*Diving deep into the crypts of physics*

# The Hilbert Book Model Project Survey

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Colophon

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The subject of this book is a purely mathematical model of physical reality.

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# The Hilbert Book Model Project survey

by Hans van Leunen

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*Summary*

This survey treats the Hilbert Book Model Project. The project concerns a well-founded, purely mathematical model of physical reality. The project relies on the conviction that physical reality owns its own kind of mathematics and that this mathematics guides and restricts the extension of the foundation to more complicated levels of the structure and the behavior of physical reality. This results in a model that more and more resembles the physical reality that humans can observe.

## Contents

# 1 The initiator of the project

The Hilbert Book Model Project is an ongoing project. Hans van Leunen is the initiator of this project. The initiator was born in the Netherlands in 1941. He will not live forever. This project will contain his scientific inheritance.

The project is introduced in a Wikiversity project [1]. In the opinion of the initiator, a Wikiversity project is a perfect way of introducing new science. It especially serves the needs of independent or retired scientific authors.

The initiator maintains a ResearchGate project that considers the Hilbert Book Model Project. The ResearchGate site supports a flexible way of discussing scientific subjects [2] [3].

The initiator has generated some documents that contain highlights as excerpts of the project, and he stored these papers on his personal e-print archive http://vixra.org/author/j_a_j_van_leunen [4].

The private website http://www.e-physics.eu contains most documents both in pdf as well as in docx format [5]. None of these documents claims copyright. Everybody is free to use the content of these papers.

## 1.1 Trustworthiness

Introducing new science always introduces controversial and unorthodox text. The Hilbert Book Model Project is an ongoing enterprise. Its content is dynamic and is revised regularly.

The content of this project is not peer-reviewed. It is the task of the author to ensure the correctness of what he writes. In the vision of the author, the reader is responsible for checking the validity of what he/she reads. The peer review process cannot cope with the dynamics of revisions and extensions that becomes possible via publishing in freely accessible e-print archives. In comparison to openly accessible publication on the internet, the peer review process is a rather slow process. In addition, it inhibits the usage of revision services, such as offered by vixra.org and by arxiv.org/

Reviewers are always biased, and they are never omniscient. The peer review process is expensive and often poses barriers to the renewal of science.

One way to check the validity of the text is to bring parts of the text to open scientific discussion sites such as ResearchGate. [2]

***The initiator challenges everybody to disprove the statements made in this report. He promises a fine bottle of XO cognac to anyone that finds a significant flaw in the presented theories.***

This challenge stands already for several years [6]. Up to so far, nobody claimed the bottle.

## 1.2 The author

Hans is born in Helmond in 1941 and visited the Eindhoven HTS in chemistry from 1957-1960.

After his military service in 1960-1963, Hans started at the Technical Highschool Eindhoven (THE) which is now called the Technical University Eindhoven (TUE) for a study in applied physics.

Hans finished this study in 1970 and then joined Philips Elcoma EOD in the development of image intensifier tubes. Later this became a department of Philips Medical Systems division.

In 1987 Hans switched to an internal software house. In 1995 Hans joined the Semiconductor division of Philips. In this period Hans designed a system for modular software generation.

In 2001 Hans retired.

From 1983 until 2006 Hans owned a software company "Technische en Wetenschappelijke Programmatuur" (TWP).

A private website treats my current activities [5].

I store my papers at a freely accessible e-print archive [4].

To investigate the foundations and the lower levels of physical reality, Hans started in 2009 a personal research project that in 2011 got its current name "The Hilbert Book Model Project."

The Hilbert Book Model is a purely mathematical unorthodox and controversial model of the foundations and the lower levels of the structure of physical reality.

## 1.3 Early encounters

I am born with a deep curiosity about my living environment. When I became aware of this, I was astonished why this environment appeared to be so complicated, and at the same time, it behaved in such a coherent way. In my childhood, I had no clue. Later some unique experiences offered me some indications. After my retirement, I started in 2009 a personal research project to discover and formulate some of the clues. The "Hilbert Book Model" is the name of my personal research project.

My interest in the structure and phenomena of physical reality started in the third year of my physics study when the configuration of quantum mechanics confronted me for the first time with its special approach. The fact that its methodology differed fundamentally from the way that physicists did classical mechanics astonished me. So, I asked my very wise lecturer on what origin this difference is based. His answer was that the superposition principle caused this difference. I was not very happy with this answer because the superposition principle was indeed part of the methodology of quantum mechanics, but in those days, I did not comprehend how that could present the main cause of the difference between the two methodologies. I decided to dive into literature, and after some search, I encountered the booklet of Peter Mittelsteadt, "Philosophische Probleme der modernen Physik" (1963). This booklet contained a chapter about quantum logic and that appeared to me to contain a more appropriate answer. Later, this appeared a far too quick conclusion. In 1936 Garrett Birkhoff and John von Neumann published a paper that described their discovery of what they called "quantum logic." [7] Quantum logic is since then in mathematical terminology known as an orthomodular lattice [8]. The relational structure of this lattice is to a large extent quite like the relational structure of classical logic. That is why the duo gave their discovery the name "quantum logic." This name was an unlucky choice because no good reason exists to consider the orthomodular lattice as a system of logical propositions. In the same paper, the duo indicated that the set of closed subspaces of a separable Hilbert space has exactly the relational structure of an orthomodular lattice. John von Neumann long doubted between Hilbert spaces and projective geometries. In the end, he selected Hilbert spaces as the best platform for developing quantum physical theories. That appears to be one of the main reasons why quantum physicists prefer Hilbert spaces as a realm in which they do their modeling of quantum physical systems. Another habit of quantum physicists also intrigued me. My lecturer thought me that all observable quantum physical quantities are eigenvalues of Hermitian operators. Hermitian operators feature real eigenvalues. When I looked around, I saw a world that had a structure that configures from a three-dimensional spatial domain and a one-dimensional and thus, scalar time domain. In the quantum physics of that time, no operator represents the time domain, and no operator was used to deliver the spatial domain in a compact fashion. After some trials, I discovered a four-dimensional number system that could provide an appropriate normal operator with an eigenspace that represented the full four-dimensional representation of my living environment. At that moment, I had not yet heard from quaternions, but an assistant professor quickly told me about the discovery of Rowan Hamilton that happened more than a century earlier. Quaternions appear to be the number system of choice for offering the structure of physical reality its powerful capabilities.

The introductory paper of Birkhoff and von Neumann already mentioned quaternions. Much later Maria Pia Solèr offered a hard prove that Hilbert spaces can only cope with members of an associative division ring. Quaternions form the most extensive associative division ring. To my astonishment, I quickly discovered that physicists preferred a spacetime structure that features a Minkowski signature instead of the Euclidean signature of the quaternions. The devised Hilbert Book Model shows that in physical reality, the Euclidean structure, as well as the spacetime structure, appear in parallel. Observers only see the spacetime structure. Physics is a science that focusses on observable information. My university, the TUE, targeted applied physics, and there was not much time nor support for diving deep into the fundamentals of quantum physics. After my study, I started a career in the high-tech industry where I joined the development of image intensifier devices. There followed my confrontation with optics and with the actual behavior of elementary particles. See: http://www.ephysics.eu/#_What_image_intensifiers reveal.

In the second part of my career, I devoted my time to establish a better way of generating software. I saw how the industry was very successful in the modular construction of hardware. The software was still developed as a monolithic system. My experiences in this trial are reported in "Story of a War Against Software Complexity"; http://vixra.org/abs/1101.0061, and "Managing the Software Generation Process"; http://vixra.org/abs/1101.0062. It taught me the power of modular design and modular construction [4].

Only after my retirement, I got enough time to dive deep into the foundations of physical reality. In 2009 after the recovery of severe disease, I started my personal research project that in 2011 got its current name "The Hilbert Book Model." For the rest of his life, the author takes the freedom to upgrade the related papers at a steady rate.

## 2 Intention

Theoretical physics still contains unresolved subjects. These deficiencies of the theory are caused by the way that physics was developed and by the attitude of the physicists that designed the current theory. Scientists take great care to secure the trustworthiness of their work, which ends in the publication of the results. They take measures to prevent that their publications get intermingled with badly prepared publications or even worse, with descriptions of fantasies. For that reason, they invented the scientific method [7]. In applied physics, the scientific method founds on observations. Applied physics flourishes because the descriptions of observations help to explore these findings, especially when formulas extend the usability of the observations beyond direct observation. In theoretical physics, this is not always possible because not all aspects of physical reality are observable. The only way of resolving this blockade is to start from a proper foundation that can be extended via trustworthy methods that rely on deduction. This approach can only be successful if the deduction process is guided and restricted such that the extensions of the foundation still describe physical reality. Thus, if a mathematical deduction is applied, then mathematics must guide and restrict this process such that a mathematically consistent extension of the model is again a valid model of physical reality. After a series of development steps, this approach must lead to a structure and behavior of the model that more and more conforms to the reality that we can observe.

This guidance and restriction are not self-evident. On the other hand, we know that when we investigate deeper, the structure becomes simpler and easier comprehensible. So, finally, we come to a fundamental structure that can be considered as a suitable foundation. The way back to more complicated levels of the structure cannot be selected freely. Mathematics must pose restrictions onto the extension of the fundamental structure. This happens to be true for a foundation that was discovered about eighty years ago by two scholars. They called their discovery quantum logic [8]. The scholar duo selected the name of this relational structure because its relational structure resembled closely the relational structure of the already known classical logic. Garrett Birkhoff was an expert in relational structures. These are sets that precisely define what relations are tolerated between the elements of the set. Mathematicians call these relational structures lattices, and they classified quantum logic as an orthomodular lattice [9]. John von Neumann was a broadly oriented scientist that together with others was searching for a platform that was suitable for the modeling of quantum mechanical systems. He long doubted between two modeling platforms. One was a projective geometry, and the other was a Hilbert space [10] [11] [12].Finally, he selected Hilbert spaces. In their introductory paper, the duo showed that quantum logic emerges into a separable Hilbert space. The set of closed subspaces inside a separable Hilbert space has exactly the relational structure of an orthomodular lattice. The union of these subspaces equals the Hilbert space. A separable Hilbert space applies an underlying vector space [13], and between every pair of vectors, it defines an inner product [14]. This inner product can only apply numbers that are taken from an associative division ring [15] [16]. In a division ring, every non-zero member owns a unique inverse. Only three suitable division rings exist. These are the real numbers, the complex numbers, and the quaternions. Depending on their dimension these number systems exist in several versions that differ in the way that Cartesian and polar coordinate systems sequence their members [18].

In the Hilbert space, operators exist that can map the Hilbert space onto itself. In this way, the operator can map some vectors along themselves. The inner product of a normalized vector with such a map produces an eigenvalue. This turns the vector into an eigenvector. Together the eigenvalues of an operator form its eigenspace. This story indicates that mathematics guides and restricts the extension of the selected foundation into more complicated levels of the structure. It shows that the scholar duo started a promising development project.

However, this initial development was not pursued much further. Axiomatic models of physical reality are not popular. Most physicists mistrust this approach. Probably these physicists consider it naïve to suspect that an axiomatic foundation can be discovered that like the way that a seed evolves in a certain type of plant, will evolve into the model of the physical reality that we can observe.

Most quantum physicists decided to take another route that much more followed the line of the physical version of the scientific method. As could be suspected this route gets hampered by the fact that not every facet of physical reality can be verified by suitable experiments.

Mainstream quantum physics took the route [20] of quantum field theory [21], which diversified into quantum electrodynamics [22] and quantum chromodynamics [23]. It bases on the principle of least action [24], the Lagrangian equation [25] and the path integral [26] However, none of these theories apply a proper foundation.

In contrast, the Hilbert Book Model Project intends to provide a purely and self-consistent mathematical model of physical reality [1] [20]. It uses the orthomodular lattice as its axiomatic foundation and applies some general characteristics of reality as guiding lines. An important ingredient is the modular design of most of the discrete objects that exist in the universe. Another difference is that the Hilbert Book Model relies on the control of coherence and binding by stochastic processes that own a characteristic function instead of the weak and strong forces and the force carriers that QFT, QED, and QCD apply [21] [22] [23].

Crucial to the Hilbert Book Model is that reality applies quaternionic Hilbert spaces as structured read-only archives of the dynamic geometric data of the discrete objects that exist in the model. The model stores these data before they can be accessed by observers. This fact makes it possible to interpret the model as the creator of the universe. The classification of modules as observers introduces two different views; the creator's view and the observer's view. Time reversal is only possible in the creator's view. It cannot be perceived by observers because observers must travel with the scanning time window.

# 3 The Hilbert Book Base Model

The Hilbert Book Model Project deviates considerably from the mainstream approaches. It tries to stay inside a purely mathematical model that can be deduced from the selected foundation. First, it designs a base model that is configured from a huge set of quaternionic separable Hilbert spaces that all share the same underlying vector space. One of these separable Hilbert spaces takes a special role and acts as a background platform. It has an infinite dimension, and it owns a unique non-separable Hilbert space that embeds its separable companion. Together these companion Hilbert spaces form the background platform of the base model. A reference operator manages the private parameter space of each separable Hilbert space. The elements of the version of the number system that the Hilbert space uses for specifying its inner products constitute this parameter space. These private parameter spaces float with their geometric center over the private parameter space of the background platform. Via the applied coordinate systems, the parameter spaces determine the symmetry of the corresponding Hilbert space. An elementary module resides on each floating separable Hilbert space. The eigenspace of a dedicated footprint operator archives the complete life story of this elementary module. After sequencing the real parts of these eigenvalues, the archive tells the life story of the point-like object as an ongoing hopping path that recurrently regenerates a coherent hop landing location swarm. The location density distribution that describes the swarm equals the square of the modulus of what physicists would call the wavefunction of the elementary module. Mainstream quantum physics calls the elementary modules elementary particles. They behave as elementary modules, but mainstream physics does not exploit that interpretation. In contrast, the Hilbert Book Model Project exploits the modular design of the model.

In fact, the sequencing defines a subspace of the underlying vector space that scans as a function of progression over the whole model. This scanning window divides the model into a historic part, a window that represents the current static status quo, and a future part. In this way, the dynamic model resembles the paging of a book in which each page tells a universe-wide story of what currently happens in this continuum. This explains the name of the Hilbert Book Model. Together with the requirement that all applied separable Hilbert spaces share the same vector space the fact that a window scans the Hilbert Book Base Model as a function of a progression parameter results in the fact that these quaternionic separable Hilbert spaces share the same real number based separable Hilbert space. After sequencing the eigenvalues, the eigenspace of the reference operator of this Hilbert space acts as a model wide proper time clock.

In contrast to the Hilbert Book Model, most other physical theories apply only a single Hilbert space that applies complex numbers for defining its inner product, or they apply a Fock space [27], which is a tensor product of complex number based Hilbert spaces. A tensor product of quaternionic Hilbert spaces [28] results in a real number based Hilbert space. In the Hilbert Book Base Model, the quaternionic separable Hilbert spaces share the same real number based Hilbert space.

The coherence of the hop landing location swarm that configures the footprint of an elementary module is ensured by the fact that the mechanism that generates the hop landing locations is a stochastic process that owns a characteristic function. This characteristic function is the Fourier transform of the location density distribution of the hop landing location swarm. The mechanism reflects the effect of the ongoing embedding of the separable Hilbert space of the elementary module into the background non-separable Hilbert space. A continuum eigenspace of a dedicated operator registers the embedding of the hop landings of all elementary modules into this continuum. The continuum corresponds to the dynamic field that physicists call the universe. This field acts as the living space of all discrete objects that exist in the universe.

## 3.1 Open questions

The suggested Hilbert Book Base Model raises some questions. The fact that the set of rational numbers is countable is used to suggest that a proper time clock exists and that this clock ticks with a fixed and model wide minimal period. The Hilbert Book Model does not offer an explanation or a suggestion for this minimal period. The known value of the frequency of the photon that is generated at the annihilation of an elementary particle offers some indication. For the electron that means a frequency of about $10^{20}$ Hertz. However, this elementary particle category exists in three known generations: electron, muon and tau.

Further, it is suggested that the private stochastic process generates a new hop landing location at each clock tick. It is possible that the stochastic process acts slower than the proper time clock and its rate differs for each generation.

Also, the mass of different type categories of elementary particles differs. Currently, the Hilbert Book Model has no detailed explanation for that difference.

# 4 Modeling dynamic fields and discrete sets

The eigenspace of a dedicated footprint operator in a quaternionic separable Hilbert space can represent the dynamic geometric data of the point-like object that resides on this Hilbert space. The eigenspace of operators in a quaternionic non-separable Hilbert space can, in addition, represent the description of a dynamic continuum. We already met the eigenspace of the reference operator, which represents the private parameter space of the Hilbert space. In the separable Hilbert space this eigenspace is countable and contains only the rational values of the version of the quaternionic number system that the separable Hilbert space can apply as eigenvalues. In the non-separable Hilbert space, the eigenspace of the reference operator also contains all the limits of the congruent series of rational values. Consequently, this eigenspace is no longer countable. In each of the applied Hilbert spaces, it is possible to use the reference operator to define a category of newly defined operators by taking for each eigenvector of the reference operator a new eigenvalue that equals the target value of a selected quaternionic function for the parameter value that equals the corresponding eigenvalue of the reference operator. In the quaternionic separable Hilbert space the new eigenspace represents the sampled field that is described by the selected quaternionic function. In the quaternionic non-separable Hilbert space the new eigenspace represents the full continuum that is described by the selected quaternionic function. Continuum eigenspaces can represent the mathematical equivalent of a dynamic physical field. The private parameter space of a quaternionic Hilbert space represents a flat field. The dynamics of a field can be described by quaternionic differential equations.

Quaternionic second order partial differential equations describe the interaction between point-like actuators and a dynamic field. Physical fields differ from mathematical fields by the fact that the value of the physical field is represented in physical units. All basic fields obey the same quaternionic differential and integral equations. The basic fields differ in their start and boundary conditions.

## 4.1 Quaternionic differential calculus

The first order partial differential equations divide the change of a field in five different parts that each represent a new field. We will represent the field change operator by a quaternionic nabla operator. This operator behaves as a quaternionic multiplier.

A quaternion can store a time-stamp in its real part and a three-dimensional spatial location in its imaginary part. The quaternionic nabla $\nabla$ acts as a quaternionic multiplying operator. Quaternionic multiplication obeys the equation

$$c = c_r + \vec{c} = ab = \left(a_r + \vec{a}\right)\left(b_r + \vec{b}\right) = a_r b_r - \left\langle \vec{a}, \vec{b} \right\rangle + a_r \vec{b} + \vec{a} b_r \pm \vec{a} \times \vec{b} \tag{4.1.1}$$

The $\pm$ sign indicates the freedom of choice of the handedness of the product rule that exists when selecting a version of the quaternionic number system. The first order partial differential follows from

$$\nabla = \left\{ \frac{\partial}{\partial \tau}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\} = \nabla_r + \vec{\nabla} \tag{4.1.2}$$

The spatial nabla $\vec{\nabla}$ is well-known as the del operator and is treated in detail in Wikipedia. [30] [31]

$$\phi = \nabla \psi = \left( \frac{\partial}{\partial \tau} + \vec{\nabla} \right)\left(\psi_r + \vec{\psi}\right) = \nabla_r \psi_r - \left\langle \vec{\nabla}, \vec{\psi} \right\rangle + \nabla_r \vec{\psi} + \vec{\nabla} \psi_r \pm \vec{\nabla} \times \vec{\psi} \tag{4.1.3}$$

The differential $\nabla \psi$ describes the change of field $\psi$. The five separate terms in the first order partial differential have a separate physical meaning. All basic fields feature this decomposition. The terms may represent new fields.

$$\phi_r = \nabla_r \psi_r - \left\langle \vec{\nabla}, \vec{\psi} \right\rangle \tag{4.1.4}$$

$$\vec{\phi} = \nabla_r \vec{\psi} + \vec{\nabla} \psi_r \pm \vec{\nabla} \times \vec{\psi} = -\vec{E} \pm \vec{B} \tag{4.1.5}$$

$\vec{\nabla} f$ is the gradient of $f$.

$\langle \vec{\nabla}, \vec{f} \rangle$ is the divergence of $\vec{f}$.

$\vec{\nabla} \times \vec{f}$ is the curl of $\vec{f}$.

The conjugate of the quaternionic nabla operator defines another type of field change.

$$\nabla^* = \nabla_r - \vec{\nabla} \tag{4.1.6}$$

$$\zeta = \nabla^* \phi = \left( \frac{\partial}{\partial \tau} - \vec{\nabla} \right) \left( \phi_r + \vec{\phi} \right) = \nabla_r \phi_r + \langle \vec{\nabla}, \vec{\phi} \rangle + \nabla_r \vec{\phi} - \vec{\nabla} \phi_r \mp \vec{\nabla} \times \vec{\phi} \tag{4.1.7}$$

## 4.2 Field excitations

Field excitations are solutions of second order partial differential equations.

One of the second order partial differential equations results from combining the two first-order partial differential equations $\phi = \nabla \psi$ and $\zeta = \nabla^* \phi$.

$$\begin{aligned} \zeta &= \nabla^* \varphi = \nabla^* \nabla \psi = \nabla \nabla^* \psi = \left( \nabla_r + \vec{\nabla} \right) \left( \nabla_r - \vec{\nabla} \right) \left( \psi_r + \vec{\psi} \right) \\ &= \left( \nabla_r \nabla_r + \langle \vec{\nabla}, \vec{\nabla} \rangle \right) \psi \end{aligned} \tag{4.2.1}$$

Integration over the time domain results in the Poisson equation

$$\rho = \langle \vec{\nabla}, \vec{\nabla} \rangle \psi \tag{4.2.2}$$

Under isotropic conditions, a very special solution of this equation is the Green's function $\frac{1}{\vec{q} - \vec{q}\,'}$ of the affected field. This solution is the spatial Dirac $\delta(\vec{q})$ pulse response of the field under strict isotropic conditions.

$$\nabla \frac{1}{\vec{q} - \vec{q}\,'} = - \frac{\left( \vec{q} - \vec{q}\,' \right)}{\left| \vec{q} - \vec{q}\,' \right|^3} \tag{4.2.3}$$

$$\langle \vec{\nabla}, \vec{\nabla} \rangle \frac{1}{\left| \vec{q} - \vec{q}\,' \right|} \equiv \left\langle \vec{\nabla}, \vec{\nabla} \frac{1}{\left| \vec{q} - \vec{q}\,' \right|} \right\rangle = - \left\langle \vec{\nabla}, \frac{\left( \vec{q} - \vec{q}\,' \right)}{\left| \vec{q} - \vec{q}\,' \right|^3} \right\rangle = 4\pi \delta \left( \vec{q} - \vec{q}\,' \right) \tag{4.2.4}$$

Under these conditions, the dynamic spherical pulse response of the field is a solution of a special form of the equation (4.2.1)

$$\left( \nabla_r \nabla_r + \langle \vec{\nabla}, \vec{\nabla} \rangle \right) \psi = 4\pi \delta \left( \vec{q} - \vec{q}\,' \right) \theta \left( \tau \pm \tau' \right) \tag{4.2.5}$$

Here $\theta(\tau)$ is a step function and $\delta(\vec{q})$ is a Dirac pulse response [33] [34].

