Completing the Slichter Triplet

Herbert Weidner[A]

Abstract: Following the discovery of a spectral line with an unusually high quality factor, exactly two other lines with comparable characteristics were found in the frequency range 45 to 130 µHz. Maybe it is the desired Slichter triplett.

Introduction

All calculations, which concern the legendary *Slichter triplet*, predict on the basis of the previously known data of the earth core that this may oscillate at about 60 µHz around its rest position in the center of the earth. The rotation of the earth causes three different natural resonances of the 1S1 translational mode to exist. Probably the metrological proof of one of the three frequencies has been successful[1]. The two missing spectral frequencies should have similar properties as the oscillation at 50.65 µHz. Presumably all frequencies of the Slichter triplet are stimulated simultaneously, have the same attenuation and are similarly modulated. Since the first spectral line (50.65 µHz) can only be detected between the years 1997 and 2000, it makes sense to look for the missing spectral lines in the same time range. The data of the station CB are sufficiently low-noise to serve as a basis for further search.

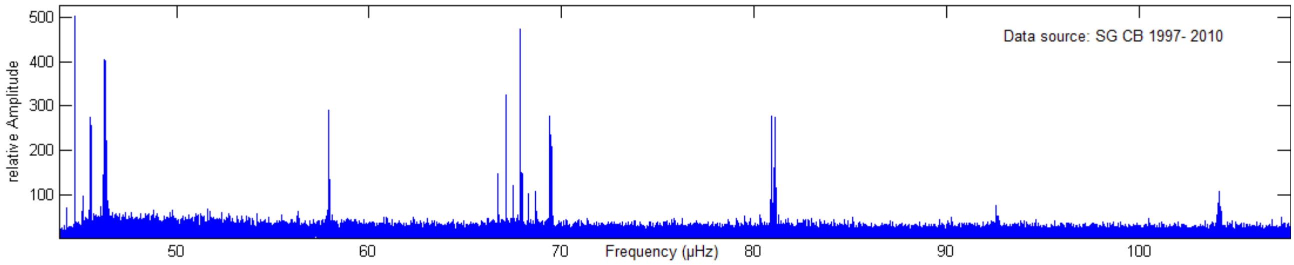

Fig 1: Low-noise spectrum from multi-year data measured by the CB gravimeter

Triplet member number 1 (m= -1)

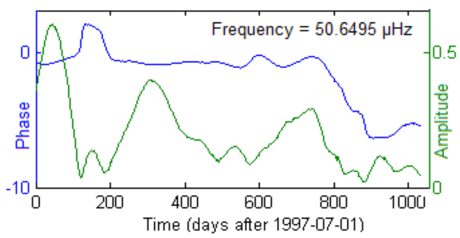

Fig 2: Trends of phase and amplitude, with Hamming window

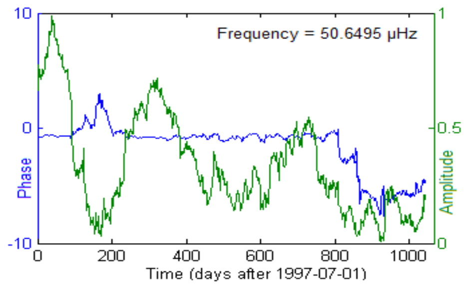

Abbildung 3: Trends of phase (blue) and amplitude (green), no window

The preparation of the data has already been described in [1] and does not need to be repeated here. In the procedure Vector-FFT, one can choose whether or not the short data series (300 data points) go through a Hamming-window before their transformation. The results differ considerably – so do the gained information. Since variant A (figure 3) does not use a window, the results are raw and unsmoothed. If "windowing" is omitted, we see a very high-frequency noise which is at least partially caused by the discontinuity at the beginning and the end of the data packet. The main task of "windowing" is to avoid the cracks. Using a window, figure 2 shows a much smoother course, as if a low pass had provided for a flattening course.

It is unclear how the signal is to be interpreted in the period from 120 to 200 hours after the beginning of the measurement. From Figure 2 it could be seen that a phase jump has occurred around π, ie that the signal was measured out of phase for 80 days. In figure 3 it looks rather as if the calculation of the phase was very inaccurate because of too small an amplitude. That can not be decided on the basis of these two pictures.

