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Classical logic and the division of zero by zero

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Abstract

Objective: The division of zero by zero turned out to be a long lasting and not ending puzzle in mathematics and physics. An end of this long discussion is not in sight. In particular zero divided by zero is treated as indeterminate thus that a result cannot be found out. It is the purpose of this publication to solve the problem of the division of zero by zero while relying on the general validity of classical logic.

Methods: A systematic re-analysis of classical logic and the division of zero by zero has been undertaken.

Results: The theorems of this publication are grounded on classical logic and Boolean algebra. There is some evidence that the problem of zero divided by zero can be solved.

Conclusion: According to classical logic, zero divided by zero is equal to one.

Keywords: Indeterminate forms, Classical logic, Zero divided by zero

1. Introduction

In general, Aristotle's unparalleled influence on the development of scientific knowledge in western world is documented especially by his contributions to classical logic too. Besides of some serious limitations of Aristotle's logic, Aristotle's logic became dominant and is still an adequate basis or our understanding science to some extent, since centuries. In point of fact, some authors are of the opinion that Aristotle himself has discovered everything there was to know about classical logic. After all, classical logic, as such at least closely related to the study of objective reality, deals with absolutely certain inferences and truths. In general, classical logic describes the most general, the simplest, the most abstract laws of objective reality. Under conditions of Aristotle's classical logic, there is no uncertainty. In contrast to classical logic, probability theory deals with uncertainties. This raises questions concerning whether there is an overlap between classical logic and probability theory at all. Without attempting to be comprehensive, it may help to sketch at least view words on this matter in this publication. Classical logic is at least closely allied with probability theory and vice versa. As such, classical logic has no meaning apart from probability theory and vice versa. It should therefore come as no surprise that there are trials to combine logic and probability theory within one and the same mathematical framework, denoted as dialectical logic. However, as already published, there are natural ways in which probability theory is the treated as an extension of classical logic to the values between +0 and +1 where probability of an event is treated as its truth value. In this context, Fuzzy logic is of no use and already refuted (Barukčić, 2017a) (Barukčić, 2017a). In particular, the relationship between classical logic and probability theory (Barukčić, 2017b) (Barukčić, 2017b) is the same as between Newtonian mechanic's and Einstein's special theory of relativity. The one passes over into the other and vice versa without any contradictions.

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2. Material and Methods

2.1. Definitions

DEFINITION 1. (NUMBER +0)

Let c denote the speed of light in vacuum, let ε_0 denote the electric constant and let μ_0 the magnetic constant, let i denote an imaginary number (Bombelli, 1579) (Bombelli, 1579). The number +0 is defined as the expression

$$\begin{aligned} +0 &\equiv \left(c^2 \times \varepsilon_0 \times \mu_0\right) - \left(c^2 \times \varepsilon_0 \times \mu_0\right) \\ &\equiv +1 - 1 \\ &\equiv +i^2 - i^2 \end{aligned} \tag{1}$$

while "=" denotes the equals sign or equality sign (Robert Recorde, 1557) (Rolle, 1690) (Recorde, 1557; Rolle, 1690) used to indicate equality and "-" (Widmann, 1489) (Pacioli, 1494) (Robert Recorde, 1557) (Widmann, 1489; Pacioli, 1494; Recorde, 1557) denotes minus signs used to represent the operations of subtraction and the notions of negative as well and "+" (Widmann, 1489; Pacioli, 1494; Recorde, 1557) denotes the plus signs used to represent the operations of addition and the notions of positive as well.

DEFINITION 2. (NUMBER +1)

Let c denote the speed of light in vacuum, let ε_0 denote the electric constant and let μ_0 the magnetic constant, let i denote an imaginary number (Bombelli, 1579). The number +0 is defined as the expression

$$+1 \equiv \left(c^2 \times \varepsilon_0 \times \mu_0\right) \equiv -i^2 \tag{2}$$

Remark 1. Quantum computing

Quantum mechanical processes can enable some new types of computation (Deutsch, 1985) (Deutsch, 1985). Soon, Benjamin Schumacher replaced "the classical idea of a binary digit with a quantum two-state system, such as the spin of electron. These quantum bits, or 'qubits', are the fundamental units of quantum information." (Schumacher, 1995) (Schumacher, 1995). A qubit is one of the simplest quantum mechanical systems. Examples: the spin of the electron (spin up and spin down), the polarization of a single photon (vertical polarization and the horizontal polarization).

DEFINITION 3. (THE SAMPLE SPACE)

Let ${}_{R}C_t$ denote the set of all the possible outcomes of a random experiment, a phenomenon in nature, a t a (random) Bernoulli trial *t*. Let ${}_{0}x_t$ denote an event, a subset of the sample space ${}_{R}C_t$. Let ${}_{0}x_t$ denote the negation of an event ${}_{0}\underline{x}_t$, another, complementary subset of the sample space ${}_{R}C_t$. In general, we define the sample space ${}_{R}C_t$ as

$${}_{R}C_t \equiv \left\{ {}_{0}x_t \quad , \quad {}_{0}\underline{x}_t \right\} \tag{3}$$

or equally as

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$$_{R}C_{t} \equiv {}_{0}x_{t} + {}_{0}\underline{x}_{t} \tag{4}$$

In other words, and according to quantum theory, the sample space $_{R}C_{t}$ at one certain Bernoulli trial t is in a state of superposition of $_{0}x_{t}$ and $_{0}\underline{x}_{t}$. Under conditions of classical logic, it is **($_{0}x_{t}$ + $_{0}\underline{x}_{t}$) = $_{R}C_{t}$ = +1**.

DEFINITION 4. (THE COMPLEX CONJUGATE SAMPLE SPACE $_{R}C_{T}^{*}$)

Let $_{R}C_{t}^{*}$ denote the complex conjugate of the sample space $_{R}C_{t}$, the set of all the possible outcomes of a random experiment et cetera. In general, we define

$$_{R}C_{t} \times {}_{R}C_{t}^{*} \equiv +1 \tag{5}$$

with the consequence that

$$_{R}C_{t}^{*} \equiv \frac{+1}{_{R}C_{t}} \tag{6}$$

DEFINITION 5. (THE EIGEN-VALUES OF $_{0}X_{T}$)

Under conditions of classical logic, $_{0}x_{t}$ can take only one of the values

$$_{0}x_{t} \equiv \{+0 \quad , \quad +1\} \tag{7}$$

DEFINITION 6. (THE EIGEN-VALUES OF $_{0}\underline{X}_{T}$)

Under conditions of classical logic, $_{0}\underline{x}_{t}$ can take only one of the values

$$_{0}\underline{x}_{t} \equiv \{+0 \quad , \quad +1\} \tag{8}$$

DEFINITION 7. (THE SIMPLE FORM OF NEGATION)