After the instant $\tau'$, this solution is described by

$$\psi = \frac{f\left(\left|\vec{q}-\vec{q}'\right| \pm c\left(\tau-\tau'\right)\vec{n}\right)}{\left|\vec{q}-\vec{q}'\right|} \tag{4.2.6}$$

The normalized vector $\vec{n}$ can be interpreted as the spin of the solution. The spherical pulse response acts either as an expanding or as a contracting spherical shock front. Over time this pulse response integrates into the Green's function. This means that the expanding pulse injects the volume of the Green's function into the field. Subsequently, the front spreads this volume over the field. The contracting shock front collects the volume of the Green's function and sucks it out of the field. The $\pm$ sign in equation (4.2.5) selects between injection and subtraction.

Apart from the spherical pulse response equation (4.2.5) supports a one-dimensional pulse response that acts as a one-dimensional shock front. This solution is described by

$$\psi = f\left(\left|\vec{q}-\vec{q}'\right| \pm c\left(\tau-\tau'\right)\vec{n}\right) \tag{4.2.7}$$

Here, the normalized vector $\vec{n}$ can be interpreted as the polarization of the solution. Shock fronts only occur in one and three dimensions. A pulse response can also occur in two dimensions, but in that case, the pulse response is a complicated vibration that looks like the result of a throw of a stone in the middle of a pond.

Equations (4.2.1) and (4.2.2) show that the operators $\frac{\partial^2}{\partial \tau^2}$ and $\left\langle \vec{\nabla}, \vec{\nabla} \right\rangle$ are valid second order partial differential operators. These operators combine in the quaternionic equivalent of the wave equation [35].

$$\varphi = \left(\frac{\partial^2}{\partial \tau^2} - \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle\right)\psi \tag{4.2.8}$$

This equation also offers one-dimensional and three-dimensional shock fronts as its solutions.

$$\psi = \frac{f\left(\left|\vec{q}-\vec{q}'\right| \pm c\left(\tau-\tau'\right)\right)}{\left|\vec{q}-\vec{q}'\right|} \tag{4.2.9}$$

$$\psi = f\left(\left|\vec{q}-\vec{q}'\right| \pm c\left(\tau-\tau'\right)\right) \tag{4.2.10}$$

These pulse responses do not contain the normed vector $\vec{n}$. Apart from pulse responses, the wave equation offers waves as its solutions [31 [35].

By splitting the field into the time-dependent part $T(\tau)$ and a location dependent part, $A(\vec{q})$, the homogeneous version of the wave equation can be transformed into the Helmholtz equation [36].

$$\frac{\partial^2 \psi}{\partial \tau^2} = \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \psi = -\omega^2 \psi \tag{4.2.11}$$

$$\psi(\vec{q}, \tau) = A(\vec{q}) T(\tau) \tag{4.2.12}$$

$$\frac{1}{T}\frac{\partial^2 T}{\partial \tau^2} = \frac{1}{A}\left\langle \vec{\nabla}, \vec{\nabla} \right\rangle A = -\omega^2 \tag{4.2.13}$$

$$\langle\vec{\nabla},\vec{\nabla}\rangle A+\omega^2 A \tag{4.2.14}$$

The time-dependent part $T(\tau)$ depends on initial conditions, or it indicates the switch of the oscillation mode. The switch of the oscillation mode means that temporarily the oscillation is stopped and instead an object is emitted or absorbed that compensates the difference in potential energy. The location-dependent part of the field $A(\vec{q})$ describes the possible oscillation modes of the field and depends on boundary conditions. The oscillations have a binding effect. They keep the moving objects within a bounded region [37].

For three-dimensional isotropic spherical conditions, the solutions have the form

$$A(r,\theta,\varphi)=\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\left\{\left(a_{lm}j_l(kr)\right)+b_{lm}Y_l^m(\theta,\varphi)\right\} \tag{4.2.15}$$

Here $j_l$ and $y_l$ are the spherical Bessel functions, and $Y_l^m$ are the spherical harmonics [38] [39]. These solutions play a role in the spectra of atomic modules.

Planar and spherical waves are the simpler wave solutions of equation**Fout! Verwijzingsbron niet gevonden.**.

$$\psi(\vec{q},\tau)=\exp\left\{\vec{n}\left(\vec{k},(\vec{q}-\vec{q}_0)-\omega\tau+\varphi\right)\right\} \tag{4.2.16}$$

$$\psi(\vec{q},\tau)=\frac{\exp\left\{\vec{n}\left(\vec{k},(\vec{q}-\vec{q}_0)-\omega\tau+\varphi\right)\right\}}{|\vec{q}-\vec{q}_0|} \tag{4.2.17}$$

A more general solution is a superposition of these basic types.

The paper treats quaternionic differential equations more extensively in chapter 14.

# 5 Photons

Photons are objects that still offer significant confusion among physicists. The mainstream interpretation is still that photons are electromagnetic waves [40]. This interpretation conflicts with the known behavior of photons. Photons that are emitted by a nearby star can be detected by a human eye. Since the space between the star and the earth does not contain waveguides, waves cannot do this trick. Electromagnetic fields require the nearby presence of electric charges. Both conditions forbid that photons are implemented by electromagnetic waves.

## 5.1 Photon structure

Photons are one-dimensional objects that are strings of equidistant energy packages, such that the string obeys the Einstein-Planck relation

$$E = h\nu \tag{5.1.1}$$

The energy packages are implemented by one-dimensional shock fronts that possess a polarization vector.

## 5.2 One-dimensional pulse responses

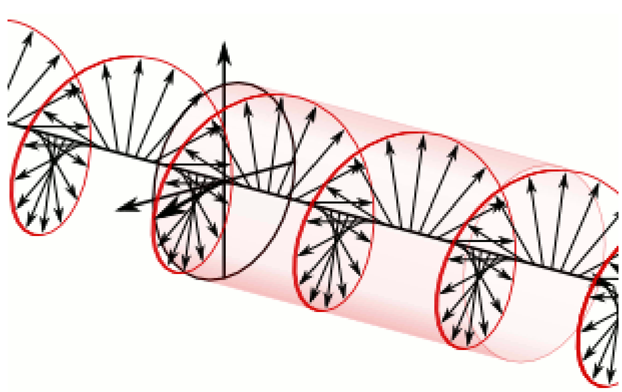
One-dimensional pulse responses that act as one-dimensional shock fronts and possess a polarization vector are solutions of the equation (4.2.5) and are described by the equation (4.2.7).

$$\psi = f\left(\left|\vec{q}-\vec{q}'\right| \pm c\left(\tau-\tau'\right)\vec{n}\right) \tag{5.1.2}$$

During travel, the front $f(\vec{q})$ keeps its shape and its amplitude. So also, during long-range trips, the shock front does not lose its integrity. The one-dimensional pulse response represents an energy package that travels with speed c through its carrier field. The energy of the package has a standard value.

In the animation of this left handed circular polarized photon, the black arrows represent the moving shock fronts [41]. The red line connects the vectors that indicate the amplitudes of the separate shock fronts. Here the picture of a guided wave is borrowed to show the similarity with such EM waves. However,

***photons are not EM waves!***



## 5.3 Photon integrity

Except for its speed, the photon emitter determines the properties of the photon. These properties are its frequency, its energy, and its polarization. The energy packages preserve their own integrity. They travel at constant speed and follow a worldline. Photon emission possesses a fixed duration. It is not an instant process. During emission, the emitter must not move and can only rotate around the direction of travel. Failing these requirements will compromise the integrity of the photon and make it impossible for a distant tiny absorber to capture the full photon. In that case, the energy packages will spray and fly to multiple locations. Consequently, they will act like dark energy objects.

The absorption of a photon by an atom requires an incredible aiming precision of the emitter. In fact, this emission can only be comprehended when it is interpreted as the time reversal of the corresponding emission process. If the absorbing atom cannot cope with the full energy of the photon, then it might absorb only part of the energy packages of the photon. The rest will stay on its route to the next absorber. Absorbing individual energy packages will result in an increase in the kinetic energy of the absorber. Absorbing the full photon or a part of it will result in an increase in the potential energy of the absorber. Usually, this results in a higher oscillation mode of one or more of the components of the absorber.

## 5.4 Light

Light is a dynamic spatial distribution of photons. Often the location density distribution of photons owns a Fourier transform. In that case, light may show wave behavior. Photons are one-dimensional particles that feature private frequency and energy. Single photons do not show wave behavior. Photons and light waves will feature different frequencies.

## 5.5 Optics

Optics is the science of imaging distributions of particles that can be characterized by a location density distribution and a corresponding Fourier transform of that location density distribution. Even though photons have a fixed non-zero spatial length, optics will treat these particles as point-like objects. Another name for the location density distribution is point spread function (PSF). Another name for the Fourier transform of the PSF is the optical transfer function (OTF) [42]. Apart from a location density distribution, the swarm of the particles is also characterized by an angular distribution and by an energy distribution. In the case of photons, the energy distribution is also a chromatic distribution.

A linearly operating imaging device can be characterized by its point spread function or alternatively by its OTF. This point spread function is an image of a point-like object. The PSF represents the blur that is introduced by the imaging device. For a homogeneous distribution of particle properties, the OTF of a chain of linearly operating imaging devices equals the product of the OTF’s of the separate devices.

The imaging properties of an imaging device may vary as a function of the location and the orientation in the imaging surface.

Without the presence of the traveling particles, the imaging devices keep their OTF. Small apertures and patterns of apertures feature an OTF. That OTF handles single particles similarly as this feature handles distributions of particles.

# 6 Modular design and construction

The discrete objects that exist in the universe show a modular design. In modular configurations, elementary particles behave as elementary modules. Together they constitute all modules that exist in the universe. Some modules constitute modular systems.

Also, photons show a modular structure.

## 6.1 Elementary modules

### 6.1.1 Symmetry-related charge

Elementary modules are very complicated objects that reside on a private platform, which possesses some of the characteristic properties of the elementary module. These properties establish the type of elementary module.

Elementary modules reside on a private Hilbert space, which uses a selected version of the quaternionic number system to specify its inner products. Consequently, the operators in this Hilbert space apply members of this version to specify its eigenvalues. The eigenspace of this operator reflects the properties of this version. Thus, the eigenspace of the reference operator reflects the symmetry of the Hilbert space. Its geometric center floats over the background parameter space. The symmetry is defined relative to the symmetry of the background platform. Mathematics can compare these differences when the axes of the Cartesian coordinate systems in these parameter spaces are parallel to each other. The model applies the Stokes theorem and the Gauss theorem to determine the effect of the symmetry differences [43] [44]. See section 16.3. The only freedoms that are left are the locations of the geometric centers of the parameter spaces and the way that the elements of the versions of the number systems are sequenced along the axes. These restrictions reduce the list of symmetry differences to a short list. It means that the elementary modules exist in a small number of different symmetry-related categories. The symmetry difference is represented by a symmetry-related charge that resides at the geometric center of the private parameter space. The opposed restrictions that determine the allowable versions of the quaternionic number system restrict the list of values of symmetry-related charges to $-3,-2,-1,0,+1,+2,+3$. The isotropic symmetry differences are represented by $-3,0,+3$

The symmetry-related charges correspond to symmetry-related fields. At the location of the charge, a source or a sink generates a corresponding potential.

The anisotropic differences spread over the three coordinate axes and are indicated by corresponding RGB color charges. If we extend this distinguishing to the real axis of the parameter spaces, then the anti-color charges add to the three RGB color charges. Further, the product rule of the quaternions introduces diversity in the handiness of the version of the number system. The polar coordinate system also allows the polar angle and the azimuth to run up or down. The range of the polar angle is π radians. The range of the azimuth is 2π radians. This freedom of choice adds to the freedom that is left by the Cartesian coordinate system.

The first conclusion is that elementary modules exist in a short list of categories that differ in their symmetry-related properties, in their angular range properties, and in their arithmetic properties.

## 6.2 Modular configuration

The elementary modules can combine into composed modules. Some modules combine into modular systems. However, not all modules can compose with arbitrary other modules. For example, symmetry-related charges that have the same sign will repel each other, while symmetry-related charges with a different sign will attract. Composition applies internal oscillation of the components of the module. This is explained in the next section. Only elementary modules with the proper angular symmetry can take part in the modular composition process. These elementary modules are called fermions. The other elementary modules are called bosons. Inside a composed module, fermions cannot share the same oscillation mode and cannot share the same angular properties, such as spin. The binding via internal oscillation must be supported by the attraction that is caused by deformation of the embedding field. The symmetry-related charges also influence the efficiency of the bond. The anisotropic elementary modules cannot themselves deform the embedding field. They must first combine into colorless hadrons before their combination can deform the embedding field. Physicists call this phenomenon color confinement.

The hop landings of isotropic elementary modules can produce spherical pulse responses that deform the embedding field. Similarly, the hop landings of hadrons can produce such spherical pulse responses.

### 6.2.1 Open question

The Hilbert Book Model does not explain why fermions feature an exclusion principle, while bosons do not possess such property. This phenomenon determines the structure of atoms and is known as the Pauli exclusion principle.

## 6.3 Stochastic control

For each elementary module, a private stochastic process generates the hop landing locations in the ongoing hopping path that recurrently regenerates the coherent hop landing location swarm that constitutes the footprint of the elementary module. Only for isotropic elementary modules, the hop landings can deform the embedding field. The footprints of anisotropic elementary modules must first combine into colorless hadrons before these footprints can deform the embedding field. This phenomenon is known as color confinement.

The type of stochastic process that generates the footprint of elementary modules owns a characteristic function that equals the Fourier transform of the location density distribution of the coherent hop landing location swarm. It is possible to interpret the stochastic process as a spatial Poisson point process in $\mathbb{R}^3$ [45]. The intensity function of this process is implemented by a spatial point spread function that equals the location density distribution of the generated hop landing location swarm. The eigenspace of the footprint operator archives the target values of a quaternionic function, whose spatial part describes the point spread function. A cyclic random distribution describes the real parts of these target values. After sequencing these real parts, the eigenspace describes the ongoing hopping path of the elementary module.

The location density distribution can be interpreted as a detection probability density distribution. If it has a Fourier transform, then a kind of uncertainty principle exists between the standard deviation of the detection probability density distribution and the standard deviation of the modulus of this Fourier transform [46]. If the standard deviation of the modulus of this Fourier transform increases, then the standard deviation of the detection probability density distribution decreases (and vice versa).

The second type of stochastic process controls composed modules. This process also owns a characteristic function. This characteristic function is a dynamic superposition of the characteristic functions of the components of the module. The superposition coefficients act as displacement generators. In this way, these coefficients control the internal positions of the components. Inside atoms, these components perform their own oscillation mode. All modules attach an extra displacement generator to their characteristic function. This displacement generator determines the location of the full module.

This analysis tells that the characteristic functions, which reside in Fourier space define the constitution of the module. In Fourier space spatial locality has no meaning. It means that the components of a module can be far apart. The phenomenon is known as entanglement [47]. Only the attracting influences of potentials can keep components closely together.

### 6.3.1 **Superposition**

The way that superposition is implemented in the Hilbert Book Model explains the most important difference between classical physics and quantum physics. Superposition of field excitations occurs in Fourier space and is controlled by the characteristic functions of stochastic processes. Color confinement inhibits the generation and subsequent superposition of the field excitation for quarks. They must first combine into colorless hadrons before they can generate the required pulse responses. Also, this combination is controlled by oscillations that are managed by the characteristic functions of the corresponding stochastic processes.

Since the definition of a composed module is defined in Fourier space, the location of the components of the modules in configuration space is not important for this definition. This definition does not depend on this location. Entanglement is the phenomenon that allows components of a module to locate far apart. This fact becomes observable when these components possess exclusive properties.

### 6.3.2 **Open questions**

The Hilbert Book Model offers no detailed explanation why the ongoing embedding of elementary modules is represented by a private stochastic process that owns a characteristic function. Similarly, the Hilbert Book Model offers no explanation for the fact that binding of modules inside composed modules is controlled by a stochastic process that owns a characteristic function that is a dynamic superposition of the characteristic functions of its components. In effect, this means that the HBM does not explain why superposition of modules is defined in Fourier space.

## 6.4 Benefits of modular design and construction

Modular design hides relations that are only relevant inside the module from the outside of the module. In this way, the modular design reduces the relational complexity of the construction of composed modules. This is further improved by the possibility to gather relations in standard interfaces. This standardization promotes the reusability of modules. The fact that composed modules can be generated from lower level modules has an enormously beneficial effect on the reduction of the relational complexity of the modular composition process.

By applying modular design, the creator has prepared the universe for modular construction, which is a very efficient way of generating new objects. However, modular configuration of objects involves the availability of modules that can be joined to become higher level modules or modular systems. This means that enough resources must be available at the proper place and the proper time. The generation of a module out of composing modules makes sense when the new module has a profitable functionality. An advantage can be that the new module or modular system has a better chance of survival in a competitive environment. In that case, stochastic modular design can easily win from monolithic design. Evolution can evolve with a pure stochastic modular design. However, as soon as intelligent species are generated as modular systems, then these individuals can take part in the control of evolution by intelligent modular design. Intelligent modular design and construction occur much faster than stochastic modular design and construction. However, intelligent modular design and construction only occur where intelligent species exist. These locations are not widespread in the universe.

### 6.4.1 **Modular hierarchy**

The modular hierarchy starts with elementary modules. Elementary modules exist in several types that differ in their basic properties.

These basic properties are their symmetry-related charge, their spin, and their regeneration cycle.

### 6.4.2 **Compound modules**

Compound modules are composed-modules for which the geometric centers of the platforms of the components coincide. The charges of the platforms of the elementary modules establish the binding of the corresponding platforms. Physicists and chemists call these compound modules atoms or atomic ions [48].

In free compound modules, the symmetry-related charges do not take part in the oscillations. The targets of the private stochastic processes of the elementary modules oscillate. This means that the hopping path of the elementary module folds around the oscillation path and the hop landing location swarm gets smeared along the oscillation path. The oscillation path is a solution of the Helmholtz equation [36]. Each fermion must use a different oscillation mode. A change of the oscillation mode goes together with the emission or the absorption of a photon. The center of emission coincides with the geometrical center of the compound module. During the emission or absorption, the oscillation mode and the hopping path halt, such that the emitted photon does not lose its integrity. Since all photons share the same emission duration, that duration must coincide with the regeneration cycle of the hop landing location swarm. Absorption cannot be interpreted so easily. In fact, it can only be comprehended as a time-reversed emission act. Otherwise, the absorption would require an incredible aiming precision for the photon.

The type of stochastic process that controls the binding of components appears to be responsible for the absorption and emission of photons and the change of oscillation modes. If photons arrive with too low energy, then the energy is spent on the kinetic energy of the common platform. If photons arrive with too high energy, then the energy is distributed over the available oscillation modes, and the rest is spent on the kinetic energy of the common platform, or it escapes into free space. The process must somehow archive the modes of the components. It can apply the private platform of the components for that purpose. Most probably the current value of the dynamic superposition coefficient is stored in the eigenspace of a special superposition operator.

#### 6.4.2.1 Open questions

The Hilbert Book Model does not reveal the fine details of the photon emission, and consequently it does not reveal the fine details of photon absorption.

### 6.4.3 Molecules

Molecules are conglomerates of compound modules that each keep their private geometrical center [49]. However, electron oscillations are shared among the compound modules. Together with the symmetry-related charges, this binds the compound modules into the molecule.

### 6.4.4 Consciousness and intelligence

In the Hilbert Book Model, all modules are considered to act as observers. That does not mean that these modules react to the perceived information in a conscious or intelligent way. In the hierarchy of modular systems, compared to intelligence, consciousness already enters at lower levels of complexity [50] [51]. However, consciousness cannot be attributed to non-living modular systems. Primitive life forms have primitive degrees of consciousness.

Intelligent species show self-reflection and can create strategies that guard their type-community or their social-community. Conscious species can also develop such guarding measures, but that is usually a result of trial and error instead of a developed strategy. The strategy is then inherited via genes.

For intelligent species, the modular design strategy of the creator can be an inspiration.

- Modular design is superior to monolithic design.
- Modular construction works economically with resources.
- It is advantageous to have access to a large number and a large diversity of suitable modules.
- Create module-type communities.
- Type communities survive far longer than the corresponding individual modules.
- Members must guard their module type community.
- Type communities may inherit and cultivate the culture of their members.
- Modular systems must care about the type communities on which they depend.
- Modular systems must care about their living environment.
- Darwin’s statement that the fittest individual will survive must be replaced by the statement that the module-type community survives that cares best for its members, its resources and its environment.

In modern human activity hardware is often designed and constructed in a modular way. In contrast, software is typically designed and constructed in a non-modular way. In comparison software is far less robust than hardware.

# 7 Dark objects and progression zigzag

The effects of the shock fronts that are caused by pulses are so tiny that no measuring instrument will ever be able to detect the presence of the single shock fronts. Thus, these field excitations can rightfully be called dark objects or more in detail dark energy and dark matter [52] [53]. These objects become noticeable in huge coherent ensembles that may contain about $10^{10}$ elements. The one-dimensional shock fronts combine in photons, and the spherical shock fronts combine in the footprints of elementary particles. They can exchange roles in pair production and pair annihilation events. For observers, these events pose interpretation problems. However, the model can interpret these events as time reversal that converts a particle into its antiparticle or vice versa. This interpretation relies on the mass-energy equivalence and on the fact that during the conversion each one-dimensional shock front is exchanged against a spherical shock front. In this interpretation, elementary particles can zigzag through the time domain. This vision suggests that elementary particles never die, but at the utmost change the direction of their life story and turn into its antiparticle. The conversion does not happen instantaneously. It takes the full regeneration cycle of the hop landing location swarm of the elementary particle. The universe-wide proper time clock ticks with a frequency of about $10^{20}$ ticks per second and the regeneration then takes about $10^{10}$ proper time clock ticks.

In huge numbers, spurious dark objects may still cause noticeable influences. The halo of dark matter around galaxies is known to produce gravitational lensing effects.