(A) 3. Feb. 2019, email: herbertweidner@gmx.de

Triplet member number 2 (m= +1)

In search of further candidates for the Slichter triplet, the range between 45 µHz and 130 µHz of the spectrum (Fig. 1) was carefully searched for spectral lines with unusual properties.

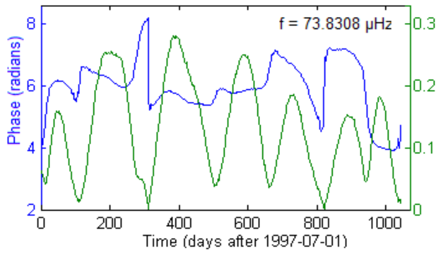

Fig 4: Trends of phase and amplitude, Hamming window

A potential candidate was found at 73.8308 µHz, whose characteristics are very similar to the spectral line near 50.65 µHz (Fig 4 and Fig 5). The phase is noisy, but nearly constant for 910 days and here again, no simple exponential decay of the amplitude is observed. During the following years until 2010, neither the phase nor the amplitude shows any regularity. As with the previously discovered spectral line (50.65 µHz), no signs were found that the strong earthquake of 2004-12-26 would have affected this vibration.

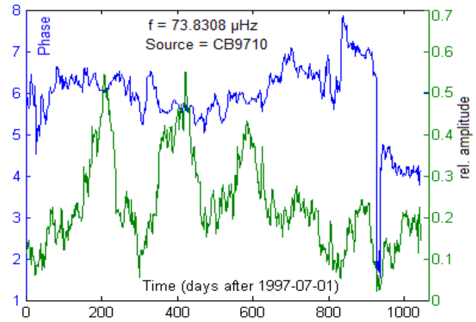

Fig 5: Trends of phase (blue) and amplitude (green), no window

The biggest difference in comparison with [1] can be found right at the beginning of the evaluation: the oscillation on the frequency 73.83 µHz seems to start a few weeks later than the one at 50.65 µHz. Given a vibration time of about four hours, this is a remarkably long delay – as long as the two observations have anything to do with each other. The present data do not allow this to be precise, because the station CB did not supply any data before 1997-07-01.

Figure 6 shows the spectrum for the first 900 days after 1997-07-01. The two properties phase and amplitude of the strong line at 73.766 µHz were determined, they differ in each respect from those of the spectral line at 73.8308 µHz. There was no attempt to suppress this line, because it does not bother. By contrast, the rather weak satellite frequencies of 73.8308 µHz are an essential component of the overall signal. These are the sidebands that determine the modulation (T ≈ 177 days, see below).

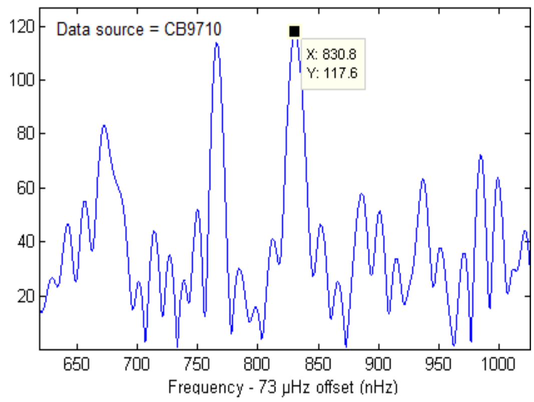

Fig 6: Spectrum of the data; The recording period is 1997 - 1999

The method “Vector FFT” [2] allows to track the evolution of phase and amplitude of any frequency. In order to determine the attenuation of the oscillation in this way, the amplification of the data was changed so that the exponential decrease of the oscillation amplitude (Fig 5, green curve) is compensated. This simultaneously affects the sideband frequencies of 73.83 µHz. The superimposition in figure 8 shows that they all behave in a similar way and therefore belong together. Due to the low signal-to-noise ratio, the best estimate of the quality factor is Q = 15000 ± 1000.

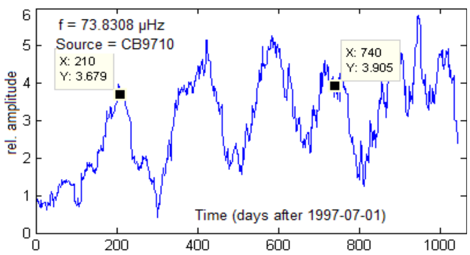

Fig 7: Sum of the colored tracks in figure 6

If you add the amplitude curve of the neighboring line at 73.766 µHz to figure 8, you can see from the very different course that both spectral lines have *no* common origin. Therefore this additive was removed.

