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# LORENTZ MAGNETIC FORCE LAW NOT PRECISELY VERIFIED

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ABSTRACT. The Lorentz magnetic force law has not been precisely verified. The experimental basis of it is in the early experiments done through the pioneers around the 1840s and 1850s; no new experiment has since been done when Hendrik Lorentz presented it in 1895 in its current form : $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$. The NIST data base of atomic mass of the various nuclides is actually the experimental data collected in a international distributed experiment to verify the Lorentz magnetic force law by using it to predict the atomic mass of nuclides. By comparing the predicted values with actual values measured using chemical methods, we could indirectly confirm the correctness of the law quantitatively to as much as 1 part in $10^7$.

## 1. INTRODUCTION

Our modern theory of electromagnetism has been formalized around the 1900; it has remained unchanged since then. It is a relativistic electromagnetism theory that combined the work of Maxwell's equations with the Lorentz force law together and unified within the framework of special relativity. There is no more any classical electromagnetism. If there is detected any electromagnetic effect at a point in space with electric and magnetic fields $\mathbf{E}, \mathbf{B}$ by an observer, the fields would not be the same relative to another different moving observers; the $\mathbf{E}, \mathbf{B}$ fields would transform due the special relativity. A moving charge $q$ with a velocity $\mathbf{v}$ near a straight current-carrying conductor would experience a magnetic force acting on $q$ obeying the magnetic force $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$. But for an observer moving at the same velocity $\mathbf{v}$ as the particle, there is no more the magnetic force; the magnetic field $\mathbf{B}$ would have transformed to an electric field $\mathbf{E}$ and the same observed force would be observed, but as $\mathbf{F} = q\mathbf{E}$.

Though the magnetic force is now considered as a purely relativistic effect, the origin of our magnetic theory began after the discovery by Oersted in 1820 that magnetism may be produce by currents. The magnetic fields around conductor currents are considered independent from any electric field that may also be around due to some

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other charged bodies; the electric and the magnetic fields are *'classically'* independent of each other. Immediately after Oersted's discovery was made known, various researchers began to work on this new phenomenon relating current with magnetism. Current-carrying conductors were found to have forces acting between them. Through experiments, Ampere and various others finally came out with laws that governed the forces acting between elemental current elements. It was from these experiments done more than 150 years ago that the current Lorentz magnetic force law was formulated. The law may, therefore, be taken to be an experimental law.

## 2. The Magnetic Field

There are three laws in electromagnetism that involves the magnetic field.

(1) The Biot-Savart Law.

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{Id\mathbf{l}\times\mathbf{r}}{r^2} \tag{1}$$

This law gives the source of the magnetic field as being due to the electric current - and to currents alone. There is no other source of magnetism. The formula is the differential form for a contribution of the magnetic field due to an infinitesimal thin line current at a point in space. To obtain the magnetic field for a complete current loop, a line integral around the loop is computed.

(2) The Faraday's Law of Induction. An electromotive force, or emf, is induced in a loop of wire when the magnetic flux $\phi$ for a surface bounded by the wire changes in time. The induced emf $E$ is given by:

$$E = -\frac{d}{dt}\oint_C \mathbf{B}\cdot d\mathbf{s} \tag{2}$$

where flux is:$\phi = \oint_C \mathbf{B}\cdot d\mathbf{s}$

(3) The Lorentz Force Law.

$$\mathbf{F} = q\mathbf{E} + q(\mathbf{v}\times\mathbf{B}) \tag{3}$$

A theory of magnetism first requires a definition of the concept of the magnetic field; only after the magnetic field is defined that laws may be proposed concerning magnetic phenomena. It is easily seen that the Faraday's law and the Lorentz force law are *'true'* physical laws; the two equations may have meaningful interpretations only when the magnetic field $\mathbf{B}$ is defined. A fundamental criterion of the scientific method is that a law in physics has to be formulated in a manner that it is verifiable through experiments. An accepted law in physics means it has been experimentally verified. Although the Biot-Savart formula is also termed a law of physics, it is not a true law as being a verifiable law. It is actually only a formula that determines how the source of magnetism comes from electric currents. In most textbooks developing magnetism, the Biot-Savart law is used as the formula to define the magnetic field, but the choice of the Biot-Savart law to define the magnetic field is not arbitrary.