***Even though the Hilbert Book Model does not consider the shock fronts as the lowest level of modules, the shock fronts together constitute all discrete objects that exist in the universe.***

The Hilbert Book model considers elementary modules as the lowest level modules. They are complicated constructs that consist of a quaternionic separable Hilbert space, a selected version of the quaternionic number system and a private stochastic process that generates their life story.

# 8 Gravity

Mainstream physics considers the origin of the deformation of our living space as an unsolved problem [54]. It presents the Higgs mechanism as the explanation why some elementary particles get their mass [55] [56]. The Hilbert Book Model relates mass to deformation of the field that represents our universe. This deformation causes the mutual attraction of massive objects [57].

## 8.1 A deforming field excitation

A spherical pulse response is a solution of a homogeneous second order partial differential equation that was triggered by an isotropic pulse. The corresponding field equation and the corresponding solution are repeated here.

$$\left(\nabla_r \nabla_r + \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle\right)\psi = 4\pi\delta\left(\vec{q} - \vec{q'}\right)\theta\left(\tau \pm \tau'\right) \tag{8.1.1}$$

Here the $\pm$ sign represents time inversion.

$$\psi = \frac{f\left(\left|\vec{q} - \vec{q'}\right| \pm c\left(\tau - \tau'\right)\vec{n}\right)}{\left|\vec{q} - \vec{q'}\right|} \tag{8.1.2}$$

The spherical pulse response integrates over time into the Green’s function of the field. The Green’s function is a solution of the Poisson equation.

$$\rho = \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \psi \tag{8.1.3}$$

The Green’s function occupies some volume.

$$g(\vec{q}) = \frac{1}{\vec{q} - \vec{q'}} \tag{8.1.4}$$

This means that locally the pulse pumps some volume into the field, or it subtracts volume out of the field. The selection between injection and subtraction depends on the sign in the step function in the equation (8.1.1). The dynamics of the spherical pulse response shows that the injected volume quickly spreads over the field. In the case of volume subtraction, the front first collects the volume and finally subtracts it at the trigger location. Gravitation considers the case in which the pulse response injects volume into the field.

Thus, locally and temporarily, the pulse deforms the field, and the injected volume persistently expands the field.

***This paper postulates that the spherical pulse response is the only field excitation that temporarily deforms the field, while the injected volume persistently expands the field.***

The effect of the spherical pulse response is so tiny and so temporarily that no instrument can ever measure the effect of a single spherical pulse response in isolation. However, when recurrently regenerated in huge numbers in dense and coherent swarms the pulse responses can cause a significant and persistent deformation that instruments can detect. This is achieved by the stochastic processes that generate the footprint of elementary modules.

The spherical pulse responses are straightforward candidates for what physicists call dark matter objects. A halo of these objects can cause gravitational lensing.

## 8.2 Gravitation potential

The gravitation potential that an elementary module causes can be approached by the convolution of the Green’s function of the field and the location density distribution of the hop landing location swarm. This approximation is influenced by the fact that the deformations, which are due to the individual pulse responses quickly fade away. Further, the density of the location distribution affects the efficiency of the deformation.

The Green’s function describes the result of a point-like pulse whose response has a mass of its own. We know how to compute the mass of a distribution of point masses [58]. At some distance of the center of the swarm, the gravitation potential can be approximated by [59]

$$g(r) \approx \frac{Gm}{r} \tag{8.2.1}$$

where $m$ is the mass of the object and $r$ equals the distance to the center of mass. Here we omit the physical units. $G$ is the gravitational constant. The fact that a distribution of point-like masses cause the gravitation potential makes this simple approximation possible.

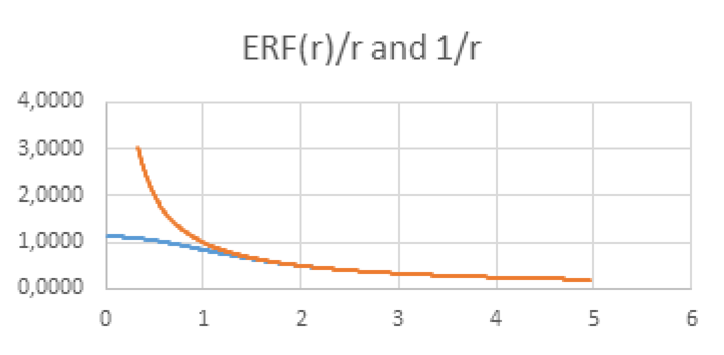
More exactly, the gravitation potential of the elementary module can be approximated by taking the convolution of the location density distribution of the hop landing location swarm. If we do this for example for a Gaussian location density distribution, then the convolution results in [60]

$$g(r) \approx Gm\frac{ERF(r)}{r} \tag{8.2.2}$$

Where $ERF(r)$ is the well-known error function. Here the gravitation potential is a perfectly smooth function that at some distance from the center equals the approximated gravitation potential that was described above in equation (8.2.1). The convolution only offers an approximation because this computation does not account for the influence of the density of the swarm and it does not compensate for the fact that the deformation by the individual pulse responses quickly fades away. Thus, the exact result depends on the duration of the recurrence cycle of the swarm.

In the example, we apply a normalized location density distribution, but the actual location density distribution might have a higher amplitude.

This might explain why some elementary module types exist in three generations [61] [62] [63].




## 8.3 Regeneration

The generation of the hopping path is an ongoing process. The generated hop landing location swarm contains a huge number of elements. Each elementary module type is controlled by a corresponding type of stochastic process. For the stochastic process, only the Fourier transform of the location density distribution of the swarm is important. Consequently, for a selected type of elementary module, it does not matter at what instant of the regeneration of the hop landing location swarm the location density distribution is determined. Thus, even when different types are bonded into composed modules, there is no need to synchronize the regeneration cycles of different types. This freedom also means that the number of elements in a hop landing location swarm may differ between elementary module types. This means that the strength of the deformation of the embedding field can differ between elementary module types. The strength of deformation relates to the mass of the elementary modules according to formula (8.2.1).

The requirement for regeneration introduces a great mystery. All generated mass appears to dilute away and must be recurrently regenerated. This fact conflicts with the conservation laws of mainstream physics. The deformation work done by the stochastic processes vanishes completely. What results is the ongoing expansion of the field. Thus, these processes must keep generating the particle to which they belong. The stochastic process accurately regenerates the hop landing location swarm, such that its rest mass stays the same.

Only the ongoing embedding of the content that is archived in the floating platform into the embedding field can explain the activity of the stochastic process. This supposes that at the instant of creation, the creator already archived the dynamic geometric data of his creatures into the eigenspaces of the footprint operators. These data consist of a scalar time-stamp and a three-dimensional spatial location. The quaternionic eigenvalues act as storage bins.

After the instant of creation, the creator left his creation alone. The set of floating separable Hilbert spaces, together with the background Hilbert space, act as a read-only repository. After sequencing the time-stamps, the stochastic processes read the storage bins and trigger the embedding of the location into the embedding field in the predetermined sequence.

### 8.3.1 Open question

If the instant of archival proceeds the passage of the window that scans the Hilbert Book Base Model as a function of progression, then the behavior of the model does not change. This indicates a freedom of the model.

## 8.4 Inertia

The relation between inertia and mass is complicated [64] [65]. It assumes that a field $\xi$ exists that tries to compensate for the change of the field when its vector part suddenly changes with time.

This special field supports the hop landing location swarm that resides on the floating platform. It reflects the activity of the stochastic process, and it floats with the platform over the background platform. It is characterized by a mass value and by the uniform velocity of the platform with respect to the background platform. The real part conforms to the deformation that the stochastic process causes. The imaginary part conforms to the speed of movement of the floating platform. The main characteristic of this field is that it tries to keep its overall change zero. We call $\xi$ the ***deformation field***.

The first order change of a field contains five terms. Mathematically, the statement that in first approximation nothing in the field $\xi$ changes, indicates that locally, the first order partial differential $\nabla\xi$ will be equal to zero.

$$\zeta = \nabla\xi = \nabla_r\xi_r - \left\langle\vec{\nabla},\vec{\xi}\right\rangle + \vec{\nabla}\xi_r + \nabla_r\vec{\xi} \pm \vec{\nabla}\times\vec{\xi} = 0 \tag{8.4.1}$$

The terms that are still eligible for change must together be equal to zero. These terms are.

$$\nabla_r\vec{\xi} + \vec{\nabla}\xi_r = 0 \tag{8.4.2}$$

In the following text plays $\vec{\xi}$ the role of the vector field and $\xi_r$ plays the role of the scalar gravitational potential of the considered object. We approximate this potential by using formula (8.2.1).

The new field $\xi = \left\{\frac{m}{r},\vec{v}\right\}$ considers a uniformly moving mass as a normal situation. It is a combination of the scalar potential $\frac{m}{r}$ and the uniform speed $\vec{v}$.

If this object accelerates, then the new field $\left\{\frac{m}{r},\vec{v}\right\}$ tries to counteract the change of the field $\dot{\vec{v}}$ by compensating this with an equivalent change of the real part $\frac{m}{r}$ of the new field. According to equation (8.4.2), this equivalent change is the gradient of the real part of the field.

$$\vec{a} = \dot{\vec{v}} = -\vec{\nabla}\left(\frac{m}{r}\right) = \frac{m\vec{r}}{|\vec{r}|^3} \tag{8.4.3}$$

This generated vector field acts on masses that appear in its realm.

Thus, if two masses $m_1$ and $m_2$ exist in each other's neighborhood, then any disturbance of the situation will cause the gravitational force

$$\vec{F}\left(\vec{r}_1 - \vec{r}_2\right) = m_1\vec{a} = \frac{m_1 m_2\left(\vec{r}_1 - \vec{r}_2\right)}{|\vec{r}_1 - \vec{r}_2|^3} \tag{8.4.4}$$

The disturbance by the ongoing expansion of the field suffices to put the gravitational force into action. The description also holds when the field $\xi$ describes a conglomerate of platforms and $m$ represents the mass of the conglomerate.

In compound modules such as ions and atoms, the field $\xi$ of a component oscillates with the deformation rather than with the platform.

Inertia bases mainly on the definition of mass that applies to the region outside the sphere where the gravitation potential behaves as the Green's function of the field. There the formula $\xi_r = \frac{m}{r}$ applies. Further, it bases in the intention of modules to keep the gravitation potential inside the mentioned sphere constant. At least that holds when this potential is averaged over the regeneration period. In that case, the overall change $\zeta$ of the deformation field $\xi$ equals zero. Next, the definition of the deformation field supposes that the swarm which causes the deformation moves as one unit. Further, the fact is used that the solutions of the homogeneous second order partial differential equation can superpose in new solutions of that same equation.

The popular sketch in which the deformation of our living space is presented by smooth dips is obviously false. The story that is represented in this paper shows the deformations as local extensions of the field, which represents the universe. In both sketches, the deformations elongate the information path, but none of the sketches explain why two masses attract each other. The above explanation founds on the habit of the stochastic process to recurrently regenerate the same time average of the gravitation potential, even when that averaged potential moves uniformly. Without the described habit of the stochastic processes, inertia would not exist.

Similar tricks can be used to explain the electrical force from the fact that the electrical field is produced by sources and sinks that can be described by the Green's function.

# 9 In the beginning

Before the stochastic processes started their action, the content of the universe was empty. It was represented by a flat field that in its spatial part was equal to the parameter space. In the beginning, a huge number of these stochastic processes started their triggering of the dynamic field that represents the universe. From that moment on the universe started expanding. This did not happen at a single point. Instead, it happened at a huge number of locations that were distributed all over the spatial part of the parameter space of the quaternionic function that describes the dynamic field.

Close to the begin of time, all distances were equal to the distances in the flat parameter space. Soon, these islands were uplifted with volume that was emitted at nearby locations. This flooding created growing distances between used locations. After some time, all parameter space locations were reached by the generated shock waves. From that moment on the universe started acting as an everywhere expanded continuum that contained deformations which in advance were very small. Where these deformations grew, the distances grew faster than in the environment. A uniform expansion appears the rule and local deformations form the exception. Deformations make the information path longer and give the idea that time ticks slower in the deformed and expanded regions. This corresponds with the gravitational red-shift of photons.

Composed modules only started to be generated after the presence of enough elementary modules. The generation of photons that reflected the signatures of atoms only started after the presence of these compound modules. However, the spurious one-dimensional shock fronts could be generated from the beginning.

This picture differs considerably from the popular scene of the big bang that started at a single location.

# 10 Life of an elementary module

An elementary module is a complicated construct. First, the particle resides on a private quaternionic separable Hilbert space that uses a selected version of the quaternionic number system to specify the inner products of pairs of Hilbert vectors and the eigenvalues of operators. The vectors belong to an underlying vector space. All elementary modules share the same underlying vector space. The selected version of the number system determines the private parameter space, which is managed by a dedicated reference operator. The coordinate systems that sequence the elements of the parameter space determine the symmetry of the Hilbert space and the elementary module inherits this symmetry. The private parameter space floats over a background parameter space that belongs to a background platform. The background platform is a separable Hilbert space that also applies the same underlying vector space. The difference in symmetry between the private parameter space and the background parameter space gives rise to a symmetry-related (electric) charge and a related color charge. The electric charge raises a corresponding symmetry-related field. The corresponding source or drain locates at the geometric center of the private parameter space.

The eigenspace of a dedicated footprint operator contains the dynamic geometric data that after sequencing of the time-stamps form the complete life-story of the elementary module. A subspace of the underlying vector space acts as a window that scans over the private Hilbert space as a function of a progression parameter that corresponds with the archived time-stamps. This subspace synchronizes all elementary modules that exist in the model.

Elementary particles are elementary modules, and together these elementary modules form all modules and modular systems that exist in the universe.

The complicated structure of elementary modules indicates that these particles never die. This does not exclude the possibility that elementary modules can zigzag over the progression parameter. Observers will perceive the progression reflection instants as pair creation and pair annihilation events. The zigzag will only become apparent in the creator's view. Thus, only the footprint of the elementary module is recurrently recreated. Its platform persists.

Probably the zigzag events correspond to an organized replacement of quaternions by two complex numbers or its reversal as is described in the Cayley-Dickson doubling [77].

A private stochastic process will recurrently regenerate the footprint of the elementary module in a cyclic fashion. During a cycle, the hopping path of the elementary module will have formed a coherent hop landing location swarm. A location density distribution describes this swarm. This location density distribution equals the Fourier transform of the characteristic function of the stochastic process that generates the hop landing locations. The location density distribution also equals the squared modulus of the wavefunction of the particle. This stochastic process mimics the mechanism that the creator applied when he created the elementary module. The stochastic process also represents the embedding of the eigenspace of the footprint operator into the continuum eigenspace of an operator that resides in the non-separable companion of the background platform. This continuum eigenspace represents the universe.

The differences between the symmetry of the private parameter space and the background parameter space give rise to symmetry-related charges that locate at the geometric center of the private parameter space. These charges give rise to symmetry-related fields. Via the geometric center of the platform, these symmetry-related fields couple to the field that represents the universe.

The kinetic energy of the platform is obtained from the effects of one-dimensional shock fronts. In many cases, these energy packages are combined in photons.

# 11 Relational structures

Lattice theory is a branch of mathematics [66].

## 11.1 Lattice

A lattice is a set of elements $a, b, c, \ldots$ that is closed for the connections $\cap$ and $\cup$. These connections obey:

- The set is ***partially ordered***.
  - This means that with each pair of elements $a, b$ belongs to an element $c$, such that $a \subset c$ and $b \subset c$.
- The set is a $\cap$ ***half lattice.***
  - This means that with each pair of elements $a, b$ an element $c$ exists, such that $c = a \cap b$.
- The set is a $\cup$ half lattice.
  - This means that with each pair of elements $a, b$ an element $c$ exists, such that $c = a \cup b$.
- The set is a lattice.
  - This means that the set is both a $\cap$ half lattice and a $\cup$ half lattice.

The following relations hold in a lattice:

$$a \cap b = b \cap a \tag{11.1.1}$$

$$(a \cap b) \cap c = a \cap (b \cap c) \tag{11.1.2}$$

$$a \cap (a \cup b) = a \tag{11.1.3}$$

$$a \cup b = b \cup a \tag{11.1.4}$$

$$(a \cup b) \cup c = a \cup (b \cup c) \tag{11.1.5}$$

$$a \cup (a \cap b) = a \tag{11.1.6}$$

The lattice has a ***partial order inclusion*** $\subset$:

$$a \subset b \Leftrightarrow a \cap b = a \tag{11.1.7}$$

## 11.2 Lattice types

A ***complementary lattice*** contains two elements $n$ and $e$ ,and with each element *a,* it contains a complementary element $a'$ such that [67]:

$$a \cap a' = n \tag{11.2.1}$$

$$a \cap n = n \tag{11.2.2}$$

$$a \cap e = a \tag{11.2.3}$$

$$a \cup a' = e \tag{11.2.4}$$

$$a \cup e = e \tag{11.2.5}$$

$$a \cup n = a \tag{11.2.6}$$

An ***orthocomplemented lattice*** contains two elements $n$ and $e$ ,and with each element *a*, it contains an element $a''$ such that [68]:

$$a \cup a^{"} = e \tag{11.2.7}$$

$$a \cap a^{"} = n \tag{11.2.8}$$

$$\left(a^{"}\right)^{"} = a \tag{11.2.9}$$

$$a \subset b \Leftrightarrow b^{"} \subset a^{"} \tag{11.2.10}$$

$e$ is the ***unity element***; $n$ is the ***null element*** of the lattice

A ***distributive lattice*** supports the distributive laws [69]:

$$a \cap (b \cup c) = (a \cap b) \cup (a \cap c) \tag{11.2.11}$$

$$a \cup (b \cap c) = (a \cup b) \cap (a \cup c) \tag{11.2.12}$$

A ***modular lattice*** supports [70]:

$$(a \cap b) \cup (a \cap c) = a \cap (b \cup (a \cap c)) \tag{11.2.13}$$

Every distributive lattice is modular.
An ***orthomodular lattice*** supports instead [71]:
There exists an element $d$ such that

$$a \subset c \Leftrightarrow (a \cup b) \cap c = a \cup (b \cap c) \cup (d \cap c) \tag{11.2.14}$$

where $d$ obeys:

$$(a \cup b) \cap d = d \tag{11.2.15}$$

$$a \cap d = n \tag{11.2.16}$$

$$b \cap d = n \tag{11.2.17}$$

$$(a \subset g)and(b \subset g) \Leftrightarrow d \subset g \tag{11.2.18}$$

In an ***atomic lattice*** holds [72]

$$\exists\{p \ni L\} \forall \{x \ni L\} \{x \subset p \Rightarrow x = n\} \tag{11.2.19}$$

$$\forall\{a \ni L\} \forall \{x \ni L\} \{(a \subseteq x \subseteq (a \cap p) \Rightarrow [(x = a) or (x = a \cap p)])\} \tag{11.2.20}$$

$p$ is an atom

## 11.3 Well known lattices

***Boolean logic***, also called classical logic, has the structure of an orthocomplemented distributive and atomic lattice [73] [74].

***Quantum logic*** has the structure of an orthocomplemented weakly modular and atomic lattice [75].
It is also called an ***orthomodular lattice*** [71].

# 12 Quaternions

Quaternions were discovered by Rowan Hamilton in 1843 [77] [76]. Later, in the twentieth century, quaternions fell in oblivion.

Hilbert spaces can only cope with number systems whose members form a divisions ring [14]. Quaternionic number systems represent the most versatile division ring. Quaternionic number systems exist in many versions that differ in the way that coordinate systems can sequence them. Quaternions can store a combination of a scalar time-stamp and a three-dimensional spatial location. Thus, they are ideally suited as storage bins for dynamic geometric data.

In this paper, we represent quaternion $q$ by a one-dimensional real part $q_r$ and a three-dimensional imaginary part $\vec{q}$ . The summation is commutative and associative

The following quaternionic multiplication rule describes most of the arithmetic properties of the quaternions.

$$c = c_r + \vec{c} = ab = \left(a_r + \vec{a}\right)\left(b_r + \vec{b}\right) = a_r b_r - \langle \vec{a}, \vec{b} \rangle + a_r \vec{b} + \vec{a} b_r \pm \vec{a} \times \vec{b} \qquad (12.1.1)$$

The $\pm$ sign indicates the freedom of choice of the handedness of the product rule that exists when selecting a version of the quaternionic number system.

A quaternionic conjugation exists

$$q^* = (q_r + \vec{q})^* = q_r - \vec{q} \qquad (12.1.2)$$

$$(ab)^* = b^* a^* \qquad (12.1.3)$$

The norm $|q|$ equals

$$|q| = \sqrt{q_r^2 + \langle \vec{q}, \vec{q} \rangle} \qquad (12.1.4)$$

$$q^{-1} = \frac{1}{q} = \frac{q}{|q|^2} \qquad (12.1.5)$$

$$q = |q| \exp\left( q_\varphi \frac{\vec{q}}{|\vec{q}|} \right) \qquad (12.1.6)$$

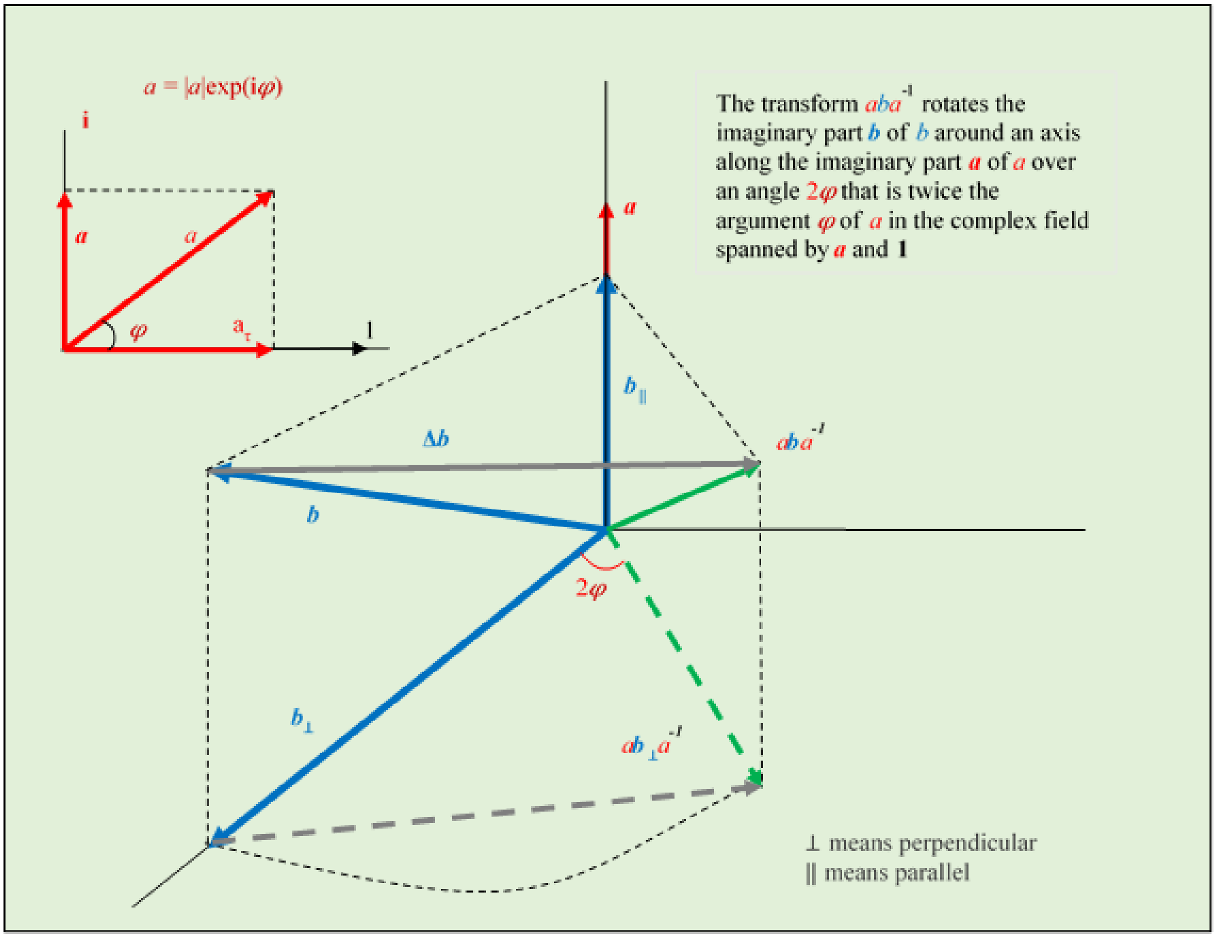
$\frac{\vec{q}}{|\vec{q}|}$ is the spatial direction of $q$.

