First Draft

The Strange Attractor Model of Turbulence and Effective Field Theories

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Abstract

Recent work has conjectured that, under general boundary conditions, non-equilibrium Renormalization Group flows are likely to end up on *strange attractors*. If this conjecture is true, effective field theories must necessarily reflect the properties of these attractors. We start from the observation that, seemingly disparate concepts such as the Berry phase, gauge potentials and the curvature tensor of General Relativity (GR), share a common geometric foundation. Developing further, we posit that the dynamics of gauge and gravitational fields may be derived from the global attributes of strange attractors. The motivation behind this ansatz is that the Navier-Stokes equations bridge the gap between fluid turbulence, on the one hand, and the mathematics of GR and gauge theory, on the other.

Key words: strange attractors, fluid turbulence, Berry phase, gauge theory, General Relativity.

1. Introduction

We have recently found that, under general boundary conditions, non-equilibrium Renormalization Group flows are prone to evolve to *strange attractors* [1, 2]. It is known that these attractors provide realistic models for the onset of chaos in nonlinear dynamics, as well as for the transition to turbulence in fluids described by the Navier-Stokes equations [3, 4]. Here we develop the idea that, seemingly disparate concepts of quantum physics and classical field theory – namely, the Berry phase, gauge potentials and the connection coefficients of General Relativity (GR) – share a common ground with fluid turbulence and its roots in the geometry of strange attractors.

2. Berry phase in quantum physics

A quantum system adiabatically transported around a closed path C in the space of external parameters acquires a non-vanishing phase (*Berry phase*, BP in short). Since BP is exclusively path-dependent, it provides key insights into the geometric structure of quantum mechanics and QFT. The BP concept is closely tied to *holonomy*, that is, the extent to which some of variables change as other variables or parameters defining a system return to their initial values [5, 6].

Consider a quantum system described by the time-independent Hamiltonian $H(t)$, whose associated eigenstate is $|\psi(t)\rangle$ and which is embedded in a slowly changing environment. After a periodic evolution of the environmental parameters ($t \to t+T$), the eigenstate returns to itself, apart from a phase angle,

$$|\psi(t)\rangle = e^{i\alpha}|\psi(0)\rangle \tag{1}$$

If ω denotes the eigenvalue of $|\psi(t)\rangle$, a generalization of the phase angle $\alpha = \omega T$ in units of $\hbar = 1$ is given by the "*dynamical phase*"

$$\gamma_d = \int_0^T \omega(t)\,dt = \int_0^T \langle\psi(t)|H(t)|\psi(t)\rangle\,dt \tag{2}$$

Berry has shown that there is a time-independent (but contour dependent) supplemental "*geometric phase*" entering the phase angle, namely,

$$\alpha = \gamma_d + \gamma(C) \tag{3}$$

where

$$\gamma(C) = \int_C \langle \psi | i\nabla\psi \rangle \, dr \tag{4}$$

The dynamical phase γ_d encodes information about the duration associated with the cyclic evolution of the complex vector $|\psi(t)\rangle$. By contrast, (4) encodes information about the geometry of the environment where the transport takes place.

3. The geometry of gauge and gravitational fields

The gauge field concept may be built from a straightforward *geometric* interpretation [7, 8]. Consider the parallel transport of a complex vector $|\psi\rangle$ round a closed rectangular loop. The difference between the value of $|\psi\rangle$ at the starting point ($|\psi\rangle_0$) and at the end point $|\psi\rangle_0 \rightarrow |\psi\rangle_f$ is given by

$$\Delta\psi = \psi_f - \psi_0 = -ig\,\Delta S^{\mu\nu} F_{\mu\nu}\psi \tag{5}$$

in which $\Delta S^{\mu\nu}$ denotes the area enclosed by the rectangle and the strength of the gauge field is

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig\left[A_\mu, A_\nu\right] \tag{6}$$

Echoing the formation of the Berry phase, the effect of parallel transport is to induce a non-vanishing rotation of $|\psi\rangle$ in internal space proportional to the strength of the gauge field. Likewise, the curvature tensor of GR may be motivated through similar arguments. Taking a vector V^{κ} on a round trip by parallel transport, the difference between the initial and final components of the vector amounts to

$$\Delta V^{\kappa} = \frac{1}{2} R^{\kappa}_{\lambda\mu\nu} V^{\lambda} \Delta S^{\mu\nu} \tag{7}$$

This equation faithfully replicates (5) and signals the presence of a gravitational field, via the curvature tensor $R^{\kappa}_{\lambda\mu\nu}$. The geometric analogy between gauge theory and General Relativity is captured in the table below.

Gauge Theory	**General Relativity**
Gauge transformation	Coordinate transformation
Gauge group	Group of coordinate transformations
Gauge potential A_{μ}	Connection coefficient $\Gamma^{\kappa}_{\mu\nu}$
Field strength $F_{\mu\nu}$	Curvature tensor $R^{\kappa}_{\lambda\mu\nu}$

Comparison between the geometry of gauge and gravitational fields.

4. The geometry of turbulence in fluid dynamics

…text to follow refs. [9-13] …

5. Conclusions and outlook

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