Cosmological Acceleration as a Consequence of Quantum de Sitter Symmetry

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Abstract

Physicists usually understand that physics cannot (and should not) explain why $c = 3 \cdot 10^8 m/s$ and $\hbar = 1.054 \cdot 10^{-34} kg \cdot m^2/s$ because c and $\hbar$ are fundamental physical quantities. At the same time they usually believe that physics should explain the value of the cosmological constant Λ. We first prove that three fundamental parameters defining transitions from more general theories to less general ones are $(c, \hbar, R)$ where R is the parameter defining contraction from the de Sitter (dS) or anti-de Sitter (AdS) algebra to the Poincare algebra. This parameter is fundamental to the same extent as c and $\hbar$. In particular, a question why R is as is does not arise, and the answer is simply that R has its value because people want to measure distances in meters. On classical level $\Lambda = \pm 3/R^2$ for dS and AdS spaces, respectively. As a consequence of the fact that quantum dS and AdS symmetries are more general than Poincare symmetry, the cosmological constant problem does not arise, Λ is necessarily not zero and there is no need to involve dark energy for explaining the cosmological acceleration. Following our previous publications, we consider a system of two free bodies in dS invariant quantum mechanics and show that in semiclassical approximation the dS repulsion is the same as in General Relativity. This result is obtained without using geometry of dS space, metric and connection but simply as a consequence of quantum dS symmetry.

Keywords: dark energy, quantum theory, de Sitter symmetry

1 Brief overview of the cosmological constant problem and dark energy

The history of General Relativity (GR) is described in a vast literature. The Lagrangian of GR is linear in Riemannian curvature R_c, but from the point of view of symmetry requirements there exist infinitely many Lagrangians satisfying such requirements. For example, $f(R_c)$ theories of gravity are widely discussed, where there can be many possibilities for choosing the function f. Then the effective gravitational constant G_{eff} can considerably differ from standard gravitational constant G. It is also argued that GR is a low energy approximation of more general theories involving higher order derivatives. The nature of gravity on quantum level is a problem, and standard canonical quantum gravity is not renormalizable. For those reasons the quantity G can be treated only as a phenomenological parameter but not fundamental one.

Let us restrict ourselves with the consideration of standard GR. Here the Einstein equations depend on two arbitrary parameters G and Λ where Λ is the cosmological constant (CC). In the formal limit of GR when matter disappears, space-time becomes Minkowski space when $\Lambda = 0$, de Sitter (dS) space when $\Lambda > 0$, and anti-de Sitter (AdS) space when $\Lambda < 0$.

Well known historical facts are that first Einstein included Λ because he believed that the Universe should be stationary, and this is possible only if $\Lambda \neq 0$. However, according to Gamow, after Friedman's results and Hubble's discovery of the Universe expansion, Einstein changed his mind and said that inclusion of Λ was the greatest blunder of his life (but there are no independent confirmations of this phrase).

The usual philosophy of GR is that curvature is created by matter and therefore Λ should be equal to zero. This philosophy has been advocated even in standard textbooks written before 1998. For example, the authors of Ref. [1] say that "...there are no convincing reasons, observational and theoretical, for introducing a nonzero value of Λ" and that "... introducing to the density of the Lagrange function a constant term which does not depend on the field state would mean attributing to space-time a principally ineradicable curvature which is related neither to matter nor to gravitational waves".

However, the data of Ref. [2] on supernovae have shown that $\Lambda > 0$ with the accuracy better than 5%, and further investigations have improved the accuracy to 1%. For reconciling this fact with the philosophy of GR, the terms with Λ in the left-hand-sides of the Einstein equations have been moved to the right-hand-sides and interpreted not as the curvature of empty space-time but as a contribution of unknown matter called dark energy. Then, as follows from the experimental value of Λ, dark energy contains approximately 70% of the energy of the Universe. At present a possible nature of dark energy is discussed in a vast literature and several experiments have been proposed.

Let us to note the following. In the formalism of GR the coordinates and curvature are needed for the description of real bodies. One of fundamental principles of physics is that definition of a physical quantity is the description on how this quantity should be measured. In the Copenhagen formulation of quantum theory measurement is an interaction with a classical object. Therefore in empty space-time nothing can be measured, and the coordinates and curvature of empty space-time have no physical meaning. This poses a problem whether the formal limit of GR when matter disappears but space-time remains is physical. Some authors (see e.g. Ref. [3]) propose approaches such that if matter disappears then space-time disappears too.