Let $_{0}\underline{x}_{t}$ denote the negation of an event/outcome/eigenvalue $_{0}x_{t}$ (i. e. anti $_{0}x_{t}$). In general, we define the negation $_{0}\underline{x}_{t}$ of an event/outcome/eigenvalue $_{0}x_{t}$ as

$$_{0}\underline{x}_{t} \equiv {}_{R}C_{t} - {}_{0}x_{t} \tag{9}$$

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Remark 2. Negation

Under conditions of classical logic 'anti $_{0}x_{t}$' passes over to 'not $_{0}x_{t}$'. Negation is a very important concept in philosophy (Newstadt, 2015) (Newstadt, 2015) and classical logic. In classical logic, negation converts only false to true and true to false. In other words, it is

$$_{0}\underline{x}_{t} \equiv \neg\ _{0}x_{t} \tag{10}$$

or

$$_{0}x_{t} \equiv \neg\ _{0}\underline{x}_{t} \tag{11}$$

where $\neg$ denotes the sign of negation of classical logic. So, if $_{0}x_{t}= +1$ (or true), then $\neg\ _{0}x_{t}= {_{0}\underline{x}_{t}}$ (pronounced 'not $_{0}x_{t}$' or equally 'anti $_{0}x_{t}$') would therefore be $_{0}\underline{x}_{t} = +0$ (false); and conversely, if $_{0}\underline{x}_{t} = +1$ (true) then $\neg\ _{0}\underline{x}_{t}= {_{0}x_{t}} = +0$ would be false. Determination and negation are related (Horn, 2001) (Horn, 2001) times. In particular, Benedict de Spinoza (1632 –1677) addressed these notions in his lost letter of June 2, 1674to his friend Jarig Jelles (Förster and Melamed, 2012) (Förster& Melamed, 2012) by the discovery that "**determinatio negatio est**" (Spinoza, 1802) (Spinoza,1802, p. 634). Hegel extended Spinoza's slogan to "**Omnis determinatio est negatio**" (Hegel, 1812) (Hegel, 1812). The relationship between $_{0}x_{t}$ and $_{0}\underline{x}_{t}$ is illustrated by the following table (Table 1).

Table 1. The relationship between $_{0}x_{t}$ and $_{0}\underline{x}_{t}$

Bernoulli trial t	$_{0}x_{t} = \neg\ _{0}\underline{x}_{t}$	$_{0}\underline{x}_{t} = \neg\ _{0}x_{t}$	$_{0}\underline{x}_{t} + {_{0}\underline{x}_{t}} = {_{R}C_{t}}$	$_{R}C_{t}$
1	+1	+0	+1 +0 = +1	+1
2	+1	+0	+1 +0 = +1	+1
3	+0	+1	+0 + 1 = +1	+1
4	+0	+1	+0 +1 = +1	+1
…	…	…	…	…

The first mathematically or algebraically formulation of the notion negation was provided to us by Georg Boole. In general, following Boole, negation in terms of algebra, can be expressed as $_{0}\underline{x}_{t} = 1 - {_{0}x_{t}}$. According to Boole, "*whatever ... is represented by the symbol x, the contrary ... will be expressed by 1 - x*" (Boole, 1854) (Boole, 1854, p. 48). Under conditions of classical logic, it is $_{R}C_{t} = 1$ but not in general. In a slightly different way, we generalize Boole's negation to **the simple general form of Boole's negation** as

$$_{0}\underline{x}_{t} \equiv {_{R}C_{t}} - {_{0}x_{t}} \tag{12}$$

From this follows that

$$_{0}x_{t} \equiv {_{R}C_{t}} - {_{0}\underline{x}_{t}} \tag{13}$$

DEFINITION 8. (THE PYTHAGOREAN THEOREM)

We define

$$a_{t}^{2} \equiv {_{R}C_{t}} \times {_{0}x_{t}} \tag{14}$$

and

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$$b_t^2 \equiv {}_{R}C_t \times {}_{0}\underline{x}_t \tag{15}$$

The Pythagorean theorem under conditions of classical logic follows as

$$\left({}_{R}C_t \times {}_{R}C_t\right) \equiv \left({}_{R}C_t \times {}_{0}x_t\right) + \left({}_{R}C_t \times {}_{0}\underline{x}_t\right) \tag{16}$$

The normalized form of the Pythagorean theorem follows as

$$\frac{\left({}_{R}C_t \times {}_{0}x_t\right)}{\left({}_{R}C_t \times {}_{R}C_t\right)} + \frac{\left({}_{R}C_t \times {}_{0}\underline{x}_t\right)}{\left({}_{R}C_t \times {}_{R}C_t\right)} \equiv +1 \tag{17}$$

2.2. Axioms

There have been many attempts to define the foundations of logic in a generally accepted manner. However, besides of an extensive discussion in the literature it is far from clear whether the truth as such is a definable notion. As generally known, axioms and rules of a publication have to be chosen carefully especially in order to avoid paradoxes and inconsistency. Thus far, for the sake of definiteness and in order to avoid paradoxes the theorems of this publication are based on the following axiom.

2.2.1. Axiom I (Lex identitatis. Principium Identitatis. Identity Law)

In general, it is

$$+1 \equiv +1 \tag{18}$$

or the superposition of +0 and +1 as one of the foundations of quantum computing

$$+1 \equiv (1+0) \times (1+0) \times (1+0) \times (\ldots) \times (1+0) \tag{19}$$

2.2.2. Axiom II

In general, it is

$$\frac{+1}{+0} \equiv +\infty \tag{20}$$

or

$$(+1) \equiv (+\infty) \times (+0) \tag{21}$$

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3. Results

THEOREM 3.1. (THE DETERMINATION OF $_{R}C_{T}$ I)

CLAIM.

From the standpoint to $_{0}x_{t}$ and due to our definitions before, $_{R}C_{t}$ is determined as

$$\frac{_{0}x_{t}}{\left(1-\left(\frac{_{0}\underline{x}_{t}}{_{R}C_{t}}\right)\right)} = {_{R}C_{t}} \tag{22}$$

PROOF.

In general, taking axiom 1 to be true, it is

$$+1 \equiv +1 \tag{23}$$

Multiplying by $_{R}C_{t}$ we obtain $1\times {_{R}C_{t}} = 1\times {_{R}C_{t}}$ or

$${_{R}C_{t}} \equiv {_{R}C_{t}} \tag{24}$$

Adding zero, the relationship does not change as such. It is

$${_{R}C_{t}} + 0 \equiv {_{R}C_{t}} \tag{25}$$

According to mathematical requirements, it is $+{_{0}\underline{x}_{t}} - {_{0}\underline{x}_{t}} = +0$ is. Rearranging equation we obtain

$${_{R}C_{t}} - {_{0}\underline{x}_{t}} + {_{0}\underline{x}_{t}} \equiv {_{R}C_{t}} \tag{26}$$

