The Hilbert Book Model Project survey

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14-3-2019

Summary

This survey treats the Hilbert Book Model Project. The project concerns a well-founded, purely mathematical model of physical reality. The project relies on the conviction that physical reality owns its own kind of mathematics and that this mathematics guides and restricts the extension of the foundation to more complicated levels of the structure and the behavior of physical reality. This results in a model that more and more resembles the physical reality that humans can observe.

Inhoud

1 The initiator of the project

The Hilbert Book Model Project is an ongoing project. Hans van Leunen is the initiator of this project. The initiator was born in the Netherlands in 1941. He will not live forever. This project will contain his scientific inheritance.

The project is introduced in a Wikiversity project. In the opinion of the initiator, a Wikiversity project is a perfect way of introducing new science. It especially serves the needs of independent or retired scientific authors.

The initiator maintains a ResearchGate project that considers the Hilbert Book Model Project. The ResearchGate site supports a flexible way of discussing scientific subjects.

The initiator has generated some documents that contain highlights as excerpts of the project, and he stored these papers on his personal e-print archive http://vixra.org/author/j_a_j_van_leunen .

The private website http://www.e-physics.eu contains most documents both in pdf as well as in docx format. None of these documents claims copyright. Everybody is free to use the content of these papers.

1.1 Trustworthiness

Introducing new science always introduces controversial and unorthodox text. The Hilbert Book Model Project is an ongoing enterprise. Its content is dynamic and is revised regularly.

The content of this project is not peer-reviewed. It is the task of the author to ensure the correctness of what he writes. In the vision of the author, the reader is responsible for checking the validity of what he/she reads. The peer review process cannot cope with the dynamics of revisions and extensions that becomes possible via publishing in freely accessible e-print archives. In comparison to openly accessible publication on the internet, the peer review process is a rather slow process. In addition, it inhibits the usage of revision services, such as offered by vixra.org and by arxiv.org/

Reviewers are always biased, and they are never omniscient. The peer review process is expensive and often poses barriers to the renewal of science.

One way to check the validity of the text is to bring parts of the text to open scientific discussion sites such as ResearchGate.

The initiator challenges everybody to disprove the statements made in this report. He promises a fine bottle of XO cognac to anyone that finds a significant flaw in the presented theories.

This challenge stands already for several years. Up to so far, nobody claimed the bottle.

1.2 The author

Hans is born in Helmond in 1941 and visited the Eindhoven HTS in chemistry from 1957-1960.

After his military service in 1060-1963, Hans started at the THE which is now called the Technical University Eindhoven (TUE) for a study in applied physics.

Hans finished this study in 1970 and then joined Philips Elcoma EOD in the development of image intensifier tubes. Later this became a department of Philips Medical Systems division.

In 1987 Hans switched to an internal software house. In 1995 Hans joined the Semiconductor division of Philips. In this period Hans designed a system for modular software generation.

In 2001 Hans retired.

From 1983 until 2006 Hans owned a software company "Technische en Wetenschappelijke Programmatuur" (TWP).

A private website treats my current activities.

I store my papers at a freely accessible e-print archive.

To investigate the foundations and the lower levels of physical reality, Hans started in 2009 a personal research project that in 2011 got its current name "The Hilbert Book Model Project."

The Hilbert Book Model is a purely mathematical unorthodox and controversial model of the foundations and the lower levels of the structure of physical reality.

1.3 Early encounters

I am born with a deep curiosity about my living environment. When I became aware of this, I was astonished why this environment appeared to be so complicated, and at the same time, it behaved in such a coherent way. In my childhood, I had no clue. Later some unique experiences offered me some indications. After my retirement, I started in 2009 a personal research project to discover and formulate some of the clues. The “Hilbert Book Model” is the name of my personal research project.

My interest in the structure and phenomena of physical reality started in the third year of my physics study when the configuration of quantum mechanics confronted me for the first time with its special approach. The fact that its methodology differed fundamentally from the way that physicists did classical mechanics astonished me. So, I asked my very wise lecturer on what origin this difference is based. His answer was that the superposition principle caused this difference. I was not very happy with this answer because the superposition principle was indeed part of the methodology of quantum mechanics, but in those days, I did not comprehend how that could present the main cause of the difference between the two methodologies. I decided to dive into literature, and after some search, I encountered the booklet of Peter Mittelsteadt, “Philosophische Probleme der modernen Physik” (1963). This booklet contained a chapter about quantum logic and that appeared to me to contain a more appropriate answer. Later, this appeared a far too quick conclusion. In 1936 Garrett Birkhoff and John von Neumann published a paper that described their discovery of what they called “quantum logic.” Quantum logic is since then in mathematical terminology known as an orthomodular lattice. The relational structure of this lattice is to a large extent quite like the relational structure of classical logic. That is why the duo gave their discovery the name “quantum logic.” This name was an unlucky choice because no good reason exists to consider the orthomodular lattice as a system of logical propositions. In the same paper, the duo indicated that the set of closed subspaces of a separable Hilbert space has exactly the relational structure of an orthomodular lattice. John von Neumann long doubted between Hilbert spaces and projective geometries. In the end, he selected Hilbert spaces as the best platform for developing quantum physical theories. That appears to be one of the main reasons why quantum physicists prefer Hilbert spaces as a realm in which they do their modeling of quantum physical systems. Another habit of quantum physicists also intrigued me. My lecturer thought me that all observable quantum physical quantities are eigenvalues of Hermitian operators. Hermitian operators feature real eigenvalues. When I looked around, I saw a world that had a structure that configures from a three-dimensional spatial domain and a one-dimensional and thus, scalar time domain. In the quantum physics of that time, no operator

represents the time domain, and no operator was used to deliver the spatial domain in a compact fashion. After some trials, I discovered a four-dimensional number system that could provide an appropriate normal operator with an eigenspace that represented the full four-dimensional representation of my living environment. At that moment, I had not yet heard from quaternions, but an assistant professor quickly told me about the discovery of Rowan Hamilton that happened more than a century earlier. Quaternions appear to be the number system of choice for offering the structure of physical reality its powerful capabilities.

The introductory paper of Birkhoff and von Neumann already mentioned quaternions. Much later Maria Pia Solèr offered a hard prove that Hilbert spaces can only cope with members of an associative division ring. Quaternions form the most extensive associative division ring. To my astonishment, I quickly discovered that physicists preferred a spacetime structure that features a Minkowski signature instead of the Euclidean signature of the quaternions. The devised Hilbert Book Model shows that in physical reality, the Euclidean structure as well as the spacetime structure appear in parallel. Observers only see the spacetime structure. Physics is a science that focusses on observable information. My university, the TUE, targeted applied physics, and there was not much time nor support for diving deep into the fundamentals of quantum physics. After my study, I started a career in the high-tech industry where I joined the development of image intensifier devices. There followed my confrontation with optics and with the actual behavior of elementary particles. See: http://www.ephysics.eu/#_What_image_intensifiers reveal.

In the second part of my career, I devoted my time to establish a better way of generating software. I saw how the industry was very successful in the modular construction of hardware. The software was still developed as a monolithic system. My experiences in this trial are reported in “Story of a War Against Software Complexity”; http://vixra.org/abs/1101.0061, and “Managing the Software Generation Process”; http://vixra.org/abs/1101.0062. It taught me the power of modular design and modular construction.

Only after my retirement, I got enough time to dive deep into the foundations of physical reality. In 2009 after the recovery of severe disease, I started my personal research project that in 2011 got its current name “The Hilbert Book Model.” For the rest of his life, the author takes the freedom to upgrade the related papers at a steady rate.

2 Intention

Theoretical physics still contains unresolved subjects. These deficiencies of the theory are caused by the way that physics was developed and by the attitude of the physicists that designed the current theory. Scientists take great care to secure the trustworthiness of their work, which ends in the publication of the results. They take measures to prevent that their publications get intermingled with badly prepared publications or even worse, with descriptions of fantasies. For that reason, they invented the scientific method. In applied physics, the scientific method founds on observations. Applied physics flourishes because the descriptions of observations help to explore these findings, especially when formulas extend the usability of the observations beyond direct observation. In theoretical physics, this is not always possible because not all aspects of physical reality are observable. The only way of resolving this blockade is to start from a proper foundation that can be extended via trustworthy methods that rely on deduction. This approach can only be successful if the deduction process is guided and restricted such that the extensions of the foundation still describe physical reality. Thus, if a mathematical deduction is applied, then mathematics must guide and restrict this process such that a mathematically consistent extension of the model is again a valid model of physical reality. After a series of development steps, this approach must lead to a structure and behavior of the model that more and more conforms to the reality that we can observe.

This guidance and restriction are not self-evident. On the other hand, we know that when we investigate deeper, the structure becomes simpler and easier comprehensible. So, finally, we come to a fundamental structure that can be considered as a suitable foundation. The way back to more complicated levels of the structure cannot be selected freely. Mathematics must pose restrictions onto the extension of the fundamental structure. This happens to be true for a foundation that was discovered about eighty years ago by two scholars. They called their discovery quantum logic. The scholar duo selected the name of this relational structure because its relational structure resembled closely the relational structure of the already known classical logic. Garrett Birkhoff was an expert in relational structures. These are sets that precisely define what relations are tolerated between the elements of the set. Mathematicians call these relational structures lattices, and they classified quantum logic as an orthomodular lattice. John von Neumann was a broadly oriented scientist that together with others was searching for a platform that was suitable for the modeling of quantum mechanical systems. He long doubted between two modeling platforms. One was a projective geometry, and the other was a Hilbert space. Finally, he selected Hilbert spaces. In their introductory paper, the duo showed that quantum logic emerges into a separable Hilbert space. The set of closed subspaces inside a separable Hilbert space has exactly the relational structure of an orthomodular lattice. The union of these subspaces equals the Hilbert space. A separable Hilbert space applies an underlying vector space, and between every pair of vectors, it defines an inner product. This inner product can only apply numbers that are taken from an associative division ring. In a division ring, every non-zero member owns a unique inverse. Only three suitable division rings exist. These are the real numbers, the complex numbers, and the quaternions. Depending on their dimension these number systems exist in several versions that differ in the way that Cartesian and polar coordinate systems sequence their members.

In the Hilbert space, operators exist that can map the Hilbert space onto itself. In this way, the operator can map some vectors along themselves. The inner product of a normalized vector with such a map produces an eigenvalue. This turns the vector into an eigenvector. Together the eigenvalues of an operator form its eigenspace. This story indicates that mathematics guides and restricts the extension of the selected foundation into more complicated levels of the structure. It shows that the scholar duo started a promising development project.

However, this initial development was not pursued much further. Axiomatic models of physical reality are not popular. Most physicists mistrust this approach. Probably these physicists consider it naïve to suspect that an axiomatic foundation can be discovered that like the way that a seed evolves in a certain type of plant, will evolve into the model of the physical reality that we can observe.

Most quantum physicists decided to take another route that much more followed the line of the physical version of the scientific method. As could be suspected this route gets hampered by the fact that not every facet of physical reality can be verified by suitable experiments.

Mainstream quantum physics took the route of quantum field theory, which diversified into quantum electrodynamics and quantum chromodynamics. It bases on the principle of least action, the Lagrangian equation and the path integral. However, none of these theories apply a proper foundation.

In contrast, the Hilbert Book Model Project intends to provide a purely and self-consistent mathematical model of physical reality. It uses the orthomodular lattice as its axiomatic foundation and applies some general characteristics of reality as guiding lines. An important ingredient is the modular design of most of the discrete objects that exist in the universe. Another difference is that the Hilbert Book Model relies on the control of coherence and binding by stochastic processes that own a characteristic function instead of the weak and strong forces and the force carriers that QFT, QED, and QCD apply.

Crucial to the Hilbert Book Model is that reality applies quaternionic Hilbert spaces as structured read-only archives of the dynamic geometric data of the discrete objects that exist in the model. The model stores these data before they can be accessed by observers. This fact makes it possible to interpret the model as the creator of the universe.

3 The Hilbert Book Base Model

The Hilbert Book Model Project deviates considerably from the mainstream approaches. It tries to stay inside a purely mathematical model that can be deduced from the selected foundation. First, it designs a base model that is configured from a huge set of quaternionic separable Hilbert spaces that all share the same underlying vector space. One of these separable Hilbert spaces takes a special role and acts as a background platform. It has an infinite dimension, and it owns a unique non-separable Hilbert space that embeds its separable companion. Together these companion Hilbert spaces form the background platform of the base model. A reference operator manages the private parameter space of each separable Hilbert space. The elements of the version of the number system that the Hilbert space uses for specifying its inner products constitute this parameter space. These private parameter spaces float with their geometric center over the private parameter space of the background platform. Via the applied coordinate systems, the parameter spaces determine the symmetry of the corresponding Hilbert space. An elementary module resides on each floating separable Hilbert space. The eigenspace of a dedicated footprint operator archives the complete life story of this elementary module. After sequencing the real parts of these eigenvalues, the archive tells the life story of the point-like object as an ongoing hopping path that recurrently regenerates a coherent hop landing location swarm. The location density distribution that describes the swarm equals the square of the modulus of what physicists would call the wavefunction of the elementary module. Mainstream quantum physics calls the elementary modules elementary particles. They behave as elementary modules, but mainstream physics does not exploit that interpretation. In contrast, the Hilbert Book Model Project exploits the modular design of the model.

In fact, the sequencing defines a subspace of the underlying vector space that scans as a function of progression over the whole model. This scanning window divides the model into a historic part, a window that represents the current static status quo, and a future part. In this way, the dynamic model resembles the paging of a book in which each page tells a universe-wide story of what currently happens in this continuum. This explains the name of the Hilbert Book Model. Together with the requirement that all applied separable Hilbert spaces share the same vector space the fact that a window scans the Hilbert Book Base Model as a function of a progression parameter results in the fact that these quaternionic separable Hilbert spaces share the same real number based separable Hilbert space. After sequencing the eigenvalues, the eigenspace of the reference operator of this Hilbert space acts as a model wide proper time clock.

In contrast to the Hilbert Book Model, most other physical theories apply only a single Hilbert space that applies complex numbers for defining its inner product, or they apply a Fock space, which is a tensor product of complex number based Hilbert spaces. A tensor product of quaternionic Hilbert spaces results in a real number based Hilbert space. In the Hilbert Book Base Model, the quaternionic separable Hilbert spaces share the same real number based Hilbert space.

The coherence of the hop landing location swarm that configures the footprint of an elementary module is ensured by the fact that the mechanism that generates the hop landing locations is a stochastic process that owns a characteristic function. This characteristic function is the Fourier transform of the location density distribution of the hop landing location swarm. The mechanism reflects the effect of the ongoing embedding of the separable Hilbert space of the elementary module into the background non-separable Hilbert space. A continuum eigenspace of a dedicated operator registers the embedding of the hop landings of all elementary modules into this continuum. The continuum corresponds to the dynamic field that physicists call the universe. This field acts as the living space of all discrete objects that exist in the universe.