A quaternion and its inverse can rotate a part of a third quaternion. The imaginary part of the rotated quaternion that is perpendicular to the imaginary part of the first quaternion is rotated over an angle that is twice the angle of the argument $\varphi$ between the real part and the imaginary part of the first quaternion. This makes it possible to shift the imaginary part of the third quaternion to a different dimension. For that reason, must $\varphi = \pi / 4$ .

Each quaternion $c$ can be written as a product of two complex numbers $a$ and $b$ of which the imaginary base vectors are perpendicular

$$
\begin{aligned}
c &= \left(a_r + a_1\vec{i}\right)\left(b_r + b_2\vec{j}\right) \\
&= a_r b_r + \left(a_1 + b_r\right)\vec{i} + \left(a_r + b_2\right)\vec{j} + a_1 b_2 \vec{k} = c_r + c_1\vec{i} + c_2\vec{j} + c_3\vec{k}
\end{aligned}
\tag{12.1.7}
$$

Where $\vec{k} = \vec{i} \times \vec{j}$



# 13 Quaternionic Hilbert spaces

Around the turn of the nineteenth century into the twentieth century David Hilbert and others developed the type of vector space that later got Hilbert's name [12].

The Hilbert space is a specific vector space because it defines an inner product for every pair of its member vectors [13].

That inner product can take values of a number system for which every non-zero member owns a unique inverse [14]. This requirement brands the number system as a division ring [14].

Only three suitable division rings exist:

- The real numbers
- The complex numbers
- The quaternions

Hilbert spaces cannot cope with bi-quaternions or octonions

## 13.1 Bra's and ket's

Paul Dirac introduced a handy formulation for the inner product that applies a bra and a ket [78].

The bra $\langle f|$ is a covariant vector, and the ket $|g\rangle$ is a contravariant vector. The inner product $\langle f|g\rangle$ acts as a metric.

For bra vectors hold

$$\langle f|+\langle g|=\langle g|+\langle f|=\langle f+g| \tag{13.1.1}$$

$$(\langle f+g|)+\langle h|=\langle f|+(\langle g+h|)=\langle f+g+h| \tag{13.1.2}$$

For ket vectors hold

$$|f\rangle+|g\rangle=|g\rangle+|f\rangle=|f+g\rangle \tag{13.1.3}$$

$$(|f+g\rangle)+|h\rangle=|f\rangle+(|g+h\rangle)=|f+g+h\rangle \tag{13.1.4}$$

For the inner product holds

$$\langle f|g\rangle=\langle g|f\rangle^* \tag{13.1.5}$$

For quaternionic numbers $\alpha$ and $\beta$ hold

$$\langle\alpha f|g\rangle=\langle g|\alpha f\rangle^*=(\langle g|f\rangle\alpha)=\alpha^*\langle f|g\rangle \tag{13.1.6}$$

$$\langle f|\beta g\rangle=\langle f|g\rangle\beta \tag{13.1.7}$$

$$\langle(\alpha+\beta)f|g\rangle=\alpha^*\langle f|g\rangle+\beta^*\langle f|g\rangle=(\alpha+\beta)^*\langle f|g\rangle \tag{13.1.8}$$

Thus

$$\alpha|f\rangle \tag{13.1.9}$$

$$\langle \alpha f | = \alpha^* \langle f | \quad (13.1.10)$$

$$|\alpha g\rangle = |g\rangle \alpha \quad (13.1.11)$$

We made a choice. Another possibility would be $\langle \alpha f | = \alpha \langle f |$ and $|\alpha g\rangle = \alpha^* |g\rangle$

In mathematics a topological space is called separable if it contains a countable dense subset; that is, there exists a sequence $\{|f_i\rangle\}_{i=\infty}^{i=0}$ of elements of the space such that every nonempty open subset of the space contains at least one element of the sequence [11] [79].

Its values on this countable dense subset determine every continuous function on the separable space $\mathfrak{H}$ [80].

The Hilbert space $\mathfrak{H}$ is separable. That means that a countable row of elements $\{|f_n\rangle\}$ exists that spans the whole space.

If $\langle f_m | f_n\rangle = \delta(m,n)$ [1 if n=m; otherwise 0], then $\{|f_n\rangle\}$ is an orthonormal base of Hilbert space $\mathfrak{H}$.

A ket base $\{|k\rangle\}$ of $\mathfrak{H}$ is a minimal set of ket vectors $|k\rangle$ that span the full Hilbert space $\mathfrak{H}$.

Any ket vector $|f\rangle$ in $\mathfrak{H}$ can be written as a linear combination of elements of $\{|k\rangle\}$.

$$|f\rangle = \sum_k |k\rangle\langle k | f\rangle \quad (13.1.12)$$

A bra base $\{\langle b|\}$ of $\mathfrak{H}^\dagger$ is a minimal set of bra vectors $\langle b|$ that span the full Hilbert space $\mathfrak{H}^\dagger$.

Any bra vector $\langle f|$ in $\mathfrak{H}^\dagger$ can be written as a linear combination of elements of $\{\langle b|\}$.

$$\langle f| = \sum_b \langle f | b\rangle\langle b| \quad (13.1.13)$$

Usually, a base selects vectors such that their norm equals 1. Such a base is called an orthonormal base

## 13.2 Operators

Operators act on a subset of the elements of the Hilbert space.

An operator $L$ is linear when for all vectors $|f\rangle$ and $|g\rangle$ for which $L$ is defined and for all quaternionic numbers $\alpha$ and $\beta$

$$L|\alpha f\rangle + L|\beta g\rangle = L|f\rangle\alpha + L|g\rangle\beta = L(|f\rangle\alpha + |g\rangle\beta) = L(|\alpha f\rangle + |\beta g\rangle) \quad (13.2.1)$$

The operator $B$ is **colinear** when for all vectors $|f\rangle$ for which $B$ is defined and for all quaternionic numbers $\alpha$ there exists a quaternionic number $\gamma$ such that

$$\alpha B|f\rangle = B|f\rangle\gamma\alpha\gamma^{-1} \equiv B|\gamma\alpha\gamma^{-1} f\rangle \quad (13.2.2)$$

If $|a\rangle$ is an eigenvector of the operator $A$ with quaternionic eigenvalue $\alpha$ ,

$$A|a\rangle = |a\rangle\alpha \tag{13.2.3}$$

then $|\beta a\rangle$ is an eigenvector of $A$ with quaternionic eigenvalue $\beta^{-1}\alpha\beta$.

$$A|\beta a\rangle = A|a\rangle\beta = |a\rangle\alpha\beta = |\beta a\rangle\beta^{-1}\alpha\beta \tag{13.2.4}$$

$A^\dagger$ is the **adjoint** of the **normal** operator $A$

$$\langle f \mid Ag\rangle = \langle fA^\dagger \mid g\rangle = \langle g \mid A^\dagger f\rangle^* \tag{13.2.5}$$

$$A^{\dagger\dagger} = A \tag{13.2.6}$$

$$(A+B)^\dagger = A^\dagger + B^\dagger \tag{13.2.7}$$

$$(AB)^\dagger = B^\dagger A^\dagger \tag{13.2.8}$$

If $A = A^\dagger$ then $A$ is a **self-adjoint** operator.

A linear operator $L$ is normal if $LL^\dagger$ exists and $LL^\dagger = L^\dagger L$

For the normal operator $N$ holds

$$\langle Nf \mid Ng\rangle = \langle NN^\dagger f \mid g\rangle = \langle f \mid NN^\dagger g\rangle \tag{13.2.9}$$

Thus

$$N = N_r + \vec{N} \tag{13.2.10}$$

$$N^\dagger = N_r - \vec{N} \tag{13.2.11}$$

$$N_r = \frac{N + N^\dagger}{2} \tag{13.2.12}$$

$$\vec{N} = \frac{N - N^\dagger}{2} \tag{13.2.13}$$

$$NN^\dagger = N^\dagger N = N_r N_r + \langle \vec{N}, \vec{N}\rangle = |N|^2 \tag{13.2.14}$$

$N_r$ is the Hermitian part of $N$.

$\vec{N}$ is the anti-Hermitian part of $N$.

For two normal operators $A$ and $B$ holds

$$AB = A_r B_r - \langle \vec{A}, \vec{B}\rangle + A_r\vec{B} + \vec{A}B_r \pm \vec{A}\times\vec{B} \tag{13.2.15}$$

For a unitary transformation $U$ holds

$$\langle Uf \mid Ug \rangle = \langle f \mid g \rangle \tag{13.2.16}$$

The closure of separable Hilbert space $\mathfrak{H}$ means that converging rows of vectors of $\mathfrak{H}$ converge to a vector in $\mathfrak{H}$.

### 13.2.1 **Operator construction**

$|f\rangle\langle g|$ is a constructed operator.

$$|g\rangle\langle f| = (|f\rangle\langle g|)^\dagger \tag{13.2.17}$$

For the orthonormal base $\{|q_i\rangle\}$ consisting of eigenvectors of the reference operator, holds

$$\langle q_n \mid q_m \rangle = \delta_{nm} \tag{13.2.18}$$

The reverse bra-ket method enables the definition of new operators that are defined by quaternionic functions.

$$\langle g \mid F \mid h \rangle = \sum_{i=1}^{N} \{ \langle g | q_i \rangle F(q_i) \langle q_i | h \rangle \} \tag{13.2.19}$$

The symbol $F$ is used both for the operator $F$ and the quaternionic function $F(q)$. This enables the shorthand

$$F \equiv |q_i\rangle F(q_i) \langle q_i| \tag{13.2.20}$$

It is evident that

$$F^\dagger \equiv |q_i\rangle F^*(q_i) \langle q_i| \tag{13.2.21}$$

For reference operator $\mathfrak{R}$ holds

$$\mathfrak{R} = |q_i\rangle q_i \langle q_i| \tag{13.2.22}$$

If $\{|q_i\rangle\}$ consists of all rational values of the version of the quaternionic number system that $\mathfrak{H}$ applies then the eigenspace of $\mathfrak{R}$ represents the private parameter space of the separable Hilbert space $\mathfrak{H}$. It is also the parameter space of the function $F(q)$ that defines the operator $F$ in the formula (13.2.20).

## 13.3 Non-separable Hilbert space

Every infinite dimensional separable Hilbert space $\mathfrak{H}$ owns a unique non-separable companion Hilbert space $\mathcal{H}$. This is achieved by the closure of the eigenspaces of the reference operator and the defined operators. In this procedure, on many occasions, the notion of the dimension of subspaces loses its sense.

***Gelfand triple*** and ***Rigged Hilbert space*** are other names for the general non-separable Hilbert spaces [81].

In the non-separable Hilbert space, for operators with continuum eigenspaces, the reverse bra-ket method turns from a summation into an integration.

$$\langle g \mid F \mid h \rangle \equiv \int \iiint \{ \langle g | q \rangle F(q) \langle q | h \rangle \} dV d\tau \tag{13.3.1}$$

Here we omitted the enumerating subscripts that were used in the countable base of the separable Hilbert space.

The shorthand for the operator $F$ is now

$$F \equiv |q\rangle F(q) \langle q| \tag{13.3.2}$$

For eigenvectors $|q\rangle$ the function $F(q)$ defines as

$$F(q) = \langle q | Fq \rangle = \int\iiint \{\langle q | q' \rangle F(q') \langle q' | q \rangle\} dV' d\tau' \tag{13.3.3}$$

The reference operator $\mathcal{R}$ that provides the continuum background parameter space as its eigenspace follows from

$$\langle g | \mathcal{R} h \rangle \equiv \int\iiint \{\langle g | q \rangle q \langle q | h \rangle\} dV d\tau \tag{13.3.4}$$

The corresponding shorthand is

$$\mathcal{R} \equiv |q\rangle q \langle q| \tag{13.3.5}$$

The reference operator is a special kind of defined operator. Via the quaternionic functions that specify defined operators, it becomes clear that every infinite dimensional separable Hilbert space owns a unique non-separable companion Hilbert space that can be considered to embed its separable companion.

The reverse bracket method combines Hilbert space operator technology with quaternionic function theory and indirectly with quaternionic differential and integral technology.

# 14 Quaternionic differential calculus

The quaternionic analysis is not so well accepted as complex function analysis [29]

## 14.1 Field equations

Maxwell equations apply the three-dimensional nabla operator in combination with a time derivative that applies coordinate time. The Maxwell equations derive from results of experiments. For that reason, those equations contain physical units.

In this treatment, the quaternionic partial differential equations apply the quaternionic nabla. The equations do not derive from the results of experiments. Instead, the formulas apply the fact that the quaternionic nabla behaves as a quaternionic multiplying operator. The corresponding formulas do not contain physical units. This approach generates essential differences between Maxwell field equations and quaternionic partial differential equations.

The quaternionic partial differential equations form a complete and self-consistent set. They use the properties of the three-dimensional spatial nabla.

The corresponding formulas are taken from Bo Thidé's EMTF book., section Appendix F4 [31].

Another online resource is Vector calculus identities [32].

The quaternionic differential equations play in a Euclidean setting that is formed by a continuum quaternionic parameter space and a quaternionic target space. The parameter space is the eigenspace of the reference operator of a quaternionic non-separable Hilbert space. The target space is eigenspace of a defined operator that resides in that same Hilbert space. The defined operator is specified by a quaternionic function that completely defines the field. Each basic field owns a private defining quaternionic function. All basic fields that are treated in this chapter are defined in this way.

Physical field theories tend to use a non-Euclidean setting, which is known as spacetime setting. This is because observers can only perceive in spacetime format. Thus, Maxwell equations use coordinate time, where the quaternionic differential equations use proper time. In both settings, the observed event is presented in Euclidean format. The hyperbolic Lorentz transform converts the Euclidean format to the perceived spacetime format. Chapter 8 treats the Lorentz transform. The Lorentz transform introduces time dilation and length contraction. Quaternionic differential calculus describes the interaction between discrete objects and the continuum at the location where events occur. Converting the results of this calculus by the Lorentz transform will describe the information that the observers perceive. Observers perceive in spacetime format. This format features a Minkowski signature. The Lorentz transform converts from the Euclidean storage format at the situation of the observed event to the perceived spacetime format. Apart from this coordinate transformation, the perceived scene is influenced by the fact that the retrieved information travels through a field that can be deformed and acts as the living space for both the observed event and the observer. Consequently, the information path deforms with its carrier field and this affects the transferred information. In this chapter, we only treat what happens at the observed event. So, we ignore the Lorentz coordinate transform, and we are not affected by the deformations of the information path.

The Hilbert Book Model archives all dynamic geometric data of all discrete creatures that exist in the model in eigenspaces of separable Hilbert spaces whose private parameter spaces float over the background parameter space, which is the private parameter space of the non-separable Hilbert space. For example, elementary particles reside on a private floating platform that is implemented by a private separable Hilbert space.

Quantum physicists use Hilbert spaces for the modeling of their theory. However, most quantum physicists apply complex-number based Hilbert spaces. Quaternionic quantum mechanics appears to represent a natural choice. Quaternionic Hilbert spaces store the dynamic geometric data in the Euclidean format in quaternionic eigenvalues that consists of a real scalar valued time-stamp and a spatial, three-dimensional location.

In the Hilbert Book Model, the instant of storage of the event data is irrelevant if it coincides with or precedes the stored time stamp. Thus, the model can store all data at an instant, which precedes all stored timestamp values. This impersonates the Hilbert Book Model as a creator of the universe in which the observable events and the observers exist. On the other hand, it is possible to place the instant of archival of the event at the instant of the event itself. It will then coincide with the archived time-stamp. In both interpretations, after sequencing the time-stamps, the repository tells the life story of the discrete objects that are archived in the model. This story describes the ongoing embedding of the separable Hilbert spaces into the non-separable Hilbert space. For each floating separable Hilbert space this embedding occurs step by step and is controlled by a private stochastic process, which owns a characteristic function. The result is a stochastic hopping path that walks through the private parameter space of the platform. A coherent recurrently regenerated hop landing location swarm characterizes the corresponding elementary object.

Elementary particles are elementary modules. Together they constitute all other modules that occur in the model. Some modules constitute modular systems. A dedicated stochastic process controls the binding of the components of the module. This process owns a characteristic function that equals a dynamic superposition of the characteristic functions of the stochastic processes that control the components. Thus, superposition occurs in Fourier space. The superposition coefficients act as gauge factors that implement displacement generators, which control the internal locations of the components. In other words, the superposition coefficients may install internal oscillations of the components. These oscillations are described by differential equations.

## 14.2 Fields

In the Hilbert Book Model fields are eigenspaces of operators that reside in the non-separable Hilbert space. Continuous or mostly continuous functions define these operators, and apart from some discrepant regions, their eigenspaces are continuums. These regions might reduce to single discrepant point-like artifacts. The parameter space of these functions is constituted by a version of the quaternionic number system. Consequently, the real number valued coefficients of these parameters are mutually independent, and the differential change can be expressed in terms of a linear combination of partial differentials. Now the total differential change $df$ of field $f$ equals

$$df = \frac{\partial f}{\partial \tau} d\tau + \frac{\partial f}{\partial x} \vec{i} dx + \frac{\partial f}{\partial y} \vec{j} dy + \frac{\partial f}{\partial z} \vec{k} dz \qquad (14.2.1)$$

In this equation, the partial differentials $\frac{\partial f}{\partial \tau}, \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial y}$ are quaternions.

The quaternionic nabla $\nabla$ assumes the ***special condition*** that partial differentials direct along the axes of the Cartesian coordinate system. Thus

$$\nabla = \sum_{i=0}^{4} \vec{e}_i \frac{\partial}{\partial x_i} = \frac{\partial}{\partial \tau} + \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \qquad (14.2.2)$$

The Hilbert Book Model assumes that the quaternionic fields are moderately changing, such that only first and second order partial differential equations describe the model. These equations can describe fields of which the continuity gets disrupted by point-like artifacts. Spherical pulse responses, one-dimensional pulse responses and Green's functions describe the reaction of the field on such disruptions.

## 14.3 Field equations

Generalized field equations hold for all basic fields. Generalized field equations fit best in a quaternionic setting.

Quaternions consist of a real number valued scalar part and a three-dimensional spatial vector that represents the imaginary part.

The multiplication rule of quaternions indicates that several independent parts constitute the product.

$$c = c_r + \vec{c} = ab = \left(a_r + \vec{a}\right)\left(b_r + \vec{b}\right) = a_r b_r - \left\langle \vec{a}, \vec{b} \right\rangle + a_r \vec{b} + \vec{a} b_r \pm \vec{a} \times \vec{b} \qquad (14.3.1)$$

The $\pm$ indicates that quaternions exist in right-handed and left-handed versions.

The formula can be used to check the completeness of a set of equations that follow from the application of the product rule.

We define the quaternionic nabla as

$$\nabla \equiv \left\{\frac{\partial}{\partial \tau}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right\} = \nabla_r + \vec{\nabla} \tag{14.3.2}$$

$$\vec{\nabla} \equiv \left\{\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right\} \tag{14.3.3}$$

$$\nabla_r \equiv \frac{\partial}{\partial \tau} \tag{14.3.4}$$

$$\phi = \phi_r + \vec{\phi} = \nabla \psi = \left(\frac{\partial}{\partial \tau} + \vec{\nabla}\right)(\psi_r + \vec{\psi}) = \nabla_r \psi_r - \langle \vec{\nabla}, \vec{\psi} \rangle + \nabla_r \vec{\psi} + \vec{\nabla} \psi_r \pm \vec{\nabla} \times \vec{\psi} \tag{14.3.5}$$

$$\phi_r = \nabla_r \psi_r - \langle \vec{\nabla}, \vec{\psi} \rangle \tag{14.3.6}$$

$$\vec{\phi} = \nabla_r \vec{\psi} + \vec{\nabla} \psi_r \pm \vec{\nabla} \times \vec{\psi} = -\vec{E} \pm \vec{B} \tag{14.3.7}$$

Further,

$\vec{\nabla} \psi_r$ is the gradient of $\psi_r$

$\langle \vec{\nabla}, \vec{\psi} \rangle$ is the divergence of $\vec{\psi}$

$\vec{\nabla} \times \vec{\psi}$ is the curl of $\vec{\psi}$

The change $\nabla \psi$ divides into five terms that each has a separate meaning. That is why these terms in Maxwell equations get different names and symbols. Every basic field offers these terms!

$$\vec{E} = -\nabla_r \vec{\psi} - \vec{\nabla} \psi_r \tag{14.3.8}$$

$$\vec{B} = \vec{\nabla} \times \psi \tag{14.3.9}$$

It is also possible to construct higher order equations. For example

$$\vec{J} = \vec{\nabla} \times \vec{B} - \nabla_r \vec{E} \tag{14.3.10}$$

The equation (14.3.6) has no equivalent in Maxwell's equations. Instead, its right part is used as a gauge.

