

Better Quantum Mechanics ?

Thoughts on a New Definition of Momentum That Makes Physics Simpler and More Consistent

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Abstract

We suggest that momentum should be redefined in order to help make physics more consistent and more logical. In this paper, we propose that there is a rest-mass momentum, a kinetic momentum, and a total momentum. This leads directly to a simpler relativistic energy momentum relation. As we point out, it is the Compton wavelength that is the true wavelength for matter; the de Broglie wavelength is mostly a mathematical artifact. This observation also leads us to a new relativistic wave equation and a new and likely better QM. Better in terms of being much more consistent and simpler to understand from a logical perspective.

Key words: momentum, kinetic momentum, rest-mass momentum, de Broglie wave, Compton wave, relativistic energy momentum relation, relativistic wave equation.

1 Introduction

Today there is no rest-mass momentum in modern physics, which leads to unnecessary complexity and even inconsistency in the field. In modern physics, the momentum for a particle with mass is given by [1]

$$p = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (1)$$

and when $v \ll c$, we can use the first term of a Taylor series expansion and approximate the momentum quite well with $p \approx mv$.

The relativistic energy momentum relation is very important in modern physics (see for example [2])

$$E^2 = p^2c^2 + (mc^2)^2 \quad (2)$$

To find the momentum of a photon, we can set the mass to zero in the last part of the equation above, solve with respect to momentum, and we get

$$p = \frac{E}{c} = \frac{\hbar}{\lambda} \quad (3)$$

Relativistic momentum is given by equation 1. In modern physics, photons are always treated as something special. They are special, but do we truly need one set of momentum equations for particles with mass and one set for photons? Based on recent analysis, we will show that this is not necessary.

For photons, the standard relativistic momentum formula does not work, so here we have defined momentum as $p = \frac{\hbar}{\lambda}$, as derived from the relativistic energy-momentum equation.

2 New Momentum Definition

We suggest that the total momentum is given by

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$$p_t = \frac{mc}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (4)$$

and that the rest-mass momentum is given by $p_r = mc$. Then a moving particle with mass has a kinetic momentum of

$$p_k = \frac{mc}{\sqrt{1 - \frac{v^2}{c^2}}} - mc \quad (5)$$

and when $v \ll c$, this can be very well approximated by the first term of a Taylor series expansion

$$p_k \approx \frac{1}{2} \frac{mv^2}{c} \quad (6)$$

In our new momentum equation, energy is always equal to momentum times the speed of light. The relationship $E = pc$ is often used in physics, but with the old version of momentum it actually only holds for photons and not for particles like electrons. Further, the relativistic momentum equation for particles with mass does not hold for photons; we are operating with two different frameworks that have been merged in a rather ad-hoc way to make the energy line up with experiments.

Our new momentum definition leads to a new relativistic energy momentum relation of

$$E = p_k c + mc^2 \quad (7)$$

That is, we have

$$E = \left(\frac{mc}{\sqrt{1 - \frac{v^2}{c^2}}} - mc \right) c + mc^2 = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (8)$$

We claim that this will also hold for photons. The key is to combine it with Haug's maximum velocity [3–7] of matter $v_{max} = c\sqrt{1 - \frac{l_p^2}{\lambda^2}}$. As discussed in previous papers, in the special case of the Planck mass particle, the maximum velocity is zero

$$v_{max} = c\sqrt{1 - \frac{l_p^2}{l_p^2}} = 0 \quad (9)$$

This sounds absurd, but in our view it represents the collision point between two photons. Recent research has been quite clear on the concept that in a photon–photon collision we likely can create matter, see [8]. This means for light there is only rest-mass momentum of the form $p = mc$, and the relativistic momentum formula and all other relativistic formulas now hold for both light and traditional matter. Modern physics often operates with two sets of rules, as a full connection made between light and matter has not been determined yet.

We also note that the Planck mass is observational time dependent and is approximately 10^{-51} kg in a one second observational time-window, but indeed has an enormous traditional value of approximately 10^{-8} kg in a one Planck time observational time window.

This leads to a new quantum probability theory that is much less mysterious than the existing quantum mechanics theory. Further, it produces one set of equations that apply equally to photons and all other matter. This stands in contrast to modern physics, which relies more on a series of mathematical tricks and complexities to compensate for the lack of a fully understood connection between photons and matter.

3 The Two Matter Waves: The de Broglie Wave and the Compton Wave

By the time of the photoelectric effect work of Einstein in 1905, it was clear that light was both a particle and a wave. In 1923, Louis de Broglie [9, 10] suggested that matter also had wave properties. He calculated the wavelength of matter from momentum and got

$$\lambda_B \approx \frac{h}{mv} \quad (10)$$

where m is the rest-mass and v is the velocity of the particle in question; this is known today as the de Broglie wavelength, or in the relativistic form

$$\lambda_B = \frac{h}{\frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}}} \quad (11)$$

Shortly after his conjecture, experimental research confirmed that matter did have wave-like properties and the de Broglie hypothesis was quickly accepted and incorporated. It was further developed later in quantum mechanics. We fully agree that matter has both a particle and a wave-like nature. Still, we think de Broglie made a serious mistake in how he calculated this wavelength. We also think there are errors in how it has been incorporated in modern physics. The de Broglie wavelength has a series of mystical properties; it is infinite for a particle when the velocity is zero, for example, and it is also linked to superluminal phase velocity.