4 Modeling dynamic fields and discrete sets

The eigenspace of a dedicated footprint operator in a quaternionic separable Hilbert space can represent the dynamic geometric data of the point-like object that resides on this Hilbert space. The eigenspace of operators in a quaternionic non-separable Hilbert space can, in addition, represent the description of a dynamic continuum. We already met the eigenspace of the reference operator, which represents the private parameter space of the Hilbert space. In the separable Hilbert space this eigenspace is countable and contains only the rational values of the version of the quaternionic number system that the separable Hilbert space can apply as eigenvalues. In the non-separable Hilbert space, the eigenspace of the reference operator also contains all the limits of the congruent series of rational values. Consequently, this eigenspace is no longer countable. In each of the applied Hilbert spaces, it is possible to use the reference operator to define a category of newly defined operators by taking for each eigenvector of the reference operator a new eigenvalue that equals the target value of a selected quaternionic function for the parameter value that equals the corresponding eigenvalue of the reference operator. In the quaternionic separable Hilbert space the new eigenspace represents the sampled field that is described by the selected quaternionic function. In the quaternionic non-separable Hilbert space the new eigenspace represents the full continuum that is described by the selected quaternionic function. Continuum eigenspaces can represent the mathematical equivalent of a dynamic physical field. The private parameter space of a quaternionic Hilbert space represents a flat field. The dynamics of a field can be described by quaternionic differential equations.

Quaternionic second order partial differential equations describe the interaction between point-like actuators and a dynamic field. Physical fields differ from mathematical fields by the fact that the value of the physical field is represented in physical units. All basic fields obey the same quaternionic differential and integral equations. The basic fields differ in their start and boundary conditions.

4.1 Quaternionic differential calculus

The first order partial differential equations divide the change of a field in five different parts that each represent a new field. We will represent the field change operator by a quaternionic nabla operator. This operator behaves as a quaternionic multiplier.

A quaternion can store a time-stamp in its real part and a three-dimensional spatial location in its imaginary part. The quaternionic nabla ∇ acts as a quaternionic multiplying operator. Quaternionic multiplication obeys the equation

$$c = c_r + \vec{c} = ab = \left(a_r + \vec{a}\right)\left(b_r + \vec{b}\right) = a_r b_r - \left\langle \vec{a}, \vec{b} \right\rangle + a_r \vec{b} + \vec{a} b_r \pm \vec{a} \times \vec{b} \tag{4.1.1}$$

The $\pm$ sign indicates the freedom of choice of the handedness of the product rule that exists when selecting a version of the quaternionic number system. The first order partial differential follows from

$$\nabla = \left\{ \frac{\partial}{\partial \tau}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\} = \nabla_r + \vec{\nabla} \tag{4.1.2}$$

The spatial nabla $\vec{\nabla}$ is well-known as the del operator and is treated in detail in Wikipedia.

$$\phi = \nabla \psi = \left(\frac{\partial}{\partial \tau} + \vec{\nabla} \right)\left(\psi_r + \vec{\psi}\right) = \nabla_r \psi_r - \left\langle \vec{\nabla}, \vec{\psi} \right\rangle + \nabla_r \vec{\psi} + \vec{\nabla} \psi_r \pm \vec{\nabla} \times \vec{\psi} \tag{4.1.3}$$

The differential $\nabla\psi$ describes the change of field ψ . The five separate terms in the first order partial differential have a separate physical meaning. All basic fields feature this decomposition. The terms may represent new fields.

$$\phi_r = \nabla_r \psi_r - \left\langle \vec{\nabla}, \vec{\psi} \right\rangle \tag{4.1.4}$$

$$\vec{\phi} = \nabla_r \vec{\psi} + \vec{\nabla}\psi_r \pm \vec{\nabla} \times \vec{\psi} = -\vec{E} \pm \vec{B} \tag{4.1.5}$$

$\vec{\nabla} f$ is the gradient of f .

$\left\langle \vec{\nabla}, \vec{f} \right\rangle$ is the divergence of $\vec{f}$.

$\vec{\nabla} \times \vec{f}$ is the curl of $\vec{f}$.

The conjugate of the quaternionic nabla operator defines another type of field change.

$$\nabla^* = \nabla_r - \vec{\nabla} \tag{4.1.6}$$

$$\zeta = \nabla^* \phi = \left(\frac{\partial}{\partial \tau} - \vec{\nabla} \right) \left(\phi_r + \vec{\phi} \right) = \nabla_r \phi_r + \left\langle \vec{\nabla}, \vec{\phi} \right\rangle + \nabla_r \vec{\phi} - \vec{\nabla} \phi_r \mp \vec{\nabla} \times \vec{\phi} \tag{4.1.7}$$

4.2 Field excitations

Field excitations are solutions of second order partial differential equations.

One of the second order partial differential equations results from combining the two first-order partial differential equations $\phi = \nabla\psi$ and $\zeta = \nabla^* \phi$.

$$\begin{aligned} \zeta &= \nabla^* \varphi = \nabla^* \nabla \psi = \nabla \nabla^* \psi = \left(\nabla_r + \vec{\nabla} \right) \left(\nabla_r - \vec{\nabla} \right) \left(\psi_r + \vec{\psi} \right) \\ &= \left(\nabla_r \nabla_r + \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \right) \psi \end{aligned} \tag{4.2.1}$$

Integration over the time domain results in the Poisson equation

$$\rho = \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \psi \tag{4.2.2}$$

Under isotropic conditions, a very special solution of this equation is the Green's function $\frac{1}{\vec{q} - \vec{q}'}$ of the affected field. This solution is the spatial Dirac $\delta\left(\vec{q}\right)$ pulse response of the field under strict isotropic conditions.

$$\nabla \frac{1}{\vec{q} - \vec{q}'} = -\frac{\left(\vec{q} - \vec{q}' \right)}{\left| \vec{q} - \vec{q}' \right|^3} \tag{4.2.3}$$

$$\left\langle \vec{\nabla},\vec{\nabla} \right\rangle \frac{1}{\left|\vec{q}-\vec{q}'\right|} \equiv \left\langle \vec{\nabla},\vec{\nabla} \frac{1}{\left|\vec{q}-\vec{q}'\right|} \right\rangle = -\left\langle \vec{\nabla}, \frac{\left(\vec{q}-\vec{q}'\right)}{\left|\vec{q}-\vec{q}'\right|^3} \right\rangle = 4\pi\delta\left(\vec{q}-\vec{q}'\right) \tag{4.2.4}$$

Under these conditions, the dynamic spherical pulse response of the field is a solution of a special form of the equation (4.2.1)

$$\left(\nabla_r \nabla_r + \left\langle \vec{\nabla},\vec{\nabla} \right\rangle\right)\psi = 4\pi\delta\left(\vec{q}-\vec{q}'\right)\theta\left(\tau \pm \tau'\right) \tag{4.2.5}$$

Here $\theta(\tau)$ is a step function and $\delta(\vec{q})$ is a Dirac pulse response. After the instant τ', this solution is described by

$$\psi = \frac{f\left(\left|\vec{q}-\vec{q}'\right| \pm c\left(\tau-\tau'\right)\vec{n}\right)}{\left|\vec{q}-\vec{q}'\right|} \tag{4.2.6}$$

The normalized vector $\vec{n}$ can be interpreted as the spin of the solution. The spherical pulse response acts either as an expanding or as a contracting spherical shock front. Over time this pulse response integrates into the Green's function. This means that the expanding pulse injects the volume of the Green's function into the field. Subsequently, the front spreads this volume over the field. The contracting shock front collects the volume of the Green's function and sucks it out of the field. The $\pm$ sign in equation (4.2.5) selects between injection and subtraction.

Apart from the spherical pulse response equation (4.2.5) supports a one-dimensional pulse response that acts as a one-dimensional shock front. This solution is described by

$$\psi = f\left(\left|\vec{q}-\vec{q}'\right| \pm c\left(\tau-\tau'\right)\vec{n}\right) \tag{4.2.7}$$

Here, the normalized vector $\vec{n}$ can be interpreted as the polarization of the solution. Shock fronts only occur in one and three dimensions. A pulse response can also occur in two dimensions, but in that case, the pulse response is a complicated vibration that looks like the result of a throw of a stone in the middle of a pond.

Equations (4.2.1) and (4.2.2) show that the operators $\frac{\partial^2}{\partial\tau^2}$ and $\left\langle \vec{\nabla},\vec{\nabla} \right\rangle$ are valid second order partial differential operators. These operators combine in the quaternionic equivalent of the wave equation.

$$\varphi = \left(\frac{\partial^2}{\partial\tau^2} - \left\langle \vec{\nabla},\vec{\nabla} \right\rangle\right)\psi \tag{4.2.8}$$

This equation also offers one-dimensional and three-dimensional shock fronts as its solutions.

$$\psi = \frac{f\left(\left|\vec{q}-\vec{q}'\right| \pm c\left(\tau-\tau'\right)\right)}{\left|\vec{q}-\vec{q}'\right|} \tag{4.2.9}$$

$$\psi = f\left(\left|\vec{q}-\vec{q}'\right| \pm c\left(\tau-\tau'\right)\right) \tag{4.2.10}$$

These pulse responses do not contain the normed vector $\vec{n}$. Apart from pulse responses, the wave equation offers waves as its solutions.

By splitting the field into the time-dependent part $T(\tau)$ and a location dependent part, $A(\vec{q})$, the homogeneous version of the wave equation can be transformed into the Helmholtz equation

$$\frac{\partial^2 \psi}{\partial \tau^2} = \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \psi = -\omega^2 \psi \tag{4.2.11}$$

$$\psi(\vec{q}, \tau) = A(\vec{q}) T(\tau) \tag{4.2.12}$$

$$\frac{1}{T} \frac{\partial^2 T}{\partial \tau^2} = \frac{1}{A} \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle A = -\omega^2 \tag{4.2.13}$$

$$\left\langle \vec{\nabla}, \vec{\nabla} \right\rangle A + \omega^2 A \tag{4.2.14}$$

The time-dependent part $T(\tau)$ depends on initial conditions, or it indicates the switch of the oscillation mode. The switch of the oscillation mode means that temporarily the oscillation is stopped and instead an object is emitted or absorbed that compensates the difference in potential energy. The location-dependent part of the field $A(\vec{q})$ describes the possible oscillation modes of the field and depends on boundary conditions. The oscillations have a binding effect. They keep the moving objects within a bounded region. See https://en.wikipedia.org/wiki/Hyperbolic_partial_differential_equation#Hyperbolic_system_and_conservation_laws.

For three-dimensional isotropic spherical conditions, the solutions have the form

$$A(r, \theta, \varphi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \left\{ \left(a_{lm} j_l(kr) \right) + b_{lm} Y_l^m(\theta, \varphi) \right\} \tag{4.2.15}$$

Here j_l and y_l are the spherical Bessel functions, and Y_l^m are the spherical harmonics. These solutions play a role in the spectra of atomic modules.

A more general solution is a superposition of these basic types.

The paper treats quaternionic differential equations more extensively in chapter 14.

5 Photons

5.1 Photon structure

Photons are one-dimensional objects that are strings of equidistant energy packages, such that the string obeys the Einstein-Planck relation

$$E = h\nu \tag{5.1.1}$$

The energy packages are implemented by one-dimensional shock fronts that possess a polarization vector.

5.2 One-dimensional pulse responses

One-dimensional pulse responses that act as one-dimensional shock fronts and possess a polarization vector are solutions of the equation (4.2.5) and are described by the equation (4.2.7).

$$\psi = f\left(\left|\vec{q} - \vec{q}'\right| \pm c(\tau - \tau')\vec{n}\right) \tag{5.1.2}$$

During travel, the front $f(\vec{q})$ keeps its shape and its amplitude. So also, during long-range trips, the shock front does not lose its integrity. The one-dimensional pulse response represents an energy package that travels with speed c through its carrier field. The energy of the package has a standard value.

5.3 Photon integrity

Except for its speed, the photon emitter determines the properties of the photon. These properties are its frequency, its energy and its polarization. The energy packages preserve their own integrity. They travel with constant speed and follow a worldline. Photon emission possesses a fixed duration. It is not an instant process. During emission the emitter must not move and can only rotate around the direction of travel. Failing these requirements will compromise the integrity of the photon and make it impossible for a tiny absorber to capture the full photon. In that case the energy packages will spray and fly to multiple locations. Consequently, they will act as dark energy objects.

The absorption of a photon by an atom requires an incredible aiming precision of the emitter. In fact this emission can only be comprehended when it is interpreted as the time reversal of the corresponding emission process. If the absorbing atom cannot cope with the full energy of the photon, then it might absorb only part of the energy packages of the photon. The rest will stay on its route to a next absorber. Absorbing individual energy packages will result in the increase of the kinetic energy of the absorber. Absorbing the full photon or a part of it, will result in an increase of the potential energy of the absorber. Usually this results in a higher oscillation mode of one of the components of the absorber.

5.4 Light

Light is a dynamic spatial distribution of photons. Often the location density distribution of photons owns a Fourier transform. In that case, light may show wave behavior. Photons are one-dimensional particles, that feature a private frequency and energy. Single photons do not show wave behavior. Photons and light waves will feature different frequencies.

5.5 Optics

Optics is the science of imaging distributions of particles that can be characterized by a location density distribution and a corresponding Fourier transform of that location density distribution. Even though photons have a fixed non-zero spatial length, optics will treat these particles as point-like

objects. Another name for the location density distribution is point spread function (PSF). Another name for the Fourier transform of the PSF is the optical transfer function (OTF). Apart from a location density distribution, the swarm of the particles is also characterized by an angular distribution and by an energy distribution. In case of photons, the energy distribution is also a chromatic distribution.

A linearly operating imaging device can be characterized by its point spread function or alternatively by its OTF. This point spread function is an image of a point-like object. The PSF represents the blur that is introduced by the imaging device. For a homogeneous distribution of particle properties, the OTF of a chain of linearly operating imaging devices equals the product of the OTF's of the separate devices.

The imaging properties of an imaging device may vary as a function of the location and the orientation in the imaging surface.

6 Modular design and construction

The discrete objects that exist in the universe show a modular design. In modular configurations, elementary particles behave as elementary modules. Together they constitute all modules that exist in the universe. Some modules constitute modular systems.

Also, photons show a modular structure.

6.1 Elementary modules

6.1.1 Symmetry-related charge

Elementary modules are very complicated objects that reside on a private platform, which possesses some of the characteristic properties of the elementary module. These properties establish the type of the elementary module.