The CC problem is as follows. In standard quantum field theory one starts from the choice of the space-time background. By analogy with the philosophy of GR, it is believed that the choice of the Minkowski background is more physical than the choice of the dS or AdS one. Here the quantity G is treated as fundamental and the value of Λ should be extracted from the vacuum expectation value of the energy-momentum tensor. The theory contains strong divergencies and a reasonable cutoff gives for Λ a value exceeding the experimental one by 120 orders of magnitude. This result is expected because in units $c = \hbar = 1$ the dimension of G is m^2, the dimension of Λ is m^{-2} and therefore one might think than Λ is of the order of $1/G$ what exceeds the experimental value by 120 orders of magnitude.

Several authors argue that the CC problem does not exists. For example, the authors of Ref. [4] titled "Why all These Prejudices Against a Constant?" note that since the solution of the Einstein equations depends on two *arbitrary phenomenological* constants G and Λ it is not clear why we should choose only a special case $\Lambda = 0$. If Λ is as small as given in Ref. [2] then it has no effect on the data in Solar System and the contribution of Λ is important only at cosmological distances. Also theorists supporting Loop Quantum Gravity say that the preferable choice of Minkowski background contradicts the background independence principle. Nevertheless, the majority of physicists working in this field believe that the CC problem does exist and the solution should be sought in the framework of dark energy, quintessence and other approaches.

2 Remarks on fundamental theories

In this section we discuss comparisons of fundamental theories. One of the known examples is the comparison of nonrelativistic theory (NT) with relativistic one (RT). One of the reasons why RT can be treated as more fundamental is that it contains a finite parameter c and NT can be treated as a special degenerate case of RT in the formal limit $c \to \infty$. Therefore, by choosing a large value of c, RT can reproduce any result of NT with a high accuracy. On the contrary, when the limit is already taken one cannot return back from NT to RT and NT cannot reproduce all results of RT. It can reproduce only results obtained when $v \ll c$. Other known examples are that classical theory is a special degenerated case of quantum one in the formal limit $\hbar \to \infty$ and RT is a special degenerate case of dS and AdS invariant theories in the formal limit $R \to \infty$ where R is the parameter of contraction from the dS or AdS algebras to the Poincare algebra (see below). A question arises whether it is possible to give a general definition when theory A is more fundamental than theory B. In view of the above examples, we propose the following

Definition: *Let theory A contain a finite parameter and theory B be obtained from theory A in the formal limit when the parameter goes to zero or infinity. Suppose that with any desired accuracy theory A can reproduce any result of theory B by choosing a value of the parameter. On the contrary, when the limit is already taken then one cannot return back to theory A and theory B cannot reproduce all results of theory A. Then theory A is more general than theory B and theory B is a special degenerate case of theory A.* A problem arises how to justify this **Definition** not only from physical but also from mathematical considerations.

In relativistic quantum theory the usual approach to symmetry on quantum level follows. Since the Poincare group is the group of motions of Minkowski space, quantum states should be described by representations of this group. This implies that the representation generators commute according to the commutation relations of the Poincare group Lie algebra:

$$
\begin{aligned}
&[P^\mu, P^\nu] = 0, \quad [P^\mu, M^{\nu\rho}] = -i(\eta^{\mu\rho}P^\nu - \eta^{\mu\nu}P^\rho), \\
&[M^{\mu\nu}, M^{\rho\sigma}] = -i(\eta^{\mu\rho}M^{\nu\sigma} + \eta^{\nu\sigma}M^{\mu\rho} - \eta^{\mu\sigma}M^{\nu\rho} - \eta^{\nu\rho}M^{\mu\sigma})
\end{aligned}
\tag{1}
$$

where $\mu, \nu = 0, 1, 2, 3$, P^μ are the operators of the four-momentum and $M^{\mu\nu}$ are the operators of Lorentz angular momenta. This approach is in the spirit of Klein's Erlangen program in mathematics.

However, as noted in Sec. 1, the notion of the background space-time is problematic and, as argued in Ref. [5], the approach should be the opposite. Each system is described by a set of linearly independent operators. By definition, the rules how they commute with each other define the symmetry algebra. In particular, *by definition*, Poincare symmetry on quantum level means that the operators commute according to Eq. (1). This definition does not involve Minkowski space at all.

Such a definition of symmetry on quantum level has been proposed in Ref. [6] and in subsequent publications of those authors. I am very grateful to Leonid Avksent'evich Kondratyuk for explaining me this definition during our collaboration. I believe that this replacement of the standard paradigm is fundamental for understanding quantum theory, and I did not succeed in finding a similar idea in the literature.