In particular, due to our definition $_{0}x_{t} = {_{R}C_{t}} - {_{0}\underline{x}_{t}}$, the equation changes to

$$+ {_{0}x_{t}} + {_{0}\underline{x}_{t}} \equiv {_{R}C_{t}} \tag{27}$$

Normalizing the relationship, it is

$$+\frac{_{0}x_{t}}{_{R}C_{t}} + \frac{_{0}\underline{x}_{t}}{_{R}C_{t}} \equiv \frac{_{R}C_{t}}{_{R}C_{t}} = +1 \tag{28}$$

or

$$+\frac{_{0}x_{t}}{_{R}C_{t}} + \frac{_{0}\underline{x}_{t}}{_{R}C_{t}} \equiv \frac{_{R}C_{t}}{_{R}C_{t}} = +1 \tag{29}$$

or

$$+\frac{_{0}x_{t}}{_{R}C_{t}} + \equiv \left(1-\left(\frac{_{0}\underline{x}_{t}}{_{R}C_{t}}\right)\right) \tag{30}$$

or

$${_{0}x_{t}} = {_{R}C_{t}} \times \left(1-\left(\frac{_{0}\underline{x}_{t}}{_{R}C_{t}}\right)\right) \tag{31}$$

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$$\frac{{}_{0}x_{t}}{\left(1-\left(\frac{{}_{0}x_{t}}{{}_{R}C_{t}}\right)\right)} = {}_{R}C_{t} \tag{32}$$

QUOD ERAT DEMONSTRANDUM.

Remark 3. (Example).

Under condition of classical logic, it is

$${}_{0}x_{t} + {}_{0}\underline{x}_{t} = {}_{R}C_{t} = +1 \tag{33}$$

Under circumstances where ${}_{0}x_{t} = +1$ it is equally $+{}_{0}\underline{x}_{t} = 0$ or

$$+1 + 0 = {}_{R}C_{t} = +1 \tag{34}$$

and we obtain

$$\frac{{}_{0}x_{t}}{\left(1-\left(\frac{{}_{0}x_{t}}{{}_{R}C_{t}}\right)\right)} = \frac{1}{\left(1-\left(\frac{0}{1}\right)\right)} = \frac{1}{(1)} = {}_{R}C_{t} = 1 \tag{35}$$

a correct result.

THEOREM 3.2. (THE DETERMINATION OF ${}_{R}C_{T}$ II)

CLAIM.

From the standpoint to ${}_{0}\underline{x}_{t}$ and due to our definitions before, ${}_{R}C_{t}$ is determined as

$$\frac{{}_{0}\underline{x}_{t}}{\left(1-\left(\frac{{}_{0}x_{t}}{{}_{R}C_{t}}\right)\right)} = {}_{R}C_{t} \tag{36}$$

PROOF.

In general, taking axiom 1 to be true, it is

$$+1 \equiv +1 \tag{37}$$

Multiplying by ${}_{R}C_{t}$ we obtain $1\times {}_{R}C_{t} = 1\times {}_{R}C_{t}$ or

$${}_{R}C_{t} \equiv {}_{R}C_{t} \tag{38}$$

Adding zero, the relationship does not change as such. It is

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$$ {}_{R}C_{t} \quad + \quad 0 \quad \equiv \quad {}_{R}C_{t} \tag{39} $$

According to mathematical requirements, it is $+{}_0\underline{x}_t - {}_0\underline{x}_t = +0$ is. Rearranging equation we obtain

$$ {}_{R}C_{t} \quad - {}_0\underline{x}_t \quad + {}_0\underline{x}_t \quad \equiv \quad {}_{R}C_{t} \tag{40} $$

In particular, due to our definition ${}_0x_t = {}_RC_t - {}_0\underline{x}_t$, the equation changes to

$$ + {}_0x_t \quad + {}_0\underline{x}_t \quad \equiv \quad {}_{R}C_{t} \tag{41} $$

Normalizing the relationship, it is

$$ + \frac{{}_0x_t}{{}_RC_t} \quad + \frac{{}_0\underline{x}_t}{{}_RC_t} \quad \equiv \quad \frac{{}_RC_t}{{}_RC_t} = +1 \tag{42} $$

or

$$ + \frac{{}_0x_t}{{}_RC_t} \quad + \frac{{}_0\underline{x}_t}{{}_RC_t} \quad \equiv \quad \frac{{}_RC_t}{{}_RC_t} = +1 \tag{43} $$

or

$$ + \frac{{}_0\underline{x}_t}{{}_RC_t} \quad + \quad \equiv \quad \left(1 - \left(\frac{{}_0x_t}{{}_RC_t}\right)\right) \tag{44} $$

or

$$ {}_0\underline{x}_t \quad = \quad {}_RC_t \times \left(1 - \left(\frac{{}_0x_t}{{}_RC_t}\right)\right) \tag{45} $$

or

$$ \frac{{}_0\underline{x}_t}{\left(1 - \left(\frac{{}_0x_t}{{}_RC_t}\right)\right)} \quad = \quad {}_RC_t \tag{46} $$

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Remark 4. (Example).

Under condition of classical logic, it is

$$ {}_0x_t \quad + \quad {}_0\underline{x}_t \quad = \quad {}_RC_t \quad = +1 \tag{47} $$

Under circumstances where ${}_0x_t = +0$ it is equally $+{}_0\underline{x}_t = +1$ or

$$ +0 \quad + \quad 1 \quad = \quad {}_RC_t \quad = +1 \tag{48} $$

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and we obtain

$$\frac{{}_{0}x_{t}}{\left(1-\left(\frac{{}_{0}x_{t}}{{}_{R}C_{t}}\right)\right)}=\frac{1}{\left(1-\left(\frac{0}{1}\right)\right)}=\frac{1}{(1)}={}_{R}C_{t}=1 \tag{49}$$

a correct result.

THEOREM 3.3. (CLASSICAL LOGIC AND THE DIVISION OF ZERO BY ZERO)

CLAIM.

In general, ${}_{R}C_{t}$ is determined as

$$\frac{{}_{0}x_{t}}{\left(1-\left(\frac{{}_{0}x_{t}}{{}_{R}C_{t}}\right)\right)}=\frac{{}_{0}x_{t}}{\left(1-\left(\frac{{}_{0}x_{t}}{{}_{R}C_{t}}\right)\right)}={}_{R}C_{t} \tag{50}$$

PROOF.

In general, taking axiom 1 to be true, it is

$$+1 \equiv +1 \tag{51}$$

Multiplying by ${}_{R}C_{t}$ we obtain $1\times {}_{R}C_{t} = 1\times {}_{R}C_{t}$ or

$${}_{R}C_{t} = {}_{R}C_{t} \tag{52}$$

According to theorem 3.1., ${}_{R}C_{t}$ is determined as

$$\frac{{}_{0}x_{t}}{\left(1-\left(\frac{{}_{0}x_{t}}{{}_{R}C_{t}}\right)\right)}={}_{R}C_{t} \tag{53}$$

According to theorem 3.2. the same ${}_{R}C_{t}$ is determined as

$$\frac{{}_{0}x_{t}}{\left(1-\left(\frac{{}_{0}x_{t}}{{}_{R}C_{t}}\right)\right)}=\frac{{}_{0}x_{t}}{\left(1-\left(\frac{{}_{0}x_{t}}{{}_{R}C_{t}}\right)\right)}={}_{R}C_{t} \tag{54}$$

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THEOREM 3.4. (CLASSICAL LOGIC AND THE DIVISION OF ZERO BY ZERO I)

CLAIM.