Elementary modules reside on a private Hilbert space, which uses a selected version of the quaternionic number system to specify its inner products. Consequently, the operators in this Hilbert space apply members of this version to specify its eigenvalues. The eigenspace of this operator reflects the properties of this version. Thus, the eigenspace of the reference operator reflects the symmetry of the Hilbert space. Its geometric center floats over the background parameter space. The symmetry is defined relative to the symmetry of the background platform. Mathematics can compare these differences if the axes of the Cartesian coordinate systems in these parameter spaces are parallel to each other. The model applies the Stokes theorem and the Gauss theorem to determine the effect of the symmetry differences. See section 16.3. The only freedoms that are left are the locations of the geometric centers of the parameter spaces and the way that the elements of the versions of the number systems are sequenced along the axes. These restrictions reduce the list of symmetry differences to a short list. It means that the elementary modules exist in a small number of different symmetry related categories. The symmetry difference is represented by a symmetry-related charge that resides at the geometric center of the private parameter space. The opposed restrictions that determine the allowable versions of the quaternionic number system restrict the list of values of symmetry-related charges to $-3,-2,-1,0,+1,+2,+3$. The isotropic symmetry differences are represented by $-3,0,+3$

The symmetry-related charges correspond to symmetry-related fields. At the location of the charge, a source or a sink generates a corresponding potential.

The anisotropic differences spread over the three coordinate axes and are indicated by corresponding RGB color charges. If we extend this distinguishing to the real axis of the parameter spaces, then the anti-color charges add to the three RGB color charges. Further, the product rule of the quaternions introduces diversity in the handiness of the version of the number system. The polar coordinate system also allows the polar angle and the azimuth to run up or down. The range of the polar angle is π radians. The range of the azimuth is 2π radians. This freedom of choice adds to the freedom that is left by the Cartesian coordinate system.

The first conclusion is that elementary modules exist in a short list of categories that differ in their symmetry-related properties, in their angular range properties, and in their arithmetic properties.

6.2 Modular configuration

The elementary modules can combine into composed modules. Some modules combine into modular systems. However, not all modules can compose with arbitrary other modules. For example, symmetry-related charges that have the same sign will repel each other, while symmetry-related charges with a different sign will attract. Composition applies internal oscillation of the components of the module. This is explained in the next section. Only elementary modules with the proper angular symmetry can take part in the modular composition process. These elementary modules are called fermions. The other elementary modules are called bosons. Fermions cannot share the same oscillation mode and cannot share the same angular properties, such as spin. The binding via internal oscillation must be supported by the attraction that is caused by deformation of the embedding field. The symmetry related charges also influence the efficiency of the bond. The anisotropic elementary modules cannot themselves deform the embedding field. They must first combine into colorless hadrons before their combination can deform the embedding field. Physicists call this phenomenon color confinement.

The hop landings of isotropic elementary modules can produce spherical pulse responses that deform the embedding field. Similarly. the hop landings of hadrons can produce such spherical pulse responses.

6.3 Stochastic control

For each elementary module, a private stochastic process generates the hop landing locations in the ongoing hopping path that recurrently regenerates the coherent hop landing location swarm that constitutes the footprint of the elementary module. Only for isotropic elementary modules, the hop landings can deform the embedding field. The footprints of anisotropic elementary modules must first combine into colorless hadrons before these footprints can deform the embedding field. This phenomenon is known as color confinement.

The type of stochastic process that generates the footprint of elementary modules owns a characteristic function that equals the Fourier transform of the location density distribution of the coherent hop landing location swarm. It is possible to interpret the stochastic process as a spatial Poisson point process in $\mathbb{R}^3$. The intensity function of this process is implemented by a spatial point spread function that equals the location density distribution of the generated hop landing location swarm. The eigenspace of the footprint operator archives the target values of a quaternionic function, whose spatial part describes the point spread function. A cyclic random distribution describes the real parts of these target values. After sequencing these real parts, the eigenspace describes the ongoing hopping path of the elementary module.

The location density distribution can be interpreted as a detection probability density distribution. If it has a Fourier transform, then a kind of uncertainty principle exists between the standard deviation of the detection probability density distribution and the standard deviation of the modulus of this Fourier transform. If the standard deviation of the modulus of this Fourier transform increases, then the standard deviation of the detection probability density distribution decreases (and vice versa).

A second type of stochastic process controls composed modules. This process also owns a characteristic function. This characteristic function is a dynamic superposition of the characteristic functions of the components of the module. The superposition coefficients act as displacement generators. In this way, these coefficients control the internal positions of the components. Inside atoms, these components perform their own oscillation mode. All modules attach an extra displacement generator to their characteristic function. This displacement generator determines the location of the full module.

This analysis tells that the characteristic functions, which reside in Fourier space define the constitution of the module. In Fourier space spatial locality has no meaning. It means that the components of a module can be far apart. The phenomenon is known as entanglement. Only the attracting influences of potentials can keep components closely together.

6.4 Benefits of modular design and construction

Modular design hides relations that are only relevant inside the module from the outside of the module. In this way, the modular design reduces the relational complexity of the construction of composed modules. This is further improved by the possibility to gather relations in standard interfaces. This standardization promotes the reusability of modules. The fact that composed modules can be generated from lower level modules has an enormously beneficial effect on the reduction of the relational complexity of the modular composition process.

By applying modular design, the creator has prepared the universe for modular construction, which is a very efficient way of generating new objects. However, modular configuration of objects involves the availability of modules that can be joined to become higher level modules or modular systems. This means that enough resources must be available at the proper place and the proper time. The generation of a module out of composing modules makes sense when the new module has a profitable functionality. An advantage can be that the new module or modular system has a better chance of survival in a competitive environment. In that case, stochastic modular design can easily win from monolithic design. Evolution can evolve with a pure stochastic modular design. However, as soon as intelligent species are generated as modular systems, then these individuals can take part in the control of evolution by intelligent modular design. Intelligent modular design and construction occur much faster than stochastic modular design and construction.

6.4.1 Modular hierarchy

The modular hierarchy starts with elementary modules. Elementary modules exist in several types that differ in their basic properties.

These basic properties are their symmetry-related charge, their spin, and their regeneration cycle.

6.4.2 Compound modules

Compound modules are composed-modules for which the geometric centers of the platforms of the components coincide. The charges of the platforms of the elementary modules establish the binding of the corresponding platforms. Physicists and chemists call these compound modules atoms or atomic ions.

In free compound modules, the symmetry-related charges do not take part in the oscillations. The targets of the private stochastic processes of the elementary modules oscillate. This means that the hopping path of the elementary module folds around the oscillation path and the hop landing location swarm gets smeared along the oscillation path. The oscillation path is a solution of the Helmholtz equation. Each fermion must use a different oscillation mode. A change of the oscillation mode goes together with the emission or the absorption of a photon. The center of emission coincides with the geometrical center of the compound module. During the emission or absorption, the oscillation mode and the hopping path halt, such that the emitted photon does not lose its integrity. Since all photons share the same emission duration, that duration must coincide with the regeneration cycle of the hop landing location swarm. Absorption cannot be interpreted so easily. In fact, it can only be comprehended as a time-reversed emission act. Otherwise, the absorption would require an incredible aiming precision for the photon.

The type of stochastic process that controls the binding of components appears to be responsible for the absorption and emission of photons and the change of oscillation modes. If photons arrive with too low energy, then the energy is spent on the kinetic energy of the common platform. If photons arrive with too high energy, then the energy is distributed over the available oscillation modes, and the rest is spent on the kinetic energy of the common platform. The process must somehow archive

the modes of the components. It can apply the private platform of the components for that purpose. Most probably the current value of the dynamic superposition coefficient is stored in the eigenspace of a special superposition operator.

6.4.3 Molecules

Molecules are conglomerates of compound modules that each keep their private geometrical center. However, electron oscillations are shared among the compound modules. Together with the symmetry-related charges, this binds the compound modules into the molecule.

6.4.4 Consciousness and intelligence

In the Hilbert Book Model, all modules are considered to act as observers. That does not mean that these modules react to the perceived information in a conscious or intelligent way. In the hierarchy of modular systems, compared to intelligence, consciousness already enters at lower levels of complexity. However, consciousness cannot be attributed to non-living modular systems. Primitive life forms have primitive degrees of consciousness.

Intelligent species show self-reflection and can create strategies that guard their type-community or their social-community. Conscious species can also develop such guarding measures, but that is usually a result of trial and error instead of a developed strategy. The strategy is then inherited via genes.

For intelligent species, the modular design strategy of the creator can be an inspiration.

- Modular design is superior to monolithic design.
- Modular construction works economically with resources.
- It is advantageous to have access to a large number and a large diversity of suitable modules.
- Create module-type communities.
- Type communities survive far longer than the corresponding individual modules.
- Members must guard their module type community.
- Type communities may inherit and cultivate the culture of their members.
- Modular systems must care about the type communities on which they depend.
- Modular systems must care about their living environment.
- Darwin's statement that the fittest individual will survive must be replaced by the statement that the module-type community survives that cares best for its members, its resources and its environment.

7 Dark objects and progression zigzag

The effects of the shock fronts that are caused by pulses are so tiny that no measuring instrument will ever be able to detect the presence of the single shock fronts. Thus, these field excitations can rightfully be called dark objects or more in detail dark energy and dark matter. These objects become noticeable in huge coherent ensembles that may contain about 10^{10} elements. The one-dimensional shock fronts combine in photons and the spherical shock fronts combine in the footprints of elementary particles. They can exchange roles in pair production and pair annihilation events. For observers these events pose interpretation problems. However, the model can interpret these events as time reversal that converts a particle into its antiparticle or vice versa. This interpretation relies on the mass-energy equivalence and on the fact that during the conversion each one-dimensional shock front is exchanged against a spherical shock front. In this interpretation elementary particles can zigzag through the time domain. This vision suggests that elementary particles never die, but at the utmost change the direction of their life story and turn into its

antiparticle. The conversion does not happen instantaneously. It takes the full regeneration cycle of the hop landing location swarm of the elementary particle. The universe wide proper time clock ticks with a frequency of about 10^{20} ticks per second and the regeneration then takes about 10^{10} proper time clock ticks.

In huge numbers, spurious dark objects may still cause noticeable influences. The halo of dark matter around galaxies is known to produce gravitational lensing effects.

Even though the Hilbert Book Model does not consider the shock fronts as the lowest level of modules, the shock fronts together constitute all discrete objects that exist in the universe.

The Hilbert Book model considers elementary modules as the lowest level modules. The are complicated constructs that consist of a quaternionic separable Hilbert space, a selected version of the quaternionic number system and a private stochastic process that generates their life story.

8 Gravitation

8.1 A deforming field excitation

A spherical pulse response is a solution of a homogeneous second order partial differential equation that was triggered by an isotropic pulse. The corresponding field equation and the solution corresponding solution are repeated here.

$$\left(\nabla_r\nabla_r + \left\langle\vec{\nabla},\vec{\nabla}\right\rangle\right)\psi = 4\pi\delta\left(\vec{q}-\vec{q'}\right)\theta\left(\tau\pm\tau'\right) \tag{8.1.1}$$

Here the $\pm$ sign represents time inversion.

$$\psi = \frac{f\left(\left|\vec{q}-\vec{q'}\right|\pm c\left(\tau-\tau'\right)\vec{n}\right)}{\left|\vec{q}-\vec{q'}\right|} \tag{8.1.2}$$

The spherical pulse response integrates over time into the Green's function of the field. The Green's function is a solution of the Poisson equation.

$$\rho = \left\langle\vec{\nabla},\vec{\nabla}\right\rangle\psi \tag{8.1.3}$$

The Green's function occupies some volume.

$$g(\vec{q}) = \frac{1}{\vec{q}-\vec{q'}} \tag{8.1.4}$$

This means that locally the pulse pumps some volume into the field, or it subtracts volume out of the field. The selection between injection and subtraction depends on the sign in the step function in the equation (8.1.1). The dynamics of the spherical pulse response shows that the injected volume quickly spreads over the field. In the case of volume subtraction, the front first collects the volume and finally subtracts it at the trigger location. Gravitation considers the case in which the pulse response injects volume into the field.

Thus, locally and temporarily, the pulse deforms the field, and the injected volume persistently expands the field.

This paper postulates that the spherical pulse response is the only field excitation that temporarily deforms the field, while the injected volume persistently expands the field.

The effect of the spherical pulse response is so tiny and so temporarily that no instrument can ever measure the effect of a single spherical pulse response in isolation. However, when recurrently regenerated in huge numbers in dense and coherent swarms the pulse responses can cause a significant and persistent deformation that instruments can detect. This is achieved by the stochastic processes that generate the footprint of elementary modules.

The spherical pulse responses are straightforward candidates for what physicists call dark matter objects. A halo of these objects can cause gravitational lensing.

8.2 Gravitation potential

The gravitation potential that an elementary module causes can be approached by the convolution of the Green's function of the field and the location density distribution of the hop landing location swarm. This approximation is influenced by the fact that the deformations, which are due to the

individual pulse responses quickly fade away. Further, the density of the location distribution affects the efficiency of the deformation.

The Green's function describes the result of a point-like pulse whose response has a mass of its own. We know how to compute the mass of a distribution of point masses. At some distance of the center of the swarm, the gravitation potential can be approximated by

$$g(r) \approx \frac{Gm}{r} \tag{8.2.1}$$

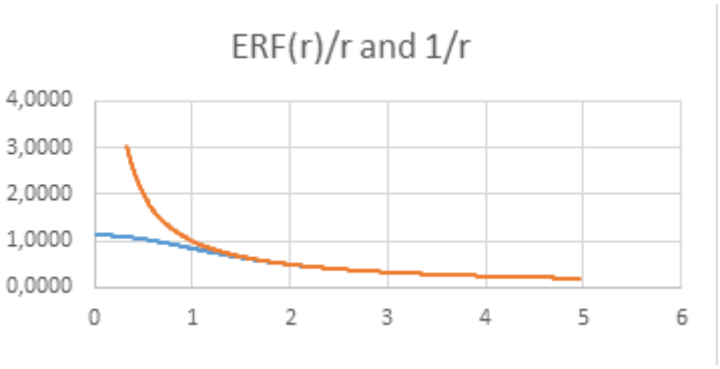
where m is the mass of the particle and r equals the distance to the center. Here we omit the physical units. G is the gravitational constant. More exactly, the gravitation potential of the elementary module can be approximated by taking the convolution of the location density distribution of the hop landing location swarm. If we do this for example for a Gaussian location density distribution, then the convolution results in

$$g(r) \approx Gm\frac{ERF(r)}{r} \tag{8.2.2}$$

Where $ERF(r)$ is the well-known error function. Here the gravitation potential is a perfectly smooth function that at some distance from the center equals the approximated gravitation potential that was described above in equation (8.2.1). The convolution only offers an approximation because this computation does not account for the influence if the density of the swarm and it does not compensate for the fact that the deformation by the individual pulse responses quickly fades away. Thus, the exact result depends on the duration of the recurrence cycle of the swarm.

In the example, the distribution might have a higher amplitude.

This might explain why some elementary module types exist in three generations.



8.3 Regeneration

The generation of the hopping path is an ongoing process. The generated hop landing location swarm contains a huge number of elements. Each elementary module type is controlled by a corresponding type of stochastic process. For the stochastic process, only the Fourier transform of the location density distribution of the swarm is important. Consequently, for a selected type of elementary module, it does not matter at what instant of the regeneration of the hop landing location swarm the location density distribution is determined. Thus, even when different types are bonded into composed modules, there is no need to synchronize the regeneration cycles of different

types. This freedom also means that the number of elements in a hop landing location swarm may differ between elementary module types. This means that the strength of the deformation of the embedding field can differ between elementary module types. The strength of deformation relates to the mass of the elementary modules according to formula (8.2.1).

