Remark on possible use of quadruple neutrosophic numbers for realistic modelling of physical systems

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Abstract

During mathematical modeling of real technical system we can meet any type and rate model uncertainty. Its reasons can be incognizance of modelers or data inaccuracy. So, classification of uncertainties, with respect to their sources, distinguishes between aleatory and epistemic ones. The aleatory uncertainty is an inherent data variation associated with the investigated system or its environment. Epistemic one is an uncertainty that is due to a lack of knowledge of quantities or processes of the system or the environment. In this short communication, we discuss quadruple neutrosophic numbers and their potential application for realistic modelling of physical systems, especially in the reliability assessment of engineering structures.

Introduction

We all know the quaternions, but quadruple neutrosophic numbers are different.

In quaternions, a+bi + cj + dk you have i^2 = j^2 = k^2 = -1 = ijk,

while on quadruple neutrosophic numbers we have:[1]

$$N = a + bT + cI + dF \text{ one has: } T^2 = T,\ I^2 = I,\ F^2 = F, \tag{1}$$

where a = known part of N, bT+cI+dF = unknown part of N, with T = degree of truth-membership, I = degree of indeterminate-membership, and F = degree of false-membership, and a, b, c, d are real (or complex) numbers, and an absorption law defined depending on expert and on application (so it varies);

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if we consider for example the neutrosophic order T > I > F, then the stronger absorbs the weaker, i.e.

$$TI = T, TF = T, \text{ and } IF = I, TIF = T. \tag{2}$$

Other orders can also be employed, for example T < I < F: (see book [1], at page 186.) Other interpretations can be given to T, I, F upon each application.

Application: statistical uncertainty and beyond

Engineers must deal with risks and uncertainties as a part of their professional work and, in particular, uncertainties are inherent to engineering models. Models play a central role in engineering. Models often represent an abstract and idealized version of the mathematical properties of a target. Using models, engineers can investigate and acquire understanding of how an object or phenomenon will perform under specified conditions.[6]

Furthermore, according to Murphy & Gardoni & Harris Jr :

> "For engineers, dealing with risk and uncertainty is an important part of their professional work. Uncertainties are involved in understanding the natural world, such as knowing whether a particular event will occur, and in knowing the performance of engineering works, such as the behavior and response of a structure or infrastructure, the variability in material properties (e.g., characteristics of soil, steel, or concrete), geometry, and external boundary conditions (e.g., loads or physical constraints). Such uncertainties produce risks. In the standard account risk is the product of a set of possible consequences and their associated probabilities of occurrence (Kaplan and Gerrick 1981), where the probabilities quantify the likelihood of occurrence of the potential consequences in light of the underlying uncertainties. One important use of models in engineering risk analysis is to quantify the likelihood or probability of the occurrence of specific events or a set of consequences. Such models are often referred to as probabilistic models to highlight their specific function to account for and quantify uncertainties."[6]

Uncertainties come in many forms, for example:

> "Uncertainty in Developing a Model

The uncertainties in developing a model are:

• Model Inexactness. This type of uncertainty arises when approximations are introduced in the formulation of a model. There are two essential problems that may arise: error in the form of the model (e.g., a linear expression is used when the actual relation is nonlinear), and missing variables (i.e., the model contains only a subset of the variables that influence the quantity of interest). …

• Mistaken Assumptions. Models are based on a set of assumptions. Uncertainties might be associated with the validity of such assumptions (e.g., problems arise when a model assumes normality or homoskedasticity when these assumptions are violated).

• Measurement Error. The parameters in a model are typically calibrated using a sample of the measured quantities of interest and the basic variables considered in the model. These observed values, however, could be inexact due to errors in the measurement devices or procedures, which then leads to errors in the calibration process. …

• Statistical Uncertainty. Statistical uncertainty arises from the sparseness of data used to calibrate a model. In particular, the accuracy of one's inferences depends on the observation sample size. The smaller the sample size, the larger is the uncertainty in the estimated values of the parameters. …However, the confidence in the model would likely increase if it was calibrated using one thousand samples. The statistical uncertainty captures our degree of confidence in a model in light of the data used to calibrate the model."[6]

With regards to statistical uncertainty, according to Ditlevsen and Madsen: [3]

"It is the purpose of any measuring method to generate information about a quantity related to the object of measurement. If the quantity is of a fluctuating nature so that it requires a probabilistic model for its description, the measuring method must make it possible to formulate quantitative information about the parameters of the chosen probabilistic model. It is obvious that a measured value of a single outcome of a non-degenerate random variable X only is sufficient for giving a crude estimate of the mean value of X and is insufficient for giving any information about the standard deviation of X. However, if a sample of X is given, that is, if measured values of a certain number of independently generated outcomes of X are given, these values can be used for calculating estimates for all parameters of the model. The reasons that such an estimation from a sample of X is possible and makes sense are to be found in the mathematical probability theory. The most elementary concepts and rules of the theory of statistics are assumed to be known to the reader. To illustrate the role of the statistical concepts in the reliability analysis it is worthwhile to repeat the most basic features of the description of the information that a sample of X of size n contains about the mean value E[X]. It

> is sufficient for our purpose to make the simplifying assumption that X has a known standard deviation D[X] = σ."

Now, it seems possible to extend it further to include not only statistical uncertainty but also modelling error etc. It can be a good application of Quadruple Neutrosophic Numbers.

Towards an improved model of engineering reliability

Few days ago, we just got an idea regarding application of symbolic Neutrosophic quadruple numbers, where we can use it to extend the notion of *standard deviation.*

As we know usually people wrote:

$$X' = x + k.\sigma \tag{3}$$

Where X mean observation, σ standard deviation, and k is usually a constant to be determined by statistical bell curve, for example 1.64 for 95% accuracy.

We can extend it by using symbolic quadruple operator:

$$X' = x \pm (k.\sigma + m.i + n.f) \tag{4}$$

Where X' stands for actual prediction from a set of observed x data, σ is standard deviation, i is indeterminacy and f falsefood.

That way modelling error (falsehood) and indeterminacy can be accounted for.

Alternatively, one can write a better expression:

$$X' = x \pm (T.\sigma + I.\sigma + F.\sigma) \tag{5}$$

where T = the truth degree of s (standard deviation), I = degree of indeterminacy about s, and F = degree of falsehood about s.

A slightly more general expression is the following:

$$X' = x \pm a\,(T.\sigma + I.\sigma + F.\sigma\,) \tag{6}$$

where T = the truth degree of s (standard deviation), I = degree of indeterminacy about s, and F = degree of falsehood about s.

Or

$$X' = x \pm (a.T.\sigma + b.I.\sigma + c.F.\sigma) \tag{7}$$

where T = the truth degree of s (standard deviation), I = degree of indeterminacy about s, and F = degree of falsehood about s, and a, b, c are constants to be determined.

That way we reintroduce quadruple Neutrosophic numbers into the whole of statistics estimate.

For further use in engineering use especially in reliability methods, readers can consult [3][4][5].

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