According to classical logic, it is

$$\frac{+0}{+0} = +1 \tag{55}$$

PROOF.

In general, taking axiom 1 to be true, it is

$$+1 \equiv +1 \tag{56}$$

Multiplying by ${}_RC_t$ we obtain $1\times {}_RC_t = 1\times {}_RC_t$ or

$${}_RC_t = {}_RC_t \tag{57}$$

or according to theorem 3.3.

$$\frac{{}_0x_t}{\left(1-\left(\frac{{}_0x_t}{{}_RC_t}\right)\right)} = \frac{{}_0\underline{x}_t}{\left(1-\left(\frac{{}_0x_t}{{}_RC_t}\right)\right)} = {}_RC_t \tag{58}$$

Under conditions of classical logic, it is ${}_RC_t = +1$. We obtain

$$\frac{{}_0x_t}{\left(1-\left(\frac{{}_0\underline{x}_t}{{}_RC_t}\right)\right)} = \frac{{}_0\underline{x}_t}{\left(1-\left(\frac{{}_0x_t}{{}_RC_t}\right)\right)} = +1 \tag{59}$$

Under conditions of classical logic were ${}_0x_t = +1$ it is equally ${}_0\underline{x}_t = +0$. We obtain

$$\frac{+1}{\left(1-\left(\frac{+0}{+1}\right)\right)} = \frac{+0}{\left(1-\left(\frac{+1}{+1}\right)\right)} = +1 \tag{60}$$

or

$$\frac{+1}{(1-0)} = \frac{+0}{(1-1)} = +1 \tag{61}$$

or

$$\frac{+1}{+1} = \frac{+0}{+0} = +1 \tag{62}$$

or

$$\frac{+0}{+0} = +1 \tag{63}$$

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THEOREM 3.5. (CLASSICAL LOGIC AND THE DIVISION OF ZERO BY ZERO II)

CLAIM.

According to classical logic, it is

$$\frac{+0}{+0} = +1 \tag{64}$$

PROOF.

In general, taking axiom 1 to be true, it is

$$+1 \equiv +1 \tag{65}$$

Multiplying by ${}_{R}C_{t}$ we obtain $1\times {}_{R}C_{t} = 1\times {}_{R}C_{t}$ or

$${}_{R}C_{t} = {}_{R}C_{t} \tag{66}$$

or according to theorem 3.3.

$$\frac{{}_{0}x_{t}}{\left(1-\left(\frac{{}_{0}x_{t}}{{}_{R}C_{t}}\right)\right)} = \frac{{}_{0}\underline{x}_{t}}{\left(1-\left(\frac{{}_{0}x_{t}}{{}_{R}C_{t}}\right)\right)} = {}_{R}C_{t} \tag{67}$$

Under conditions of classical logic, it is ${}_{R}C_{t} = +1$. We obtain

$$\frac{{}_{0}x_{t}}{\left(1-\left(\frac{{}_{0}x_{t}}{{}_{R}C_{t}}\right)\right)} = \frac{{}_{0}\underline{x}_{t}}{\left(1-\left(\frac{{}_{0}x_{t}}{{}_{R}C_{t}}\right)\right)} = +1 \tag{68}$$

Under conditions of classical logic were ${}_{0}x_{t} = +0$ it is equally ${}_{0}\underline{x}_{t} = +1$, we obtain

$$\frac{+0}{\left(1-\left(\frac{+1}{+1}\right)\right)} = \frac{+1}{\left(1-\left(\frac{+0}{+1}\right)\right)} = +1 \tag{69}$$

or

$$\frac{+0}{(1-1)} = \frac{+1}{(1-0)} = +1 \tag{70}$$

or

$$\frac{+0}{+0} = \frac{+1}{+1} = +1 \tag{71}$$

or

$$\frac{+0}{+0} = +1 \tag{72}$$

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THEOREM 3.6. (THE DIVISION OF ZERO BY ZERO AND AXIOM 1)

Let +X denote any (mathematical) object thus that +X-X = + 0. Let +Y denote any (mathematical) object thus that +Y-Y = + 0. In point of fact, even +X=+Y is possible.

CLAIM.

As long as (+0 /+0) = + 1, it is equally true that

$$+1 \quad \equiv \quad +1 \tag{73}$$

PROOF.

In general, if it is true that

$$\frac{+0}{+0} \quad = \quad +1 \tag{74}$$

we have to face some consequences? Especially, it should not be possible to derive any logical contradiction out of this relationship. As defined above, it is +X-X=+ 0 and +Y-Y=+0. Substituting into equation, we obtain

$$\frac{+X-X}{+Y-Y} \quad = \quad +1 \tag{75}$$

or

$$+X-X \quad = \quad +Y-Y \tag{76}$$

or

$$+X+Y \quad = \quad +Y+X \tag{77}$$

or

$$+X \quad = \quad +X \tag{78}$$

Thus far, especially if +X=+1, it follows that

$$+1 \quad \equiv \quad +1 \tag{79}$$

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Remark 5.

This theorem has not been able to derive a logical contradiction from the equation (+0/+0) = +1.

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THEOREM 3.7. (EINSTEIN'S NORMALIZED MASS ENERGY EQUIVALENCE RELATIONSHIP)

Let m_0 denote the "rest-mass" as measured by the co-moving observer at a certain (period or point in) time t, let m_R denotes the "relativistic-mass" as measured by the stationary observer at a same or simultaneous (period or point in) time t, let v be the relative constant velocity between the co-moving and the stationary observer, let c be the speed of the light in vacuum.

CLAIM.

Einstein's mass-energy equivalence relationship can be normalized as

$$\frac{(m_0)^2}{(m_R)^2} + \frac{(v)^2}{(c)^2} = +1 \tag{80}$$

PROOF.

According to Einstein's special theory of relativity, it is

$$m_0 = \sqrt[2]{1-\frac{v^2}{c^2}} \times m_R \tag{81}$$

Rearranging equation, it is

$$(m_0)^2 = \left(1-\frac{v^2}{c^2}\right) \times (m_R)^2 \tag{82}$$

or

$$\frac{(m_0)^2}{(m_R)^2} = \left(1-\frac{v^2}{c^2}\right) \tag{83}$$

and at the end

$$\frac{(m_0)^2}{(m_R)^2} + \frac{(v)^2}{(c)^2} = +1 \tag{84}$$

QUOD ERAT DEMONSTRANDUM.

Remark 6.