The German mathematician Hermann Grassmann proposed in 1845 an elementary force law exerted by a current element $id\mathbf{s}_1$ on another element $id\mathbf{s}_2$[1]; in modern symbols:

$$\mathbf{F} = id\mathbf{s}_2 \times \frac{id\mathbf{s}_1 \times \hat{\mathbf{r}}}{r^2} \tag{4}$$

It is from this law that we have our present Lorentz magnetic force law. The second cross-product term gave us our current Biot-Savart law. It is from the first cross product term that Hendrik Lorentz (1895) came up with the magnetic force on a moving charge to replace a current element. So the current law for the magnetic force may be traced to the experiments done in the early days from the 1820s onward.

There is an alternative to the development of the basic of a relativistic magnetostatic which has a slight variation from what is described above. A typical example is in the textbook by Purcell[2]. The difference is in the definition of the magnetic field. Instead of the Biot-Savart law being the definition of the magnetic field, Purcell takes the Lorentz force law (3) to be the starting relation to define the magnetic field. At any point in space where a charge $q$ moving with a velocity $\mathbf{v}$ experiences any electromagnetic force, it is assumed that the force may be separated into an electric component $q\mathbf{E}$ which is not dependent on $\mathbf{v}$ and a magnetic component $q(\mathbf{v} \times \mathbf{B})$ which is dependent only on the motion of the charge, .i.e on $\mathbf{v}$. It is from the relation :$\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$ that the vector $\mathbf{B}$ is defined. Such a form of definition for $\mathbf{B}$ to be the magnetic field is rather curious. It is tantamount to we defining how nature should behave concerning a magnetic phenomenon. For the charge $q$ moving with various magnitude and direction of motion, nature would provide us with a definite vector $\mathbf{B}$ that satisfy the relation :$\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$. But how can we be sure that such a vector exist? Purcell[2](pg 174) provides the answer:

> *In the following pages we'll see how this comes about. It will turn out that a field B with these properties **must** exist if the forces between electric charges obey the postulates of special relativity. Seen from this point of view, magnetic forces are a relativistic aspect of charge in motion.*

It seems that Purcell's development of the magnetic theory has eliminated the need for any experimental verification of the Lorentz magnetic force relation. Purcell has the proof that the relation :$\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$ is implied in the relativistic treatment of the forces between electric charges. The relation is now an exact relation by definition. But is it so?

In actual fact, Purcell's approach changes nothing concerning a need for the magnetic force to be experimentally verified. What the alternative approach has done is to push the *'burden-of-proof'* from the Lorentz magnetic force law (3) to that of the Biot-Savart law (1). The Biot-Savart law would not be a *'classical'* definition of the magnetic field anymore; it would become a true physical law that need to be verified experimentally. The verification is to show that the RHS of the equation (1) would indeed lead to a vector field equal to the defined magnetic field in: $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$.

The Biot-Savart law gives the formula to compute a vector field of position $\mathbf{B}_e$ (due to electric current) in the space around some given electric current configuration. Purcell's magnetic field definition is: $\mathbf{F} = q(\mathbf{v} \times \mathbf{B}_d)$, $\mathbf{B}_d$ being the defined magnetic field vector. The experimental verification of the Biot-Savart law now means we have to verify the equality of two vector fields of space positions:

$$\mathbf{B}_e = \mathbf{B}_d \tag{5}$$

It is inconceivable that any direct experiment may be designed to verify the validity of the identity in (5), not even for a limited region of space around the simplest of electric current configuration - a straight current-carrying conductor. What conceivably may be done is to replace $\mathbf{B}_d$ with $\mathbf{B}_e$ to arrive at:

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B}_e) \tag{6}$$

We can then indirectly verify (5) if we can verify (6). After going one full circle, Purcell's alternative definition again comes back to an experimental test of (6) which is the same statement of the Lorentz magnetic force law (3). So there is no escaping from the need to treat the Lorentz magnetic force law as a verifiable law.

