“Fuzzy time”, a Solution of Unexpected Hanging Paradox

(a Fuzzy interpretation of Quantum Mechanics)

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Abstract. Although Fuzzy logic and Fuzzy Mathematics is a widespread subject and there is a vast literature about it, yet the use of Fuzzy issues like Fuzzy sets [18] and Fuzzy numbers was relatively rare in time concept. This could be seen in the Fuzzy time series [2],[13],[14]. In addition, some attempts are done in fuzzifying Turing Machines [1] but seemingly there is no need to fuzzify time. Throughout this article, we try to change this picture and show why it is helpful to consider the instants of time as Fuzzy numbers.

Introduction

In physics, though there are revolutionary ideas on the time concept like B theories in contrast to A theory [9] also about central concepts like space, momentum… it is a long time that these concepts are changed, but time is considered classically in all well-known and established physics theories. Seemingly, we stick to the classical time concept in all fields of science and we have a vast inertia to change it.

Our goal in this article is to provide some bases why it is rational and reasonable to change and modify this picture. Here, the central point is the modified version of “Unexpected Hanging” paradox as it is described in [3],[4],[5],[6]. As it is shown in [5], [6] this modified version leads us to a contradiction and based on that it is presented there why some problems in Theory of Computation are not solved yet. To resolve the difficulties arising there, we have two choices. Either “choosing” a new type of Logic like “Paraconsistent Logic” to tolerate contradiction or changing and improving the time concept and consequently to modify the “Turing Computational Model”. Throughout this paper, we select the second way for benefiting from saving some aspects of Classical Logic.

Classically, in Physics the considered time model is real numbers R. In these types of models, any two intervals (a,b) and (c,d) in which b is bigger than c are disjoint intervals that shows the bounded intervals are usually disjoint sets (with probability 1). In an improved model type, which does not hold for the last property, the paradox is solved easily. In the case where the instants of time are not real numbers but the fuzzy numbers which are not finite support functions and are defined from positive to negative infinity, the paradox will not hold since we have not any separate or disjoint intervals.

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Since we have no two disjoint intervals the paradox argument will not take shape.

For instance, consider the first argument step in Unexpected Hanging Paradox. In the first step, we suppose that it is not executed since Saturday. Here, we have no disjoint interval of time, so we are not capable to have a similar phrase and argument in this model as we had in paradox by considering classical time, therefore the argument will not work in the new model. This solves the difficulties about the paradox, both the origin and the modified version of the paradox.

This picture is somehow similar to the time concept as described by Brouwer [16], [17] when he introduces the time instants as engaged links of a chain, although Brouwerian concept of time will not solve this problem, Since the support of the associated functions to this theory are not infinite.

In sequel, since our computation model is based on the physics theory, the following question is a plausible one behind it:

"Is there any Physics based explanation for considering the time instants as

fuzzy numbers?"

We proposed two different ways to solve this problem as explained in [5], [6]. Either, directly to change the Logic which leads us to a type of Logic such as Paraconsistent logic or changing the time concept and consequently Turing Computational Model to a new concept, as the above-mentioned explanation.

Interestingly, considering time as fuzzy number leads us to employing Fuzzy Mathematics and Fuzzy Logic. Indeed, depending on the definition of logical connectives we have different types of Intuitionistic Logic [11].

In sum, in both cases we are able to consider a modification in the associated Logic. More precisely, these two different paradox solving ways could lead us to two different approaches in Logic: Paraconsistent Logic & Intuitionism. Nevertheless, it is remarkable that in the second way it is not essential to change Classical Logic, we simply change the standard model of time.

Now we face one important question:

In the case when we accept Fuzzy time what will be the convenient Theory of Physics? If we have no right answer to this question, then our endeavors are in vain. Indeed, a convenient theory of Physics is required.

Here, we try to propound a Physics theory based on that. Actually, our main target is Quantum Mechanics since the "Fuzzy Time" concept at the first glance seems more problematic there.

In addition, we show that Quantum Mechanics could be considered as an opportunity to calculate Fuzzy number associated to the instants of time.