Our goal is to compare four theories: classical (i.e. non-quantum) theory, nonrelativistic quantum theory, relativistic quantum theory and dS or AdS quantum theory. All those theories are described by representations of the symmetry algebra containing ten linearly independent operators A_α ($\alpha = 1, 2, ...10$): four energy-momentum operators, three angular momentum operators and three Galilei or Lorentz boost operators. For definiteness we assume that the operators A_α where $\alpha = 1, 2, 3, 4$ refer to energy-momentum operators, the operators A_α where $\alpha = 5, 6, 7$ refer to angular momentum operators and the operators A_α where $\alpha = 8, 9, 10$ refer to Galilei or Lorentz boost operators. Let $[A_\alpha, A_\beta] = ic_{\alpha\beta\gamma}A_\gamma$ where summation over repeated indices is assumed. In the theory of Lie algebras the quantities $c_{\alpha\beta\gamma}$ are called the structure constants.

Let S_0 be a set of (α, β) pairs such that $c_{\alpha\beta\gamma} = 0$ for all values of γ and S_1 be a set of (α, β) pairs such that $c_{\alpha\beta\gamma} \neq 0$ at least for some values of γ. Since $c_{\alpha\beta\gamma} = -c_{\beta\alpha\gamma}$ it suffices to consider only such (α, β) pairs where $\alpha < \beta$. If $(\alpha, \beta) \in S_0$ then the operators A_α and A_β commute while if $(\alpha, \beta) \in S_1$ then they do not commute.

Let (S_0^A, S_1^A) be the sets (S_0, S_1) for theory A and (S_0^B, S_1^B) be the sets (S_0, S_1) for theory B. As noted above, we will consider only theories where $\alpha, \beta = 1, 2, ...10$. Then one can prove the following

Statement: *Let theory A contain a finite parameter and theory B be obtained from theory A in the formal limit when the parameter goes to zero or infinity. If the sets S_0^A and S_0^B are different and $S_0^A \subset S_0^B$ (what equivalent to $S_1^B \subset S_1^A$ if the* *sets S_1^A and S_1^B are different) then theory A is more general than theory B and theory B is a special degenerate case of theory A.*

Proof: Let $\tilde{S}$ be the set of (α, β) pairs such that $(\alpha, \beta) \in S_1^A$ and $(\alpha, \beta) \in S_0^B$. Then in theory B $c_{\alpha\beta\gamma} = 0$ for any γ. One can choose the parameter such that in theory A all the quantities $c_{\alpha\beta\gamma}$ are arbitrarily small. Therefore by choosing a value of the parameter theory A can reproduce any result of theory B with any desired accuracy. When the limit is already taken then, in theory B, $[A_\alpha, A_\beta] = 0$ for all $(\alpha, \beta) \in \tilde{S}$. This means that the operators A_α and A_β become fully independent and therefore there is no way to return to the situation when they do not commute. Therefore for theories A and B the conditions of **Definition** are satisfied.

It is sometimes stated that the expressions in Eq. (1) are not general enough because they are written in the system of units $c = \hbar = 1$. Let us consider this problem in more details. The operators $M^{\mu\nu}$ in Eq. (1) are dimensionless. In particular, standard angular momentum operators $(J_x, J_y, J_z) = (M^{12}, M^{31}, M^{23})$ are dimensionless and satisfy the commutation relations

$$[J_x, J_y] = iJ_z, \quad [J_z, J_x] = iJ_y, \quad [J_y, J_z] = iJ_x \tag{2}$$

If one requires that the operators $M^{\mu\nu}$ should have the dimension $kg \cdot m^2/sec$ then they should be replaced by $M^{\mu\nu}/\hbar$, respectively. In that case the new commutation relations will have the same form as in Eqs. (1) and (2) but the right-hand-sides will contain the additional factor $\hbar$.

The result for the components of angular momentum depends on the system of units. As shown in quantum theory, in units $\hbar = 1$ the result is given by a half-integer $0, \pm 1/2, \pm 1, ...$ We can reverse the order of units and say that in units where the angular momentum is a half-integer l, its value in $kg \cdot m^2/sec$ is $1.05457162 \cdot 10^{-34} \cdot l \cdot kg \cdot m^2/s$. Which of those two values has more physical significance? In units where the angular momentum components are half-integers, the commutation relations (2) do not depend on any parameters. Then the meaning of l is clear: it shows how big the angular momentum is in comparison with the minimum nonzero value 1/2. At the same time, the measurement of the angular momentum in units $kg \cdot m^2/s$ reflects only a historic fact that at macroscopic conditions on the Earth in the period between the 18th and 21st centuries people measured the angular momentum in such units.