## 3. Verification Of The Lorentz Magnetic Force Law

The Lorentz magnetic force law may be considered an experimental law as it was formulated based on the early experiments around the 1840s. It may be said to be a verified law, but the experimental basis of the law is already more than 150 years old. Since then, no modern test of the law has ever been conducted. We do not have any knowledge on how precise the statement of the law is quantitatively. On the contrary, the Coulomb's law, first formulated in 1785, has been verified in modern times to a high degree of certainty [3](1971):

> *Expressed as a deviation from Coulomb's law of the form $1/r^{2+q}$, our experiment gives $q = (2.7 \pm 3.1)10^{-16}$.*

This shows the Coulomb's law obeys the inverse-square relation good to 1 part in $10^{16}$. Newton's law of universal gravitation, too, is said to have been verified accurate to 1 part in $10^{10}$ for earth-moon distances.

A law of physics may be verified directly or indirectly. Quite often because of the very nature of the formulation, a direct test of a law may not be possible. An example of a law that may be verified directly is the Coulomb's law derived experimentally using a torsion balance. On the other hand, the Newton's three laws of motion cannot be verified directly through experiments. In actual fact, the three laws are just proposed axioms or hypothesis underlying Newton's theory of motion. As a clarification, the second law is a definition for the concept of force that is fully equivalent to defining force through the relation : $force \propto invariant\_mass \times acceleration$; a definition is never testable through an experiment. Newtons three laws of motion, therefore, cannot be verified directly through experiments. Newton's laws together with his proposed law of universal gravitation forms Newtonian mechanics. The Newton's laws of motion may be verified to be correct only indirectly through the predictions of Newtonian mechanics. The predicted motions of planets around the sun through the application of Newtonian mechanics has been verified to be correct when compared to the empirical data collected for the motions of the various planets.

Let's examine how the Lorentz magnetic force law: $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$ may be verified. Any direct test of it would present great difficulties:

- Measurement of force on a moving charge. Rarely is it possible to have a means to directly measure force on a charge particle that has motion, .e.g that of an electron.
- The various variations of velocity need to be measured for magnitude as well as direction within some region of space within the magnetic field.
- The magnetic field within some region of space need to be directly measured or computed from first principle. If the source of the magnetic field is from an electric current configuration, it may only be computed from first principle, .i.e. based on the Biot-Savart law. If the source of the magnetic field is from a permanent magnet, the field strength has to be directly measured; but there is no known instrument that could measure magnetic field strength based on the Biot-Savart law.

As can be seen, any relation involving the magnetic field is bound to present great difficulties for experimental verification.

An indirect verification of the Lorentz magnetic force may be done within some restricted conditions. If a charge particle moves in a uniform magnetic field and the plane of motion is perpendicular to the magnetic field, the particle's trajectory would be a perfect circle.

Various techniques have employed such circular motion, e.g. as in mass spectrometry. If an electron moves in such a manner entering the fixed uniform magnetic field region $\mathbf{B}$ with a velocity $\mathbf{v}$, it will trace a circular arc with radius $r$. The equation of circular motion is:

$$\begin{aligned} F &= evB = mv^2/r; \\ v &= (eB/m)r; \end{aligned} \tag{7}$$

The electron could be allowed to trace a certain arc length in vacuum within the field $B$; it then leaves the magnetic field and travels in a straight line and be allowed to hit a screen. The velocity $v$ may be obtained through time-of-flight measurements; $r$ could easily be found through the geometry of the trajectory and the known point it hits the screen. It is seen that $v$ varies linearly with $r$. If a set of data points of $(v_i, r_i)$ is obtained, it could be used to examine how well the points fit the linear relation.

Though such an experiment is simple in theory, there could be great difficulties. The experiment would not be able to establish such a linear relation precisely unless the measurements for $v$ and $r$ could be done with great precision. The field $B$, too, need to be steady and uniform to a high degree of precision within the path where it matters. There is no report of any such experiments being attempted.

## 4. The NIST Data Base Of Atomic Mass

The various difficulties as outlined above may be the reason why, till now, no experiment has ever been done to verify the Lorentz magnetic force law in order to have some numbers to indicate its accuracy. The physics community still has need to use the relation $F = q(\mathbf{v}\times\mathbf{B})$; but when it is used, it is simply taken to be an exact relation ignoring the fact that there may be uncertainty to its accuracy. This is the present undesired situation. In fact, there is a way - a relatively simple way - for the Lorentz magnetic force law to be verified to a fairly high degree of accuracy - to as much as 1 part to $10^7$.