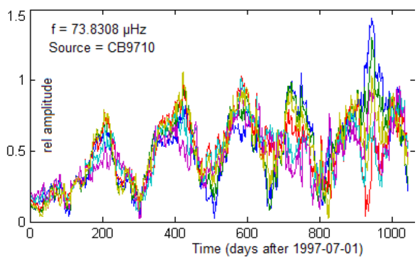

Fig 8: Superposition of the amplitudes of carrier and sideband frequencies

The good match of the colored individual amplitudes in figure 8 are a strong indication that they can be treated as a single package. Therefore, in figure 7, their sum is shown, showing that the spectral line is 73.8308 µHz amplitude modulated (there are no phase jumps as in 3S2 [3]). The modulation frequency is 1/(177 days) and differs significantly from the modulation frequency of 50.65 µHz [1]. One reason for this difference is certainly that the observers sit on top of the rotating earth crust and measure the vibration of the solid core without being rigidly attached to it.

Triplet member number 3 (m = 0)

So far, two spectral lines with similar properties have been discovered, which could have a common cause: 50.6496 µHz and 73.8308 µHz. Is there a third line? In which area should you search for the missing spectral line? What does the theory say?

For a spherical non-rotating Earth model, the seismic normal mode 1S1 degenerates and we have a single spectral line. Because of rotation and ellipticity, a degenerate eigenfrequency is split into three distinct eigenfrequencies. Dahlen (1968) proposed that an eigenfrequency f0 obeys a quadratic splitting law: $f_m = f_0(1+a+bm+cm^2)$ with m= –1 or m=0 oder m= +1. The splitting parameters a, b and c depend on the Earth model. In [4], many possible and unproven solutions are listed in tables 2 and 3. Table 3 shows that adjacent values differ by about 8.5%. However, the two frequencies found so far differ by 46%, which is far beyond any estimates that can be found in literature. If the two frequencies found so far are $f_{m=-1}$ and $f_{m=+1}$, we expext $f_{m=0}$ near 61 µHz.

Variant A: FFT without windowing.

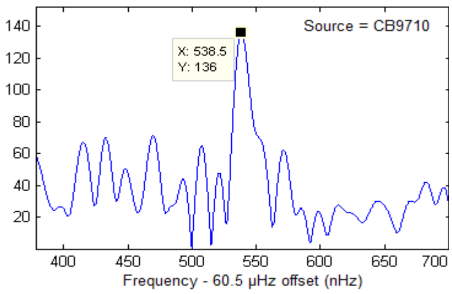

Fig 9: Spectrum of the data; The recording period is 1997 - 1999

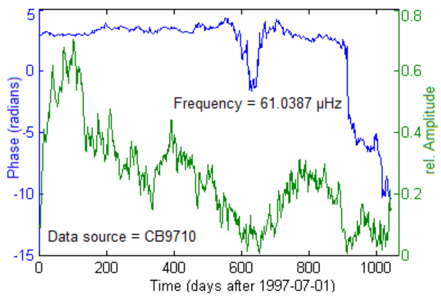

Fig 10: Trends of phase (blue) and amplitude (green), no window

There, the search began and was immediately successful, as can be seen in figure 10. The phase is constant for more than 900 days, except for a short fluctuation due to low amplitude. During the following years until 2010, neither the phase nor the amplitude shows any regularity. No signs were found that the strong earthquake of 2004-12-26 would have affected this vibration.

The signal-to-noise ratio of the spectral line at 61.0387 µHz (figure 9) is significantly better than the SNR of the other two lines.