We conclude that for quantum theory itself the quantity $\hbar$ is not needed. However, it is needed for the transition from quantum theory to classical one: we introduce $\hbar$, then the operators $M^{\mu\nu}$ have the dimension $kg \cdot m^2/sec$, and since the right-hand-sides of Eqs. (1) and (2) in this case contain an additional factor $\hbar$, all the commutation relations disappear in the formal limit $\hbar \to 0$. Therefore in classical theory the set S_1 is empty and all the (α, β) pairs belong to S_0. Since in quantum theory there exist (α, β) pairs such that the operators A_α and A_β do not commute then in quantum theory the set S_1 is not empty and, as follows from **Statement**, classical theory is the special degenerate case of quantum one in the formal limit $\hbar \to 0$. Since in classical theory all operators commute with each other then in this theory operators are not needed and one can work only with physical quantities. A question why $\hbar$ is as is does not arise since the answer is: because people want to measure angular momenta in $kg \cdot m^2/sec$.

Consider now the relation between RT and NT. If we introduce the Lorentz boost operators $L^j = M^{0j}$ $(j = 1, 2, 3)$ then Eqs. (1) can be written as

$$
\begin{aligned}
&[P^0, P^j] = 0, \quad [P^j, P^k] = 0, \quad [J^j, P^0] = 0, \quad [J^j, P^k] = i\epsilon_{jkl}P^l, \\
&[J^j, J^k] = i\epsilon_{jkl}J^l, \quad [J^j, L^k] = i\epsilon_{jkl}L^l, \quad [L^j, P^0] = iP^j
\end{aligned}
\tag{3}
$$

$$
[L^j, P^k] = i\delta_{jk}P^0, \quad [L^j, L^k] = -i\epsilon_{jkl}J^l \tag{4}
$$

where $j, k, l = 1, 2, 3$, ϵ_{jkl} is the fully asymmetric tensor such that $\epsilon_{123} = 1$, δ_{jk} is the Kronecker symbol and a summation over repeated indices is assumed. If we now define the energy and Galilei boost operators as $E = P^0c$ and $G^j = L^j/c$ $(j = 1, 2, 3)$, respectively then the new expressions in Eqs. (3) will have the same form while instead of Eq. (4) we will have

$$
[G^j, P^k] = i\delta_{jk}E/c^2, \quad [G^j, G^k] = -i\epsilon_{jkl}J^l/c^2 \tag{5}
$$

Note that in relativistic theory itself the quantity c is not needed. One can choose $c = 1$ and treat velocities v as dimensionless quantities such that $v \leq 1$ if tachyons are not taken into account. One needs c only for transition from RT to NT: when we introduce c then the dimension of velocities becomes m/s and instead of the operators P^0 and L^j we work with the operators E and G^j, respectively. If M is the Casimir operator for the Poincare algebra defined such that $M^2c^4 = E^2 - \mathbf{P}^2c^2$ then in the formal limit $c \to \infty$ the first expression in Eq. (5) becomes $[G^j, P^k] = i\delta_{jk}M$ while the commutators in the second expression become zero. Therefore in NT the (α, β) pairs with $\alpha, \beta = 8, 9, 10$ belong to S_0 while in RT they belong to S_1. Therefore, as follows from **Statement**, NT is a special degenerate case of RT in the formal limit $c \to \infty$. The question why $c = 3 \cdot 10^8 m/s$ and not, say $c = 7 \cdot 10^9 m/s$ does not arise since the answer is: because people want to measure c in m/s.

In his famous paper "Missed Opportunities" [7] Dyson notes that RT is more fundamental than NT and dS and AdS theories are more fundamental than RT not only from physical but also from pure mathematical considerations. Poincare group is more symmetric than Galilei one and the transition from the former to the latter at $c \to \infty$ is called contraction. Analogously dS and AdS groups are more symmetric than Poincare one and the transition from the former to the latter at $R \to \infty$ (described below) also is called contraction. At the same time, since dS and AdS groups are semisimple they have a maximum possible symmetry and cannot be obtained from more symmetric groups by contraction. However, since we treat symmetry not from the point of view of a group of motion for the corresponding background space but from the point of view of commutation relations in the symmetry algebra, we will discuss the relations between the dS and AdS algebra on one hand and the Poincare algebra on the other.