At present, nearly all atomic mass measurements are made with the Penning trap mass spectrometer - a highly sophisticated piece of equipment. It has resolution that may be as high as 1 part in $10^{11}$. There is a brief description of its principle in another paper of the author [4]. From the equation of motion of a trapped ion within the small cell space of the Penning trap, we can arrive at an equation with an angular frequency $\omega_c$ instead of the usual parameters of velocity $v$ and radius $r$: $m = \dfrac{qB}{\omega_c}$. With the electric and magnetic fields environment unchanged for two trap charged particles with masses $m_2$, $m_1$, the relative mass with the same electron charge would be:

$$\frac{m_2}{m_1} = \frac{\omega_1}{\omega_2} \tag{8}$$

This relation (8) enables the relative atomic mass of two particles to be found through measurements of the emitted characteristic frequencies of the oscillating particles. It is because of the high resolution with which we are able to measure frequencies that the Penning trap can give great precision in its measurements.

However, there is an issue with the supposed *'measured mass'* as measured with the Penning trap. The principle underlying the working of the Penning trap as given in the equation (8) is based strictly on the Lorentz magnetic force law: $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$ - in any way that the magnetic force law fails, the working of the Penning trap fails. Whenever the Penning trap is used to measure the relative mass of ions of two nuclides, what it actually does is nothing but an experiment - an experiment to verify the validity and accuracy of the Lorentz magnetic force law.

A piece of equipment - the Penning trap - is designed based on the Lorentz magnetic force law. The equipment is capable of measuring the relative atomic mass of two nuclides. By comparing the measured figure with the actual relative atomic mass of the two nuclides, we would know how accurate the Lorentz magnetic force law is, or even about its validity. Such a measurement has to be viewed as an experiment as the Lorentz magnetic force law has yet to be verified to have the accuracy to that of the precision of the Penning trap. So the huge NIST data base of the atomic mass is actually the collected predicted values of atomic masses according to the Lorentz magnetic force law. It is a collection of experimental data collected through a distributed experiment done through international collaboration from the various research centers around the world.

> *The huge NIST data base of atomic masses of nuclides are predicted values according to the Lorentz magnetic force law obtained through experiments carried out with the Penning trap.*

4.1. **True Atomic Mass Of Nuclides.** Until now, no conclusion could be made on the experiment concerning the accuracy or the validity of the Lorentz magnetic force law. Currently, there is no data of any sort of the actual atomic masses of the known nuclides. The physics community is using the NIST data as actual values of atomic mass instead of the data being just predicted values. But we do have the method to measure the true atomic mass of the nuclides. Before the invention of mass spectrometry in the 20th century, the chemists had measured atomic weights using chemical analysis. But because the elements found in nature are usually in a mixture of various isotopes of the elements, the old atomic weights data are of little use today. Furthermore, the precision of those early measurements are low.

Today, we have very accurate chemical balances that could even measure 1 gram accurate to 1 part in $10^7$. Also we have the technique to separate pure isotopes of most elements. Chemical analysis of compounds to determined the composition by weights of the elements is considered a relatively easy task with today's sophisticated chemical methods. As the predicted atomic masses from the NIST data have greater precision than than 1 part in $10^7$, a comparison of the predicted values with the actual values found through chemical analysis would allow us to conclude if the predictions of the Lorentz magnetic force law are correct.

> *The Lorentz magnetic force law could be verified indirectly through its prediction of the atomic mass of the various nuclides. If the NIST predicted values are in agreement with the actual values obtained through chemical analysis, the Lorentz magnetic force law would then be experimentally verified to at least 1 part in $10^7$.*

## 5. CONCLUSION

The Lorentz magnetic force law has not been precisely verified. The experimental basis of it is in the early experiments done through the pioneers around the 1840s and 1850s; no new experiment has since been done when Hendrik Lorentz presented it in 1895 in its current form : $\mathbf{F} = q(\mathbf{v}\times\mathbf{B})$. The NIST data base of atomic mass of the various nuclides is actually the experimental data collected in a international distributed experiment to verify the Lorentz magnetic force law by using it to predict the atomic mass of nuclides. By comparing the predicted values with actual values measured using chemical methods, we could indirectly confirm the correctness of the law quantitatively to as much as 1 part in $10^7$.

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