The requirement for regeneration introduces a great mystery. All generated mass appears to dilute away and must be recurrently regenerated. This fact conflicts with the conservation laws of mainstream physics. The deformation work done by the stochastic processes vanishes completely. What results is the ongoing expansion of the field. Thus, these processes must keep generating the particle to which they belong.

Only the ongoing embedding of the content that is archived in the floating platform into the embedding field can explain the activity of the stochastic process. This supposes that at the instant of creation, the creator already archived the dynamic geometric data of his creatures into the eigenspaces of the footprint operators. These data consist of a scalar time-stamp and a three-dimensional spatial location. The quaternionic eigenvalues act as storage bins.

After the instant of creation, the creator left his creation alone. The set of floating separable Hilbert spaces, together with the background Hilbert space act as a read-only repository. After sequencing the time-stamps, the stochastic processes read the storage bins and trigger the embedding of the location into the embedding field.

8.4 Inertia

The relation between inertia and mass is complicated. It assumes that a field exists that tries to compensate for the change of the field when its vector part suddenly changes with time.

This special field supports the hop landing location swarm that resides on the floating platform. It reflects the activity of the stochastic process, and it floats with the platform over the background platform. It is characterized by a mass value and by the uniform velocity of the platform with respect to the background platform. The real part conforms to the deformation that the stochastic process causes. The imaginary part conforms to the moving deformation. The main characteristic of this field is that it tries to keep its overall change zero. We call ξ the ***deformation field***.

The first order change of a field contains five terms. Mathematically, the statement that in first approximation nothing in the field ξ changes indicates that locally, the first order partial differential $\nabla\xi$ will be equal to zero.

$$\zeta = \nabla\xi = \nabla_r\xi_r - \left\langle \vec{\nabla}, \vec{\xi} \right\rangle + \vec{\nabla}\xi_r + \nabla_r\vec{\xi} \pm \vec{\nabla}\times\vec{\xi} = 0 \tag{8.4.1}$$

The terms that are still eligible for change must together be equal to zero. These terms are.

$$\nabla_r\vec{\xi} + \vec{\nabla}\xi_r = 0 \tag{8.4.2}$$

In the following text plays $\vec{\xi}$ the role of the vector field and ξ_r plays the role of the scalar gravitational potential of the considered object. We approximate this potential by using formula (8.2.1).

The new field $\xi = \left\{ \frac{m}{r}, \vec{v} \right\}$ considers a uniformly moving mass as a normal situation. It is a combination of the scalar potential $\frac{m}{r}$ and the uniform speed $\vec{v}$.

If this object accelerates, then the new field $\left\{ \frac{m}{r}, \vec{v} \right\}$ tries to counteract the change of the field $\dot{\vec{v}}$ by compensating this with an equivalent change of the real part $\frac{m}{r}$ of the new field. This equivalent change is the gradient of the real part of the field.

$$-\vec{\nabla}\left(\frac{m}{r} \right) = \frac{m\vec{r}}{\left|\vec{r}\right|^3} \tag{8.4.3}$$

This generated vector field acts on masses that appear in its realm.

Thus, if two masses m_1 and m_2 exist in each other's neighborhood, then any disturbance of the situation will cause the gravitational force

$$\vec{F}\left(\vec{r}_1 - \vec{r}_2\right) = \frac{m_1 m_2 \left(\vec{r}_1 - \vec{r}_2\right)}{\left|\vec{r}_1 - \vec{r}_2\right|^3} \tag{8.4.4}$$

The disturbance by the ongoing expansion of the field suffices to put the gravitational force into action. The description also holds when the field ξ describes a conglomerate of platforms and m represents the mass of the conglomerate.

In compound modules such as ions and atoms, the field ξ of a component oscillates with the deformation rather than with the platform.

Inertia bases mainly on the definition of mass that applies to the region outside the sphere where the gravitation potential behaves as the Green's function of the field. There the formula $\xi_r = \frac{m}{r}$ applies. Further, it bases in the intention of modules to keep the gravitation potential inside the mentioned sphere constant. At least that holds when this potential is averaged over the regeneration period. In that case, the overall change ζ of the deformation field ξ equals zero. Next, the definition of the deformation field supposes that the swarm which causes the deformation moves as one unit. Further, the fact is used that the solutions of the homogeneous second order partial differential equation can superpose in new solutions of that same equation.

The popular sketch in which the deformation of our living space is presented by smooth dips is obviously false. The story that is represented in this paper shows the deformations as local extensions of the field, which represents the universe. In both sketches, the deformations elongate the information path, but none of the sketches explain why two masses attract each other. The above explanation founds on the habit of the stochastic process to recurrently regenerate the same time average of the gravitation potential, even when that averaged potential moves uniformly. Without the described habit of the stochastic processes, inertia would not exist.

Similar tricks can be used to explain the electrical force from the fact that the electrical field is produced by sources and sinks that can be described by the Green's function.

9 Black holes

9.1 Geometry

Black holes are regions of the field that are encapsulated by a surface that cannot be passed by spherical shock fronts. Only the shock fronts that locate at the border of the region can add volume to the region. Thus, the increase of the volume of that region is restricted by the surface of the encapsulation. This differs from free space, where stochastic processes can inject volume anywhere. Black holes represent the most efficient packaging of volume that stochastic processes can achieve.

Black holes are characterized by a Schwarzschild radius. It is the radius where the escape speed of massive objects equals light speed. The gravitational energy U of a massive object with mass m in a gravitation field of an object with mass M is

$$U = -\frac{GMm}{r} \tag{8.4.5}$$

The escape velocity follows from the initial energy $½mv^2$ of the object with mass m and velocity v.

$$½mv_0^2 - \frac{GMm}{r_0} = 0 \tag{8.4.6}$$

This results in

$$v_0 = \sqrt{2\frac{GM}{r_0}} \tag{8.4.7}$$

Due to the kinetic energy equivalence $h\nu = \frac{1}{2}m\nu^2$,this means for photons

$$U = -\frac{2GMh\nu}{rc^2} \tag{8.4.8}$$

The frequency ν of the photon changes with the radius r

$$h\nu = h\nu_0 - \frac{2GMh\nu_0}{rc^2} \tag{8.4.9}$$

This formula describes the gravitational redshift of photons. The radius at which the frequency ν has reduced to zero is the Schwarzschild radius r_s

$$r_s = \frac{2GM}{c^2} \tag{8.4.10}$$

9.2 The border of the black hole

For a non-rotating neutral black hole, photons cannot pass the sphere with the Schwarzschild radius r_s.

At the Schwarzschild radius, the escape velocity of massive objects equals the light speed.

It also means that one-dimensional shock fronts and spherical shock fronts cannot escape the sphere.

Spherical shock fronts can only add volume to the black hole when their actuator hovers over the region of the black hole. The injection increases the Schwarzschild radius. The injection also increases the mass M. An increase in the Schwarzschild radius means an increase in the volume of this sphere. This is like the injection of volume into the volume of the field that occurs via the pulses that generate the elementary modules. However, in this case, the volume stays within the Schwarzschild sphere. In both cases, the volume of the field expands.

The HBM postulates that the geometric center of an elementary module cannot enter the region of the black hole. This means that part of the active region of the stochastic process that produces the footprint of the elementary module can hover over the region of the black hole. In this overlap region, the pulses can inject volume into the black hole. Otherwise, no stochastic process could inject volume into the black hole.

The Schwarzschild sphere contains unstructured volume. No modules exist within that sphere.

The Bekenstein bound relates the Schwarzschild black hole to its entropy.

$$S \le \frac{\kappa ER}{hc} \Rightarrow S = \frac{\kappa ER}{hc} = \frac{2\kappa GM^2}{hc} \tag{8.5.1}$$

This indicates that the entropy S is proportional to the area of the black hole. This only holds for the entropy at the border of the black hole.

10 In the beginning

Before the stochastic processes started their action, the volume content of the universe was empty. It could be represented by a flat field. In the beginning, a huge number of these processes started their triggering of the dynamic field that represents the universe. From that moment on the universe started expanding. This did not happen at a single point. Instead, it happened at a huge number of locations that were distributed all over the spatial part of the parameter space of the quaternionic function that describes the dynamic field.

Close to the begin of time, all distances were equal to the distances in the flat parameter space. Soon, these islands were uplifted with volume that was emitted at nearby locations. This flooding created growing distances between used locations. After some time, all parameter space locations were reached by the generated shock waves. From that moment on the universe started acting as an everywhere expanded continuum that contained deformations which in advance were very small. Where these deformations grew, the distances grew faster than in the environment. A uniform expansion appears the rule and local deformations form the exception. Deformations make the information path longer and give the idea that time ticks slower in the deformed and expanded regions. This corresponds with the gravitational red-shift of photons.

Composed modules only started to be generated after the presence of enough elementary modules. The generation of photons that reflected the signatures of atoms only started after the presence of these compound modules.

This picture differs considerably from the popular scene of the big bang that started at a single location.

11 Life of an elementary module

An elementary module is a complicated construct. First, the particle resides on a private quaternionic separable Hilbert space that uses a selected version of the quaternionic number system to specify the inner products of pairs of Hilbert vectors and the eigenvalues of operators. The vectors belong to an underlying vector space. All elementary modules share the same underlying vector space. The selected version of the number system determines the private parameter space, which is managed by a dedicated reference operator. The coordinate systems that sequence the elements of the parameter space determine the symmetry of the Hilbert space and the elementary module inherits this symmetry. The private parameter space floats over a background parameter space that belongs to a background platform. The background platform is a separable Hilbert space that also applies the same underlying vector space. The difference in symmetry between the private parameter space and the background parameter space gives rise to a symmetry related (electric) charge and a related color charge. The electric charge raises a corresponding symmetry related field. The corresponding source or drain locates at the geometric center of the private parameter space.

The eigenspace of a dedicated footprint operator contains the dynamic geometric data that after sequencing of the time-stamps form the complete life-story of the elementary module. A subspace of the underlying vector space acts as a window that scans over the private Hilbert space as a function of a progression parameter that corresponds with the archived time-stamps. This subspace synchronizes all elementary modules that exist in the model.

Elementary modules are elementary modules, and together these elementary modules form all modules and modular systems that exist in the universe.

The complicated structure of elementary modules indicates that these particles never die. This does not exclude the possibility that elementary modules can zigzag over the progression parameter. Observers will perceive the progression reflection instants as pair creation and pair annihilation events. The zigzag will only become apparent in the creator’s view. Thus, only the footprint of the elementary module is recurrently recreated. Its platform persists.

A private stochastic process will recurrently regenerate the footprint of the elementary module in a cyclic fashion. During a cycle, the hopping path of the elementary module will have formed a coherent hop landing location swarm. A location density distribution describes this swarm. This location density distribution equals the Fourier transform of the characteristic function of the stochastic process that generates the hop landing locations. The location density distribution also equals the squared modulus of the wavefunction of the particle. This stochastic process mimics the mechanism that the creator applied when he created the elementary module. The stochastic process also represents the embedding of the eigenspace of the footprint operator into the continuum eigenspace of an operator that resides in the non-separable companion of the background platform. This continuum eigenspace represents the universe.

The differences between the symmetry of the private parameter space and the background parameter space give rise to symmetry-related charges that locate at the geometric center of the private parameter space. These charges give rise to symmetry-related fields. Via the geometric center of the platform, these symmetry related fields couple to the field that represents the universe.

The kinetic energy of the platform is obtained from the effects of one-dimensional shock fronts. In many cases, these energy packages are combined in photons.

12 Relational structures

12.1 Lattice

A lattice is a set of elements $a,b,c,...$ that is closed for the connections $\cap$ and $\cup$. These connections obey:

- The set is ***partially ordered***.
  - This means that with each pair of elements a,b belongs an element c, such that $a \subset c$ and $b \subset c$.
- The set is a $\cap$ ***half lattice.***
  - This means that with each pair of elements a,b an element c exists, such that $c = a \cap b$.
- The set is a $\cup$ half lattice.
  - This means that with each pair of elements a,b an element c exists, such that $c = a \cup b$.
- The set is a lattice.
  - This means that the set is both a $\cap$ half lattice and a $\cup$ half lattice.

The following relations hold in a lattice:

$$a \cap b = b \cap a \tag{11.1.1}$$

$$(a \cap b) \cap c = a \cap (b \cap c) \tag{11.1.2}$$

$$a \cap (a \cup b) = a \tag{11.1.3}$$

$$a \cup b = b \cup a \tag{11.1.4}$$

$$(a \cup b) \cup c = a \cup (b \cup c) \tag{11.1.5}$$

$$a \cup (a \cap b) = a \tag{11.1.6}$$

The lattice has a ***partial order inclusion*** $\subset$:

$$a \subset b \Leftrightarrow a \cap b = a \tag{11.1.7}$$

12.2 Lattice types

A ***complementary lattice*** contains two elements n and e ,and with each element *a* it contains a complementary element a'

such that:

$$a \cap a' = n \tag{11.2.1}$$

$$a \cap n = n \tag{11.2.2}$$

$$a \cap e = a \tag{11.2.3}$$

$$a \cup a' = e \tag{11.2.4}$$

$$a \cup e = e \tag{11.2.5}$$

$$a \cup n = a \tag{11.2.6}$$

An ***orthocomplemented lattice*** contains two elements n and e ,and with each element *a,* it contains an element a'' such that:

$$a \cup a'' = e \tag{11.2.7}$$

$$a \cap a'' = n \tag{11.2.8}$$

$$\left(a''\right)'' = a \tag{11.2.9}$$

$$a \subset b \Leftrightarrow b'' \subset a'' \tag{11.2.10}$$

e is the ***unity element***; n is the ***null element*** of the lattice

A ***distributive lattice*** supports the distributive laws:

$$a \cap \left(b \cup c\right) = \left(a \cap b\right) \cup \left(a \cap c\right) \tag{11.2.11}$$

$$a \cup \left(b \cap c\right) = \left(a \cup b\right) \cap \left(a \cup c\right) \tag{11.2.12}$$

A ***modular lattice*** supports:

$$\left(a \cap b\right) \cup \left(a \cap c\right) = a \cap \left(b \cup \left(a \cap c\right)\right) \tag{11.2.13}$$

A ***weak modular lattice*** supports instead:

There exists an element d such that

$$a \subset c \Leftrightarrow \left(a \cup b\right) \cap c = a \cup \left(b \cap c\right) \cup \left(d \cap c\right) \tag{11.2.14}$$

where d obeys:

$$\left(a \cup b\right) \cap d = d \tag{11.2.15}$$

$$a \cap d = n \tag{11.2.16}$$

$$b \cap d = n \tag{11.2.17}$$

$$\left(a \subset g\right) and \left(b \subset g\right) \Leftrightarrow d \subset g \tag{11.2.18}$$

In an ***atomic lattice*** holds

$$\exists \{p \ni L\} \forall \{x \ni L\} \{x \subset p \Rightarrow x = n\} \tag{11.2.19}$$

$$\forall \{a \ni L\} \forall \{x \ni L\} \left\{\left(a \subseteq x \subseteq \left(a \cap p\right) \Rightarrow \left[\left(x = a\right) or \left(x = a \cap p\right)\right]\right)\right\} \tag{11.2.20}$$

p is an atom

12.3 Well known lattices

Classical logic has the structure of an orthocomplemented distributive modular and atomic lattice.