By analogy with the definition of Poincare symmetry on quantum level, the definition of dS symmetry on quantum level should not involve the fact that the dS group is the group of motions of dS space. Instead, *the definition* is that the operators M^{ab} ($a,b = 0,1,2,3,4,\ M^{ab} = -M^{ba}$) describing the system under consideration satisfy the commutation relations *of the dS Lie algebra* so(1,4), *i.e.*,

$$[M^{ab}, M^{cd}] = -i(\eta^{ac}M^{bd} + \eta^{bd}M^{ac} - \eta^{ad}M^{bc} - \eta^{bc}M^{ad}) \tag{6}$$

where η^{ab} is the diagonal metric tensor such that $\eta^{00} = -\eta^{11} = -\eta^{22} = -\eta^{33} = -\eta^{44} = 1$. The *definition* of AdS symmetry on quantum level is given by the same equations but $\eta^{44} = 1$.

With such a definition of symmetry on quantum level, dS and AdS symmetries are more natural than Poincare symmetry. In the dS and AdS cases all the ten representation operators of the symmetry algebra are angular momenta while in the Poincare case only six of them are angular momenta and the remaining four operators represent standard energy and momentum. If we *define* the operators P^μ as $P^\mu = M^{4\mu}/R$ where R is a parameter with the dimension *length* then in the formal limit when $R \to \infty$, $M^{4\mu} \to \infty$ but the quantities P^μ are finite, Eqs. (6) become Eqs. (1). This procedure is called contraction and in the given case it is the same for the dS or AdS symmetry. As follows from Eqs. (1) and (6), if $\alpha, \beta = 1,2,3,4$ then the (α, β) pairs belong to S_0 in RT and to S_1 in dS and AdS theories. Therefore, as follows from **Statement**, RT is indeed a special degenerate case of dS and AdS theories in the formal limit when $R \to \infty$.

One of the consequences is that the CC problem described in Sec. 1 does not exist because its formulation is based on the incorrect assumption that RT is more fundamental than dS and AdS theories. We will also see below that in classical approximation R becomes the radius of dS space.

Note that the operators in Eq. (6) do not depend on R at all. This quantity is needed only for transition from dS quantum theory to Poincare quantum theory. In full analogy with the above discussion of quantities $\hbar$ and c a question why R is as is does not arise and the answer is: because people want to measure distances in meters.

On classical level, dS space is usually treated as the four-dimensional hypersphere in the five-dimensional space such that

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 - x_0^2 = R'^2 \tag{7}$$

where R' is the radius of dS space and at this stage it is not clear whether or not R' coincide with R. Transformations from the dS group are usual and hyperbolic rotations of this space. They can be parametrized by usual and hyperbolic angles and do not depend on R'. In particular, if instead of x_a we introduce the quantities $\xi_a = x_a/R'$ then the dS space can be represented as a set of points

$$\xi_1^2 + \xi_2^2 + \xi_3^2 + \xi_4^2 - \xi_0^2 = 1 \tag{8}$$

Therefore in classical dS theory itself the quantity R' is not needed at all. It is needed only for transition from dS space to Minkowski one: we choose

R' in meters, then the curvature of this space is $\Lambda = 3/R'^2$ and a vicinity of the point $x_4 = R'$ or $x_4 = -R'$ becomes Minkowski space in the formal limit $R' \to \infty$. Analogous remarks are valid for the transition from AdS theory to Poincare one, and in this case $\Lambda = -3/R'^2$.

We have proved that all the three discussed comparisons satisfy the conditions formulated in **Definition** above. Namely, the more general theory contains a finite parameter and the less general theory can be treated as a special degenerate case of the former in the formal limit when the parameter goes to zero or infinity. The more general theory can reproduce all results of the less general one by choosing some value of the parameter. On the contrary, when the limit is already taken one cannot return back from the less general theory to the more general one.

In Refs. [8, 9] we considered properties of dS quantum theory and gave arguments that dS symmetry is more natural than Poincare one. However, the above discussion proves that dS and AdS symmetries are not only more natural than Poincare symmetry but more fundamental. In particular, R is fundamental to the same extent as $\hbar$ and c and therefore R **must be finite**.

In the literature the notion of the $c\hbar G$ cube of physical theories is sometimes used. The meaning is that any relativistic theory should contain c, any quantum theory should contain $\hbar$ and any gravitation theory should contain G. The more fundamental a theory is the greater number of those parameters it contains. In particular, relativistic quantum theory of gravity is the most fundamental because it contains all the three parameters c, $\hbar$ and G while nonrelativistic classical theory without gravitation is the least fundamental because it contains none of those parameters.