The equation before can be extended in a more general way (Barukčić, 2013) (Barukčić, 2016) (Barukčić, 2013; Barukčić, 2016) to a relativistic energy-momentum relation as

$$\frac{(m_0)^2 \times (c)^2 \times (c)^2}{(m_R)^2 \times (c)^2 \times (c)^2} + \frac{(v)^2 \times (m_R)^2 \times (c)^2}{(c)^2 \times (m_R)^2 \times (c)^2} = +1 \tag{85}$$

or as

$$\frac{(E_0)^2}{(E_R)^2} + \frac{(p_R)^2 \times (c)^2}{(E_R)^2} = \frac{(E_0)^2}{(E_R)^2} + \frac{(E_{Electro-mag.\ wave})^2}{(E_R)^2} = +1 \tag{86}$$

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THEOREM 3.8. (THE DEFINITION OF THE NUMBER +1 BY EINSTEIN'S SPECIAL THEORY OF RELATIVITY I)

Let m_0 denote the "rest-mass" as measured by the co-moving observer at a certain (period or point in) time t, let m_R denotes the "relativistic-mass" as measured by the stationary observer at a same or simultaneous (period or point in) time t, let v be the relative constant velocity between the co-moving and the stationary observer, let c be the speed of the light in vacuum.

CLAIM.

Einstein's special relativity theory defines and determines the number +1 as

$$\frac{(v)^2}{(c)^2 \times \left(1 - \frac{(m_0)^2}{(m_R)^2}\right)} = +1 \tag{87}$$

PROOF.

In general, taking axiom 1 to be true, it is

$$+1 \equiv +1 \tag{88}$$

According to one of our theorems before, it is equally

$$\frac{(m_0)^2}{(m_R)^2} + \frac{(v)^2}{(c)^2} = +1 \tag{89}$$

or

$$\frac{(m_0)^2}{(m_R)^2} + \frac{(v)^2}{(c)^2} = +1 \tag{90}$$

or equally

$$\frac{(v)^2}{(c)^2} = 1 - \frac{(m_0)^2}{(m_R)^2} \tag{91}$$

Under conditions were a division is possible and allowed, we obtain

$$\frac{(v)^2}{(c)^2 \times \left(1 - \frac{(m_0)^2}{(m_R)^2}\right)} = +1 \tag{92}$$

QUOD ERAT DEMONSTRANDUM.

Remark 7.

Einstein's mass–energy equivalence arose originally from a paradox described by Henri Poincaré (Poincaré, 1900) (Poincaré, 1900). In his publication on 21 November 1905 entitled as "*Does the inertia of a body depend upon its energy-content?*" Einstein proposed to consider the following: "*Gibt ein Körper die Energie L in From von Strahlung ab, so verkleinert sich seine Masse um L/V*^{2}" (Einstein, 1905) (Einstein, 1905).

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THEOREM 3.9. (THE DEFINITION OF THE NUMBER +1 BY EINSTEIN'S SPECIAL THEORY OF RELATIVITY II)

Let m_0 denote the "rest-mass" as measured by the co-moving observer at a certain (period or point in) time t, let m_R denotes the "relativistic-mass" as measured by the stationary observer at a same or simultaneous (period or point in) time t, let *v* be the relative constant velocity between the co-moving and the stationary observer, let *c* be the speed of the light in vacuum.

CLAIM.

Einstein's special relativity theory defines and determines the number +1 in another way as

$$\frac{(m_0)^2}{(m_R)^2 \times \left(1 - \frac{(v)^2}{(c)^2}\right)} = +1 \tag{93}$$

PROOF.

In general, taking axiom 1 to be true, it is

$$+1 \equiv +1 \tag{94}$$

According to one of our theorems before, it is equally

$$\frac{(m_0)^2}{(m_R)^2} + \frac{(v)^2}{(c)^2} = +1 \tag{95}$$

or

$$\frac{(m_0)^2}{(m_R)^2} + \frac{(v)^2}{(c)^2} = +1 \tag{96}$$

or equally

$$\frac{(m_0)^2}{(m_R)^2} = 1 - \frac{(v)^2}{(c)^2} \tag{97}$$

Under conditions were a division is possible and allowed, we obtain

$$\frac{(m_0)^2}{(m_R)^2 \times \left(1 - \frac{(v)^2}{(c)^2}\right)} = +1 \tag{98}$$

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THEOREM 3.10. (EINSTEIN'S SPECIAL THEORY OF RELATIVITY UNDER CONDITIONS WERE V=0)

Let m_0 denote the "rest-mass" as measured by the co-moving observer at a certain (period or point in) time t, let m_R denotes the "relativistic-mass" as measured by the stationary observer at a same or simultaneous (period or point in) time t, let *v* be the relative constant velocity between the co-moving and the stationary observer, let *c* be the speed of the light in vacuum.

CLAIM.

Under conditions were the relative velocity (between a co-moving and a stationary observer) is **v = 0**, Einstein's special relativity theory does not collapse. Under these circumstances it is

$$\frac{(m_0)^2}{(m_R)^2} = +1 \tag{99}$$

PROOF.

In general, taking axiom 1 to be true, it is

$$+1 \equiv +1 \tag{100}$$

According to one of our theorems before, it is equally

$$\frac{(m_0)^2}{(m_R)^2} + \frac{(v)^2}{(c)^2} = +1 \tag{101}$$

Under conditions were the relative velocity (between a co-moving and a stationary observer) is v = 0, it is

$$\frac{(m_0)^2}{(m_R)^2} + \frac{(0)^2}{(c)^2} = +1 \tag{102}$$

or equally

$$\frac{(m_0)^2}{(m_R)^2} = +1 \tag{103}$$

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THEOREM 3.11. (EINSTEIN'S SPECIAL THEORY OF RELATIVITY UNDER CONDITIONS WERE V=C)

Let m_0 denote the "rest-mass" as measured by the co-moving observer at a certain (period or point in) time t, let m_R denotes the "relativistic-mass" as measured by the stationary observer at a same or simultaneous (period or point in) time t, let v be the relative constant velocity between the co-moving and the stationary observer, let c be the speed of the light in vacuum.

CLAIM.

Under conditions were the relative velocity (between a co-moving and a stationary observer) is **v = c**, Einstein's special relativity theory does not collapse. Under these circumstances it is

$$\frac{(m_0)^2}{(m_R)^2} = +0 \tag{104}$$

PROOF.

In general, taking axiom 1 to be true, it is

$$+1 \equiv +1 \tag{105}$$

According to one of our theorems before, it is equally

$$\frac{(m_0)^2}{(m_R)^2} + \frac{(v)^2}{(c)^2} = +1 \tag{106}$$

Under conditions were the relative velocity (between a co-moving and a stationary observer) is v = c, it is

$$\frac{(m_0)^2}{(m_R)^2} + \frac{(c)^2}{(c)^2} = +1 \tag{107}$$

or equally

$$\frac{(m_0)^2}{(m_R)^2} + 1 = +1 \tag{108}$$

Under conditions were the relative velocity (between a co-moving and a stationary observer) is v = c, it is equally valid that

$$\frac{(m_0)^2}{(m_R)^2} = +0 \tag{109}$$

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THEOREM 3.12. (EINSTEIN'S SPECIAL THEORY OF RELATIVITY AND THE DIVISION OF ZERO BY ZERO I)

Let m_0 denote the "rest-mass" as measured by the co-moving observer at a certain (period or point in) time t, let m_R denotes the "relativistic-mass" as measured by the stationary observer at a same or simultaneous (period or point in) time t, let v be the relative constant velocity between the co-moving and the stationary observer, let c be the speed of the light in vacuum.