Two special second-order partial differential equations use the terms $\frac{\partial^2 \psi}{\partial \tau^2}$ and $\langle \vec{\nabla}, \vec{\nabla} \rangle \psi$

$$\phi = \left\{\frac{\partial^2}{\partial \tau^2} - \langle \vec{\nabla}, \vec{\nabla} \rangle \right\} \psi \tag{14.3.11}$$

$$\rho = \left\{\frac{\partial^2}{\partial \tau^2} + \langle \vec{\nabla}, \vec{\nabla} \rangle \right\} \psi \tag{14.3.12}$$

The equation (14.3.11) is the quaternionic equivalent of the wave equation [35].

The equation (14.3.12) can be divided into two first-order partial differential equations.

$$\chi = \nabla^* \varphi = \nabla^* \nabla \psi = \nabla \nabla^* \psi = \left(\nabla_r + \vec{\nabla}\right)\left(\nabla_r - \vec{\nabla}\right)\left(\psi_r + \vec{\psi}\right) = \left(\nabla_r \nabla_r + \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle\right)\psi \quad (14.3.13)$$

This composes from $\chi = \nabla^* \varphi$ and $\varphi = \nabla \psi$

$\dfrac{\partial^2}{\partial \tau^2} - \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle$ is the quaternionic equivalent of d'Alembert's operator $\square$.

The operator $\dfrac{\partial^2}{\partial \tau^2} + \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle$ does not yet have an accepted name.

The Poisson equation equals

$$\rho = \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \psi \quad (14.3.14)$$

A very special solution of this equation is the Green's function $\dfrac{1}{\vec{q} - \vec{q'}}$ of the affected field

$$\nabla \frac{1}{\vec{q} - \vec{q'}} = -\frac{\left(\vec{q} - \vec{q'}\right)}{\left|\vec{q} - \vec{q'}\right|^3} \quad (14.3.15)$$

$$\left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \frac{1}{\left|\vec{q} - \vec{q'}\right|} \equiv \left\langle \vec{\nabla}, \vec{\nabla} \frac{1}{\left|\vec{q} - \vec{q'}\right|} \right\rangle = -\left\langle \vec{\nabla}, \vec{\nabla} \frac{\left(\vec{q} - \vec{q'}\right)}{\left|\vec{q} - \vec{q'}\right|^3} \right\rangle = 4\pi \delta\left(\vec{q} - \vec{q'}\right) \quad (14.3.16)$$

The spatial integral over Green's function is a volume.

(14.3.11) offers a dynamic equivalent of the Green's function, which is a spherical shock front. It can be written as

$$\psi = \frac{f\left(\left|\vec{q} - \vec{q'}\right| - c\left(\tau - \tau'\right)\right)}{\left|\vec{q} - \vec{q'}\right|} \quad (14.3.17)$$

A one-dimensional type of shock front solution is

$$\psi = \vec{f}\left(\left|\vec{q} - \vec{q'}\right| - c\left(\tau - \tau'\right)\right) \quad (14.3.18)$$

The equation (14.3.11) is famous for its wave type solutions

$$\nabla_r \nabla_r \psi = \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \psi = -\omega^2 \psi \quad (14.3.19)$$

Periodic harmonic actuators cause the appearance of waves,

Planar and spherical waves are the simpler wave solutions of this equation.

$$\psi\left(\vec{q}, \tau\right) = \exp\left\{\vec{n}\left(\vec{k}, \left(\vec{q} - \vec{q}_0\right) - \omega\tau + \varphi\right)\right\} \quad (14.3.20)$$

$$\psi\left(\vec{q}, \tau\right) = \frac{\exp\left\{\vec{n}\left(\vec{k}, \left(\vec{q} - \vec{q}_0\right) - \omega\tau + \varphi\right)\right\}}{\left|\vec{q} - \vec{q}_0\right|} \quad (14.3.21)$$

The Helmholtz equation considers the quaternionic function that defines the field separable [36].

$$\psi(q_r, \vec{q}) = A(\vec{q})T(q_r) \tag{14.3.22}$$

$$\frac{\langle \vec{\nabla}, \vec{\nabla} \rangle A}{A} = \frac{\nabla_r \nabla_r T}{T} = -k^2 \tag{14.3.23}$$

$$\langle \vec{\nabla}, \vec{\nabla} \rangle A = -k^2 A \tag{14.3.24}$$

$$\nabla_r \nabla_r T = -k^2 T \tag{14.3.25}$$

For three-dimensional isotropic spherical conditions, the solutions have the form

$$A(r, \theta, \varphi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \left\{ \left( a_{lm} j_l(kr) \right) + b_{lm} Y_l^m(\theta, \varphi) \right\} \tag{14.3.26}$$

Here $j_l$ and $y_l$ are the spherical Bessel functions, and $Y_l^m$ are the spherical harmonics. These solutions play a role in the spectra of atomic modules [38] [39].

A more general solution is a superposition of these basic types.

(14.3.12) offers a dynamic equivalent of the Green's function, which is a spherical shock front. It can be written as

$$\psi = \frac{f\left(\vec{q} - \vec{q'} + c(\tau - \tau')\right)}{\left|\vec{q} - \vec{q'}\right|} \tag{14.3.27}$$

A one-dimensional type of shock front solution is

$$\psi = \vec{f}\left(\vec{q} - \vec{q'} + c(\tau - \tau')\right) \tag{14.3.28}$$

Equation (14.3.12) offers no waves as part of its solutions.

During travel, the amplitude and the lateral direction $\frac{\vec{f}}{\left|\vec{f}\right|}$ of the one-dimensional shock fronts are fixed. The longitudinal direction is along $\frac{\vec{q} - \vec{q'}}{\left|\vec{q} - \vec{q'}\right|}$.

The shock fronts that are triggered by point-like actuators are the tiniest field excitations that exist. The actuator must fulfill significant restricting requirements. For example, a perfectly isotropic actuator must trigger the spherical shock front. The actuator can be a quaternion that belongs to another version of the quaternionic number system than the version, which the background platform applies. The symmetry break must be isotropic. Electrons fulfill this requirement. Neutrinos do not break the symmetry but have other reasons why they cause a valid trigger. Quarks break symmetry, but not in an isotropic way.

# 15 Line, surface and volume integrals

## 15.1 Line integrals

The curl can be presented as a line integral [85]

$$\left\langle \vec{\nabla}\times\vec{\psi},\vec{n}\right\rangle \equiv \lim_{A\to 0}\left(\frac{1}{A}\oint_C \left\langle \vec{\psi},d\vec{r}\right\rangle\right) \tag{15.1.1}$$

## 15.2 Surface integrals

With respect to a local part of a closed boundary that is oriented perpendicular to vector $\vec{n}$ the partial differentials relate as

$$\vec{\nabla}\psi = -\left\langle \vec{\nabla},\vec{\psi}\right\rangle + \vec{\nabla}\psi_r \pm \vec{\nabla}\times\vec{\psi} \Leftrightarrow \vec{n}\psi = -\left\langle \vec{n},\vec{\psi}\right\rangle + \vec{n}\psi_r \pm \vec{n}\times\vec{\psi} \tag{15.2.1}$$

This is exploited in the surface-volume integral equations that are known as Stokes and Gauss theorems [43] [44].

$$\iiint \vec{\nabla}\psi\, dV = \oiint \vec{n}\psi\, dS \tag{15.2.2}$$

$$\iiint \left\langle \vec{\nabla},\vec{\psi}\right\rangle dV = \oiint \left\langle \vec{n},\vec{\psi}\right\rangle dS \tag{15.2.3}$$

$$\iiint \vec{\nabla}\times\vec{\psi}\, dV = \oiint \vec{n}\times\vec{\psi}\, dS \tag{15.2.4}$$

$$\iiint \vec{\nabla}\psi_r\, dV = \oiint \vec{n}\psi_r\, dS \tag{15.2.5}$$

This result turns terms in the differential continuity equation into a set of corresponding integral balance equations.

The method also applies to other partial differential equations. For example

$$\vec{\nabla}\times\left(\vec{\nabla}\times\vec{\psi}\right) = \vec{\nabla}\left\langle \vec{\nabla},\vec{\psi}\right\rangle - \left\langle \vec{\nabla},\vec{\nabla}\right\rangle\vec{\psi} \Leftrightarrow \vec{\nabla}\times\left(\vec{\nabla}\times\vec{\psi}\right) = \vec{n}\left\langle \vec{n},\vec{\psi}\right\rangle - \left\langle \vec{n},\vec{n}\right\rangle\vec{\psi} \tag{15.2.6}$$

$$\iiint_V \left\{\vec{\nabla}\times\left(\vec{\nabla}\times\vec{\psi}\right)\right\} dV = \oiint_S \left\{\vec{\nabla}\left\langle \vec{\nabla},\vec{\psi}\right\rangle\right\} dS - \oiint_S \left\{\left\langle \vec{\nabla},\vec{\nabla}\right\rangle\vec{\psi}\right\} dS \tag{15.2.7}$$

One dimension less, a similar relation exists.

$$\iint_S \left(\left\langle \vec{\nabla}\times\vec{a},\vec{n}\right\rangle\right) dS = \oint_C \left\langle \vec{a},d\vec{l}\right\rangle \tag{15.2.8}$$

## 15.3 Using volume integrals to determine the symmetry-related charges

In its simplest form in which no discontinuities occur in the integration domain $\Omega$ the generalized Stokes theorem runs as

$$\int_\Omega d\omega = \int_{\partial\Omega} \omega = \oint_\Omega \omega \tag{15.3.1}$$

We separate all point-like discontinuities from the domain $\Omega$ by encapsulating them in an extra boundary. Symmetry centers represent spherically ordered parameter spaces in regions $H_n^x$ that float on a background parameter space $\mathfrak{R}$. The boundaries $\partial H_n^x$ separate the regions from the domain $H_n^x$. The regions $H_n^x$ are platforms for local discontinuities in basic fields. These fields are continuous in domain $\Omega - H$ .

$$H = \bigcup_{n=1}^{N} H_n^x \tag{15.3.2}$$

The symmetry centers $\mathfrak{S}_n^x$ are encapsulated in regions $H_n^x$, and the encapsulating boundary $\partial H_n^x$ is not part of the disconnected boundary, which encapsulates all continuous parts of the quaternionic manifold $\omega$ that exists in the quaternionic model.

$$\int_{\Omega - H} d\omega = \int_{\partial\Omega \cup \partial H} \omega = \int_{\partial\Omega} \omega - \sum_{k=1}^{N} \int_{\partial H_n^x} \omega \tag{15.3.3}$$

In fact, it is sufficient that $\partial H_n^x$ surrounds the current location of the elementary module. We will select a boundary, which has the shape of a small cube of which the sides run through a region of the parameter spaces where the manifolds are continuous.

If we take everywhere on the boundary the unit normal to point outward, then this reverses the direction of the normal on $\partial H_n^x$ which negates the integral. Thus, in this formula, the contributions of boundaries $\{\partial H_n^x\}$ are subtracted from the contributions of the boundary $\partial\Omega$. This means that $\partial\Omega$ also surrounds the regions $\{\partial H_n^x\}$

***This fact renders the integration sensitive to the ordering of the participating domains***.

Domain $\Omega$ corresponds to part of the background parameter space $\mathfrak{R}$. As mentioned before the symmetry centers $\mathfrak{S}_n^x$ represent encapsulated regions $\{\partial H_n^x\}$ that float on the background parameter space $\mathfrak{R}$. The Cartesian axes of $\mathfrak{S}_n^x$ are parallel to the Cartesian axes of background parameter space $\mathfrak{R}$. Only the orderings along these axes may differ.

Further, the geometric center of the symmetry center $\mathfrak{S}_n^x$ is represented by a floating location on parameter space $\mathfrak{R}$.

The symmetry center $\mathfrak{S}_n^x$ is characterized by a private symmetry flavor. That symmetry flavor relates to the Cartesian ordering of this parameter space. With the orientation of the coordinate axes fixed, eight independent Cartesian orderings are possible.

The consequence of the differences in the symmetry flavor on the subtraction can best be comprehended when the encapsulation $\partial H_n^x$ is performed by a ***cubic space form*** that is aligned along the Cartesian axes that act in the background parameter space. Now the six sides of the cube contribute differently to the effects of the encapsulation when the ordering of $H_n^x$ differs from the Cartesian ordering of the reference parameter space $\mathfrak{R}$. Each discrepant axis ordering corresponds to one-third of the surface of the cube. This effect is represented by the ***symmetry-related charge***, which includes the ***color charge*** of the symmetry center. It is easily comprehensible related to the algorithm which below is introduced for the computation of the symmetry-related charge. Also, the relation to the color charge will be clear. ***Thus, this effect couples the ordering of the local parameter spaces to the symmetry-related charge of the encapsulated elementary module.*** The differences with the ordering of the surrounding parameter space determines the value of the symmetry-related charge of the object that resides inside the encapsulation!

## 15.4 Symmetry flavor

The Cartesian ordering of its private parameter space determines the symmetry flavor of the platform [18]. For that reason, this symmetry is compared with the reference symmetry, which is the symmetry of the background parameter space. Four arrows indicate the symmetry of the platform. The background is represented by:

⬆⬆⬆⬆

Now the symmetry-related charge follows in three steps.

1. Count the difference of the spatial part of the symmetry of the platform with the spatial part of the symmetry of the background parameter space.
2. If the handedness changes from **R** to **L**, then switch the sign of the count.
3. Switch the sign of the result for anti-particles.

| **Symmetry flavor** | | | | | |
|---|---|---|---|---|---|
| Ordering<br>x y z τ | sequence | Handedness<br>Right/Left | Color<br>charge | Electric<br>charge<br>* 3 | Symmetry type. |
| ⬆⬆⬆⬆ | 0 | **R** | N | +0 | neutrino |
| ⬇⬆⬆⬆ | 1 | **L** | R | −1 | down quark |
| ⬆⬇⬆⬆ | 2 | **L** | G | −1 | down quark |
| ⬇⬇⬆⬆ | 3 | **L** | B | −1 | down quark |
| ⬆⬆⬇⬆ | 4 | **R** | B | +2 | up quark |
| ⬇⬆⬇⬆ | 5 | **R** | G | +2 | up quark |
| ⬆⬇⬇⬆ | 6 | **R** | R | +2 | up quark |
| ⬇⬇⬇⬆ | 7 | **L** | N | −3 | electron |
| ⬆⬆⬆⬇ | 8 | **R** | N | +3 | positron |
| ⬇⬆⬆⬇ | 9 | **L** | R | −2 | Anti-up quark |
| ⬆⬇⬆⬇ | 10 | **L** | G | −2 | anti-up quark |
| ⬇⬇⬆⬇ | 11 | **L** | B | −2 | anti-up quark |
| ⬆⬆⬇⬇ | 12 | **R** | B | +1 | anti-down quark |
| ⬇⬆⬇⬇ | 13 | **R** | R | +1 | anti-down quark |
| ⬆⬇⬇⬇ | 14 | **R** | G | +1 | anti-down quark |
| ⬇⬇⬇⬇ | 15 | **L** | N | −0 | anti-neutrino |

The suggested particle names that indicate the symmetry type are borrowed from the Standard Model. In the table, compared to the standard model, some differences exist with the selection of the anti-predicate. All considered particles are elementary fermions. The freedom of choice in the polar coordinate system might determine the spin [19]. The azimuth range is 2π radians, and the polar angle range is π radians. Symmetry breaking means a difference between the platform symmetry and the symmetry of the background. Neutrinos do not break the symmetry. Instead, they may cause conflicts with the handedness of the multiplication rule.

## 15.5 Derivation of physical laws

The quaternionic equivalents of Ampère's law are

$$\vec{J} \equiv \vec{\nabla}\times\vec{B} = \nabla_r \vec{E} \Leftrightarrow \vec{J} \equiv \vec{n}\times\vec{B} = \nabla_r \vec{E} \qquad (15.5.1)$$

$$\iint_S \left\langle \vec{\nabla}\times\vec{B}, \vec{n} \right\rangle dS = \oint_C \left\langle \vec{B}, d\vec{l} \right\rangle = \iint_S \left\langle \vec{J} + \nabla_r \vec{E}, \vec{n} \right\rangle dS \qquad (15.5.2)$$

The quaternionic equivalents of Faraday's law are:

$$\nabla_r \vec{B} = \vec{\nabla}\times\left(\nabla_r \vec{\psi}\right) = -\vec{\nabla}\times\vec{E} \Leftrightarrow \nabla_r \vec{B} = \vec{n}\times\left(\nabla_r \vec{\psi}\right) = -\vec{\nabla}\times\vec{E} \qquad (15.5.3)$$

$$\oint_c \left\langle \vec{E}, d\vec{l} \right\rangle = \iint_S \left\langle \vec{\nabla} \times \vec{E}, \vec{n} \right\rangle dS = -\iint_S \left\langle \nabla_r \vec{B}, \vec{n} \right\rangle dS \tag{15.5.4}$$

$$\vec{J} = \vec{\nabla} \times \left( \vec{B} - \vec{E} \right) = \vec{\nabla} \times \vec{\varphi} - \nabla_r \vec{\varphi} = \vec{v}\rho \tag{15.5.5}$$

$$\iint_S \left\langle \vec{\nabla} \times \vec{\varphi}, \vec{n} \right\rangle dS = \oint_C \left( \left\langle \vec{\varphi}, d\vec{l} \right\rangle \right) = \iint_S \left\langle \vec{v}\rho + \nabla_r \vec{\varphi}, \vec{n} \right\rangle dS \tag{15.5.6}$$

The equations (15.5.4) and (15.5.6) enable the derivation of the ***Lorentz force*** [82].

$$\vec{\nabla} \times \vec{E} = -\nabla_r \vec{B} \tag{15.5.7}$$

$$\frac{d}{d\tau} \iint_S \left\langle \vec{B}, \vec{n} \right\rangle dS = \iint_{S(\tau_0)} \left\langle \dot{\vec{B}}(\tau_0), \vec{n} \right\rangle ds + \frac{d}{d\tau} \iint_{S(\tau)} \left\langle \vec{B}(\tau_0), \vec{n} \right\rangle ds \tag{15.5.8}$$

The Leibniz integral equation states [83]

$$\begin{aligned} &\frac{d}{dt} \iint_{S(\tau)} \left\langle \vec{X}(\tau_0), \vec{n} \right\rangle dS \\ &= \iint_{S(\tau_0)} \left\langle \dot{\vec{X}}(\tau_0) + \left\langle \vec{\nabla}, \vec{X}(\tau_0) \right\rangle \vec{v}(\tau_0), \vec{n} \right\rangle dS - \oint_{C(\tau_0)} \left\langle \vec{v}(\tau_0) \times \vec{X}(\tau_0), d\vec{l} \right\rangle \end{aligned} \tag{15.5.9}$$

With $\vec{X} = \vec{B}$ and $\left\langle \vec{\nabla}, \vec{B} \right\rangle = 0$ follows

$$\begin{aligned} &\frac{d\Phi_B}{d\tau} = \frac{d}{d\tau} \iint_{S(\tau)} \left\langle \dot{\vec{B}}(\tau), \vec{n} \right\rangle dS = \iint_{S(\tau_0)} \left\langle \vec{B}(\tau_0), \vec{n} \right\rangle dS - \oint_{C(\tau_0)} \left\langle \vec{v}(\tau_0) \times \vec{B}(\tau_0), d\vec{l} \right\rangle \\ &= -\oint_{C(\tau_0)} \left\langle E(\tau_0), d\vec{l} \right\rangle - \oint_{C(\tau_0)} \left\langle \vec{v}(\tau_0) \times \vec{B}(\tau_0), d\vec{l} \right\rangle \end{aligned} \tag{15.5.10}$$

The electromotive force (EMF) $\varepsilon$ equals [84]

$$\begin{aligned} &\varepsilon = \oint_{C(\tau_0)} \left\langle \frac{\vec{F}(\tau_0)}{q}, d\vec{l} \right\rangle = -\frac{d\Phi_B}{d\tau}\bigg|_{\tau=\tau_0} \\ &= \oint_{C(\tau_0)} \left\langle \vec{E}(\tau_0), d\vec{l} \right\rangle + \oint_{C(\tau_0)} \left\langle \vec{v}(\tau_0) \times \vec{B}(\tau_0), d\vec{l} \right\rangle \end{aligned} \tag{15.5.11}$$

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} \tag{15.5.12}$$

# 16 Polar coordinates

In polar coordinates, the nabla delivers different formulas.

In pure spherical conditions, the Laplacian reduces to:

$$\left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \psi = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \psi}{\partial r} \right) \tag{16.1.1}$$

The Green's function blurs the location density distribution of the hop landing location swarm of an elementary particle. If the location density distribution has the form of a Gaussian distribution, then the blurred function is the convolution of this location density distribution and the Green's function. The Gaussian distribution is

$$\rho(r) = \frac{1}{\left(\sigma\sqrt{2\pi}\right)^3} \exp\left( -\frac{r^2}{2\sigma^2} \right) \tag{16.1.2}$$

The shape of the deformation of the field for this ***example*** is given by:

$$\mathfrak{T}(r) = \frac{ERF\left( -\frac{r}{\sigma\sqrt{2}} \right)}{4\pi r} \tag{16.1.3}$$

In this function, every trace of the singularity of the Green's function has disappeared. It is due to the distribution and the huge number of participating hop locations. This shape is just an example. Such extra potentials add a local contribution to the field that acts as the living space of modules and modular systems. The shown extra contribution is due to the local elementary module that the swarm represents. Together, a myriad of such bumps constitutes the living space.

# 17 Lorentz transform

## 17.1 The transform

The shock fronts move with speed $c$. In the quaternionic setting, this speed is unity.

$$x^2 + y^2 + z^2 = c^2\tau^2 \tag{17.1.1}$$

Swarms of spherical pulse response triggers move with lower speed $v$ .