However, as noted in Sec. 1, since the nature of gravity is not clear yet, the quantity G is not fundamental. As follows from the above discussion, the three fundamental parameters are $(c, \hbar, R)$, and, in contrast to usual statements, the situation is the opposite: relativistic theory should not contain c, quantum theory should not contain $\hbar$ and dS or AdS theories should not contain R. Those three parameters are needed only for transitions from more general theories to less general ones. The most general dS and AdS quantum theories do not contain dimensionful quantities at all while the least general nonrelativistic classical theory contains three dimensional quantities (kg, m, s).

3 A system of two bodies in quantum dS theory

Since experimental data indicate that $\Lambda > 0$, in what follows we will consider only the dS theory and will not consider the AdS one. Our next goal is to show that classical equations of motions for a system of two free macroscopic bodies on dS space follow from quantum dS quantum mechanics in semiclassical approximation. We will assume that the distance between the bodies is much greater than the sizes of the bodies and the bodies do not have anomalously large internal angular momenta. Then from the formal point of view the motion of two bodies as a whole can be described by the same formulas as the motion of two elementary particles with zero spin. We will follow technical results described in Refs. [8, 9].

In quantum dS theory elementary particles are described by irreducible representations (IRs) of the dS algebra. As shown in Refs [8, 9], one can explicitly construct IRs of the dS algebra describing elementary particles.

It is known that in Poincare theory any massive IR can be implemented in the Hilbert space of functions $\chi(\mathbf{v})$ on the Lorenz 4-velocity hyperboloid with the points $v = (v_0, \mathbf{v})$, $v_0 = (1+\mathbf{v}^2)^{1/2}$ such that $\int |\chi(\mathbf{v})|^2 d\rho(\mathbf{v}) < \infty$ and $d\rho(\mathbf{v}) = d^3\mathbf{v}/v_0$ is the Lorenz invariant volume element. For positive energy IRs the value of energy is $E = mv_0$ where m is the particle mass *defined as the positive square root* $(E^2 - \mathbf{P}^2)^{1/2}$. Therefore for massive IRs, $m > 0$ by definition.

However, as shown by Mensky in his excellent book on induced representations [10], in contrast to Poincare theory, IRs in dS theory can be implemented only on two Lorenz hyperboloids, i.e. the Hilbert space for such IRs consist of sets of two functions $(\chi_1(\mathbf{v}), \chi_2(\mathbf{v}))$ such that

$$\int (|\chi_1(\mathbf{v})|^2 + |\chi_2(\mathbf{v})|^2) d\rho(\mathbf{v}) < \infty$$

As shown in Refs. [8, 9], in Poincare limit one dS IR splits into two IRs of the Poincare algebra with positive and negative energies and, as argued in those references, this implies that one IR of the dS algebra describes a particle and its antiparticle simultaneously. Since in the present paper we do not deal with antiparticles and neglect spin effects, we give only expressions for the action of the operators on the upper hyperboloid in the case of zero spin [8, 9]:

$$\begin{aligned} &\mathbf{J} = l(\mathbf{v}), \quad \mathbf{L} = -iv_0\frac{\partial}{\partial\mathbf{v}}, \quad \mathbf{B} = m_{dS}\mathbf{v} + i[\frac{\partial}{\partial\mathbf{v}} + \mathbf{v}(\mathbf{v}\frac{\partial}{\partial\mathbf{v}}) + \frac{3}{2}\mathbf{v}] \\ &\mathcal{E} = m_{dS}v_0 + iv_0(\mathbf{v}\frac{\partial}{\partial\mathbf{v}} + \frac{3}{2}) \end{aligned} \tag{9}$$

where $\mathbf{B} = \{M^{41}, M^{42}, M^{43}\}$, $\mathbf{l}(\mathbf{v}) = -i\mathbf{v} \times \partial/\partial\mathbf{v}$, $\mathcal{E} = M^{40}$ and m_{dS} is a positive quantity.

This implementation of the IR is convenient for the transition to Poincare limit. Indeed, the operators of the Lorenz algebra in Eq. (9) are the same as in the IR of the Poincare algebra. Suppose that the limit of m_{dS}/R when $R \to \infty$ is finite and denote this limit as m. Then in the limit $R \to \infty$ we get standard expressions for the operators of the IR of the Poincare algebra where m is standard mass, $E = \mathcal{E}/R = mv_0$ and $\mathbf{P} = \mathbf{B}/R = m\mathbf{v}$. For this reason m_{dS} has the meaning of the dS mass. In contrast to m, m_{dS} is dimensionless. Since Poincare symmetry is a special case of dS one, m_{dS} is more fundamental than m. Since Poincare symmetry works with a high accuracy, the value of R is supposed to be very large.