CLAIM.

Under conditions of Einstein's special relativity were v = 0, it is

$$\frac{+0}{+0} \equiv +1 \tag{110}$$

PROOF.

In general, taking axiom 1 to be true, it is

$$+1 \equiv +1 \tag{111}$$

Einstein's special relativity defines the number +1 on the one hand as

$$\frac{(m_0)^2}{(m_R)^2 \times \left(1 - \frac{(v)^2}{(c)^2}\right)} = +1 \tag{112}$$

and on the other hand, equally as

$$\frac{(m_0)^2}{(m_R)^2 \times \left(1 - \frac{(v)^2}{(c)^2}\right)} = \frac{(v)^2}{(c)^2 \times \left(1 - \frac{(m_0)^2}{(m_R)^2}\right)} = +1 \tag{113}$$

Under conditions, were v = 0, it is $((m_0^2)/(m_R^2)) = 1$. The equation above can be rearranged as

$$\frac{1}{\left(1 - \frac{(0)^2}{(c)^2}\right)} = +1 = \frac{(v)^2}{(c)^2 \times \left(1 - \frac{(m_0)^2}{(m_R)^2}\right)} = +1 \tag{114}$$

or as

$$\frac{1}{\left(1 - \frac{(0)^2}{(c)^2}\right)} = +1 = \frac{(0)^2}{(c)^2 \times (1-1)} = \frac{(0)^2}{(c)^2 \times (0)} = +1 \tag{115}$$

and at the end

$$\frac{+0}{+0} \equiv +1 \tag{116}$$

QUOD ERAT DEMONSTRANDUM.

Remark 8.

Einstein's special relativity can be tested by experiments and allows and demands the division of zero by zero. According to it is ((+0)/ (+0)) = +1.

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THEOREM 3.13. (EINSTEIN'S SPECIAL THEORY OF RELATIVITY AND THE DIVISION OF ZERO BY ZERO II)

Let m_0 denote the "rest-mass" as measured by the co-moving observer at a certain (period or point in) time t, let m_R denotes the "relativistic-mass" as measured by the stationary observer at a same or simultaneous (period or point in) time t, let *v* be the relative constant velocity between the co-moving and the stationary observer, let c be the speed of the light in vacuum.

CLAIM.

Under conditions of Einstein's special relativity were v = c, it is

$$\frac{+0}{+0} \equiv +1 \tag{117}$$

PROOF.

In general, taking axiom 1 to be true, it is

$$+1 \equiv +1 \tag{118}$$

Einstein's special relativity defines the number +1 on the one hand as

$$\frac{(m_0)^2}{(m_R)^2 \times \left(1 - \frac{(v)^2}{(c)^2}\right)} = +1 \tag{119}$$

and on the other hand, equally as

$$\frac{(m_0)^2}{(m_R)^2 \times \left(1 - \frac{(v)^2}{(c)^2}\right)} = \frac{(v)^2}{(c)^2 \times \left(1 - \frac{(m_0)^2}{(m_R)^2}\right)} = +1 \tag{120}$$

Under conditions, were v = c it is it is $((m_0{}^2)/(m_R{}^2)) = 0$ and the equation above can be rearranged as

$$\frac{(m_0)^2}{(m_R)^2 \times \left(1 - \frac{(v)^2}{(c)^2}\right)} = +1 = \frac{(c)^2}{(c)^2 \times (1 - 0)} = +1 = +1 \tag{121}$$

Under conditions, were v = c, it is $((m_0{}^2)/(m_R{}^2)) = 0$ and it is equally

$$(+0)\frac{+1}{\left(1 - \frac{(c)^2}{(c)^2}\right)} = \frac{+0}{+0} = \frac{(c)^2}{(c)^2 \times (1 - 0)} = +1 = +1 \tag{122}$$

and at the end

$$\frac{+0}{+0} \equiv +1 \tag{123}$$

QUOD ERAT DEMONSTRANDUM.

Remark 9.

Einstein's special relativity demands the division of zero by zero even under conditions were v = c. Under these conditions, we must consider the pure electro-magnetic energy/wave. According to Einstein's special relativity it is again ((+0)/ (+0)) = +1.

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THEOREM 3.14. (THE GENERALLY NORMALIZED FROM OF THE PYTHAGOREAN THEOREM)

The Pythagorean theorem, more or less attributed to Pythagoras (ca. 570 BC-ca. 495 BC), was already known by the Old Babylonians (ca. 1900-1600 B.C.E) more than a millennium before Pythagoras (Maor, 2010) (Maor, 2010) who used this relation to solve some geometric problems. The Pythagorean theorem is still one of the fundamental relations in Euclidean geometry and equally one of the most famous statements in mathematics, and is defined itself as $(a_t)^2 + (b_t)^2 = ({}_RC_t)^2$, where ${}_RC_t$ represents the length of the hypotenuse of a right-angled triangle and a_t and b_t the lengths of the triangle's other two sides. According to Euclid's Elements, Book I, Proposition 47 it is: "*In right-angled triangles the square on the side opposite the right angle equals the sum of the squares on the sides containing the right angle.*" (Euclid, Heath, and Heiberg, 1908) (Euclid, Heath, and Heiberg, 1908)

CLAIM.

The generally normalized form of the Pythagorean theorem (Barukčić, 2013) (Barukčić, 2013) is given by

$$\frac{a_t^2}{{}_RC_t^2} + \frac{b_t^2}{{}_RC_t^2} = +1 \tag{124}$$

PROOF.

In general, taking axiom 1 to be true, it is

$$+1 \equiv +1 \tag{125}$$

Multiplying by ${}_RC_t^2$, the length of the hypotenuse of a right-angled triangle squared, we obtain $1\times {}_RC_t^2 = 1\times {}_RC_t^2$ or

$${}_RC_t^2 = {}_RC_t^2 \tag{126}$$

The Pythagorean theorem can be expressed as the Pythagorean equation as

$$a_t^2 + b_t^2 = {}_RC_t^2 \tag{127}$$

Finally, rearranging equation, it is

$$\frac{a_t^2}{{}_RC_t^2} + \frac{b_t^2}{{}_RC_t^2} = \frac{{}_RC_t^2}{{}_RC_t^2} = +1 \tag{128}$$

QUOD ERAT DEMONSTRANDUM.

Remark 10.