For the geometric centers of these swarms still holds:

$$x^2 + y^2 + z^2 - c^2\tau^2 = x'^2 + y'^2 + z'^2 - c^2\tau'^2 \tag{17.1.2}$$

If the locations $\{x, y, z\}$ and $\{x', y', z'\}$ move with uniform relative speed $v$, then

$$ct' = ct\cosh(\omega) - x\sinh(\omega) \tag{17.1.3}$$

$$x' = x\cosh(\omega) - ct\sinh(\omega) \tag{17.1.4}$$

$$\cosh(\omega) = \frac{\exp(\omega) + \exp(-\omega)}{2} = \frac{c}{\sqrt{c^2 - v^2}} \tag{17.1.5}$$

$$\sinh(\omega) = \frac{\exp(\omega) - \exp(-\omega)}{2} = \frac{v}{\sqrt{c^2 - v^2}} \tag{17.1.6}$$

$$\cosh(\omega)^2 - \sinh(\omega)^2 = 1 \tag{17.1.7}$$

This is a hyperbolic transformation that relates two coordinate systems.

This transformation can concern two platforms $P$ and $P'$ on which swarms reside and that move with uniform relative speed .

However, it can also concern the storage location $P$ that contains a timestamp $t$ and spatial location $\{x, y, z\}$ and platform $P'$ that has coordinate time $t$ and location $\{x', y', z'\}$ .

In this way, the hyperbolic transform relates two individual platforms on which the private swarms of individual elementary particles reside.

It also relates the stored data of an elementary particle and the observed format of these data for the elementary particle that moves with speed  relative to the background parameter space.

The Lorentz transform converts a Euclidean coordinate system consisting of a location $\{x, y, z\}$ and proper time stamps $\tau$ into the perceived coordinate system that consists of the spacetime coordinates $\{x', y', z', ct'\}$ in which $t'$ plays the role of proper time. The uniform velocity $v$ causes time dilation $\Delta t' = \frac{\Delta\tau}{\sqrt{1 - \frac{v^2}{c^2}}}$ and length contraction $\Delta L' = \Delta L\sqrt{1 - \frac{v^2}{c^2}}$

## 17.2 Minkowski metric

Spacetime is ruled by the Minkowski metric.

In flat field conditions, proper time τ is defined by

$$\tau = \pm \frac{\sqrt{c^2t^2 - x^2 - y^2 - z^2}}{c} \tag{17.2.1}$$

And in deformed fields, still

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 \tag{17.2.2}$$

Here $ds$ is the spacetime interval and $d\tau$ is the proper time interval. $dt$ is the coordinate time interval

## 17.3 Schwarzschild metric

Polar coordinates, the Minkowski metric converts to the Schwarzschild metric. The proper time interval $d\tau$ obeys [ 89] [90]

$$c^2 d\tau^2 = \left(1 - \frac{r_s}{r}\right) c^2 dt^2 - \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - r^2 \left(d\theta^2 + \sin^2 d\varphi^2\right) \tag{17.3.1}$$

Under pure isotropic conditions the last term on the right side vanishes.

In the environment of a black hole, the formula $r_s$ stands for the Schwarzschild radius.

$$r_s = \frac{2GM}{c^2} \tag{17.3.2}$$

The variable $r$ equals the distance to the point-like mass $M$ .

# 18 Black holes

Black holes are regions from which nothing, not even photons can escape. Consequently, no information exists about the interior of a black hole. Only something is known about the direct environment of the black hole [86].

## 18.1 Geometry

Black holes are regions of the field that are encapsulated by a surface that cannot be passed by spherical shock fronts. Only the shock fronts that locate at the border of the region can add volume to the region. Thus, the increase of the volume of that region is restricted by the surface of the encapsulation. This differs from free space, where stochastic processes can inject volume anywhere. Black holes represent the most efficient packaging that stochastic processes can achieve.

Black holes are characterized by a Schwarzschild radius. [87] [88] It is supposed to be the radius where the escape speed of massive objects equals light speed. The gravitational energy $U$ of a massive object with mass $m$ in a gravitation field of an object with mass $M$ is

$$U = -\frac{GMm}{r} \tag{18.1.1}$$

In non-relativistic conditions, the escape velocity follows from the initial energy $½mv^2$ of the object with mass $m$ and velocity $v$. At the border, the kinetic energy is consumed by the gravitation energy.

$$½mv_0^2 - \frac{GMm}{r_0} = 0 \tag{18.1.2}$$

This results in escape velocity $v_0$

$$v_0 = \sqrt{2\frac{GM}{r_0}} \tag{18.1.3}$$

It looks as if the Schwarzschild radius can be obtained by taking the speed of light for the escape velocity. Apart from the fact that this condition can never be tested experimentally, this violates the non-relativity conditions. If we replace $½mv^2$ by the energy equivalent of the rest mass $mc^2$, then the wrong formula for the Schwarzschild radius results.

We try another route and use the fact that photons cannot pass the Schwarzschild radius. Next, we investigate the gravitational redshift of photons.

$$h\nu = h\nu_0\left(1 - \frac{r_s}{r}\right) = \frac{h\nu_0}{r}\left(r - r_s\right) \tag{18.1.4}$$

The Schwarzschild radius is the radius at which the energy and thus the frequency $\nu$ of the photon has reduced to zero.

Due to gravitation, the frequency $\nu$ of photons with original frequency $\nu_0$ changes with the distance $r$ to a point mass $M$ .

$$h\nu = h\nu_0 - \frac{2GMh\nu_0}{rc^2} = h\nu_0\left(1 - \frac{r_s}{r}\right) \tag{18.1.5}$$

Formula (18.1.5) describes the gravitational redshift of photons. The radius at which the frequency $\nu$ has reduced to zero is the Schwarzschild radius $r_s$

$$r_s = \frac{2GM}{c^2} \tag{18.1.6}$$

## 18.2 The border of the black hole

For a non-rotating neutral black hole, photons cannot pass the sphere with the Schwarzschild radius $r_s$ .

It also means that one-dimensional shock fronts and spherical shock fronts cannot pass this radius of this sphere. First, we consider what happens if a spherical pulse response injects geometric volume into the region of the black hole.

Spherical shock fronts can only add volume to the black hole when their actuator hovers over the region of the black hole. The injection increases the Schwarzschild radius. The injection also increases the mass $M$ . An increase in the Schwarzschild radius means an increase in the geometric volume of this sphere. This is like the injection of volume into the volume of the field that occurs via the pulses that generate the elementary modules. However, in this case, the volume stays within the Schwarzschild sphere. According to the formula of the Schwarzschild radius the volume of the enclosed sphere is not proportional to the mass of the sphere. The mass is proportional to the radius. In both cases, the volume of the field expands, but something different happens.

The HBM postulates that the geometric center of an elementary module cannot enter the region of the black hole. This means that part of the active region of the stochastic process that produces the footprint of the elementary module can hover over the region of the black hole. In this overlap region, the pulses can inject volume into the black hole. Otherwise, the stochastic process cannot inject volume into the black hole.

According to the HBM, the Schwarzschild sphere contains unstructured geometric volume. No modules exist within that sphere.

## 18.3 An alternative explanation

The two modes in which spherical pulse responses can operate offers a second interpretation. This explanation applies the volume sucking mode of the spherical pulse response. This mode removes the volume of the Green’s function from the local field that modifies the environment of the location of the pulse into a continuum, such that only the rational value of the location of the pulse results. A large series of such pulse responses will turn the local continuum into a discrete set of rational location values. Thus, within the region of the black hole, the pulses ***turn the continuum field into a sampled field***. Inside that discrete set, oscillation is no longer possible and shock fonts do not occur. The elementary particles cannot develop in that region. However, the pulses appear to extend the black hole region not in a similar way as the volume injection pulses in empty space would do. In both cases, these pulses can extend the mass of the region. But in the black hole region the mass increment is proportional to the radius of the sphere, while in free space the mass increment is proportional to the injected volume.

In the next chapter a more sensible explanation is given that introduces mixed fields, which contain closed regions, which do not contain a continuum, but instead a compact discrete set of rational numbers.

## 18.4 The Bekenstein bound

The Bekenstein bound relates the Schwarzschild black hole to its entropy.

$$S \le \frac{\kappa E R}{hc} \Rightarrow S = \frac{\kappa E R}{hc} = \frac{2\kappa G M^2}{hc} \tag{18.3.1}$$

This indicates that the entropy $S$ is proportional to the area of the black hole. This only holds for the entropy at the border of the black hole.

# 19 Mixed fields

Usually a dynamic field is a continuum eigenspace of a normal operator that resides in a quaternionic non-separable Hilbert space. In a quaternionic separable Hilbert space the field is countable and is a sampled field that consists only of the rational target values of the quaternionic function that defines the eigenspace of the operator. This function uses the eigenspace of the reference operator as its parameter space.

If the set of rational numbers in a version of the quaternionic number system is convoluted with the Green's function of a quaternionic field, then the corresponding continuum quaternionic number system results. Thus, adding the geometric volume of the Green's function to a rational number converts its environment into a continuum. In reverse, sucking the volume in the surround of a rational number that is embedded in a continuum will turn the rational number into its naked value. This can only happen at a border that separates the continuum from a discrete set. It will move the rational number from the continuum to the discrete set.

It is possible to define functions that are continuous in most of the parameter space, but that takes only discrete values in one or more closed regions of the parameter space. In the non-separable Hilbert space, the closed region corresponds to a subspace that encloses a separable Hilbert space. The surface that encloses the closed region must be a continuum. However, its interior only contains a discrete set. All converging series of elements of this set must, if the limit exists, have this limit in the enclosing surface. This surface has a minimal area that corresponds to the geometric volume of the enclosed region. We can interpret the shift of a rational number from a discrete set to a nearby continuum as the embedding of a separable Hilbert space into a non-separable Hilbert space. The reverse of this procedure is also possible.

A mechanism that injects geometric volume into this region must steal this volume from the surrounding continuum. If this mechanism applies point-sized pulses, then the injection inserts a rational number and the corresponding geometric volume increases. This inserted geometric volume relates to the volume of the Green's function of the continuum. We use the word "relates to" instead of "is proportional to" because the relation is not proportionality. This is explained by Birkhoff's theorem [89] [90].

In its simplest shape the region is a sphere and the radius of the sphere is proportional to the mass of the region.

Shock fronts and waves cannot pass the border of the enclosed region and cannot exist inside this region.

The enclosed region deforms the continuous part of the field. This deformation relates to the geometric volume of the enclosed region and thus relates to the number of injected rational numbers. The deformation corresponds to the mass property of the enclosed region. According to equation (8.2.1), the mass $M$ determines the gravitation potential energy of mass $m$ at distant $r$ from the center of the region.

$$U(r) \approx \frac{GMm}{r} \tag{19.1.1}$$

Due to gravitation, a photon that started from a long distance and approaches the region the contained energy reduces when the gravitation potential increases. Consequently, its frequency reduces from the initial frequency $\nu_0$ to the frequency $\nu$ at distance $r$ from the center of the region.

$$h\nu = h\nu_0\left(1-\frac{r_s}{r}\right) = \frac{h\nu_0}{r}(r-r_s) \tag{19.1.2}$$

In that case, the radius at which the frequency $\nu$ has reduced to zero is the Schwarzschild radius $r_s$

$$r_s = \frac{2GM}{c^2} \tag{19.1.3}$$

## 19.1 Open questions

Mixed fields can contain regions that only contain a set of rational quaternions. The region is encapsulated by a surface that represents a continuum. This border separates the discrete region from a continuum. The encapsulated region behaves as a black hole. The continuum can contain a series of such regions. It is not clear whether and how these regions can merge.

It is possible that the continuum is surrounded by a continuous border that separates it from a discrete region. This discrete region can contain a series of regions that are surrounded by a continuous border and that contain a continuum. In this way a multiverse can be established.

Inside the discrete regions. information transfer is blocked

# 20 Material penetrating field

## 20.1 Field equations

Basic fields can penetrate homogeneous regions of the material. Within these regions, the fields get crumpled. Consequently, the average speed of spherical fronts, One-dimensional fronts, and waves diminish, or these vibrations just get dampened away. The basic field that we consider here is a smoothed version $\vec{\psi}$ of the original field $\psi$ that penetrates the material.

$$\vec{\phi} = \nabla_r \vec{\psi} + \vec{\nabla} \psi_r \pm \vec{\nabla} \times \vec{\psi} = -\vec{E} \pm \vec{B} \tag{20.1.1}$$

$$\vec{\varphi} = \nabla_r \vec{\psi} + \vec{\nabla} \psi_r \pm \vec{\nabla} \times \vec{\psi} = -\vec{\mathfrak{E}} \pm \vec{\mathfrak{B}} \tag{20.1.2}$$

The first order partial differential equation does not change much. The separate terms in the first order differential equations must be corrected by a material-dependent factor and extra material dependent terms appear.

These extra terms correspond to polarization $\vec{P}$ and magnetization $\vec{M}$ of the material, and the factors concern the permittivity $\varepsilon$ and the permeability $\mu$ of the material. This results in corrections in the $\vec{\mathfrak{E}}$ and the $\vec{\mathfrak{B}}$ field and the average speed of one-dimensional fronts and waves reduces from 1 to $\frac{1}{\sqrt{\varepsilon\mu}}$.

$$D = \varepsilon \vec{\mathfrak{E}} + \vec{P} \tag{20.1.3}$$

$$\vec{H} = \frac{1}{\mu} \vec{\mathfrak{B}} - \vec{M} \tag{20.1.4}$$

$$\rho_b = -\left\langle \vec{\nabla}, \vec{P} \right\rangle \tag{20.1.5}$$

$$\rho_f = -\left\langle \vec{\nabla}, \vec{D} \right\rangle \tag{20.1.6}$$

$$\vec{J}_b = \vec{\nabla} \times \vec{M} + \nabla_r \vec{P} \tag{20.1.7}$$

$$\vec{J}_f = \vec{\nabla} \times \vec{H} - \nabla_r \vec{D} \tag{20.1.8}$$

$$\rho = \frac{1}{\varepsilon} \left\langle \vec{\nabla}, \vec{\mathfrak{E}} \right\rangle = \rho_b + \rho_f \tag{20.1.9}$$

$$\vec{J} = \frac{1}{\mu} \vec{\nabla} \times \vec{\mathfrak{B}} - \frac{\varepsilon}{\mu} \nabla_r \vec{\mathfrak{E}} = \vec{J}_b + \vec{J}_f \tag{20.1.10}$$

$$\vec{\phi} = \vec{\mathfrak{E}} - \vec{\mathfrak{B}} = \frac{1}{\varepsilon} \left( \vec{D} - \vec{P} \right) - \mu \left( \vec{H} + \vec{M} \right) \tag{20.1.11}$$

The subscript $_b$ signifies bounded. The subscript $_f$ signifies free.

The homogeneous second order partial differential equations hold for the smoothed field $\psi$.

$$\left\{ \nabla_r \nabla_r \pm v^2 \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \right\} \psi = 0 \tag{20.1.12}$$

## 20.2 Pointing vector

The **Poynting vector** represents the directional energy flux density (the rate of energy transfer per unit area) of a basic field. The quaternionic equivalent of the Poynting vector is defined as:

$$\vec{S} = \vec{E} \times \vec{H} \tag{20.2.1}$$

$u$ is the electromagnetic energy density for linear, nondispersive materials, given by

$$u = \frac{\langle \vec{E}, \vec{B} \rangle + \langle \vec{B}, \vec{H} \rangle}{2} \tag{20.2.2}$$

$$\frac{\partial u}{\partial \tau} = -\langle \vec{\nabla}, \vec{S} \rangle - \langle \vec{J}_f, \vec{E} \rangle \tag{20.2.3}$$

# 21 Action

The set of basic fields that occur in the model form a system. These fields interact at a finite number of discrete locations. The symmetry-related $\mathfrak{A}^x$ fields always attach to the geometrical center of a dedicated symmetry center. The $\mathfrak{C}$ field attaches at a stochastically determined location somewhere in the vicinity of this geometric center. However, integrated over the regeneration cycle of the corresponding particle the averaged attachment point coincides with the geometric center of the symmetry center. Thus, in these averaged conditions the two fields can be considered as being superposed. In the averaged mode the $\mathfrak{C}$ field has weak extrema. The $\mathfrak{A}^x$ fields always have strong extrema. In the averaged mode the fields can be superposed into a new field $\mathfrak{F}$ that shares the symmetry center related extrema.

The path of the geometric center of the symmetry center is following the least action principle. This is not the hopping path along which the corresponding particle can be detected.

The coherent location swarm $\{a_i^x\}$ also represents a hopping path. Its coherence means that the swarm owns a continuous location density distribution that characterizes this swarm. A more far-reaching coherence requirement is that the characterizing continuous location density distribution also has a Fourier transform. Therefore, the swarm owns a displacement generator. This means that at first approximation the swarm moves as one unit. These facts have much impact on the hopping path and on the movement of the underlying symmetry center. The displacement generator that characterizes part of the dynamic behavior of the symmetry center is represented by the momentum operator $\vec{p}$ . This displacement generator describes the movement of the swarm as one unit. It describes the movement of the platform that carries the elementary particle. On the platform, the hopping path is closed. In the embedding field, the platform moves.

We suppose that momentum $\vec{p}$ is constant during the particle generation cycle. Every hop gives a contribution to the path. These contributions can be divided into three steps per contributing hop:

1. Change to Fourier space. This involves inner product $\langle \vec{a}_i | \vec{p} \rangle$
2. Evolve during an infinitesimal progression step into the future.
   a. Multiply with the corresponding displacement generator $\vec{p}$
   b. The generated step in configuration space is $(\vec{a}_{i+1} - \vec{a}_i)$.
   c. The action contribution in Fourier space is $\langle \vec{p}, \vec{a}_{i+1} - \vec{a}_i \rangle$.
   d. This combines in a unitary factor $\exp(\langle \vec{p}, \vec{a}_{i+1} - \vec{a}_i \rangle)$
3. Change back to configuration space. This involves inner product $\langle \vec{p} | \vec{a}_{i+1} \rangle$
   a. The term contributes a factor $\langle \vec{a}_i | \vec{p} \rangle \exp(\langle \vec{p}, \vec{a}_{i+1} - \vec{a}_i \rangle) \langle \vec{p} | \vec{a}_{i+1} \rangle$.
4.

Two subsequent steps give:

$$\begin{aligned}
&\langle \vec{a}_i | \vec{p} \rangle \exp(\langle \vec{p}, \vec{a}_{i+1} - \vec{a}_i \rangle) \langle \vec{p} | \vec{a}_{i+1} \rangle \langle \vec{a}_{i+1} | \vec{p} \rangle \exp(\langle \vec{p}, \vec{a}_{i+2} - \vec{a}_i \rangle) \langle \vec{p} | \vec{a}_{i+2} \rangle \\
&= \langle \vec{a}_i | \vec{p} \rangle \exp(\langle \vec{p}, \vec{a}_{i+1} - \vec{a}_i \rangle) \exp(\langle \vec{p}, \vec{a}_{i+2} - \vec{a}_{i+1} \rangle) \langle \vec{p} | \vec{a}_{i+2} \rangle \\
&= \langle \vec{a}_i | \vec{p} \rangle \exp(\langle \vec{p}, \vec{a}_{i+2} - \vec{a}_i \rangle) \langle \vec{p} | \vec{a}_{i+2} \rangle
\end{aligned} \tag{21.1.1}$$

The terms in the middle turn into unity. The other terms also join.
Over a full particle generation cycle with N steps this results in:

$$\prod_{i=1}^{N-1}\langle\vec{a}_i|\vec{p}\rangle\exp\left(\langle\vec{p},\vec{a}_{i+1}-\vec{a}_i\rangle\right)\langle\vec{p}|\vec{a}_{i+1}\rangle$$
$$=\langle\vec{a}_1|\vec{p}\rangle\exp\left(\langle\vec{p},\vec{a}_N-\vec{a}_1\rangle\right)\langle\vec{p}|\vec{a}_N\rangle$$
$$=\langle\vec{a}_1|\vec{p}\rangle\exp\left(\sum_{i=2}^{N}\langle\vec{p},\vec{a}_{i+1}-\vec{a}_i\rangle\right)\langle\vec{p}|\vec{a}_N\rangle \tag{21.1.2}$$
$$=\langle\vec{a}_1|\vec{p}\rangle\exp(L)\langle\vec{p}|\vec{a}_N\rangle$$

$$L\,d\tau=\sum_{i=2}^{N}\langle\vec{p},\vec{a}_{i+1}-\vec{a}_i\rangle=\langle\vec{p},d\vec{q}\rangle \tag{21.1.3}$$

$$L=\langle\vec{p},\dot{\vec{q}}\rangle \tag{21.1.4}$$

$L$ is known as the Lagrangian.

Equation (21.1.4) holds for the special condition in which $\vec{p}$ is constant. If $\vec{p}$ is not constant, then the Hamiltonian $H$ varies with location.

$$\frac{\partial H}{\partial q_i}=-\dot{p}_i \tag{21.1.5}$$

$$\frac{\partial H}{\partial p_i}=\dot{q}_i \tag{21.1.6}$$

$$\frac{\partial L}{\partial q_i}=\dot{p}_i \tag{21.1.7}$$

$$\frac{\partial L}{\partial \dot{q}_i}=-\dot{p}_i \tag{21.1.8}$$

$$\frac{\partial H}{\partial \tau}=-\frac{\partial L}{\partial \tau} \tag{21.1.9}$$

$$\frac{d}{d\tau}\frac{\partial L}{\partial \dot{q}_i}=\frac{\partial L}{\partial q_i} \tag{21.1.10}$$

$$H+L=\sum_{i=1}^{3}\dot{q}_i\,p_i \tag{21.1.11}$$

Here we used proper time $\tau$ rather than coordinate time $t$.

This procedure derives the Lagrangian and the Hamilton equations from the stochastic hopping path. Each term in the series shows that the displacement generator forces the combination of terms to generate a closed hopping path on the platform that carries the elementary particle. The only term that is left is the displacement generation of the whole hop landing location swarm. That term describes the movement of the platform.

# 22 Dirac equation

In its original form, the Dirac equation for the free electron and the free positron is formulated by using complex number based spinors and matrices [91] [92]. That equation can be split into two equations, one for the electron and one for the positron. The matrices implement the functionality of a bi-quaternionic number system. Bi-quaternions do not form a division ring. Thus, Hilbert spaces cannot cope with bi-quaternionic eigenvalues. The Dirac equation plays an important role in mainstream physics.

## 22.1 The Dirac equation in original format

In its original form, the Dirac equation is a complex equation that uses spinors, matrices, and partial derivatives.

Dirac was searching for a split of the Klein-Gordon equation into two first order differential equations.

$$\frac{\partial^2 f}{\partial t^2}-\frac{\partial^2 f}{\partial x^2}-\frac{\partial^2 f}{\partial y^2}-\frac{\partial^2 f}{\partial z^2}=-m^2 f \qquad (22.1.1)$$

$$\left(\nabla_r\nabla_r-\left\langle\vec{\nabla},\vec{\nabla}\right\rangle\right)f=\Box f=-m^2 f \qquad (22.1.2)$$

Here $\Box=\left(\nabla_r\nabla_r-\left\langle\vec{\nabla},\vec{\nabla}\right\rangle\right)$ is the d'Alembert operator.