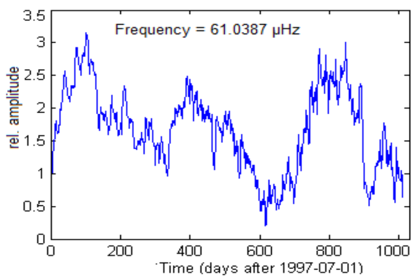

Fig 11: Sum of the colored tracks in figure 10

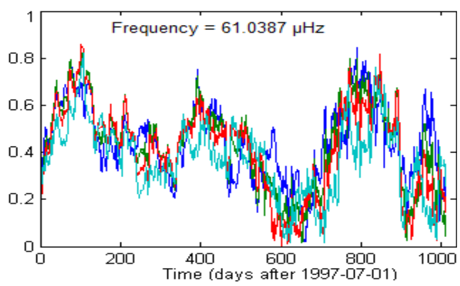

Fig 12: Superposition of the amplitudes of carrier and sideband frequencies

As before, the amplification of the data was changed so that the exponential decrease of the oscillation amplitude (Fig 10, green curve) is compensated. This simultaneously affects the sideband frequencies of 61.0387 µHz. The superimposition in figure 12 shows that they all behave in a similar way and therefore belong together. Due to the low signal-to-noise ratio, the best estimate of the quality factor is Q = 15000 ± 1000.

The good match of the colored individual amplitudes in figure 12 is a strong indication that they can be treated as a single package. Therefore, in figure 11, their sum is shown, showing that the spectral line near 61.0387 μHz is amplitude modulated. The modulation frequency is approximately 1/(360 days) and almost corresponds to the year length.

Variant B: Using a Hamming window before FFT.

The use of the window smooths the curves and some details differ from those in variant A. The effective shortening of the window causes other details to become visible (Fig 13). A comparison with figure 10 shows: the short disappearance of the signal 610 hours after the start of the measurement does not produce a continuous phase deviation without a window. If a window is used (figure 13), the same disturbance generates a phase jump of ≈ 2π. It is unpredictable whether the brief suspension of the signal produces a permanent phase shift or not. Regardless of the application of a hamming window, after about 900 hours, the phase constance disappears and the spectral line disappears in the noise. The spectrum (not shown) is not significantly deformed when using a hamming windows.

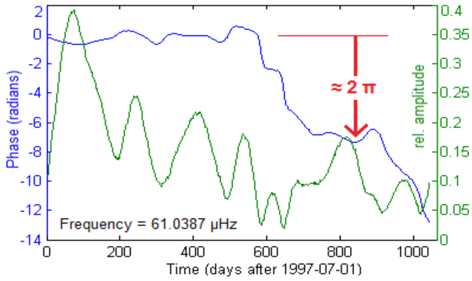

Fig 13: Trends of phase and amplitude, with Hamming window

Summary

In the low-noise data series of the SG Canberra, three spectral lines were discovered at 50.6496 ± 0.0002 μHz, 61.0387 ± 0.0002 μHz and 73.8308 ± 0.0002 μHz. The error width follows from the selected frequency resolution of the FFT (131072 bins). The properties of all three lines agree in important points, which is why it can be assumed that all three lines are generated by a common source.

- The amplitudes are approximately identical and are just above the noise.
- Each of the three lines is attenuated with approximately the same time constant (Q ≈ 14000) and disappears in the noise 900 days after the start of measurement (1997-07-01).
- Each of the three spectral lines is amplitude-modulated, with the modulation cycles differing significantly, each lasting several hundred days. Higher modulation frequencies can not be detected because of the poor SNR.
- The most important point: during an unusually long period of almost three years, the phase is extraordinarily constant.
- The distance of the two "satellites" from the central frequency is almost identical.
- It can not be determined when and why the vibrations start. The latest date is probably the 1997-07-01, when the SG station Canberra started to work. Next, it must be checked if other stations have registered enough low-noise data from earlier periods.
- Neither phase nor amplitude of the three spectral lines are affected by the strong earthquake of 2004-12-26. As they they appear and disappear simultaneously, there seems to be another mechanism of excitation.
- Careful tests have shown that there is no other spectral line in the frequency range between 45 and 120 μHz, which has comparable properties as stated above. During the search, the constancy of the phase was checked very carefully.

So far, several theoretical models have been developed, how to calculate the frequencies of the Slichter triplett and there are also many solutions. But not one of them has been confirmed on the basis of experimental data.

Acknowledge

Thanks to all operators for gathering SG data and thanks the IGETS data base at GFZ Potsdam for storing the data.

[1] H. Weidner, Weak Signal Detection in Geophysical Data using Vector-FFT, 2019
[2] H. Weidner, Frequency Measurements in Noisy Records using Vector-FFT, 2019
[3] H. Weidner, Phase Inversion of the 3S2 Oscillation of the Earth, 2018
[4] S. Rosat, Y. Rogister, D. Crossley, J. Hinderer, A search for the Slichter Triplet with Superconducting Gravimeters: impact of the density jump at the inner core boundary, 2005