According to the generally normalized form of the Pythagorean theorem we must consider the following situations. If $a_t^2 = 0$ then $b_t^2 = {}_RC_t^2$ or

$$\frac{0}{{}_RC_t^2} + \frac{b_t^2}{{}_RC_t^2} = +1 \tag{129}$$

If $b_t^2 = 0$ then $a_t^2 = {}_RC_t^2$ or

$$\frac{a_t^2}{{}_RC_t^2} + \frac{0}{{}_RC_t^2} = +1 \tag{130}$$

In other words, the Pythagorean theorem is defined even under these conditions.

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THEOREM 3.15. (THE DEFINITION OF THE NUMBER +1 BY THE PYTHAGOREAN THEOREM II)

CLAIM.

The Pythagorean theorem determines the number +1 as

$$\frac{a_t^2}{\left({}_RC_t^2\right) \times \left(1 - \frac{b_t^2}{{}_RC_t^2}\right)} = +1 \tag{131}$$

PROOF.

In general, taking axiom 1 to be true, it is

$$+1 \equiv +1 \tag{132}$$

According to the theorem before, the Pythagorean theorem defines the number +1 as

$$\frac{a_t^2}{{}_RC_t^2} + \frac{b_t^2}{{}_RC_t^2} = +1 \tag{133}$$

Rearranging equation, we obtain

$$\frac{a_t^2}{{}_RC_t^2} = 1 - \frac{b_t^2}{{}_RC_t^2} \tag{134}$$

or

$$\frac{a_t^2}{\left({}_RC_t^2\right) \times \left(1 - \frac{b_t^2}{{}_RC_t^2}\right)} = +1 \tag{135}$$

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THEOREM 3.16. (THE DEFINITION OF THE NUMBER +1 BY THE PYTHAGOREAN THEOREM II)

CLAIM.

The Pythagorean theorem determines the number +1 as

$$\frac{b_t^2}{\left({}_{R}C_t^2\right) \times \left(1 - \frac{a_t^2}{{}_{R}C_t^2}\right)} = +1 \tag{136}$$

PROOF.

In general, taking axiom 1 to be true, it is

$$+1 \equiv +1 \tag{137}$$

According to the theorem before, the Pythagorean theorem defines the number +1 as

$$\frac{a_t^2}{{}_{R}C_t^2} + \frac{b_t^2}{{}_{R}C_t^2} = +1 \tag{138}$$

Rearranging equation, we obtain

$$\frac{b_t^2}{{}_{R}C_t^2} = 1 - \frac{a_t^2}{{}_{R}C_t^2} \tag{139}$$

or

$$\frac{b_t^2}{\left({}_{R}C_t^2\right) \times \left(1 - \frac{a_t^2}{{}_{R}C_t^2}\right)} = +1 \tag{140}$$

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THEOREM 3.17. (THE DIVISION OF ZERO BY ZERO ACCORDING TO THE PYTHAGOREAN THEOREM I)

CLAIM.

The Pythagorean theorem determines the division of zero by zero as

$$\frac{+0}{+0} = +1 \tag{141}$$

PROOF.

In general, taking axiom 1 to be true, it is

$$+1 \equiv +1 \tag{142}$$

The Pythagorean theorem defines the number +1 as

$$\frac{a_t^2}{\left({}_RC_t^2\right) \times \left(1 - \frac{b_t^2}{{}_RC_t^2}\right)} = +1 \tag{143}$$

and equally as

$$\frac{a_t^2}{\left({}_RC_t^2\right) \times \left(1 - \frac{b_t^2}{{}_RC_t^2}\right)} = \frac{b_t^2}{\left({}_RC_t^2\right) \times \left(1 - \frac{a_t^2}{{}_RC_t^2}\right)} = +1 \tag{144}$$

Under conditions, where $a_t^2=0$, it is equally $b_t^2 = {}_RC_t^2$. The equation before changes to

$$\frac{a_t^2}{\left({}_RC_t^2\right) \times \left(1 - \frac{b_t^2}{{}_RC_t^2}\right)} = \frac{{}_RC_t^2}{\left({}_RC_t^2\right) \times \left(1 - \frac{0^2}{{}_RC_t^2}\right)} = \frac{+1}{+1} = +1 \tag{145}$$

or to

$$\frac{0^2}{\left({}_RC_t^2\right) \times \left(1 - \frac{{}_RC_t^2}{{}_RC_t^2}\right)} = \frac{0}{\left({}_RC_t^2\right) \times (1-1)} = \frac{+0}{+0} = \frac{{}_RC_t^2}{\left({}_RC_t^2\right) \times \left(1 - \frac{0^2}{{}_RC_t^2}\right)} = \frac{+1}{+1} = +1 \tag{146}$$

or to

$$\frac{+0}{+0} = +1 \tag{147}$$

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THEOREM 3.18. (THE DIVISION OF ZERO BY ZERO ACCORDING TO THE PYTHAGOREAN THEOREM II)

CLAIM.

The Pythagorean theorem determines the division of zero by zero as

$$\frac{+0}{+0} = +1 \tag{148}$$

PROOF.

In general, taking axiom 1 to be true, it is

$$+1 \equiv +1 \tag{149}$$

The Pythagorean theorem defines the number +1 as

$$\frac{a_t^2}{\left({}_RC_t^2\right) \times \left(1 - \frac{b_t^2}{{}_RC_t^2}\right)} = +1 \tag{150}$$

and equally as

$$\frac{a_t^2}{\left({}_RC_t^2\right) \times \left(1 - \frac{b_t^2}{{}_RC_t^2}\right)} = \frac{b_t^2}{\left({}_RC_t^2\right) \times \left(1 - \frac{a_t^2}{{}_RC_t^2}\right)} = +1 \tag{151}$$

Under conditions, where $b_t^2=0$, it is equally $a_t^2 = {}_RC_t^2$. The equation before changes to

$$\frac{a_t^2}{\left({}_RC_t^2\right) \times \left(1 - \frac{b_t^2}{{}_RC_t^2}\right)} = \frac{+0}{\left({}_RC_t^2\right) \times \left(1 - \frac{{}_RC_t^2}{{}_RC_t^2}\right)} = \frac{+0}{+0} = +1 \tag{152}$$

or to

$$\frac{{}_RC_t^{\,2}}{\left({}_RC_t^{\,2}\right) \times \left(1 - \frac{0^2}{{}_RC_t^{\,2}}\right)} = \frac{+1}{1-0} = \frac{+1}{+1} = \frac{+0}{\left({}_RC_t^{\,2}\right) \times \left(1 - \frac{{}_RC_t^{\,2}}{{}_RC_t^{\,2}}\right)} = \frac{+0}{+0} = +1 \tag{153}$$

or to

$$\frac{+0}{+0} = +1 \tag{154}$$

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Remark 11.

According to the Pythagorean theorem it is (+0)/(+0) = +1.

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THEOREM 3.19. (TODAY'S MULTIPLICATION BY ZERO IS SELF-CONTRADICTORY)

Following the lead of the principles of classical logic, it is appropriate to focus on what it means that from something which is obviously wrong cannot follow something which is obviously true assumed that there are no technical errors. Particularly, technically correct and allowed logical or mathematical operations cannot result in **true statements being deduced from false statements**. In this sense, today's rules concerning the multiplication by zero are completely useless and must be abandoned.