Dirac used a combination of matrices and spinors in order to reach this result. He applied the Pauli matrices in order to simulate the behavior of vector functions under differentiation [93].

The unity matrix $I$ and the Pauli matrices $\sigma_1,\sigma_2,\sigma_3$ are given by

$$I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\sigma_1=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix},\sigma_2=\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix},\sigma_3=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} \qquad (22.1.3)$$

Here $i=\sqrt{-1}$ .For one of the potential orderings of the quaternionic number system, the Pauli matrices together with the unity matrix $I$ relate to the quaternionic base vectors 1*, **i**, **j** and **k***

$$1\Rightarrow I,\vec{i}\Rightarrow i\sigma_1,\vec{j}\Rightarrow i\sigma_2,\vec{k}\Rightarrow i\sigma_3 \qquad (22.1.4)$$

This results in the multiplication rule

$$\sigma_1\sigma_2-\sigma_2\sigma_1=2i\sigma_3,\sigma_2\sigma_3-\sigma_3\sigma_2=2i\sigma_1,\sigma_3\sigma_1-\sigma_1\sigma_3=2i\sigma_2 \qquad (22.1.5)$$

$$\sigma_1\sigma_1=\sigma_2\sigma_2=\sigma_3\sigma_3=I \qquad (22.1.6)$$

The different ordering possibilities of the quaternionic number system correspond to different symmetry flavors. Half of these possibilities offer a right-handed external vector product. The other half offer a left-handed external vector product.

We will regularly use:

$$i\left(\vec{\sigma},\vec{\nabla}\right)=\vec{\nabla} \qquad (22.1.7)$$

With

$$p_\mu=-i\nabla_\mu \qquad (22.1.8)$$

follow

$$p_\mu\sigma_\mu=-ie_\mu\nabla_\mu \qquad (22.1.9)$$

$$\left\langle p,\sigma\right\rangle=-i\vec{\nabla} \qquad (22.1.10)$$

## 22.2 Dirac's formulation

The original Dirac equation uses 4x4 matrices $\vec{\alpha}$ and $\beta$.

$\vec{\alpha}$ and $\beta$ *are* matrices that implement the bi-quaternion arithmetic behavior including the possible symmetry flavors of bi-quaternionic number systems and continuums.

$$\alpha_1 = \begin{bmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{bmatrix} = -i \begin{bmatrix} 0 & \vec{i} \\ \vec{i} & 0 \end{bmatrix} \tag{22.2.1}$$

$$\alpha_2 = \begin{bmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{bmatrix} = -i \begin{bmatrix} 0 & \vec{j} \\ \vec{j} & 0 \end{bmatrix} \tag{22.2.2}$$

$$\alpha_3 = \begin{bmatrix} 0 & \sigma_3 \\ \sigma_3 & 0 \end{bmatrix} = -i \begin{bmatrix} 0 & \vec{k} \\ \vec{k} & 0 \end{bmatrix} \tag{22.2.3}$$

$$\beta = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{22.2.4}$$

$$\beta\beta = I \tag{22.2.5}$$

The interpretation of the Pauli matrices as a representation of a special kind of angular momentum has led to the half-integer eigenvalue of the corresponding spin operator.

Dirac's selection leads to

$$\left( p_r - \langle \vec{\alpha}, \vec{p} \rangle - \beta mc \right) \{\varphi\} = 0 \tag{22.2.6}$$

$\{\varphi\}$ is a four-component spinor, which splits into

$$\left( p_r - \langle \vec{\alpha}, \vec{p} \rangle - \beta mc \right) \varphi_A = 0 \tag{22.2.7}$$

and

$$\left( p_r - \langle \vec{\alpha}, \vec{p} \rangle + \beta mc \right) \varphi_B = 0 \tag{22.2.8}$$

$\varphi_A$ and $\varphi_B$ are two component spinors. Thus, the original Dirac equation splits into:

$$\left( \nabla_r - \vec{\nabla} - imc \right) \varphi_A = 0 \tag{22.2.9}$$

$$\left( \nabla_r - \vec{\nabla} + imc \right) \varphi_B = 0 \tag{22.2.10}$$

This split does not lead easily to a second order partial differential equation that looks like the Klein Gordon equation.

## 22.3 Relativistic formulation

Instead of Dirac's original formulation, usually, the relativistic formulation is used.

That formulation applies gamma matrices, instead of the alpha and beta matrices. This different choice influences the form of the equations that result in the two-component spinors.

$$\gamma_1 = \begin{bmatrix} 0 & \sigma_1 \\ -\sigma_1 & 0 \end{bmatrix} = -i \begin{bmatrix} 0 & \vec{i} \\ -\vec{i} & 0 \end{bmatrix} \tag{22.3.1}$$

$$\gamma_2 = \begin{bmatrix} 0 & \sigma_2 \\ -\sigma_2 & 0 \end{bmatrix} = -i \begin{bmatrix} 0 & \vec{j} \\ -\vec{j} & 0 \end{bmatrix} \tag{22.3.2}$$

$$\gamma_3 = \begin{bmatrix} 0 & \sigma_3 \\ -\sigma_3 & 0 \end{bmatrix} = -i \begin{bmatrix} 0 & \vec{k} \\ -\vec{k} & 0 \end{bmatrix} \tag{22.3.3}$$

$$\gamma_0 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{22.3.4}$$

Thus

$$\begin{aligned} &\gamma_\mu = \gamma_0 \alpha_\mu ; \mu = 1,2,3 \\ &\gamma_0 = \beta \end{aligned} \tag{22.3.5}$$

Further

$$\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{22.3.6}$$

The matrix $\gamma_5$ anti-commutes with all other gamma matrices.

Several different sets of gamma matrices are possible. The choice above leads to a “Dirac equation” of the form

$$\left(i\gamma^\mu \nabla^\mu - mc\right)\left\{\varphi\right\} = 0 \tag{22.3.7}$$

More extended:

$$\left(\gamma_0 \frac{\partial}{\partial t} + \gamma_1 \frac{\partial}{\partial x} + \gamma_2 \frac{\partial}{\partial y} + \gamma_3 \frac{\partial}{\partial z} - \frac{m}{ih}\right)\left\{\varphi\right\} = 0 \tag{22.3.8}$$

$$\left(\gamma_0 \frac{\partial}{\partial t} + \left\langle \vec{\gamma}, \vec{\nabla} \right\rangle - \frac{m}{ih}\right)\left\{\varphi\right\} = 0 \tag{22.3.9}$$

$$\left(\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \frac{\partial}{\partial t} + \begin{bmatrix} 0 & \left\langle \vec{\sigma}, \vec{\nabla} \right\rangle \\ -\left\langle \vec{\sigma}, \vec{\nabla} \right\rangle & 0 \end{bmatrix} - \frac{m}{ih} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) \begin{bmatrix} \varphi_A \\ \varphi_B \end{bmatrix} = 0 \tag{22.3.10}$$

$$\left(i \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \frac{\partial}{\partial t} + \begin{bmatrix} 0 & \vec{\nabla} \\ -\vec{\nabla} & 0 \end{bmatrix} + \frac{m}{h} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) \begin{bmatrix} \varphi_A \\ \varphi_B \end{bmatrix} = 0 \tag{22.3.11}$$

$$i \frac{\partial}{\partial t} \varphi_A + \vec{\nabla} \varphi_B + \frac{m}{h} \varphi_A = 0 \tag{22.3.12}$$

$$i \frac{\partial}{\partial t} \varphi_B + \vec{\nabla} \varphi_A - \frac{m}{h} \varphi_B = 0 \tag{22.3.13}$$

Also, this split does not easily lead to a second order partial differential equation that looks like the Klein Gordon equation.

## 22.4 A better choice

Another interpretation of the Dirac approach replaces $\gamma_0$ with $\gamma_5$:

$$\left(\gamma_5\frac{\partial}{\partial t}+\gamma_1\frac{\partial}{\partial x}+\gamma_2\frac{\partial}{\partial y}+\gamma_3\frac{\partial}{\partial z}-\frac{m}{ih}\right)\{\varphi\}=0 \tag{22.4.1}$$

$$\left(\gamma_5\frac{\partial}{\partial t}+\langle\vec{\gamma},\vec{\nabla}\rangle-\frac{m}{ih}\right)\{\varphi\}=0 \tag{22.4.2}$$

$$\left(\begin{bmatrix}0&1\\1&0\end{bmatrix}\frac{\partial}{\partial t}+\begin{bmatrix}0&\langle\vec{\sigma},\vec{\nabla}\rangle\\-\langle\vec{\sigma},\vec{\nabla}\rangle&0\end{bmatrix}-\frac{m}{ih}\begin{bmatrix}1&0\\0&1\end{bmatrix}\right)\begin{bmatrix}\varphi_A\\\varphi_B\end{bmatrix}=0 \tag{22.4.3}$$

$$\left(i\begin{bmatrix}0&1\\1&0\end{bmatrix}\frac{\partial}{\partial t}+\begin{bmatrix}0&\vec{\nabla}\\-\vec{\nabla}&0\end{bmatrix}+\frac{m}{h}\begin{bmatrix}1&0\\0&1\end{bmatrix}\right)\begin{bmatrix}\varphi_A\\\varphi_B\end{bmatrix}=0 \tag{22.4.4}$$

$$i\frac{\partial}{\partial t}\varphi_B+\vec{\nabla}\varphi_B+\frac{m}{h}\varphi_A=0 \tag{22.4.5}$$

$$i\frac{\partial}{\partial t}\varphi_A-\vec{\nabla}\varphi_A+\frac{m}{h}\varphi_B=0 \tag{22.4.6}$$

This version invites splitting of the four-component spinor equation into two equations for two-component spinors:

$$\left(i\frac{\partial}{\partial t}+\vec{\nabla}\right)\varphi_B=-\frac{m}{h}\varphi_A \tag{22.4.7}$$

$$\left(i\frac{\partial}{\partial t}-\vec{\nabla}\right)\varphi_A=-\frac{m}{h}\varphi_B \tag{22.4.8}$$

This looks far more promising. We can insert the right part of the first equation into the left part of the second equation.

$$\left(i\frac{\partial}{\partial t}-\vec{\nabla}\right)\left(i\frac{\partial}{\partial t}+\vec{\nabla}\right)\varphi_A=\frac{m^2}{h^2}\varphi_A \tag{22.4.9}$$

$$\left(\frac{\partial^2}{\partial t^2}+\langle\vec{\nabla},\vec{\nabla}\rangle\right)\varphi_A=-\frac{m^2}{h^2}\varphi_A \tag{22.4.10}$$

$$\left(i\frac{\partial}{\partial t}+\vec{\nabla}\right)\left(i\frac{\partial}{\partial t}-\vec{\nabla}\right)\varphi_B=\frac{m^2}{h^2}\varphi_B \tag{22.4.11}$$

$$\left(\frac{\partial^2}{\partial t^2}+\langle\vec{\nabla},\vec{\nabla}\rangle\right)\varphi_B=-\frac{m^2}{h^2}\varphi_B \tag{22.4.12}$$

This is what Dirac wanted to achieve. The two first order differential equations couple into a second order differential equation, but that equation is not equivalent to the Klein Gordon equation. It is equivalent to the equation (4.2.1).

The nabla operator acts differently onto the two-component spinors $\psi_A$ and $\psi_B$.

## 22.5 The Dirac nabla

The Dirac nabla $\mathfrak{D}$ differs from the quaternionic nabla $\nabla$ .

$$\nabla = \left\{ \frac{\partial}{\partial \tau}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\} = \nabla_r + \vec{\nabla} \tag{22.5.1}$$

$$\nabla^* = \nabla_r - \vec{\nabla} \tag{22.5.2}$$

$$\nabla\nabla^* = \nabla^*\nabla = \nabla_r\nabla_r - \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \tag{22.5.3}$$

$$\mathfrak{D} = \left\{ i\frac{\partial}{\partial \tau}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\} = i\nabla_r + \vec{\nabla} \tag{22.5.4}$$

$$\mathfrak{D}^* = i\nabla_r - \vec{\nabla} \tag{22.5.5}$$

$$\mathfrak{D}\mathfrak{D}^* = \mathfrak{D}^*\mathfrak{D} = -\nabla_r\nabla_r - \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle = -\nabla^*\nabla \tag{22.5.6}$$

# 23 Low dose rate imaging

The author started his career in the high-tech industry in the development of image intensifier devices. His job was to help to optimize the imaging quality of these image intensifier devices. This concerned both image intensifiers for night vision applications and x-ray image intensifiers that were aimed at medical applications. Both types of devices target low dose rate application conditions. These devices achieve image intensification in quite different ways. Both types can be considered to operate in a linear way. The qualification of the image intensifier aided the recognition capability is because human image perception is optimized for low dose rate conditions.

At low dose rates, the author never perceived waves in the intensified images. At the utmost, he saw hail storms of impinging discrete particles and the corresponding detection patterns can simulate interference patterns. The conclusion is that the waves that might be present in the observed image are probability waves. Individual photons are perceived as detected quanta. They are never perceived as waves.

## 23.1 Intensified image perception

When I entered my new job, the head of the department confronted me with a remarkable relationship that observers of intensified images had discovered and that he used in order to optimize the imaging quality of image intensifier devices. It appeared that perceptibility increases when the dose rate increases. It also increased when the surface of the observed detail increases. As expected, it increases when the object contrast increases. Temporal integration also had a positive effect on the perception of relatively static objects. Phosphors that are applied as scintillators or as electron-to-photon convertors cause a significant temporal integration. The rate at which the perceptibility increases seems to indicate that the perceived quanta are generated by spatial Poisson point processes. Thus, increasing the quantum detection capability should be the prime target of the image intensifier developers. However, for X-ray image intensifiers increasing gamma quantum detection capability usually conflicts with keeping sufficient imaging sharpness for perceiving small details. Thus, the second level target for image intensifying devices is getting the imaging sharpness at an acceptable level. The variance of the quantum intensification factor reduces the signal to noise ratio, and that effect must be compensated by increasing the dose rate. This is unwanted. Further, the quantum intensification must be high enough in order to trigger the image receiver.

In the intensification chain, also some attenuations take place. These attenuations can be represented by binomial processes. For example, photocathodes and scintillation layers do not reach the full hundred percent detection capability. In addition, input windows and input screens just absorb part of the impinging quanta. A primary Poisson process combines with one or more binomial processes in order to form a new Poisson process that offers a lower quantum production efficiency. A very important binomial process is represented by the spatial spread function that is the result of imaging blur. Imaging blur can be characterized by the optical transfer function, which is the Fourier transform of the spatial spread function.

Another important fact is that not only the existence of an object must be decided by the receiver. The detected object must also be recognized by the receiver. For intensified image recognition some very complicated processes in the visual trajectory of the receiver become decisive. The recognition process occurs in stages, and at every stage, the signal to noise ratio plays a decisive role. If the level of the signal to noise ratio is too low, then the signal transfer is blocked.

# 24 Human perception

## 24.1 Information encoding

With respect to the visual perception, the human visual trajectory closely resembles the visual trajectory of all vertebrates. This was discovered by Hubel and Weisel [94]. They got a Noble price for their work.

The sensitivity of the human eye covers a huge range. The visual trajectory implements several special measures that help to extend that range. At high dose rates, the pupil of the eye acts as a diaphragm that partly closes the lens and, in this way, it increases the sharpness of the picture on the retina. At such dose rates, the cones perform the detection job. The cones are sensitive to colors and offer a quick response. In unaided conditions, the rods take over at low dose rates, and they do not differentiate between colors. In contrast to the cones, the rods apply a significant integration time. This integration diminishes the effects of quantum noise that becomes noticeable at low dose rates. The sequence of optimizations does not stop at the retina. In the trajectory from the retina to the fourth cortex of the brain, several dedicated decision centers decode the received image by applying masks that trigger on special aspects of the image. For example, a dedicated mask can decide whether the local part of the image is an edge, in which direction this edge is oriented and in which direction the edge moves. Other masks can discern circular spots. Via such masks, the image is encoded before the information reaches the fourth cortex. Somewhere in the trajectory, the information of the right eye crosses the information that is contained in the left eye. The difference is used to construct a three-dimensional vision. Quantum noise can easily disturb the delicate encoding process. That is why the decision centers do not pass their information when its signal to noise ratio is below a given level. That level is influenced by the physical and mental condition of the observer. At low dose rates, this signal to noise ratio barrier prevents a psychotic view. The higher levels of the brain thus do not receive a copy of the image that was detected at the retina. Instead, that part of the brain receives a set of quite trustworthy encoded image data that will be deciphered in an associative way. ***It is expected that other parts of the brain for a part act in a similar noise blocking way.***

The evolution of the vertebrates must have installed this delicate visual data processing subsystem in a period in which these vertebrates lived in rather dim circumstances, where the visual perception of low dose rate images was of vital importance.

This indicates that the signal to noise ratio in the image that arrives at the eye’s pupil has a significant influence on the perceptibility of the low dose image. At high dose rates, the signal to noise ratio hardly plays a role. In those conditions, the role of the spatial blur is far more important.

It is fairly easy to measure the signal to noise ratio in the visual channel by applying a DC meter and an RMS meter. However, at very low dose rates, the damping of both meters might pose problems. What quickly becomes apparent is the relation of the signal to noise ratio and the number of the quanta that participate in the signal. The measured relation is typical for stochastic quantum generation processes that are classified as Poisson processes.

It is also easy to comprehend that when the signal is spread over a spatial region, the number of quantal that participate per surface unit is diminishing. Thus spatial blur has two influences. It lowers the local signal, and on the other hand, it increases the integration surface. Lowering the signal decreases the number of quanta. Enlarging the integration surface will increase the number of involved quanta. Thus, these two effects partly compensate each other. An optimum perceptibility condition exists that maximizes the signal to noise ratio in the visual trajectory.

## 24.2 Blur

The blur is caused by the Point Spread Function. This function represents a spatially varying binomial process that attenuates the efficiency of the original Poisson process. This creates a new Poisson process that features a spatially varying efficiency. Several components in the imaging chain may contribute to the Point Spread Function such that the effective Point Spread Function equals the convolution of the Point Spread Functions of the components. Mathematically it can be shown that for linear image processors the Optical Transfer Functions form an easier applicable characteristic than the Point Spread Functions because the Fourier transform that converts the Point Spread Function into the Optical Transfer Function converts the convolutions into simple multiplications.

The Optical Transfer Function is influenced by several factors. Examples are the color distribution, the angular distribution and the phase homogeneity of the impinging radiation. Also veiling glare may hamper the imaging quality.

## 24.3 Detective quantum efficiency

The fact that the signal to noise ratio appears to be a deciding factor in the perception process has led to a second way of characterizing the relevant influences. The Detective Quantum Efficiency (DQE) characterizes the efficiency of the usage of the available quanta. It compares the actual situation with the hypothetical situation in which all generated quanta would be used in the information channel. The measured signal noise ratio is compared to the ideal situation in which the stochastic generator is a Poisson process, and no binomial processes will attenuate that primary Poisson process. This means that blurring and temporal integration must play no role in the determination of the idealized reference detector that is used in specifying the DQE and the measured device will be compared to quantum detectors that will capture all available quanta. It also means that intensification processes will not add extra relative variance to the signal of the idealized detector. The application of microchannel plates will certainly add extra relative variance. This effect will be accounted for as a deterioration of the detection efficiency and not as a change of the stochastic process from a Poisson process to an exponential process. Mathematically this is an odd procedure, but it is a valid approach when the measurements are used to evaluate the perceptibility objectively.

## 24.4 Quantum Physics

The fact that the objective qualification of perceptibility can be performed by the Optical Transfer Function in combination with the Detective Quantum Efficiency indicates that the generation of the quanta is governed by a Poisson process that is coupled to a series of binomial process and secondary Poisson processes, where some of the binomial processes are implemented by spatial Point Spread Functions, and others are spatially uniform attenuators.

The processes that generate the primary quanta are considered to belong to the category of the inhomogeneous spatial Poisson point processes. These are processes that are applied by mechanisms that produce the locations of elementary particles, or they are processes that control the distribution of photons during the emission of these information messengers.

# 25 How the brain works

## 25.1 Preprocessing

A study on how the environment is observed and interpreted should start with an investigation of how the sense-organs and the brain cooperate. Between the sense-organs and the brain exists a series of pre-processors that encode and pre-interpret the incoming signals. This process also performs some noise filtering, such that later stages of the processing are not bothered by misinformation. For that reason the pre-processors act as decision centers where the signal transfer is blocked when the signal to noise ratio stays underneath a given level, e.g., 2.3 (Crozier's law. The level may differ in different persons.). In this way, the visual trajectories run via a cross-over to the cortex. The cross-over encodes and adds depth information. After a series of additional pre-processing steps, the signal arrives in the fourth cortex layer. Here about four square millimeters is devoted to the direct environment of each receptor of the fovea. In this area, a complete geometric encoding of the local geometry and dynamics of the perceived picture is presented. This includes whether the detected detail is a line or an edge or another form, in which direction it is positioned and whether the detail moves. (See the papers of Hubel and Wiesel on the visual trajectory and the visual cortex for more detailed information) [94].

## 25.2 Processing

Thus, the brain does not work with a pictorial copy of the picture that is received on the fovea. In further steps, the encoded map is interpreted. That part of the brain tries to associate the details of the map with remembered and recognized items. When dynamics is considered, then it must also be considered that the eyes are continuously scanning the input scene.

## 25.3 Image intensification

I studied visual perception because I needed this to specify useful measuring standards for night vision and X-ray imaging equipment (~1975). Many of the known visual illusions are due to the pre-processing in the visual trajectory. The viewing chain includes lenses, image intensifier tubes and either a camera or the human visual system. This last component includes the eyeball. The object is noisy and can be considered as a Poisson process. With respect to the noise the optical components act as binomial processes. Their point spread functions act as integration area. Image intensification is usually a Poisson process, but channel plates are characterized by an exponential distribution rather than by a Poisson distribution. Chains that include Poisson processes and binomial processes can be considered as one generalized Poisson process. Thus, imaging chains that include channel plates are more difficult to characterize.

## 25.4 Imaging quality characteristics

When the imaging chain can be characterized by a Poisson process, then its quantum detection efficiency can be characterized by the Detective Quantum efficiency (DQE). Its optical imaging quality can be characterized by the Optical Transfer Function (OTF). With inhomogeneous light imaging, it is sufficient to use the modulus, the Modulation Transfer Function (MTF). The MTF of the chain is the product of the MTF's of the components of the imaging chain.