Quantum logic has the structure of an orthocomplemented weakly modular and atomic lattice.

It is also called an ***orthomodular lattice***.

Both lattices are atomic lattices.

13 Quaternions

Hilbert spaces can only cope with number systems whose members form a divisions ring. Quaternionic number systems represent the most versatile division ring. Quaternionic number systems exist in many versions that differ in the way that coordinate systems can sequence them. Quaternions can store a combination of a scalar time-stamp and a three-dimensional spatial location. Thus, they are ideally suited as storage bins for dynamic geometric data.

In this paper, we represent quaternion q by a one-dimensional real part q_r and a three-dimensional imaginary part $\vec{q}$. The summation is commutative and associative

The following quaternionic multiplication rule describes most of the arithmetic properties of the quaternions.

$$c = c_r + \vec{c} = ab = \left(a_r + \vec{a}\right)\left(b_r + \vec{b}\right) = a_r b_r - \left\langle \vec{a}, \vec{b} \right\rangle + a_r \vec{b} + \vec{a} b_r \pm \vec{a} \times \vec{b} \tag{12.1.1}$$

The $\pm$ sign indicates the freedom of choice of the handedness of the product rule that exists when selecting a version of the quaternionic number system.

A quaternionic conjugation exists

$$q^* = (q_r + \vec{q})^* = q_r - \vec{q} \tag{12.1.2}$$

$$\left(ab\right)^* = b^* a^* \tag{12.1.3}$$

The norm $\left|q\right|$ equals

$$\left|q\right| = \sqrt{q_r^2 + \left\langle \vec{q}, \vec{q} \right\rangle} \tag{12.1.4}$$

$$q^{-1} = \frac{1}{q} = \frac{q}{\left|q\right|^2} \tag{12.1.5}$$

$$q = \left|q\right| \exp\left(q_\varphi \frac{\vec{q}}{\left|\vec{q}\right|} \right) \tag{12.1.6}$$

$\frac{\vec{q}}{\left|\vec{q}\right|}$ is the spatial direction of q .

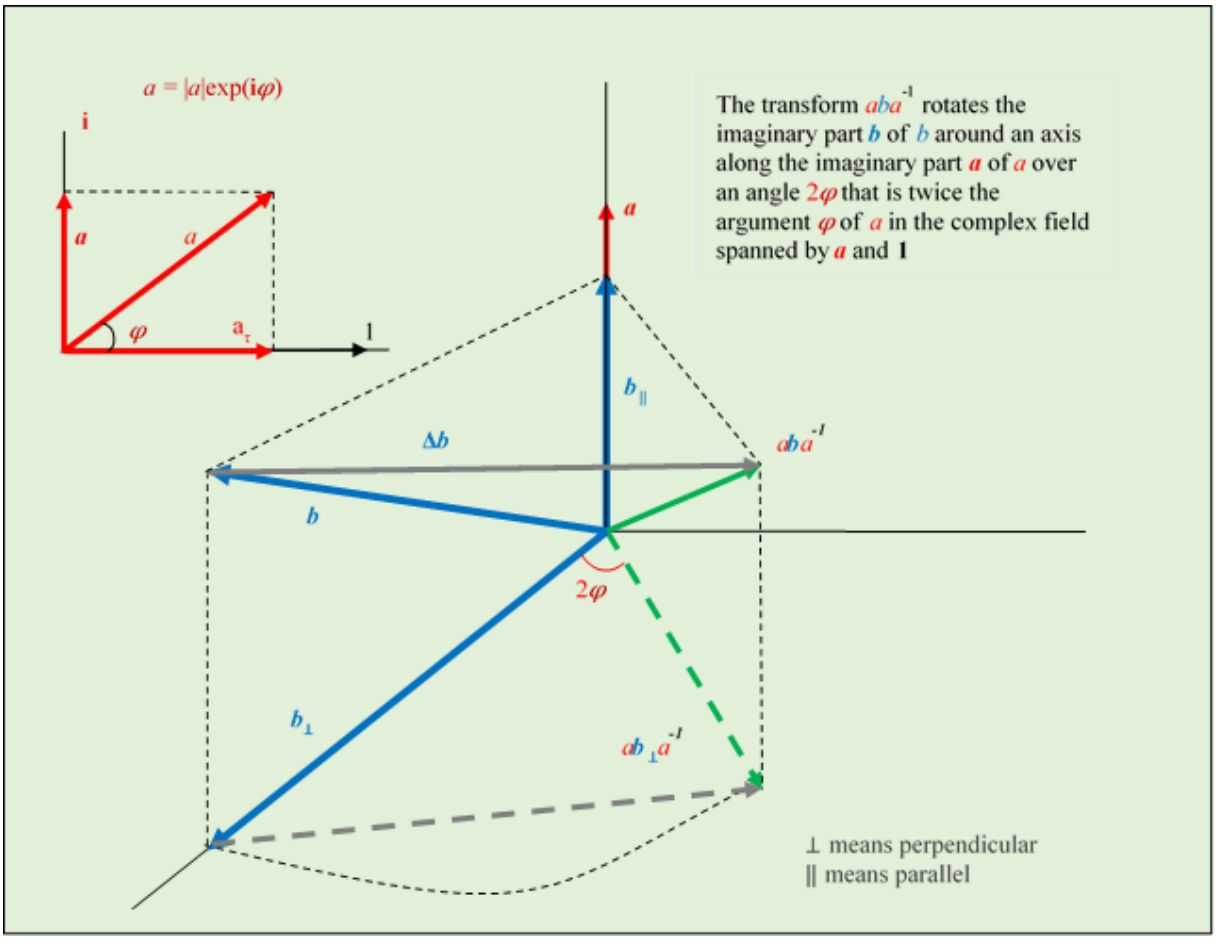
A quaternion and its inverse can rotate a part of a third quaternion. The imaginary part of the rotated quaternion that is perpendicular to the imaginary part of the first quaternion is rotated over an angle that is twice the angle of the argument φ between the real part and the imaginary part of the first

quaternion. This makes it possible to shift the imaginary part of the third quaternion to a different dimension. For that reason, must $\varphi = \pi / 4$.

Each quaternion c can be written as a product of two complex numbers a and b of which the imaginary base vectors are perpendicular

$$\begin{aligned} c &= \left(a_r + a_1\vec{i}\right)\left(b_r + b_2\vec{j}\right) \\ &= a_r b_r + \left(a_1 + b_r\right)\vec{i} + \left(a_r + b_2\right)\vec{j} + a_1 b_2 \vec{k} = c_r + c_1\vec{i} + c_2\vec{j} + c_3\vec{k} \end{aligned} \tag{12.1.7}$$

Where $\vec{k} = \vec{i} \times \vec{j}$



14 Quaternionic Hilbert spaces

Around the turn of the nineteenth century into the twentieth century David Hilbert and others developed the type of vector space that later got Hilbert's name.

The Hilbert space is a particular vector space because it defines an inner product for every pair of its member vectors.

That inner product can take values of a number system for which every non-zero member owns a unique inverse. This requirement brands the number system as a division ring.

Only three suitable division rings exist:

- The real numbers

- The complex numbers
- The quaternions

Hilbert spaces cannot cope with bi-quaternions or octonions

14.1 Bra's and ket's

Paul Dirac introduced a handy formulation for the inner product that applies a bra and a ket.

The bra $\langle f|$ is a covariant vector, and the ket $|g\rangle$ is a contravariant vector. The inner product $\langle f \mid g\rangle$ acts as a metric.

For bra vectors hold

$$\langle f|+\langle g|=\langle g|+\langle f|=\langle f+g| \tag{13.1.1}$$

$$\big(\langle f+g|\big)+\langle h|=\langle f|+\big(\langle g+h|\big)=\langle f+g+h| \tag{13.1.2}$$

For ket vectors hold

$$|f\rangle+|g\rangle=|g\rangle+|f\rangle=|f+g\rangle \tag{13.1.3}$$

$$\big(|f+g\rangle\big)+|h\rangle=|f\rangle+\big(|g+h\rangle\big)=|f+g+h\rangle \tag{13.1.4}$$

For the inner product holds

$$\langle f \mid g\rangle=\langle g \mid f\rangle^{*} \tag{13.1.5}$$

For quaternionic numbers α and β hold

$$\langle \alpha f \mid g\rangle=\langle g \mid \alpha f\rangle^{*}=\big(\langle g \mid f\rangle\alpha\big)=\alpha^{*}\langle f \mid g\rangle \tag{13.1.6}$$

$$\langle f \mid \beta g\rangle=\langle f \mid g\rangle\beta \tag{13.1.7}$$

$$\big\langle(\alpha+\beta)\, f \mid g\big\rangle=\alpha^{*}\langle f \mid g\rangle+\beta^{*}\langle f \mid g\rangle=(\alpha+\beta)^{*}\langle f \mid g\rangle \tag{13.1.8}$$

Thus

$$\alpha|f\rangle \tag{13.1.9}$$

$$\langle \alpha f|=\alpha^{*}\langle f| \tag{13.1.10}$$

$$|\alpha g\rangle=|g\rangle\alpha \tag{13.1.11}$$

We made a choice. Another possibility would be $\langle \alpha f|=\alpha\langle f|$ and $|\alpha g\rangle=\alpha^{*}|g\rangle$

In mathematics a topological space is called separable if it contains a countable dense subset; that is, there exists a sequence $\{|f_i\rangle\}_{i=\infty}^{i=0}$ of elements of the space such that every nonempty open subset of the space contains at least one element of the sequence.

Its values on this countable dense subset determine every continuous function on the separable space $\mathfrak{H}$.

The Hilbert space $\mathfrak{H}$ is separable. That means that a countable row of elements $\{|f_n\rangle\}$ exists that spans the whole space.

If $\langle f_m | f_n \rangle = \delta(m,n)$ [1 if n=m; otherwise 0], then $\{|f_n\rangle\}$ is an orthonormal base of Hilbert space $\mathfrak{H}$.

A ket base $\{|k\rangle\}$ of $\mathfrak{H}$ is a minimal set of ket vectors $|k\rangle$ that span the full Hilbert space $\mathfrak{H}$.

Any ket vector $|f\rangle$ in $\mathfrak{H}$ can be written as a linear combination of elements of $\{|k\rangle\}$.

$$|f\rangle = \sum_k |k\rangle\langle k|f\rangle \tag{13.1.12}$$

A bra base $\{\langle b|\}$ of $\mathfrak{H}^\dagger$ is a minimal set of bra vectors $\langle b|$ that span the full Hilbert space $\mathfrak{H}^\dagger$.

Any bra vector $\langle f|$ in $\mathfrak{H}^\dagger$ can be written as a linear combination of elements of $\{\langle b|\}$.

$$\langle f| = \sum_b \langle f|b\rangle\langle b| \tag{13.1.13}$$

Usually, a base selects vectors such that their norm equals 1. Such a base is called an orthonormal base

14.2 Operators

Operators act on a subset of the elements of the Hilbert space.

An operator L is linear when for all vectors $|f\rangle$ and $|g\rangle$ for which L is defined and for all quaternionic numbers α and β

$$L|\alpha f\rangle + L|\beta g\rangle = L|f\rangle\alpha + L|g\rangle\beta = L\left(|f\rangle\alpha + |g\rangle\beta\right) = L\left(|\alpha f\rangle + |\beta g\rangle\right) \tag{13.2.1}$$

The operator B is **colinear** when for all vectors $|f\rangle$ for which B is defined and for all quaternionic numbers α there exists a quaternionic number γ such that

$$\alpha B|f\rangle = B|f\rangle\gamma\alpha\gamma^{-1} \equiv B|\gamma\alpha\gamma^{-1}f\rangle \tag{13.2.2}$$

If $|a\rangle$ is an eigenvector of the operator A with quaternionic eigenvalue α,

$$A|a\rangle = |a\rangle\alpha \tag{13.2.3}$$

then $|\beta a\rangle$ is an eigenvector of A with quaternionic eigenvalue $\beta^{-1}\alpha\beta$.

$$A|\beta a\rangle = A|a\rangle\beta = |a\rangle\alpha\beta = |\beta a\rangle\beta^{-1}\alpha\beta \tag{13.2.4}$$

$A^\dagger$ is the **adjoint** of the **normal** operator A

$$\langle f | Ag\rangle = \langle fA^\dagger | g\rangle = \langle g | A^\dagger f\rangle^* \tag{13.2.5}$$

$$A^{\dagger\dagger} = A \tag{13.2.6}$$

$$\left(A+B\right)^{\dagger} = A^{\dagger} + B^{\dagger} \tag{13.2.7}$$

$$\left(AB\right)^{\dagger} = B^{\dagger}A^{\dagger} \tag{13.2.8}$$

If $A = A^{\dagger}$ then A is a **self-adjoint** operator.

A linear operator L is normal if $LL^{\dagger}$ exists and $LL^{\dagger} = L^{\dagger}L$

For the normal operator N holds

$$\left\langle Nf \mid Ng \right\rangle = \left\langle NN^{\dagger} f \mid g \right\rangle = \left\langle f \mid NN^{\dagger} g \right\rangle \tag{13.2.9}$$

Thus

$$N = N_r + \vec{N} \tag{13.2.10}$$

$$N^{\dagger} = N_r - \vec{N} \tag{13.2.11}$$

$$N_r = \frac{N + N^{\dagger}}{2} \tag{13.2.12}$$

$$\vec{N} = \frac{N - N^{\dagger}}{2} \tag{13.2.13}$$

$$NN^{\dagger} = N^{\dagger}N = N_r N_r + \left\langle \vec{N}, \vec{N} \right\rangle = \left|N\right|^2 \tag{13.2.14}$$

N_r is the Hermitian part of N.

$\vec{N}$ is the anti-Hermitian part of N.

For two normal operators A and B holds

$$AB = A_r B_r - \left\langle \vec{A}, \vec{B} \right\rangle + A_r \vec{B} + \vec{A} B_r \pm \vec{A} \times \vec{B} \tag{13.2.15}$$

For a unitary transformation U holds

$$\left\langle Uf \mid Ug \right\rangle = \left\langle f \mid g \right\rangle \tag{13.2.16}$$

The closure of separable Hilbert space $\mathfrak{H}$ means that converging rows of vectors of $\mathfrak{H}$ converge to a vector in $\mathfrak{H}$.

