Comment on "Flat rotation curve without dark matter: the generalized Newton's law of gravitation"

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Abstract: In a relatively recent article by Arbab I. Arbab (Astrophys. Space Sci. **355**, 343, 2015), the author claimed that the problem of flat rotation curve could be solved in the context of the so-called'generalized Newton's law of gravitation' without resorting to the dark matter. However, the present comment proves more conclusively that the proposed formulae and the calculations are physico-mathematically wrong. Consequently, the author's claim is mathematically and physically highly questionable.

Keywords: generalized Newton's law of gravitation, gravitomagnetism, flat rotation curve

1. Introduction

In previous comment [1] on the article entitled '*The generalized Newton's law of gravitation*' by Arbab I. Arbab, we have identified the main reason that eventually led the author to the erroneous formulae, which was the deliberate confusion between the gravitation and electromagnetism through the superfluous idea of gravitomagnetism.

In the present comment we have focused our attention on the supposed application of the wrongly generalized Newton's law to the problem of flat rotation curve.

In his original article '*Flat rotation curve without dark matter: the generalized Newton's law of gravitation*'[2], Arbab I. Arbab claimed that the problem of flat rotation curve may be solved in the context of the 'generalized Newton's law of gravitation' without requiring the dark matter. And, step by step, he derived twenty-one erroneous equations from two wrong equations (1) and (7) in **Ref.**[2]. In what follows we will see the proofs of the author's fatal errors and their source.

2. Proofs of the author's fatal errors and their source

Now, we arrive at our main subject namely the scrutiny of the paper under consideration '*Flat rotation curve without dark matter: the generalized Newton's law of gravitation*' [2]. Recall that our first major objection is that the author failed to derive the correct formulae supposed to be a consequence of the so-called *generalized Newton's law of gravitation* to solve the problem of flat rotation curve without requiring the dark matter. In order to make our scrutiny more comprehensible, we are obliged to rewrite the author's central claims, word by word. In Section 2 (page 1) entitled 'Gravitomagnetic force', the author wrote**:** *In the generalized Newton's law of gravitation developed by Arbab (2010, 2012), one has*

$$F = -\frac{GMm}{r^2} - \frac{\pi m v^4}{v_c^2 r}, \tag{1}$$

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where v_c is some characteristic velocity. This force is the gravitational analogue of Lorentz force of electromagnetism. The second term in Eq.(1) accounts for the gravitomagnetic force arising from the motion of the orbiting mass, m. For ordinary velocities Eq.(1) reduces to the ordinary Newton's law of gravitation. However, since we are interested in the behavior of matter at very large distances where the object (star) speed is so big, the situation will be different, as we will describe below. For a circular motion, one has

$$\frac{mv^2}{r} = \frac{GMm}{r^2} + \frac{\pi m v^4}{v_c^2 r}. \tag{2}$$

Equation (2) is solved to give

$$v^2 = \frac{v_c^2}{2\pi}\left(1 \pm \sqrt{1 - \frac{4\pi GM}{v_c^2 r}}\right). \tag{3}$$

First, the reader can easily notice that contrary to the author's claim, for a circular motion, Eq.(2) cannot be deduced from Eq.(1) because of the explicit presence of the minus sign (–) in Eq.(1).

It is clear, the minus sign on the RHS of Eq.(1) cannot be magically canceled just by replacing F with $mv^2 r^{-1}$. Consequently, Eq.(3) is mathematically and physically meaningless since it is a solution to an erroneous Eq.(2) for v^2.

– The source of the author's fatal errors

In the **Ref.**[3] the author wrote Eq.(1) in the following form

$$F = \frac{GMm}{r^2} - \frac{mv^4}{c^2 r}, \tag{i}$$

where c is the light speed in vacuum.