Consider the non-relativistic approximation when $|\mathbf{v}| \ll 1$. If we wish to work with units where the dimension of velocity is *meter/sec*, we should replace $\mathbf{v}$ by $\mathbf{v}/c$. If $\mathbf{p} = m\mathbf{v}$ then it is clear from the expression for $\mathbf{B}$ in Eq. (9) that $\mathbf{p}$ becomes the real momentum $\mathbf{P}$ only in the limit $R \to \infty$. At this stage we do not have any coordinate space yet. However, if we assume that semiclassical approximation is valid, then, by analogy with standard quantum mechanics, we can *define* the position operator $\mathbf{r}$ as $i\partial/\partial\mathbf{p}$.

In classical approximation we can treat $\mathbf{p}$ and $\mathbf{r}$ as usual vectors. Then as follows from Eq. (9)

$$\mathbf{P} = \mathbf{p} + mc\mathbf{r}/R, \quad H = \mathbf{p}^2/2m + c\mathbf{pr}/R, \quad \mathbf{L} = -m\mathbf{r} \tag{10}$$

where $H = E - mc^2$ is the classical nonrelativistic Hamiltonian. As follows from these expressions,

$$H(\mathbf{P}, \mathbf{r}) = \frac{\mathbf{P}^2}{2m} - \frac{mc^2\mathbf{r}^2}{2R^2} \tag{11}$$

The last term in Eq. (11) is the dS correction to the non-relativistic Hamiltonian. It is interesting to note that the non-relativistic Hamiltonian depends on c although it is usually believed that c can be present only in relativistic theory. This illustrates the fact mentioned in the preceding section that the transition to nonrelativistic theory understood as $|\mathbf{v}| \ll 1$ is more physical than that understood as $c \to \infty$. The presence of c in Eq. (11) is a consequence of the fact that this expression is written in standard units. In nonrelativistic theory c is usually treated as a very large quantity. Nevertheless, the last term in Eq. (11) is not large since we assume that R is very large.

As follows from Eq. (11) and the Hamilton equations, in dS theory a free body moves with the acceleration given by

$$\mathbf{a} = \mathbf{r}c^2/R^2 \tag{12}$$

where $\mathbf{a}$ and $\mathbf{r}$ are the acceleration and the radius vector of the particle, respectively. Since R is very large, the acceleration is not negligible only at cosmological distances when $|\mathbf{r}|$ is of the order of R.

Following our results in Refs. [8, 9], we now consider whether the result (12) is compatible with GR. As noted in the preceding section, the dS space is a four-dimensional manifold in the five-dimensional space defined by Eq. (7). In the formal limit $R' \to \infty$ the action of the dS group in a vicinity of the point $(0, 0, 0, 0, x_4 = R')$ becomes the action of the Poincare group on Minkowski space. With this parameterization the metric tensor on dS space is

$$g_{\mu\nu} = \eta_{\mu\nu} - x_\mu x_\nu/(R'^2 + x_\rho x^\rho) \tag{13}$$

where $\mu, \nu, \rho = 0, 1, 2, 3$, $\eta_{\mu\nu}$ is the Minkowski metric tensor, and a summation over repeated indices is assumed. It is easy to calculate the Christoffel symbols in the approximation where all the components of the vector x are much less than R': $\Gamma_{\mu,\nu\rho} = -x_\mu\eta_{\nu\rho}/R'^2$. Then a direct calculation shows that in the nonrelativistic approximation the equation of motion for a single particle is the same as in Eq. (12) if $R' = R$.

Another way to show that Eq. (12) is compatible with GR follows. The known result of GR is that if the metric is stationary and differs slightly from the

Minkowskian one then in the nonrelativistic approximation the curved space-time can be effectively described by a gravitational potential $\varphi(\mathbf{r}) = (g_{00}(\mathbf{r}) - 1)/2c^2$. We now express x_0 in Eq. (7) in terms of a new variable t as $x_0 = t + t^3/6R'^2 - t\mathbf{x}^2/2R'^2$. Then the expression for the interval becomes

$$ds^2 = dt^2(1 - \mathbf{r}^2/R'^2) - d\mathbf{r}^2 - (\mathbf{r}d\mathbf{r}/R')^2 \tag{14}$$

Therefore, the metric becomes stationary and $\varphi(\mathbf{r}) = -\mathbf{r}^2/2R'^2$ in agreement with Eq. (12) if $R' = R$.