14.2.1 Operator construction

$\left| f \right\rangle \left\langle g \right|$ is a constructed operator.

$$\left| g \right\rangle \left\langle f \right| = \left(\left| f \right\rangle \left\langle g \right| \right)^{\dagger} \tag{13.2.17}$$

For the orthonormal base $\{|q_i\rangle\}$ consisting of eigenvectors of the reference operator holds

$$\langle q_n \mid q_m \rangle = \delta_{nm} \tag{13.2.18}$$

The reverse bra-ket method enables the definition of new operators that are defined by quaternionic functions.

$$\langle g \mid F \mid h \rangle = \sum_{i=1}^{N} \{\langle g|q_i\rangle F(q_i)\langle q_i|h\rangle\} \tag{13.2.19}$$

The symbol F is used both for the operator F and the quaternionic function $F(q)$. This enables the shorthand

$$F \equiv |q_i\rangle F(q_i)\langle q_i| \tag{13.2.20}$$

It is evident that

$$F^{\dagger} \equiv |q_i\rangle F^{*}(q_i)\langle q_i| \tag{13.2.21}$$

For reference operator $\mathfrak{R}$ holds

$$\mathfrak{R} = |q_i\rangle q_i \langle q_i| \tag{13.2.22}$$

If $\{|q_i\rangle\}$ consists of all rational values of the version of the quaternionic number system that $\mathfrak{H}$ applies then the eigenspace of $\mathfrak{R}$ represents the private parameter space of the separable Hilbert space $\mathfrak{H}$. It is also the parameter space of the function $F(q)$ that defines the operator F in the formula (13.2.20).

14.3 Non-separable Hilbert space

Every infinite dimensional separable Hilbert space $\mathfrak{H}$ owns a unique non-separable companion Hilbert space $\mathcal{H}$. This is achieved by the closure of the eigenspaces of the reference operator and the defined operators. In this procedure, on many occasions, the notion of the dimension of subspaces loses its sense.

Gelfand triple and ***Rigged Hilbert space*** are other names for the general non-separable Hilbert spaces.

In the non-separable Hilbert space, for operators with continuum eigenspaces, the reverse bra-ket method turns from a summation into an integration.

$$\langle g \mid F \mid h \rangle \equiv \int\int\int\int \{\langle g|q\rangle F(q)\langle q|h\rangle\} dV d\tau \tag{13.3.1}$$

Here we omitted the enumerating subscripts that were used in the countable base of the separable Hilbert space.

The shorthand for the operator F is now

$$F \equiv |q\rangle F(q)\langle q| \tag{13.3.2}$$

For eigenvectors $|q\rangle$ the function $F(q)$ defines as

$$F(q)=\langle q\,|\,Fq\rangle=\int\iiint\{\langle q\,|\,q'\rangle F(q')\langle q'\,|\,q\rangle\}dV'd\tau' \tag{13.3.3}$$

The reference operator $\mathcal{R}$ that provides the continuum background parameter space as its eigenspace follows from

$$\langle g\,|\,\mathcal{R}h\rangle\equiv\int\iiint\{\langle g|q\rangle q\langle q|h\rangle\}dVd\tau \tag{13.3.4}$$

The corresponding shorthand is

$$\mathcal{R}\equiv|q\rangle q\langle q| \tag{13.3.5}$$

The reference operator is a special kind of defined operator. Via the quaternionic functions that specify defined operators, it becomes clear that every infinite dimensional separable Hilbert space owns a unique non-separable companion Hilbert space that can be considered to embed its separable companion.

The reverse bracket method combines Hilbert space operator technology with quaternionic function theory and indirectly with quaternionic differential and integral technology.

15 Quaternionic differential calculus

15.1 Field equations

Maxwell equations apply the three-dimensional nabla operator in combination with a time derivative that applies coordinate time. The Maxwell equations derive from results of experiments. For that reason, those equations contain physical units.

In this treatment, the quaternionic partial differential equations apply the quaternionic nabla. The equations do not derive from the results of experiments. Instead, the formulas apply the fact that the quaternionic nabla behaves as a quaternionic multiplying operator. The corresponding formulas do not contain physical units. This approach generates essential differences between Maxwell field equations and quaternionic partial differential equations.

The quaternionic partial differential equations form a complete and self-consistent set. They use the properties of the three-dimensional spatial nabla.

The corresponding formulas are taken from Bo Thidé's EMTF book., section Appendix F4.

Another online resource is Vector calculus identities.

The quaternionic differential equations play in a Euclidean setting that is formed by a continuum quaternionic parameter space and a quaternionic target space. The parameter space is the eigenspace of the reference operator of a quaternionic non-separable Hilbert space. The target space is eigenspace of a defined operator that resides in that same Hilbert space. The defined operator is specified by a quaternionic function that completely defines the field. Each basic field owns a private defining quaternionic function. All basic fields that are treated in this chapter are defined in this way.

Physical field theories tend to use a non-Euclidean setting, which is known as spacetime setting. This is because observers can only perceive in spacetime format. Thus, Maxwell equations use coordinate time, where the quaternionic differential equations use proper time. In both settings, the observed event is presented in Euclidean format. The hyperbolic Lorentz transform converts the Euclidean format to the perceived spacetime format. Chapter 8 treats the Lorentz transform. The Lorentz transform introduces time dilation and length contraction. Quaternionic differential calculus describes the interaction between discrete objects and the continuum at the location where events occur. Converting the results of this calculus by the Lorentz transform will describe the

information that the observers perceive. Observers perceive in spacetime format. This format features a Minkowski signature. The Lorentz transform converts from the Euclidean storage format at the situation of the observed event to the perceived spacetime format. Apart from this coordinate transformation, the perceived scene is influenced by the fact that the retrieved information travels through a field that can be deformed and acts as the living space for both the observed event and the observer. Consequently, the information path deforms with its carrier field and this affects the transferred information. In this chapter, we only treat what happens at the observed event. So, we ignore the Lorentz coordinate transform, and we are not affected by the deformations of the information path.

The Hilbert Book Model archives all dynamic geometric data of all discrete creatures that exist in the model in eigenspaces of separable Hilbert spaces whose private parameter spaces float over the background parameter space, which is the private parameter space of the non-separable Hilbert space. For example, elementary particles reside on a private floating platform that is implemented by a private separable Hilbert space.

Quantum physicists use Hilbert spaces for the modeling of their theory. However, most quantum physicists apply complex-number based Hilbert spaces. Quaternionic quantum mechanics appears to represent a natural choice. Quaternionic Hilbert spaces store the dynamic geometric data in the Euclidean format in quaternionic eigenvalues that consists of a real scalar valued time-stamp and a spatial, three-dimensional location.

In the Hilbert Book Model, the instant of storage of the event data is irrelevant if it coincides with or precedes the stored time stamp. Thus, the model can store all data at an instant, which precedes all stored timestamp values. This impersonates the Hilbert Book Model as a creator of the universe in which the observable events and the observers exist. On the other hand, it is possible to place the instant of archival of the event at the instant of the event itself. It will then coincide with the archived time-stamp. In both interpretations, after sequencing the time-stamps, the repository tells the life story of the discrete objects that are archived in the model. This story describes the ongoing embedding of the separable Hilbert spaces into the non-separable Hilbert space. For each floating separable Hilbert space this embedding occurs step by step and is controlled by a private stochastic process, which owns a characteristic function. The result is a stochastic hopping path that walks through the private parameter space of the platform. A coherent recurrently regenerated hop landing location swarm characterizes the corresponding elementary object.

Elementary particles are elementary modules. Together they constitute all other modules that occur in the model. Some modules constitute modular systems. A dedicated stochastic process controls the binding of the components of the module. This process owns a characteristic function that equals a dynamic superposition of the characteristic functions of the stochastic processes that control the components. Thus, superposition occurs in Fourier space. The superposition coefficients act as gauge factors that implement displacement generators, which control the internal locations of the components. In other words, the superposition coefficients may install internal oscillations of the components. These oscillations are described by differential equations.

15.2 Fields

In the Hilbert Book Model fields are eigenspaces of operators that reside in the non-separable Hilbert space. Continuous or mostly continuous functions define these operators, and apart from some discrepant regions, their eigenspaces are continuums. These regions might reduce to single discrepant point-like artifacts. The parameter space of these functions is constituted by a version of the quaternionic number system. Consequently, the real number valued coefficients of these parameters are mutually independent, and the differential change can be expressed in terms of a linear combination of partial differentials. Now the total differential change df of field f equals

$$df = \frac{\partial f}{\partial \tau} d\tau + \frac{\partial f}{\partial x} \vec{i} dx + \frac{\partial f}{\partial y} \vec{j} dy + \frac{\partial f}{\partial z} \vec{k} dz \tag{14.2.1}$$

In this equation, the partial differentials $\frac{\partial f}{\partial \tau}, \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial y}$ are quaternions.

The quaternionic nabla ∇ assumes the ***special condition*** that partial differentials direct along the axes of the Cartesian coordinate system. Thus

$$\nabla = \sum_{i=0}^{4} \vec{e}_i \frac{\partial}{\partial x_i} = \frac{\partial}{\partial \tau} + \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \tag{14.2.2}$$

The Hilbert Book Model assumes that the quaternionic fields are moderately changing, such that only first and second order partial differential equations describe the model. These equations can describe fields of which the continuity gets disrupted by point-like artifacts. Warps, clamps and Green's functions describe the reaction of the field on such disruptions.

15.3 Field equations

Generalized field equations hold for all basic fields. Generalized field equations fit best in a quaternionic setting.

Quaternions consist of a real number valued scalar part and a three-dimensional spatial vector that represents the imaginary part.

The multiplication rule of quaternions indicates that several independent parts constitute the product.

$$c = c_r + \vec{c} = ab = \left(a_r + \vec{a}\right)\left(b_r + \vec{b}\right) = a_r b_r - \left\langle \vec{a}, \vec{b} \right\rangle + a_r \vec{b} + \vec{a} b_r \pm \vec{a} \times \vec{b} \tag{15.1.1}$$

The $\pm$ indicates that quaternions exist in right-handed and left-handed versions.

The formula can be used to check the completeness of a set of equations that follow from the application of the product rule.

We define the quaternionic nabla as

$$\nabla \equiv \left\{ \frac{\partial}{\partial \tau}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\} = \nabla_r + \vec{\nabla} \tag{15.1.2}$$

$$\vec{\nabla} \equiv \left\{ \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\} \tag{15.1.3}$$

$$\nabla_r \equiv \frac{\partial}{\partial \tau} \tag{15.1.4}$$

$$\phi = \phi_r + \vec{\phi} = \nabla \psi = \left(\frac{\partial}{\partial \tau} + \vec{\nabla} \right)\left(\psi_r + \vec{\psi} \right) = \nabla_r \psi_r - \left\langle \vec{\nabla}, \vec{\psi} \right\rangle + \nabla_r \vec{\psi} + \vec{\nabla} \psi_r \pm \vec{\nabla} \times \vec{\psi} \tag{15.1.5}$$

$$\phi_r = \nabla_r \psi_r - \left\langle \vec{\nabla}, \vec{\psi} \right\rangle \tag{15.1.6}$$

$$\vec{\phi} = \nabla_r \vec{\psi} + \vec{\nabla} \psi_r \pm \vec{\nabla} \times \vec{\psi} = -\vec{E} \pm \vec{B} \tag{15.1.7}$$

Further,

$\vec{\nabla}\psi_r$ is the gradient of ψ_r

$\left\langle \vec{\nabla}, \vec{\psi} \right\rangle$ is the divergence of $\vec{\psi}$

$\vec{\nabla} \times \vec{\psi}$ is the curl of $\vec{\psi}$

The change $\nabla\psi$ divides into five terms that each has a separate meaning. That is why these terms in Maxwell equations get different names and symbols. Every basic field offers these terms!

$$\vec{E} = -\nabla_r \vec{\psi} - \vec{\nabla}\psi_r \tag{15.1.8}$$

$$\vec{B} = \vec{\nabla} \times \psi \tag{15.1.9}$$

It is also possible to construct higher order equations. For example

$$\vec{J} = \vec{\nabla} \times \vec{B} - \nabla_r \vec{E} \tag{15.1.10}$$

The equation (15.1.6) has no equivalent in Maxwell's equations. Instead, its right part is used as a gauge.

Two special second-order partial differential equations use the terms $\frac{\partial^2 \psi}{\partial \tau^2}$ and $\left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \psi$

$$\phi = \left\{ \frac{\partial^2}{\partial \tau^2} - \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \right\} \psi \tag{15.1.11}$$

$$\rho = \left\{ \frac{\partial^2}{\partial \tau^2} + \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \right\} \psi \tag{15.1.12}$$

The equation (15.1.11) is the quaternionic equivalent of the wave equation.

The equation (15.1.12) can be divided into two first-order partial differential equations.

$$\chi = \nabla^* \varphi = \nabla^* \nabla \psi = \nabla \nabla^* \psi = \left(\nabla_r + \vec{\nabla} \right) \left(\nabla_r - \vec{\nabla} \right) \left(\psi_r + \vec{\psi} \right) = \left(\nabla_r \nabla_r + \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \right) \psi \tag{15.1.13}$$

This composes from $\chi = \nabla^* \varphi$ and $\varphi = \nabla \psi$

$\frac{\partial^2}{\partial \tau^2} - \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle$ is the quaternionic equivalent of d'Alembert's operator $\square$.

The operator $\frac{\partial^2}{\partial \tau^2} + \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle$ does not yet have an accepted name.

The Poisson equation equals

$$\rho = \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \psi \tag{15.1.14}$$

A very special solution of this equation is the Green's function $\frac{1}{\vec{q} - \vec{q'}}$ of the affected field

$$\nabla \frac{1}{\vec{q}-\vec{q}'} = -\frac{\left(\vec{q}-\vec{q}'\right)}{\left|\vec{q}-\vec{q}'\right|^3} \tag{15.1.15}$$

$$\left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \frac{1}{\left|\vec{q}-\vec{q}'\right|} \equiv \left\langle \vec{\nabla}, \vec{\nabla} \frac{1}{\left|\vec{q}-\vec{q}'\right|} \right\rangle = -\left\langle \vec{\nabla}, \vec{\nabla} \frac{\left(\vec{q}-\vec{q}'\right)}{\left|\vec{q}-\vec{q}'\right|^3} \right\rangle = 4\pi\delta\left(\vec{q}-\vec{q}'\right) \tag{15.1.16}$$

The spatial integral over Green's function is a volume.