It seems that the author has simply modified the second term on RHS of Eq.(i) to get Eq.(1), thus the explicit presence of the minus sign (–) before the first term in Eq.(1) may be just a typographical error. This assertion is also supported by the fact that the Newton's law of gravitation in scalar form $F = GMmr^{-2}$ is generally written without the minus sign, that's at variance with the same law written in vector form $\mathbf{F} = -GMmr^{-3}\mathbf{r}$. In fact, F is the magnitude of **F**.

Now, if we assume that the correct expression for Eq.(1) is

$$F = \frac{GMm}{r^2} - \frac{\pi m v^4}{v_c^2 r}. \tag{**1**}$$

In this case, Eq.(2) becomes

$$\frac{mv^2}{r} = \frac{GMm}{r^2} - \frac{\pi m v^4}{v_c^2 r}, \tag{**2**}$$

and its solution for v^2 is of the form

$$v^2 = \frac{v_c^2}{2\pi}\left(-1 \pm \sqrt{1+\frac{4\pi GM}{v_c^2 r}}\right) = -\frac{v_c^2}{2\pi}\left(1 \mp \sqrt{1+\frac{4\pi GM}{v_c^2 r}}\right). \quad (\mathbf{3})$$

Now, let's consider the case when, $(4\pi GM/v_c^2) < r$. Hence, by binomial series expansion of the square root in Eq.(**3**) and after neglecting the higher orders terms, we get the following two solutions

$$v_-^2 \approx \frac{GM}{r}\ , \quad (\mathbf{4})$$

and

$$v_+^2 \approx -\frac{v_c^2}{\pi}\left(1+\frac{\pi GM}{v_c^2 r}\right), \quad (\mathbf{5})$$

As we can remark, the solution (**4**) is just the classical expression of the squared orbital velocity and the solution (**5**) is mathematically and physically meaningless.

In Section 3 (page 1) entitled 'Dark matter potential energy' the author wrote**:** *If we now substitute Eq. (3) in Eq. (1), we will get*

$$F = -\frac{mv_c^2}{2\pi r}\left(1 \pm \sqrt{1-\frac{4\pi GM}{v_c^2 r}}\right). \quad (7)$$

Despite the fact that Eq.(3) itself is incorrect, we can easily show that Eq.(7) is also completely wrong. That's contrary to the author's claim, the substitution of Eq.(3) in Eq.(1) cannot lead to Eq.(7).

– Proof: by substituting Eq.(3) in Eq.(1), we find

$$F = -\frac{GMm}{r^2} - \frac{\pi m v^4}{v_c^2 r} = -\frac{GMm}{r^2} - \frac{\pi m}{v_c^2 r}\left[\frac{v_c^2}{2\pi}\left(1 \pm \sqrt{1-\frac{4\pi GM}{v_c^2 r}}\right)\right]^2,$$

$$F = -\frac{GMm}{r^2} - \frac{mv_c^2}{4\pi r}\left[1 \pm \sqrt{1+\frac{4\pi GM}{v_c^2 r}}\right]^2. \quad (ii)$$

It is clear, the expression of Eq.(ii) proves more convincingly the incorrectness of Eq.(7).
Therefore, all the equations derived from the erroneous Eq.(7) are mathematically and physically meaningless.

3. Conclusion

We have scrutinized the paper '*Flat rotation curve without dark matter: the generalized Newton's law of gravitation*' [2] and proved that this paper is physico-mathematically incorrect. The paper contains fatal errors. Consequently, the so-called 'The generalized Newton's law of gravitation' and

its extension and application cannot be considered as an intellectual and scientific contribution to the science in general and to the gravitational physics in particular as the paper is exceedingly questionable.

Furthermore, in his acknowledgements, the author wrote: «*... I would also like to thank the anonymous referees for their critical comments and suggestions*.» accordingly, the legitimate question that should be asked is: Why were the anonymous referees unable to identify the author's fatal mistakes?

References

[1] https://www.researchgate.net/publication/280945568

[2] A.I. Arbab, Astrophys. Space Sci. **355**, 343, (2015)

[3] A.I. Arbab, Astrophys. Space Sci. **325**, 37 (2010)