_P}{p_P R}\right)^2 + \left(\frac{p^1_{12} l_P}{p_P R}\right)^2 + \left(\frac{p^1_{13} l_P}{p_P R}\right)^2 \\
T_{22} &= -\left(\frac{p^2_{20} l_P}{p_P R}\right)^2 + \left(\frac{p^2_{21} l_P}{p_P R}\right)^2 + \left(\frac{p^2_{22} l_P}{p_P R}\right)^2 + \left(\frac{p^2_{23} l_P}{p_P R}\right)^2 \\
T_{33} &= -\left(\frac{p^3_{30} l_P}{p_P R}\right)^2 + \left(\frac{p^3_{31} l_P}{p_P R}\right)^2 + \left(\frac{p^3_{32} l_P}{p_P R}\right)^2 + \left(\frac{p^3_{33} l_P}{p_P R}\right)^2
\end{aligned}
$$

From it follows plane wave solutions so i get wave function vector as:

$$
\psi_\mu = \begin{pmatrix} -\Psi_0 e^{ix^a k_{a0} - i\omega_0 t} & \Psi_0 e^{ix^a k_{a1} - i\omega_1 t} & \Psi_0 e^{ix^a k_{a2} - i\omega_2 t} & \Psi_0 e^{ix^a k_{a3} - i\omega_3 t} \end{pmatrix}
$$

There is relation between energy tensor and wave numbers of wave function that has to be fulfilled, they have to be equal:

$$
\begin{aligned}
T_{00} &= \omega_0^2 - k_{10}^2 - k_{20}^2 - k_{30}^2 \\
T_{11} &= -\omega_1^2 + k_{11}^2 + k_{21}^2 + k_{31}^2 \\
T_{22} &= -\omega_2^2 + k_{12}^2 + k_{22}^2 + k_{32}^2 \\
T_{33} &= -\omega_3^2 + k_{13}^2 + k_{23}^2 + k_{33}^2
\end{aligned}
$$

Those are simplest solutions to wave equation, from them i can calculate metric tensor. thus geometry of spacetime for given wave function vector.

Many system equation and measurement

If i have one system equation is in really simple form as expressed in first chapter. But it can be extended to many system using tensor product. First i write probability for one system, it's just sum of vector wave function components with it's complex conjugate:

$$P = \int_{x_1,t_1}^{x_2,t_2} \sum_{\mu=0}^{3} \psi_\mu \psi_\mu^* d^4x$$

Probability tells what is change of particle being in position of spactime x_1, t_1 to x_2, t_2 where x has three components and d^4x means spacial and time components. Whole probability has to be equal to one so that integral for whole spacetime is one:

$$P = \int_X \sum_{\mu=0}^{3} \psi_\mu \psi_\mu^* d^4x = 1$$

For many system i use tensor product and change Laplace operator to be sum for many coordinates, first operator is sum of operators for each particle so that i can write first part of equation as:

$$(\Delta_1 + \Delta_2 ... + \Delta_n)\left(\psi_{\mu_1} \otimes \psi_{\mu_2} ... \otimes \psi_{\mu_n}\right) - T^{\gamma_1...\gamma_n}_{\gamma_1...\gamma_n\mu_1...\mu_n\nu_1...\nu_n}\left(\psi^{\nu_1} \otimes \psi^{\nu_2} ... \otimes \psi^{\nu_n}\right) = 0$$

From it i can write second part of field equation that is equality between metric tensor and energy tensor by:

$$T^{\gamma_1...\gamma_n}_{\gamma_1...\gamma_n\mu_1...\mu_n\nu_1...\nu_n}\left(\psi^{\nu_1} \otimes \psi^{\nu_2} ... \otimes \psi^{\nu_n}\right) - g_{\mu_1\nu_1}...g_{\mu_n\nu_n}\left(\psi^{\nu_1} \otimes \psi^{\nu_2} ... \otimes \psi^{\nu_n}\right) = 0$$

Those are many systems field equations, now i can write probability for n system state by just extending vector sum to many indexes:

$$P = \int_{x_1,t_1}^{x_2,t_2} ... \int_{x_{1n},t_{1n}}^{x_{2n},t_{2n}} \sum_{\mu=0}^{3} \psi_\mu \psi_\mu^* ... \sum_{\mu_n=0}^{3} \psi_{\mu_n} \psi_{\mu_n}^* d^4x_1 ... dx_n^4$$

References and Notes

1. *Energy Momentum Relation*

   https://en.wikipedia.org/wiki/Energy-momentum_relation