(15.1.11) offers a dynamic equivalent of the Green's function, which is a spherical shock front. It can be written as

$$\psi = \frac{f\left(\left|\vec{q}-\vec{q}'\right| - c\left(\tau-\tau'\right)\right)}{\left|\vec{q}-\vec{q}'\right|} \tag{15.1.17}$$

A one-dimensional type of shock front solution is

$$\psi = \vec{f}\left(\left|\vec{q}-\vec{q}'\right| - c\left(\tau-\tau'\right)\right) \tag{15.1.18}$$

The equation (15.1.11) is famous for its wave type solutions

$$\nabla_r \nabla_r \psi = \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \psi = \omega\psi = \exp\left(2\pi\vec{x}\tau\right) \tag{15.1.19}$$

Periodic harmonic actuators cause the appearance of waves,

The Helmholtz equation considers the quaternionic function that defines the field separable.

$$\psi\left(q_r, \vec{q}\right) = A\left(\vec{q}\right)T\left(q_r\right) \tag{15.1.20}$$

$$\frac{\left\langle \vec{\nabla}, \vec{\nabla} \right\rangle A}{A} = \frac{\nabla_r \nabla_r T}{T} = -k^2 \tag{15.1.21}$$

$$\left\langle \vec{\nabla}, \vec{\nabla} \right\rangle A = -k^2 A \tag{15.1.22}$$

$$\nabla_r \nabla_r T = -k^2 T \tag{15.1.23}$$

For three-dimensional isotropic spherical conditions, the solutions have the form

$$A\left(r,\theta,\varphi\right) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \left\{ \left(a_{lm} j_l\left(kr\right)\right) + b_{lm} Y_l^m\left(\theta,\varphi\right) \right\} \tag{15.1.24}$$

Here j_l and y_l are the spherical Bessel functions, and Y_l^m are the spherical harmonics. These solutions play a role in the spectra of atomic modules.

A more general solution is a superposition of these basic types.

(15.1.12) offers a dynamic equivalent of the Green's function, which is a spherical shock front. It can be written as

$$\psi = \frac{f\left(\vec{q} - \overrightarrow{q'} + c\left(\tau - \tau'\right)\right)}{\left|\vec{q} - \overrightarrow{q'}\right|} \tag{15.1.25}$$

A one-dimensional type of shock front solution is

$$\psi = \vec{f}\left(\vec{q} - \overrightarrow{q'} + c\left(\tau - \tau'\right)\right) \tag{15.1.26}$$

Equation (15.1.12) offers no waves as part of its solutions.

During travel, the amplitude and the lateral direction $\frac{\vec{f}}{\left|\vec{f}\right|}$ of the one-dimensional shock fronts are fixed. The longitudinal direction is along $\frac{\vec{q} - \overrightarrow{q'}}{\left|\vec{q} - \overrightarrow{q'}\right|}$.

The shock fronts that are triggered by point-like actuators are the tiniest field excitations that exist. The actuator must fulfill significant restricting requirements. For example, a perfectly isotropic actuator must trigger the spherical shock front. The actuator can be a quaternion that belongs to another version of the quaternionic number system than the version, which the background platform applies. The symmetry break must be isotropic. Electrons fulfill this requirement. Neutrinos do not break the symmetry but have other reasons why they cause a valid trigger. Quarks break symmetry, but not in an isotropic way.

16 Line, surface and volume integrals

16.1 Line integrals

The curl can be presented as a line integral

$$\left\langle \vec{\nabla} \times \vec{\psi}, \vec{n} \right\rangle \equiv \lim_{A \to 0} \left(\frac{1}{A} \oint_C \left\langle \vec{\psi}, d\vec{r} \right\rangle \right) \tag{16.1.1}$$

16.2 Surface integrals

With respect to a local part of a closed boundary that is oriented perpendicular to vector $\vec{n}$ the partial differentials relate as

$$\vec{\nabla}\psi = -\left\langle \vec{\nabla}, \vec{\psi} \right\rangle + \vec{\nabla}\psi_r \pm \vec{\nabla} \times \vec{\psi} \Leftrightarrow \vec{n}\psi = -\left\langle \vec{n}, \vec{\psi} \right\rangle + \vec{n}\psi_r \pm \vec{n} \times \vec{\psi} \tag{16.2.1}$$

This is exploited in the surface-volume integral equations that are known as Stokes and Gauss theorems.

$$\iiint \vec{\nabla}\psi dV = \oiint \vec{n}\psi dS \tag{16.2.2}$$

$$\iiint \left\langle \vec{\nabla}, \vec{\psi} \right\rangle dV = \oiint \left\langle \vec{n}, \vec{\psi} \right\rangle dS \tag{16.2.3}$$

$$\iiint \vec{\nabla} \times \vec{\psi} dV = \oiint \vec{n} \times \vec{\psi} dS \tag{16.2.4}$$

$$\iiint \vec{\nabla}\psi_r dV = \oiint \vec{n}\psi_r dS \tag{16.2.5}$$

This result turns terms in the differential continuity equation into a set of corresponding integral balance equations.

The method also applies to other partial differential equations. For example

$$\vec{\nabla}\times\left(\vec{\nabla}\times\vec{\psi}\right)=\vec{\nabla}\left\langle\vec{\nabla},\vec{\psi}\right\rangle-\left\langle\vec{\nabla},\vec{\nabla}\right\rangle\vec{\psi}\Leftrightarrow\vec{\nabla}\times\left(\vec{\nabla}\times\vec{\psi}\right)=\vec{n}\left\langle\vec{n},\vec{\psi}\right\rangle-\left\langle\vec{n},\vec{n}\right\rangle\vec{\psi} \tag{16.2.6}$$

$$\iiint_V\left\{\vec{\nabla}\times\left(\vec{\nabla}\times\vec{\psi}\right)\right\}dV=\oiint_S\left\{\vec{\nabla}\left\langle\vec{\nabla},\vec{\psi}\right\rangle\right\}dS-\oiint_S\left\{\left\langle\vec{\nabla},\vec{\nabla}\right\rangle\vec{\psi}\right\}dS \tag{16.2.7}$$

One dimension less, a similar relation exists.

$$\iint_S\left(\left\langle\vec{\nabla}\times\vec{a},\vec{n}\right\rangle\right)dS=\oint_C\left\langle\vec{a},d\vec{l}\right\rangle \tag{16.2.8}$$

16.3 Using volume integrals to determine the symmetry related charges

In its simplest form in which no discontinuities occur in the integration domain Ω the generalized Stokes theorem runs as

$$\int_\Omega d\omega=\int_{\partial\Omega}\omega=\oint_\Omega\omega \tag{16.3.1}$$

We separate all point-like discontinuities from the domain Ω by encapsulating them in an extra boundary. Symmetry centers represent spherically ordered parameter spaces in regions H_n^x that float on a background parameter space $\mathfrak{R}$. The boundaries ∂H_n^x separate the regions from the domain H_n^x. The regions H_n^x are platforms for local discontinuities in basic fields. These fields are continuous in domain $\Omega-H$.

$$H=\bigcup_{n=1}^{N}H_n^x \tag{16.3.2}$$

The symmetry centers $\mathfrak{S}_n^x$ are encapsulated in regions H_n^x and the encapsulating boundary ∂H_n^x is not part of the disconnected boundary, which encapsulates all continuous parts of the quaternionic manifold ω that exists in the quaternionic model.

$$\int_{\Omega-H}d\omega=\int_{\partial\Omega\cup\partial H}\omega=\int_{\partial\Omega}\omega-\sum_{k=1}^{N}\int_{\partial H_n^x}\omega \tag{16.3.3}$$

In fact, it is sufficient that ∂H_n^x surrounds the current location of the elementary module. We will select a boundary, which has the shape of a small cube of which the sides run through a region of the parameter spaces where the manifolds are continuous.

If we take everywhere on the boundary the unit normal to point outward, then this reverses the direction of the normal on ∂H_n^x which negates the integral. Thus, in this formula, the contributions of boundaries $\left\{\partial H_n^x\right\}$ are subtracted from the contributions of boundary $\partial\Omega$. This means that $\partial\Omega$ also surrounds the regions $\left\{\partial H_n^x\right\}$

This fact renders the integration sensitive to the ordering of the participating domains.

Domain Ω corresponds to part of the background parameter space $\mathfrak{R}$. As mentioned before the symmetry centers $\mathfrak{S}_n^x$ represent encapsulated regions $\{\partial H_n^x\}$ that float on the background parameter space $\mathfrak{R}$. The Cartesian axes of are parallel to the Cartesian axes of background parameter space $\mathfrak{R}$. Only the orderings along these axes may differ.

Further, the geometric center of symmetry center $\mathfrak{S}_n^x$ is represented by a floating location on parameter space $\mathfrak{R}$.

The symmetry center $\mathfrak{S}_n^x$ is characterized by a private symmetry flavor. That symmetry flavor relates to the Cartesian ordering of this parameter space. With the orientation of the coordinate axes fixed, eight independent Cartesian orderings are possible.

The consequence of the differences in the symmetry flavor on the subtraction can best be comprehended when the encapsulation ∂H_n^x is performed by a ***cubic space form*** that is aligned along the Cartesian axes that act in the background parameter space. Now the six sides of the cube contribute different to the effects of the encapsulation when the ordering of H_n^x differs from the Cartesian ordering of the reference parameter space $\mathfrak{R}$. Each discrepant axis ordering corresponds to one third of the surface of the cube. This effect is represented by the ***symmetry related charge***, which includes the ***color charge*** of the symmetry center. It is easily comprehensible related to the algorithm which below is introduced for the computation of the symmetry related charge. Also, the relation to the color charge will be clear. ***Thus, this effect couples the ordering of the local parameter spaces to the symmetry related charge of the encapsulated elementary module.*** The differences with the ordering of the surrounding parameter space determines the value of the symmetry related charge of the object that resides inside the encapsulation!

16.4 Symmetry flavor

The Cartesian ordering of its private parameter space determines the symmetry flavor of the platform. For that reason, this symmetry is compared with the reference symmetry, which is the symmetry of the background parameter space. Four arrows indicate the symmetry of the platform. The background is represented by:

Now the symmetry-related charge follows in three steps.

1. Count the difference of the spatial part of the symmetry of the platform with the spatial part of the symmetry of the background parameter space.
2. If the handedness changes from **R** to **L**, then switch the sign of the count.
3. Switch the sign of the result for anti-particles.

Symmetry flavor

Ordering x y z τ	sequence	Handedness Right/Left	Color charge	Electric charge * 3	Symmetry type.
	⓪	**R**	N	+0	neutrino
	①	**L**	R	−1	down quark
	②	**L**	G	−1	down quark
	③	**L**	B	−1	down quark
	④	**R**	B	+2	up quark
	⑤	**R**	G	+2	up quark
	⑥	**R**	R	+2	up quark

	⑦	**L**	N	−3	electron
	⑧	**R**	N	+3	positron
	⑨	**L**	R	−2	anti-up quark
	⑩	**L**	G	−2	anti-up quark
	⑪	**L**	B	−2	anti-up quark
	⑫	**R**	B	+1	anti-down quark
	⑬	**R**	R	+1	anti-down quark
	⑭	**R**	G	+1	anti-down quark
	⑮	**L**	N	−0	anti-neutrino

The suggested particle names that indicate the symmetry type are borrowed from the Standard Model. In the table, compared to the standard model, some differences exist with the selection of the anti-predicate. All considered particles are elementary fermions. The freedom of choice in the polar coordinate system might determine the spin. The azimuth range is 2π radians, and the polar angle range is π radians. Symmetry breaking means a difference between the platform symmetry and the symmetry of the background. Neutrinos do not break the symmetry. Instead, they may cause conflicts with the handedness of the multiplication rule.

16.5 Derivation of physical laws

The quaternionic equivalents of Ampère's law are

$$\vec{J} \equiv \vec{\nabla} \times \vec{B} = \nabla_r \vec{E} \Leftrightarrow \vec{J} \equiv \vec{n} \times \vec{B} = \nabla_r \vec{E} \tag{16.5.1}$$

$$\iint_S \left\langle \vec{\nabla} \times \vec{B}, \vec{n} \right\rangle dS = \oint_C \left\langle \vec{B}, d\vec{l} \right\rangle = \iint_S \left\langle \vec{J} + \nabla_r \vec{E}, \vec{n} \right\rangle dS \tag{16.5.2}$$

The quaternionic equivalents of Faraday's law are:

$$\nabla_r \vec{B} = \vec{\nabla} \times \left(\nabla_r \vec{\psi} \right) = -\vec{\nabla} \times \vec{E} \Leftrightarrow \nabla_r \vec{B} = \vec{n} \times \left(\nabla_r \vec{\psi} \right) = -\vec{\nabla} \times \vec{E} \tag{16.5.3}$$

$$\oint_c \left\langle \vec{E}, d\vec{l} \right\rangle = \iint_S \left\langle \vec{\nabla} \times \vec{E}, \vec{n} \right\rangle dS = -\iint_S \left\langle \nabla_r \vec{B}, \vec{n} \right\rangle dS \tag{16.5.4}$$

$$\vec{J} = \vec{\nabla} \times \left(\vec{B} - \vec{E} \right) = \vec{\nabla} \times \vec{\varphi} - \nabla_r \vec{\varphi} = \vec{v} \rho \tag{16.5.5}$$

$$\iint_S \left\langle \vec{\nabla} \times \vec{\varphi}, \vec{n} \right\rangle dS = \oint_C \left(\left\langle \vec{\varphi}, d\vec{l} \right\rangle \right) = \iint_S \left\langle \vec{v} \rho + \nabla_r \vec{\varphi}, \vec{n} \right\rangle dS \tag{16.5.6}$$

The equations (16.5.4) and (16.5.6) enable the derivation of the ***Lorentz force***.

$$\vec{\nabla} \times \vec{E} = -\nabla_r \vec{B} \tag{16.5.7}$$

$$\frac{d}{d\tau} \iint_S \left\langle \vec{B}, \vec{n} \right\rangle dS = \iint_{S(\tau_0)} \left\langle \dot{\vec{B}}(\tau_0), \vec{n} \right\rangle ds + \frac{d}{d\tau} \iint_{S(\tau)} \left\langle \vec{B}(\tau_0), \vec{n} \right\rangle ds \tag{16.5.8}$$

The Leibniz integral equation states

$$\begin{aligned}&\frac{d}{dt}\iint\limits_{S(\tau)}\left\langle \vec{X}(\tau_0),\vec{n}\right\rangle dS\\&=\iint\limits_{S(\tau_0)}\left\langle \dot{\vec{X}}(\tau_0)+\left\langle \vec{\nabla},\vec{X}(\tau_0)\right\rangle \vec{v}(\tau_0),\vec{n}\right\rangle dS-\oint\limits_{C(\tau_0)}\left\langle \vec{v}(\tau_0)\times\vec{X}(\tau_0),d\vec{l}\right\rangle\end{aligned}\tag{16.5.9}$$

With $\vec{X}=\vec{B}$ and $\left\langle \vec{\nabla},\vec{B}\right\rangle=0$ follows

$$\begin{aligned}&\frac{d\Phi_B}{d\tau}=\frac{d}{d\tau}\iint\limits_{S(\tau)}\left\langle \dot{\vec{B}}(\tau),\vec{n}\right\rangle dS=\iint\limits_{S(\tau_0)}\left\langle \vec{B}(\tau_0),\vec{n}\right\rangle dS-\oint\limits_{C(\tau_0)}\left\langle \vec{v}(\tau_0)\times\vec{B}(\tau_0),d\vec{l}\right\rangle\\&=-\oint\limits_{C(\tau_0)}\left\langle E(\tau_0),d\vec{l}\right\rangle-\oint\limits_{C(\tau_0)}\left\langle \vec{v}(\tau_0)\times\vec{B}(\tau_0),d\vec{l}\right\rangle\end{aligned}\tag{16.5.10}$$

The electromotive force (EMF) ε equals

$$\begin{aligned}\varepsilon&=\oint\limits_{C(\tau_0)}\left\langle \frac{\vec{F}(\tau_0)}{q},d\vec{l}\right\rangle=-\left.\frac{d\Phi_B}{d\tau}\right|_{\tau=\tau_0}\\&=\oint\limits_{C(\tau_0)}\left\langle \vec{E}(\tau_0),d\vec{l}\right\rangle+\oint\limits_{C(\tau_0)}\left\langle \vec{v}(\tau_0)\times\vec{B}(\tau_0),d\vec{l}\right\rangle\end{aligned}\tag{16.5.11}$$

$$\vec{F}=q\vec{E}+q\vec{v}\times\vec{B}\tag{16.5.12}$$

17 Polar coordinates

In polar coordinates, the nabla delivers different formulas.