CLAIM.

Todays' understanding of the multiplication by zero is logically and mathematically inconsistent because the same can change a statement which is obviously wrong (+1=+0) into a statement which is which is obviously true.

PROOF.

In general, our starting statement is

$$+\mathbf{1} \equiv +\mathbf{0} \tag{155}$$

and as such obviously not true. It should not be possible in the absence of any technical errors to deduce a true statement from such a false one. Adding +2 on both sides of the equation, it is

$$+1+2 \equiv +0+2 \tag{156}$$

or

$$+3 \equiv +2 \tag{157}$$

Multiplying equation by +0, we obtain

$$(+3) \times (+0) \equiv (+2) \times (+0) \tag{158}$$

According to today's rule of the multiplication by zero this is identical with

$$(+0) \equiv (+0) \tag{159}$$

or

$$(+1-1) \equiv (+1-1) \tag{160}$$

Today's rules of the multiplication by zero enables that **a statement which is obviously wrong (+1=+0) can be changed without any technical errors into a statement which is obviously true** or

$$+\mathbf{1} \equiv +\mathbf{1} \tag{161}$$

QUOD ERAT DEMONSTRANDUM.

Remark 12.

A consistent logical or mathematical operation is one that does not entail any contradiction. Consistently with the theorem above is that from contradictory premises or statements (+1=+0), anything follows (*ex contradictione sequitur quodlibet (ECSQ)*). In other words, whatever is claimed, its contradiction is also true. The more from a theorem or a theory containing a true contradiction, everything as true as well as everything as false can be deduced the more such theorems and theories must be identified and labeled with a contradiction. Historically, *ex contradictione sequitur quodlibet* (or *the Principle of Explosion*) is ascribed to **William of Soissons**, a 12th century French logician who lived in Paris. Karl Popper made similar claims in a different context: "*We see from this that if a theory contains a contradiction, then it entails everything, and therefore, indeed, nothing [...]. A theory which involves a contradiction is therefore entirely useless as a theory*". (Popper, 2002) (Popper, 2002, p. 429). Today's rules concerning the multiplication by zero are logically inconsistent. New techniques which remove today's inconsistency as associated with the rules of the multiplication by zero are necessary.

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THEOREM 3.20. (THE NEW RULE OF THE MULTIPLICATION BY ZERO)

CLAIM.

The multiplication by zero is logically and mathematically consistent if the same does not change a statement which is obviously wrong (+1=+0) into a statement which is which is obviously true.

PROOF.

In general, our starting statement is

$$\mathbf{+1} \equiv \mathbf{+0} \tag{162}$$

and as such obviously not true. It should not be possible in the absence of technical errors to deduce a true statement from such a false one. Adding +2 on both sides of the equation, it is

$$+1+2 \equiv +0+2 \tag{163}$$

or

$$+3 \equiv +2 \tag{164}$$

Multiplying equation by +0, we obtain

$$(+3) \times (+0) \equiv (+2) \times (+0) \tag{165}$$

The new rule of the multiplication by zero is that

$$(+3) \times (+0) \equiv (+2) \times (+0) \tag{166}$$

stays that what it is and does not collapse into +0 = +0. Dividing by zero, it is

$$(+3) \times \frac{(+0)}{(+0)} \equiv (+2) \times \frac{(+0)}{(+0)} \tag{167}$$

or

$$(+3) \times \frac{(+1)}{(+0)} \times (+0) \equiv (+2) \times \frac{(+1)}{(+0)} \times (+0) \tag{168}$$

or according to axiom 2

$$(+3) \times (+\infty) \times (+0) \equiv (+2) \times (+\infty) \times (+0) \tag{169}$$

According to our axiom 2, it is $0\times\infty=1$. We obtain

$$(+3) \times (+1) \equiv (+2) \times (+1) \tag{170}$$

or

$$+3-2 \equiv +2-2 \tag{171}$$

The new rule of the multiplication by zero assures that **a statement which is obviously wrong (+1=+0) stays in the absence of any technical errors that what it is, obviously wrong** or

$$\mathbf{+1} \equiv \mathbf{+0} \tag{172}$$

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4. Discussion

The Division of zero by zero has a very long history. The issue of division by zero is documented in literature at least since the times of Aristotle. The concept of zero and the symbol of zero appears to have travelled from the Mesopotamians via the Greeks to India. Historically, especially the Hindu mathematicians of ancient India like Aryabhatta (476 - 550 AD), Brahmagupta (598-665 AD), Bhāskara II (1114-1185 AD) and others came up with a concept of zero. Brahamagupta (628), an outstanding Indian mathematician and astronomer of the 7th Century, in his *Brahmasphula siddhanta* was of the opinion that that 0/0 = 0 (Paolli, 2017), while Bhāskara II defined in *Bijaganita* that n / 0 = ∞ (Ufuoma, 2017).

And yet, despite a long history of debate going back to Aristotle himself, the problem of the division of zero by zero is still not solved. In the present time Barukčić and Barukčić (Barukčić and Barukčić, 2016), (Barukčić and Barukčić, 2016) supported by Paolilli (Paolilli, 2017) (Paolilli, 2017) and Mwangi (Patrick Mwangi, 2018) (Mwangi, 2018), comes to the conclusion that 0/0=1, while other authors (Anderson, Völker, and Adams, 2007) (Bergstra, Hirshfeld, and Tucker, 2009)(Bergstra and Ponse, 2015) (Matsuura and Saitoh, 2016) (Michiwaki, Saitoh, and Yamada, 2016) (Anderson, Völker, and Adams, 2007; Bergstra, Hirshfeld, and Tucker, 2009; Matsuura and Saitoh, 2016; Michiwaki, Saitoh, and Yamada, 2016) are not of this opinion. May be that the position of these authors are compatible with paraconsistent logic. On the very strong end of the spectrum of logic, paraconsistent logics is claiming that some contradictions are really true. Needless to say, all approaches to paraconsistency simply deny *ex contradictione quodlibet*. In other words, paraconsistent logic is a logical system which rejects *the principle of explosion* (da Costa, 1974) (da Costa, 1974) or *ex contradictione sequitur quodlibet* (Latin, “from a contradiction, anything follows”) while trying to deal with contradictions. The term ‘paraconsistent’ was coined by *Francisco Miró Quesada* at the Third Latin-American symposium on Mathematical Logic, Campinas, Brazil, July 11-17, 1976 (Quesada, 1977) (Quesada, 1977). Even if we must acknowledge the objective existence of contradictions in nature (Barukčić, 2019) (Barukčić, 2019) this does not justifies the existence of inconsistencies. In point of fact, every time when a co-moving observer performs some measurements, he will find that something is *either the one or not the one* but not both at the same time while a stationary observer can but must not find that *something is equally both, the one and not the one* (Barukčić, 2019) (Barukčić, 2019).

5. Conclusion

A division of zero by zero is possible and defined. Zero divided by zero is one.

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