We conclude that in classical limit the parameter R defining contraction from quantum dS symmetry to quantum Poincare symmetry indeed equals the radius of dS space.

Consider now a system of two free bodies in dS space. Let $(\mathbf{r}_i, \mathbf{a}_i)$ $(i = 1, 2)$ be their radius vectors and accelerations, respectively. Then Eq. (12) is valid for each particle if $(\mathbf{r}, \mathbf{a})$ is replaced by $(\mathbf{r}_i, \mathbf{a}_i)$, respectively. Now if we define the relative radius vector $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ and the relative acceleration $\mathbf{a} = \mathbf{a}_1 - \mathbf{a}_2$ then they will satisfy the same Eq. (12) which shows that the dS antigravity is repulsive.

Let us now consider a system of two free bodies in the framework of the representation of the dS algebra. The particles are described by the variables $\mathbf{P}_j$ and $\mathbf{r}_j$ $(j = 1, 2)$. Define standard nonrelativistic variables

$$\begin{aligned} &\mathbf{P}_{12} = \mathbf{P}_1 + \mathbf{P}_2, \quad \mathbf{q} = (m_2\mathbf{P}_1 - m_1\mathbf{P}_2)/(m_1 + m_2) \\ &\mathbf{R}_{12} = (m_1\mathbf{r}_1 + m_2\mathbf{r}_2)/(m_1 + m_2), \quad \mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2 \end{aligned} \tag{15}$$

Then, as follows from Eq. (10), in the nonrelativistic approximation the two-particle quantities $\mathbf{P}$, $\mathbf{E}$ and $\mathbf{L}$ are given by

$$\mathbf{P} = \mathbf{P}_{12}, \quad E = M + \frac{\mathbf{P}_{12}^2}{2M} - \frac{Mc^2\mathbf{R}_{12}^2}{2R^2}, \quad \mathbf{L} = -M\mathbf{R}_{12} \tag{16}$$

where

$$M = M(\mathbf{q}, \mathbf{r}) = m_1 + m_2 + H_{nr}(\mathbf{r}, \mathbf{q}), \quad H_{nr}(\mathbf{r}, \mathbf{q}) = \frac{\mathbf{q}^2}{2m_{12}} - \frac{m_{12}c^2\mathbf{r}^2}{2R^2} \tag{17}$$

and m_{12} is the reduced two-particle mass. Here the operator M acts in the space of functions $\chi(\mathbf{q})$ such that $\int |\chi(\mathbf{q})|^2 d^3\mathbf{q} < \infty$ and $\mathbf{r}$ acts in this space as $\mathbf{r} = i\partial/\partial\mathbf{q}$.

It now follows from Eq. (9) that M has the meaning of the two-body mass and therefore $M(\mathbf{q}, \mathbf{r})$ is the internal two-body Hamiltonian. Then, by analogy with the derivation of Eq. (12), it can be shown from the Hamilton equations that in semiclassical approximation the relative acceleration is given by the same expression (12) but now $\mathbf{a}$ is the relative acceleration and $\mathbf{r}$ is the relative radius vector.

4 Discussion and conclusion

The fact that two free bodies have a relative acceleration is known for cosmologists considering dS symmetry on classical level. This effect is called the dS antigravity.

The term antigravity in this context means that the particles repulse rather than attract each other. In the case of the dS antigravity the relative acceleration of two free particles is proportional (not inversely proportional!) to the distance between them. As shown above, this classical result is a direct consequence of GR.

At the same time, since GR is pure classical theory and quantum theory is more general than classical one a problem arises whether the cosmological acceleration can be obtained from quantum theory in semiclassical approximation. Following our results in Refs. [8, 9], it is shown in the present paper that this is the case. The result for the cosmological acceleration has been obtained without using dS space, its metric, connection etc. This result is simply a consequence of standard dS quantum mechanics of two free bodies and the calculation does not involve any geometry. The fact that $\Lambda \neq 0$ is a consequence of dS symmetry on quantum level: since dS symmetry is more general than Poincare one then R is finite, on classical level $\Lambda = 3/R^2$ *must* be nonzero, and the problem why Λ is as is does not arise. This has nothing to do with gravity, existence or nonexistence of dark energy and with the problem whether or not empty space-time should be necessarily flat.

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