In pure spherical conditions, the Laplacian reduces to:

$$\left\langle \vec{\nabla},\vec{\nabla}\right\rangle\psi=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right)\tag{17.1.1}$$

The Green's function blurs the location density distribution of the hop landing location swarm of an elementary particle. If the location density distribution has the form of a Gaussian distribution, then the blurred function is the convolution of this location density distribution and the Green's function. The Gaussian distribution is

$$\rho(r)=\frac{1}{\left(\sigma\sqrt{2\pi}\right)^3}\exp\left(-\frac{r^2}{2\sigma^2}\right)\tag{17.1.2}$$

The shape of the deformation of the field for this ***example*** is given by:

$$\mathfrak{T}(r)=\frac{ERF\left(-\frac{r}{\sigma\sqrt{2}}\right)}{4\pi r}\tag{17.1.3}$$

In this function, every trace of the singularity of the Green's function has disappeared. It is due to the distribution and the huge number of participating hop locations. This shape is just an example. Such extra potentials add a local contribution to the field that acts as the living space of modules and modular systems. The shown extra contribution is due to the local elementary

module that the swarm represents. Together, a myriad of such bumps constitutes the living space.

18 Material penetrating field

18.1 Field equations

Basic fields can penetrate homogeneous regions of the material. Within these regions, the fields get crumpled. Consequently, the average speed of warps, clamps, and waves diminish, or these vibrations just get dampened away. The basic field that we consider here is a smoothed version $\vec{\psi}$ of the original field ψ that penetrates the material.

$$\vec{\phi} = \nabla_r \vec{\psi} + \vec{\nabla}\psi_r \pm \vec{\nabla} \times \vec{\psi} = -\vec{E} \pm \vec{B} \tag{18.1.1}$$

$$\vec{\varphi} = \nabla_r \vec{\psi} + \vec{\nabla}\psi_r \pm \vec{\nabla} \times \vec{\psi} = -\vec{\mathfrak{C}} \pm \vec{\mathfrak{B}} \tag{18.1.2}$$

The first order partial differential equation does not change much. The separate terms in the first order differential equations must be corrected by a material-dependent factor and extra material dependent terms appear.

These extra terms correspond to polarization $\vec{P}$ and magnetization $\vec{M}$ of the material, and the factors concern the permittivity ε and the permeability μ of the material. This results in corrections in the $\vec{\mathfrak{E}}$ and the $\vec{\mathfrak{B}}$ field and the average speed of warps and waves reduces from 1 to $\frac{1}{\sqrt{\varepsilon\mu}}$.

$$D = \varepsilon\vec{\mathfrak{E}} + \vec{P} \tag{18.1.3}$$

$$\vec{H} = \frac{1}{\mu}\vec{\mathfrak{B}} - \vec{M} \tag{18.1.4}$$

$$\rho_b = -\left\langle \vec{\nabla}, \vec{P} \right\rangle \tag{18.1.5}$$

$$\rho_f = -\left\langle \vec{\nabla}, \vec{D} \right\rangle \tag{18.1.6}$$

$$\vec{J}_b = \vec{\nabla} \times \vec{M} + \nabla_r \vec{P} \tag{18.1.7}$$

$$\vec{J}_f = \vec{\nabla} \times \vec{H} - \nabla_r \vec{D} \tag{18.1.8}$$

$$\rho = \frac{1}{\varepsilon}\left\langle \vec{\nabla}, \vec{\mathfrak{E}} \right\rangle = \rho_b + \rho_f \tag{18.1.9}$$

$$\vec{J} = \frac{1}{\mu}\vec{\nabla} \times \vec{\mathfrak{B}} - \frac{\varepsilon}{\mu}\nabla_r \vec{\mathfrak{E}} = \vec{J}_b + \vec{J}_f \tag{18.1.10}$$

$$\vec{\phi} = \vec{\mathfrak{E}} - \vec{\mathfrak{B}} = \frac{1}{\varepsilon}\left(\vec{D} - \vec{P}\right) - \mu\left(\vec{H} + \vec{M}\right) \tag{18.1.11}$$

The subscript $_b$ signifies bounded. The subscript $_f$ signifies free.

The homogeneous second order partial differential equations hold for the smoothed field ψ.

$$\left\{\nabla_r \nabla_r \pm v^2 \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle\right\} \psi = 0 \tag{18.1.12}$$

18.2 Pointing vector

The **Poynting vector** represents the directional energy flux density (the rate of energy transfer per unit area) of a basic field. The quaternionic equivalent of the Poynting vector is defined as:

$$\vec{S} = \vec{E} \times \vec{H} \tag{18.2.1}$$

u is the electromagnetic energy density for linear, nondispersive materials, given by

$$u = \frac{\left\langle \vec{E}, \vec{B} \right\rangle + \left\langle \vec{B}, \vec{H} \right\rangle}{2} \tag{18.2.2}$$

$$\frac{\partial u}{\partial \tau} = -\left\langle \vec{\nabla}, \vec{S} \right\rangle - \left\langle \vec{J}_f, \vec{E} \right\rangle \tag{18.2.3}$$

19 Lorentz transform

The shock fronts move with speed c. In the quaternionic setting, this speed is unity.

$$x^2 + y^2 + z^2 = c^2 \tau^2 \tag{19.1.1}$$

Swarms of clamp triggers move with lower speed v .

For the geometric centers of these swarms still holds:

$$x^2 + y^2 + z^2 - c^2 \tau^2 = x'^2 + y'^2 + z'^2 - c^2 \tau'^2 \tag{19.1.2}$$

If the locations $\{x, y, z\}$ and $\{x', y', z'\}$ move with uniform relative speed v, then

$$ct' = ct \cosh(\omega) - x \sinh(\omega) \tag{19.1.3}$$

$$x' = x \cosh(\omega) - ct \sinh(\omega) \tag{19.1.4}$$

$$\cosh(\omega) = \frac{\exp(\omega) + \exp(-\omega)}{2} = \frac{c}{\sqrt{c^2 - v^2}} \tag{19.1.5}$$

$$\sinh(\omega) = \frac{\exp(\omega) - \exp(-\omega)}{2} = \frac{v}{\sqrt{c^2 - v^2}} \tag{19.1.6}$$

$$\cosh(\omega)^2 - \sinh(\omega)^2 = 1 \tag{19.1.7}$$

This is a hyperbolic transformation that relates two coordinate systems.

This transformation can concern two platforms P and P' on which swarms reside and that move with uniform relative speed .

However, it can also concern the storage location P that contains a timestamp τ and spatial location $\{x, y, z\}$ and platform P' that has coordinate time t and location $\{x', y', z'\}$.

In this way, the hyperbolic transform relates two individual platforms on which the private swarms of individual elementary particles reside.

It also relates the stored data of an elementary particle and the observed format of these data for the elementary particle that moves with speed relative to the background parameter space.

The Lorentz transform converts a Euclidean coordinate system consisting of a location $\{x, y, z\}$ and proper time stamps τ into the perceived coordinate system that consists of the spacetime coordinates $\{x', y', z', ct'\}$ in which t' plays the role of proper time. The uniform velocity v causes time dilation $\Delta t' = \frac{\Delta \tau}{\sqrt{1-\frac{v^2}{c^2}}}$ and length contraction $\Delta L' = \Delta L\sqrt{1-\frac{v^2}{c^2}}$

20 Action

The set of basic fields that occur in the model form a system. These fields interact at a finite number of discrete locations. The symmetry related $\mathfrak{A}^x$ fields always attach to the geometrical center of a dedicated symmetry center. The $\mathfrak{C}$ field attaches at a stochastically determined location somewhere in the vicinity of this geometric center. However, integrated over the regeneration cycle of the corresponding particle the averaged attachment point coincides with the geometric center of the symmetry center. Thus, in these averaged conditions the two fields can be considered as being superposed. In the averaged mode the $\mathfrak{C}$ field has weak extrema. The $\mathfrak{A}^x$ fields always have strong extrema. In the averaged mode the fields can be superposed into a new field $\mathfrak{F}$ that share the symmetry center related extrema.

The path of the geometric center of the symmetry center is following the least action principle. This is not the hopping path along which the corresponding particle can be detected.

The coherent location swarm $\left\{a_i^x\right\}$ also represents a hopping path. Its coherence means that the swarm owns a continuous location density distribution that characterizes this swarm. A more far reaching coherence requirement is that the characterizing continuous location density distribution also has a Fourier transform. Therefore, the swarm owns a displacement generator. This means that at first approximation the swarm moves as one unit. These facts have much impact on the hopping path and on the movement of the underlying symmetry center. The displacement generator that characterizes part of the dynamic behavior of the symmetry center is represented by the momentum operator $\vec{p}$. This displacement generator describes the movement of the swarm as one unit. It describes the movement of the platform that carries the elementary particle. On the platform the hopping path is closed. In the embedding field the platform moves.

We suppose that momentum $\vec{p}$ is constant during the particle generation cycle. Every hop gives a contribution to the path. These contributions can be divided into three steps per contributing hop:

1. Change to Fourier space. This involves inner product $\left\langle \vec{a}_i \middle| \vec{p} \right\rangle$
2. Evolve during an infinitesimal progression step into the future.
    a. Multiply with the corresponding displacement generator $\vec{p}$
    b. The generated step in configuration space is $\left(\vec{a}_{i+1} - \vec{a}_i\right)$.
    c. The action contribution in Fourier space is $\left\langle \vec{p}, \vec{a}_{i+1} - \vec{a}_i \right\rangle$.
    d. This combines in a unitary factor $\exp\left(\left\langle \vec{p}, \vec{a}_{i+1} - \vec{a}_i \right\rangle\right)$
3. Change back to configuration space. This involves inner product $\left\langle \vec{p} \middle| \vec{a}_{i+1} \right\rangle$
    a. The term contributes a factor $\left\langle \vec{a}_i \middle| \vec{p} \right\rangle \exp\left(\left\langle \vec{p}, \vec{a}_{i+1} - \vec{a}_i \right\rangle\right) \left\langle \vec{p} \middle| \vec{a}_{i+1} \right\rangle$.
4.

Two subsequent steps give:

$$\begin{aligned}
&\langle \vec{a}_i | \vec{p} \rangle \exp\left(\langle \vec{p}, \vec{a}_{i+1} - \vec{a}_i \rangle\right) \langle \vec{p} | \vec{a}_{i+1} \rangle \langle \vec{a}_{i+1} | \vec{p} \rangle \exp\left(\langle \vec{p}, \vec{a}_{i+2} - \vec{a}_i \rangle\right) \langle \vec{p} | \vec{a}_{i+2} \rangle \\
&= \langle \vec{a}_i | \vec{p} \rangle \exp\left(\langle \vec{p}, \vec{a}_{i+1} - \vec{a}_i \rangle\right) \exp\left(\langle \vec{p}, \vec{a}_{i+2} - \vec{a}_{i+1} \rangle\right) \langle \vec{p} | \vec{a}_{i+2} \rangle \\
&= \langle \vec{a}_i | \vec{p} \rangle \exp\left(\langle \vec{p}, \vec{a}_{i+2} - \vec{a}_i \rangle\right) \langle \vec{p} | \vec{a}_{i+2} \rangle
\end{aligned} \tag{20.1.1}$$

The terms in the middle turn into unity. The other terms also join.

Over a full particle generation cycle with N steps this results in:

$$\begin{aligned}
&\prod_{i=1}^{N-1} \langle \vec{a}_i | \vec{p} \rangle \exp\left(\langle \vec{p}, \vec{a}_{i+1} - \vec{a}_i \rangle\right) \langle \vec{p} | \vec{a}_{i+1} \rangle \\
&= \langle \vec{a}_1 | \vec{p} \rangle \exp\left(\langle \vec{p}, \vec{a}_N - \vec{a}_1 \rangle\right) \langle \vec{p} | \vec{a}_N \rangle \\
&= \langle \vec{a}_1 | \vec{p} \rangle \exp\left(\sum_{i=2}^{N} \langle \vec{p}, \vec{a}_{i+1} - \vec{a}_i \rangle\right) \langle \vec{p} | \vec{a}_N \rangle \\
&= \langle \vec{a}_1 | \vec{p} \rangle \exp(L) \langle \vec{p} | \vec{a}_N \rangle
\end{aligned} \tag{20.1.2}$$

$$L d\tau = \sum_{i=2}^{N} \langle \vec{p}, \vec{a}_{i+1} - \vec{a}_i \rangle = \langle \vec{p}, d\vec{q} \rangle \tag{20.1.3}$$

$$L = \langle \vec{p}, \dot{\vec{q}} \rangle \tag{20.1.4}$$

L is known as the Lagrangian.

Equation (20.1.4) holds for the special condition in which $\vec{p}$ is constant. If $\vec{p}$ is not constant, then the Hamiltonian H varies with location.

$$\frac{\partial H}{\partial q_i} = -\dot{p}_i \tag{20.1.5}$$

$$\frac{\partial H}{\partial p_i} = \dot{q}_i \tag{20.1.6}$$

$$\frac{\partial L}{\partial q_i} = \dot{p}_i \tag{20.1.7}$$

$$\frac{\partial L}{\partial \dot{q}_i} = -\dot{p}_i \tag{20.1.8}$$

$$\frac{\partial H}{\partial \tau} = -\frac{\partial L}{\partial \tau} \tag{20.1.9}$$

$$\frac{d}{d\tau} \frac{\partial L}{\partial \dot{q}_i} = \frac{\partial L}{\partial q_i} \tag{20.1.10}$$

$$H + L = \sum_{i=1}^{3} \dot{q}_i p_i \tag{20.1.11}$$

Here we used proper time τ rather than coordinate time t.

This procedure derives the Lagrangian and the Hamilton equations from the stochastic hopping path. Each term in the series shows that the displacement generator forces the combination of terms to generate a closed hopping path on the platform that carries the elementary particle. The only term that is left is the displacement generation of the whole hop landing location swarm. That term describes the movement of the platform.