## 25.5 The vision of noisy images

The intensification of image intensifiers is such that at low radiation levels the output image is formed by large numbers of separate light dots that together give the impression of a snowy picture. The visual trajectory contains a sequence of pre-processors that each performs a part of the encoding of the object. At its input, the visual cortex gets an encoded image rather than an optical image of the perceived scene. This encoded image is further encoded and interpreted in channels higher in the brain. This is done by associating the elements of the encoded image that is entering the visual cortex. The folded visual cortex offers about four square millimeters for the encoding of the environment of each separate receptor in the fovea. The pre-processors act as decision centers. When the offered signal to noise ratio is too low, then nothing is passed.

This is a general principle in the encoding process and also governs the association of encoded data in other parts of the brain.

The research resulted in a significant contribution of our laboratory to the world standards for the measurement of the OTF and the DQE.

## 25.6 Information association

The associative nature of the process is common for all kinds of objects and parts of objects. That includes objects that did not enter through one of the sense-organs. For example, a house is not stored in the brain as a complete concept. It is stored as a series of details that can be associated with the concept. If a sufficient number of these details are detected, then a decision center in the brain decides that the whole concept is present. In this way, not only a particular house can be recognized, but the process can also recognize a series of objects that resemble the original house. It classifies houses. By adding details that can be associated with it, the concept of a house can be widened. The resulting information, i.e., the information that passed the decision center, is used for further reasoning. Together with other details, the same details can also be used to detect other concepts by a different association. When the association act still produces too much noise, then the information is not produced, and further reasoning is neither disturbed nor triggered by this fact. High enough in the hierarchy individuals can be discerned. The brain is not static. The network of communication paths and decision centers is dynamically adapted to changing needs.

## 25.7 Noise filter

The decision level for the signal to noise ratio may vary from person to person. If the level becomes too low, then the person may start hallucinating. Further, the level may be influenced by body owned messenger stuff, drugs, poisons, and medicines.

## 25.8 Reasoning

The brain is capable of performing complex reasoning. However, it must be trained to perform the reasoning in a logical way. For example, it must learn that the start from a false presumption can cause the deduction of any conclusion, just or false. When a path of reasoning is helpful, then it is stored in a similar way as an observation. Not the reasoning itself is stored, but the details that are part of the reasoning path. Also, here association of the details and a suitable noise threshold plays its role. The reasoning can be identified as a theory and its concept can be widened. The brain can also generate new details that together with existing details can act as a reasonable theory. Even noise can generate such signals. These details can be perceived as a dream or as a newly invented theory. It depends on whether the theory is accepted as realistic. That means that the brain must be capable of testing the realism of a theory. This testing can be improved by training. The brain can forget stored details and stored concepts. This holds for objects as well as theories. Valuable concepts are regularly refreshed and become better remembered.

## 25.9 Other species

Hubel and Wiesel did their experiments on several kinds of vertebrates, such as goldfishes, cats and humans. Their main target was visual perception. Where the handling of the signals of sense organs in the brains is quite similar for all vertebrates, the handling of paths of reasoning by humans is superior in comparison to other vertebrates.

## 25.10 Humans

Humans have an advantage over other vertebrates. Apart from direct observation the theories and the concepts of things can also be retrieved by communication with other parties. This occurs by education, discussion, reading books, papers or journals, seeing films or videos or surfing the internet. These media can also act as a reference medium that extends the storage capacity of the brain.

## 25.11 Science

Mathematics is a particularly helpful tool that extends the capability of the brain to perform reasoning in a logical and precise way. Physics extends this capability further with a focus on observables. Philosophy adds self-reflection and focuses on the why and how of existence. Every branch of science adds to the capabilities of the individuals and to the effectiveness of the community.

## 25.12 Physical reality

Our brain has limited storage capacity. We cannot comprehend things that have an enormous complexity. However, we can detect regularities. Our brain is optimized to detect regularities. The laws of physics appear regularly in our observations or can be deduced from regularly returning observations. More complex laws are derived using tools and in combination with other people. Nature is not only controlled by laws. It is also controlled by boundary conditions. These boundary conditions may be caused by the influence of items that lay beyond the reach of our direct observations. The number and complexity of boundary conditions far outgrow the number of recognized laws of nature. The laws of nature play a role in our theories. However, the boundary conditions play a much smaller role. This is because the laws of nature that we detect treat a simplified version of the environment. In this abstraction, the boundary conditions play no real role. This is another reason why our theories differ from physical reality.

## 25.13 Theories

These deliberations learn that theories are a product of our mind. They can be used as a looking glass that helps in the observation and interpretation of physical reality. However, it is false to interpret the theories as or as part of physical reality. When a theory fits, then it is congruent, to some extent, with physical reality. That does not say that we as human beings and the environment from which we take our observations are not part of reality. It says that what our brain produces is another thing than physical reality.

## 25.14 Inventions of the human mind

Infinity is typically an invention by the human mind. There exist strong indications that nature does not support infinity. In the same sense, unlimited precision real numbers are prohibited in the physical universe. However, we can embed the results of our observations in a model that includes infinities and unlimited precision. For example, classical mechanics and field theories use these concepts. Quantum mechanics shows us that as soon as we introduce unlimited precision, we are immediately confronted with Heisenberg's uncertainty principle. We need infinity and unlimited precision in order to resolve the paradoxes that otherwise creep into our theories. We use theories that are in direct conflict with each other. One forbids infinity; the other theory uses and requires it. This says at least one thing; none of the theories describes physical reality correctly. Thus none of the theories can replace the concept of physical reality. Still, it appears useful to use both theories side by side. It means that great care must be taken with the interpretation of the theories.

## 25.15 History

Mathematical theories and physical theories tend to build upon the results of other exact theories. After some generations, a very complex building is obtained. After a while, it becomes humanly impossible to check whether the building elements are correct and whether the binding is done correctly. So, complex exact theories should be questioned.

## 25.16 Dreams

In this sense, only when we study our own dreams, fantasies or theories, then we observe these items and the dreams; fantasies and theories become part of "physical reality." If the theory is congruent with a part of physical reality, it will become useful as a view on physical reality.

## 25.17 Addendum

I measured/calculated only up to the fourth layer of the visual cortex. Hubel and Wiesel did the pinching. We did perception experiments and developed and built equipment that worked optimally with that part of the visual tract. During that investigation, several disciplines that were considered advanced at that time (1970-1987) were used and expanded. For example, together with Wolfgang Wittenstein I wrote most of the STANAG on the measurement of the optical transfer function (OTF and its modulus the MTF) of electron optical appliances. Later I took this NATO standard to the ISO standardization committee that transferred it into an equivalent standard for optical equipment. Next, I was also involved in raising the corresponding IEC and DIN standards. Parallel to this I also took part in the creation of the IEC and DIN standards for the measurement of the detective quantum efficiency (DQE). The research of the imaging channel starting from the radiation source and ending in the visual cortex resulted in a useful perception model that we used to improve our products. The standardized measuring methods enabled us to communicate the superior imaging quality of our products to our customers in a reliable and trustworthy way.

Personally, it offered me deep insight into the relation between optics and quantum physics. I learned to handle Fourier transforms into an environment where the idealized Fourier theory does not fit. The measured multidimensional Fourier transform has a restricted validity not only due to the spatial non-uniformity of the imaging properties. The measuring result also depends on the angular and chromatic distribution of the radiation and on the homogeneity of that radiation. Part of the imaging chain consisted of glass lenses. Another part contained electron lenses and fiber plates. Intermediate imaging surfaces consist of phosphors that convert gamma quanta or electrons into light flashes. Other surfaces are covered with photocathode layers that convert detected quanta into electrons, which are sent into the electron optical lens system until they reach the phosphor layer. The investigated appliances were image intensifier tubes for night vision purposes and X-ray image intensifier tubes that are used in medical diagnostic equipment. In this way, I got a deep insight into the behavior of quanta and experienced out of first hand that ALL information comes to us in the form of a noisy cloud of quanta. Only in masses, these quanta can be interpreted as a continuous wave of radiation.

# 26 Physical creation story

The fundamental consideration of physical reality quickly leads to a story of creation, in which the whole course of creation of what occurs in the universe is told.

## 26.1 Motivation

More and more my theories about the structure and the behavior of the universe became looking like a story of the creation of the universe. This is an alarming development because of scientists and especially physicists averse to religious concepts inside scientific documents. Still, I decided to write my theory in the form of a creation story. It is mixed with many mathematical and physical concepts because I wanted to generate a paper that is scientifically justified. I cannot avoid treating what can be accomplished with Hilbert spaces and number systems. They play an essential role in the theory as ways to store the dynamic geometric data that describe the life story of tiny objects. The same holds for shock fronts. These are field excitations that appear to constitute all other objects.

Already at the instant of creation, the creator appears to have archived all dynamic geometric data of all discrete objects in a read-only repository. In this repository, every elementary particle owns a private book that contains its full life story. Elementary particles behave as elementary modules, and together they constitute all other modules. Some modules constitute modular systems. My readers are intelligent modular systems. Free-thinkers will be disillusioned by the fact that everything is already determined in the repository. The creator fools them by applying stochastic processes in the generation of the footprints of the elementary particles.

By installing a few ground rules, the creator generated a very complicated universe that contains intelligent creatures. This creation process took more than thirteen billion years, but the result exists in front of our nose.

By showing that he is a modular designer and a modular constructor, the creator presents his intelligent creatures an interesting example.

## 26.2 Justification

This story is not about religion. It concerns the creation of the universe. If a creator is mentioned, then this concerns a creating abstract object and not an individual that creates.

The universe is a field in which we live. The field can be deformed by the embedding of massive objects and is a carrier of radiation, of which a part can be observed with the naked eye.

The physical reality is represented by this field and what happens in this field.

What appears in this field is at the time of creation stored in an abstract storage medium. This storage medium is here called the Hilbert Book Model. The HBM consists of many separate books that each describe the history of an elementary particle and a background platform that archives the history of the universe in another way. Each part of the model describes the genesis, the past, the present and the future of the described subject. The present is a window that runs across all the books.

## 26.3 Creation

This historiography gives the opportunity to speak about a story of creation. In fact, the model itself is the creator of the situation.

Elementary particles are described in a mathematical storage medium known as a quaternionic separable Hilbert space. A Hilbert space is a special vector space that provides an inner product for each pair of vectors. Quaternions are arithmetic numbers composed of a scalar and a three-dimensional vector. Therefore, they are ideally suited as a storage bin for a time stamp and a three-dimensional location. The quaternions give the number value to the inner product of the corresponding vector pair. The separable Hilbert space contains operators that describe the map of the Hilbert space onto itself and can store rational quaternions into storage bins that are attached to Hilbert vectors. The numbers are called eigenvalues, and the corresponding vectors are called eigenvectors. Together, the eigenvalues form the eigenspace of the operator.

Quaternionic number systems exist in many versions that differ in the way that Cartesian and polar coordinate systems rank their members. Each quaternionic separable Hilbert space chooses its own version of the number system and maintains that choice in the eigenspace of a special reference operator. In this way, the Hilbert space owns a private parameter space. The private separable Hilbert spaces of elementary particles hover with the geometric center of their parameter space over the parameter space of the background platform. By using this parameter space and a set of continuous quaternionic functions, a series of newly defined operators can be specified. The new defined operator reuses the eigenvectors of the reference operator and replaces the corresponding eigenvalue by the target value of the selected function by using the original eigenvalue as the parameter value. This newly defined operator contains in its eigenspace a field that is defined by the function. The field is a continuum. The eigenspaces of the operators are countable. Thus, the eigenspace of the new operator contains the sampled values of the field. In fact, the private parameter space is also a sampled continuum. The eigenspace of the reference operator contains only the rational elements of the selected version of the number system.

Nothing prevents all applied separable Hilbert spaces from sharing the same underlying vector space. We assume that the background platform is a quaternionic separable Hilbert space which contains infinitely many dimensions. This possesses a unique non-separable partner Hilbert space that supports operators, which possess continuous eigenspaces. These eigenspaces are therefore complete fields. Such eigenspaces are not countable. One of these operators owns an eigenspace that contains the field, which represents the universe. This field is deformed by the embedding of the hop landings of the elementary particles. The locations of the hop landings are stored in the eigenspace of the footprint operator in the private Hilbert space of the corresponding elementary particle. After sorting the timestamps, the footprint operator's eigenspace describes the entire lifecycle of the elementary particle as one continued hopping path. That hopping path recurrently generates a swarm of hop landing locations. A location density distribution describes the swarm. Because the particle is point-shaped, this is a detection probability density distribution. This is equal to the square of the modulus of the what physicists call the wavefunction of the elementary particle. The hop landing location swarm represents the particle.

## 26.4 Dynamics

At the time of the creation, the creator let a private stochastic process determine the hop landing locations of each elementary particle. This process is a combination of a Poisson process and a binomial process. A point spread function controls the binomial process. The stochastic process possesses a characteristic function that causes the production of a coherent swarm. It is the Fourier transform (the spatial spectrum) of the detection probability density distribution. As a result, the point spread function is equal to the location density distribution of the produced swarm. This design ensures that when the characteristic function becomes wider, the point spread function becomes narrower. In this way, the creator gives his creatures the impression that he does not determine the hopping path. He leaves some freedom to the objects that are formed by the elementary particles. However, in the beginning, the entire lifecycle of all elementary particles is already archived in their private storage medium. After that archival, nothing changes in this storage medium. The archive can only be read. Since the timestamps are stored together with the locations, the corresponding Hilbert book contains the entire life chronicles of the elementary particle.

The version of the quaternionic number system that the private Hilbert space of the elementary particle selects determines the symmetry of the private Hilbert space and of the elementary particle. This is characterized by an electric charge that houses in the geometric center of the particle platform's parameter space. The axes of all Cartesian coordinate systems must be parallel or perpendicular to each other. The geometric center may differ and may even move. Only the ranking along the axles may differ in direction. The electrical charge turns out to be a consequence of the difference between the symmetry of the gliding platform and the symmetry of the background platform. Because only a small number of versions of the quaternionic number are allowed, there exist very few different electrical charges. As a result, electrical charges can occur in the proportions -3,-2,-1, 0, 1, 2, and 3.

The separable Hilbert space of the background platform is naturally embedded in the non-separable Hilbert space. This is because both Hilbert spaces have the same symmetry. The embedding does not cause any disruption of symmetry. This does not apply to the embedding of the footprints of the elementary particles, because their Hilbert spaces possess a deviating symmetry. When embedding, only isotropic disturbances of the symmetry can cause an isotropic disturbance. Such a disturbance may temporarily deform the embedding field.

The swarm of hop landing locations can generate a swarm of spherical pulse responses. Only an isotropic pulse causes a spherical shock front. This shock front integrates over time into the Green's function of the field. This function has volume, and the pulse response injects this volume into the field. The shock front then spreads this volume over the field. As a result, the initial deformation of the field is rapidly flowing away. The stochastic process must continue to deliver new pulses to achieve a significant and permanent deformation. To get an impression of the deformation we must convolute the location density distribution of the hop landing location swarm with the Green's function of the field. Convolution blurs the image of the swarm. This does not give a correct picture, because the overlap of the spherical shock fronts depends on the spatial density of the swarm and on the time that the shock fronts need to overlap sufficiently. Far from the geometrical center of the swarm, the deformation is like the shape of the Green's function. The two functions still differ in a factor. This factor indicates the strength of the deformation. The factor is proportional to the mass of the particle. In fact, this is the method by which the scholars determine the mass of an object.

## 26.5 Modularity

Elementary particles behave as elementary modules. Together they form all the other modules that occur in the universe. Some modules constitute modular systems.

The composite modules and the modular systems are also controlled by a stochastic process. This is a different type of process than the type of process that regulates the footprint of the elementary particle. This second type controls the composition of the object. This type of stochastic process also possesses a characteristic function. This characteristic function is a dynamic superposition of the characteristic functions of the components of the compound object. The superposition coefficients act as displacement generators. They determine the internal positions of the components. The characteristic function connects to an additional displacement generator that regulates the movement of the whole module. This means that the composed module moves as one unit. The binding of the components is reinforced by the deformation of the embedding field and by the attraction of the electrical charges of the elementary particles.

This description shows that superposition takes place in the Fourier space. So what a composite module or modular system determines, is captured in the Fourier space. Locality does not play a role In Fourier space. This sketches the phenomenon that scholars call entanglement. In principle, the binding within a composite module is to a large extent established in the Fourier space. The parts can, therefore, be far apart. For properties of components, for which an exclusion principle applies, this can have remarkable consequences.

All modules act as observers and can perceive phenomena. Elementary particles are very primitive observers. All observers receive their information through the field in which they are embedded. The observed event has a timestamp. For the observer that timestamp locates in the past. As a result, the information is stored in the Euclidean format in the storage medium in a storage bin that contains a timestamp and a three-dimensional location. By the observer, that information is perceived in space-time coordinates. A hyperbolic Lorentz transformation describes the conversion of the Euclidean storage coordinates into the perceived spacetime coordinates. The hyperbolic Lorentz transformation adds time interval dilatation and length compression. The deformation of the embedding field also deforms the path through which the information is transported. This also influences the transported information.

## 26.6 Illusion

At the instant of creation, the creator fills the storage bins of the footprint operators. The contents of this store won't change anymore. The later events in the embedding field also have no influence on the archive. Since the creator uses stochastic processes to fill the footprint storage, intelligent observers will get the impression that they still possess free will. The embedding of the footprints follows step by step the time-stamped locations that were generated by the stochastic processes and were archived in the eigenspace of the footprint operator. The observer should not be fatalistic and think that his behavior does not matter because everything is already determined. The reverse is true. The behavior of each module has consequences because each perceived event affects the observer. This fact affects the future in an almost causal fashion. The stochastic disturbance is relatively small.

## 26.7 Cause

The driving force behind the dynamics of the universe is the continual embedding of the hop landing locations of the elementary particles into the field that represents the universe.

It seems as if the continuous deformation of the field seems to come out of nowhere and that the individual deformations then disappear quickly by the flooding away of the inserted volume. The expansion persists.

The shock-fronts play an essential role because the spherical fronts spread the volume into the field. The one-dimensional shock fronts carry information and also carry extra movement energy. They move the platforms on which elementary particles travel through the universe.

## 26.8 Begin to end

The universe is a field, and that field can be described by a quaternionic function. In the begin of its existence, the stochastic processes that produce particle hop landings that inject volume into this field had not yet done any work. Thus, the field did not yet contain any spatial volume. However, spread over the full spatial part of the parameter space, a myriad of stochastic processes immediately started to deform and expand the spatial part of the field. The deformations fade away but are quickly repeated by the recurrently regenerated hop location swarms. This produced a bumpy look of the early universe. This is not in accordance with the usual interpretation of the begin of the universe, which is sketched as a big bang at a single location.

Black holes are special phenomena. They are bordered areas in which volume can only be added by widening the border. No shock front can pass this edge. The black holes wipe elementary particles together at their border. Part of the footprint of the elementary particles, hovers over the black hole region and the pulses extend the volume that is enclosed by the border into all directions. The BH is filled by the equivalent of storage capacity, which is no longer devoted to the modular design and modular construction process. Inside the BH region, these processes are impossible. Also, the elementary particles that are stitched at the border will be prevented from generating higher order modules.

This characterizes the event horizon of the BH. It also indicates what the end of the history of the universe will be. A huge BH uniformly filled with spatial volume. The platforms of the elementary particles can still cling at the outside of the border of the final BH.

## 26.9 Lessons

After the instant of creation, the creator does no longer care for his creatures. This creator is not a merciful God. It makes no sense to beg this creator for special benefits. For the creator, everything is determined. Due to the application of the stochastic processes, the intelligent observers still get the illusion that they have a free will. For their perception, all their actions cause a sensible result. The uncertainty that is introduced by the stochastic processes is relatively small.

The creator is a modular designer and a modular constructor. For his intelligent creatures, this makes an important example. Modular construction is very economical with its resources and provides relatively fast usable and reliable results. This method of working creates its own rules. It makes sense to have a large number and a large variety of suitable modules at hand. It even makes sense to create communities of module types and communities of modular system types.

The lessons, which the creator teaches his intelligent creatures, follow from the modular design of the creation. The lesson is not the survival of the fittest. The survival of the module type-community is more important than the survival of the individual modular system. It makes sense to care for the module type-community that the individual belongs to. It also makes sense to care for the module type-communities on which your module type-community depends. That then demands to properly secure the module communities of which one depends. It has much sense to care for the habitat of your module type-community.

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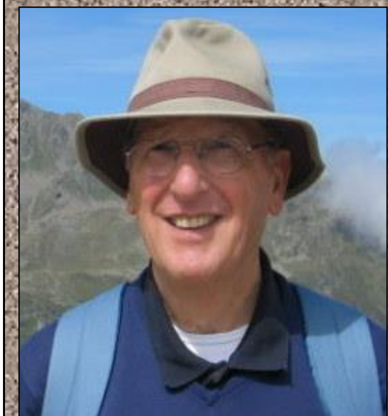

This survey treats the Hilbert Book Model Project. The project concerns a well-founded, purely mathematical model of physical reality. The project relies on the conviction that physical reality owns its own kind of mathematics and that this mathematics guides and restricts the extension of the foundation to more complicated levels of the structure and the behavior of physical reality. This results in a model that more and more resembles the physical reality that humans can observe.

The book is written by a retired physicist.

Msc Hans van Leunen

He started the Hilbert Book Model Project when he was 70 years.

To feed his curiosity, Hans dived deep into the crypts of physical reality. He discovered that more than eighty years ago, two scholars already discovered a suitable foundation of a mathematical model of the structure and behavior of physical reality. They called their discovery quantum logic. The Hilbert Book Model Project explores this foundation by extending this structure to higher levels of the structure of the model and adds dynamics to the Hilbert Book Base Model.

This approach is unorthodox and unconventional. It enters an area where many aspects cannot be verified by experiments and must be deduced by trustworthy mathematical methods. In this way, the project discovered new mathematics and new physics.

The Hilbert Book Base Model appears to offer a very powerful and flexible modeling environment for physical theories.

The model extensively applies quaternionic Hilbert space technology, and quaternionic integral and differential calculus. The project extensively exploits the capabilities of the existing versions of the quaternionic number system.

The project explores the obvious modular design of the objects that exist in the universe.

In contrast to mainstream physics the Hilbert Book Model applies stochastic processes instead of forces and force carriers to control the coherence and the binding of modules.