Quantum Mechanics Explanation

In this chapter, we show that if we consider Fuzzy time, it means we have a classical interpretation of Quantum Mechanics. In the literature there are attempts to connects Fuzzy Concepts and Quantum Mechanics [7],[12] in different ways, but here the way is to connect "Fuzzy time" and "Quantum Mechanics" in order to find a plausible fuzzy numbers for time instances and to compute it.

In quantum mechanics, the corner stone of the Theory is based on the wave function (Schrodinger Equation):

$$i\hbar \frac{\sigma}{\sigma(t)}\ \Psi(\vec{r},t) =[\frac{-h^2}{2m}\ \nabla^2 + V(r,t)]\ \Psi(\vec{r},t)$$

From this equation, we find the particle probability associated function as described in Quantum Mechanics.

This gives us a distribution while time is considered as a classical concept (Classical Model of quantum mechanics). We show that this distribution could be obtained by considering time as a Fuzzy concept, we called it either the second Model or second interpretation in contrast to the first model (Quantum mechanics Model).

To describe the second Model, first we define a distribution on R^4, the function $X(t,x,y,z){:}\, R^4 \to R$ probability fucnction. Mathematically, this is equal to ψ (wave function) which by our claim should be derived from Schrödinger equation but it is remarkable to mention ψ is defined in the first Model.

α_{q_1,q_2}=$\alpha(q_1,q_2)$

$$q_1 = (t,x,yz)$$

$q_2 = (t',x',y',z')$

α_{q_1,q_2} or $\alpha(q_1,q_2)$is the distribution probability of going from q_1 to q_2. Hence:

$X(t,x,y,z) = \iiint_{-\infty}^{+\infty}\alpha(q_1,q_2)dt'dx'dy'dz'$=

$$\int_{z'=-\infty}^{z'=+\infty} \int_{y'=-\infty}^{y'=+\infty} \int_{x'=-\infty}^{x'=+\infty} \int_{t'=-\infty}^{t'=+\infty} X(x'-x, y'-y, z'-z, t'-t) f(x'-x, y'-y, z'-z, t' - t) dt'dx'dy'dz'$$

(*).

Since $X(t, x, y, z) = \Psi(x, y, z, t)$ (they are mathematically equal even though they are defined in two different models).

So, we have the following system of equations:

1.$i\hbar \frac{\sigma}{\sigma(t)} \Psi(\vec{r}, t) =[\frac{-h^2}{2m} \nabla^2 + V(r, t)] \Psi(\vec{r}, t)$

$$\Psi(\vec{r}, t) = \Psi(x, y, z, t) =$$

2. $\int_{z'=-\infty}^{z'=+\infty} \int_{y'=-\infty}^{y'=+\infty} \int_{x'=-\infty}^{x'=+\infty} \int_{t'=-\infty}^{t'=+\infty} \Psi(x'-x, y'-y, z'-z, t'-t) f(x' - x, y'-y, z'-z, t' - t) dt'dx'dy'dz' = \Psi(t, x, y, z)$

We get Ψ from equation 1 and we solve the second problem. The second problem is a three-dimensional Volterra equation, we find function f by solving that.
By following theorem, we conclude that f has an infinite support.

Let $\{C_i\}_{i \epsilon N}$ be a set of equi-center circles C_i is a circle with radius i and $\psi_i = \psi \upharpoonright C_i - C_{i-1}$.
f_i Is a continues function derived from ψ_i by solving the equation:

$$X(t, x, y, z) =$$

$$\int_{z'=-\infty}^{z'=+\infty} \int_{y'=-\infty}^{y'=+\infty} \int_{x'=-\infty}^{x'=+\infty} \int_{t'=-\infty}^{t'=+\infty} X(x'-x, y'-y, z'-z, t'-t) f_i(x'-x, y'-y, z'-z, t' - t) dt'dx'dy'dz'$$

Theorem: By above conditions we have
1. For each *i* , f_i is strictly positive.($f_i > 0$).
2. $f = \sum_{i=1}^{\infty} \quad f_i$ is a continuous function and the solution of (*).

CONCLUSION: f is strictly positive and its support is)-∞, +∞(.

The above conclusion shows the second model satisfies our expectation of the concept of time as a fuzzy number and this solves the problem with the paradox.

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