In 1923, working at around the same time as de Broglie, Compton [11] discovered a wave related to electrons – the so-called Compton wavelength that is given by

$$\lambda_c = \frac{h}{mc} \quad (12)$$

And for a moving particle

$$\lambda_c = \frac{h}{\frac{mc}{\sqrt{1-\frac{v^2}{c^2}}}} \quad (13)$$

and when $v \ll c$ this can be very well approximated with the first term of a Taylor series expansion, $\lambda_c \approx \frac{h}{mc}$.

The Compton wavelength of an electron has been measured in many experiments; it is a short wavelength of about $2.4263102367 \times 10^{-12}$ m (2014 NIST CODATA) and fits perfectly with theory. No one has measured the length of the de Broglie wavelength, even though some have claimed to do so. If one knows the Compton wavelength, however, one can easily find the mass of the electron, since the mass is related to the Compton wavelength

$$m_e = \frac{h}{\lambda_c} \frac{1}{c} = \frac{\hbar}{\lambda_c} \frac{1}{c} \quad (14)$$

This means the mass of an elementary particle can be found by measuring the Compton wavelength of the particle, as has been done experimentally with electrons, see [12]. Still, the de Broglie wavelength is a mathematical function of the physical Compton wavelength, namely

$$\lambda_B = \lambda_c \frac{c}{v} \quad (15)$$

So, we can indirectly measure the de Broglie wavelength from the physical Compton wavelength. Further, the link between mass and Compton time frequency has been explored and supported by recent experimental research. Dolce and Perali [13] conclude that “*the rest-mass of a particle is associated to a rest periodicity known as Compton periodicity*”, see also [14].

We claim that the de Broglie wavelength is to a large degree a mathematical artifact, but will discuss this more in section 6. Notice also that the Compton wavelength of an electron is calculated by dividing the Planck constant by $mc = \frac{h}{\lambda} \frac{1}{c} = \frac{h}{\lambda}$. Strangely, mc is the momentum of a photon, but not the momentum of anything with rest-mass, according to standard physics. Still, the photon momentum definition is used to calculate a measurable wavelength that is directly linked to the mass of elementary particles. Why should there be two different matter waves, the de Broglie and the Compton wave? Naturally, it is strange say that we can predict a consistent wavelength of matter with mass by dividing the Planck constant by a photon-like momentum for matter. On the other hand, if we say there exists a total momentum equal to our newly introduced momentum, namely

$$p_t = \frac{mc}{\sqrt{1-\frac{v^2}{c^2}}} \quad (16)$$

then the Compton wavelength is simply the Planck constant divided by the total momentum, just as the idea of de Broglie, but with a correct momentum. It is identical to the Compton formula, but here we have a simple explanation for what the components are. In addition, our wave formula holds for photons as long as we use the maximum velocity formula for matter; it is zero for a photon. That is, the Compton wavelength of a photon is

$$\lambda = \frac{h}{p_t} = \frac{h}{\frac{mc}{\sqrt{1-\frac{v^2}{c^2}}}} = \frac{h}{mc} = \frac{h}{\frac{h}{\lambda_c} \frac{1}{c}} = \frac{h}{p} \quad (17)$$

4 Inconsistencies and Mystical Interpretations in Modern Physics Related to Non-Optimal Momentum Definition

The relativistic energy mass relation is again given by

$$E = \sqrt{p^2 c^2 + (mc^2)^2} \quad (18)$$

It is important to realize that this indirectly allows negative energy, negative mass, and negative momentum, since we must have $E = \sqrt{(\pm p)^2 c^2 + (\pm mc^2)^2}$. This has been a significant challenge, that opens for such things as negative energy, negative mass, and negative momentum, that have never been observed. However, there is much speculation in modern physics about negative mass, negative energy, and even negative probabilities to arrive at a fully consistent theory. For example, Dirac [15] had interesting discussions concerning how negative probabilities show up in quantum mechanics:

Thus the two undesirable things, negative energy and negative probability, always occur together.

– Paul Dirac, 1942

Pauli, Feynman, [16, 17] and many others also speculated on negative probabilities. Negative probabilities actually make no logical sense, just as negative matter and negative energy defy common sense and logic. We would claim that this is all rooted in an incorrect definition of momentum, which is not physical, but simply a mathematical non-optimal defined “derivative.” The relativistic energy mass relation is one of the cornerstones in quantum mechanics. A number of relativistic quantum mechanics equations, such as the Klein–Gordon equation, for example, are directly linked to the relativistic energy momentum relation. This relation gives the correct energy, but it is unnecessarily complex, as it is based on an ill-specified momentum. Further, negative energy states coming out from quantum mechanics (e.g., the relativistic energy momentum equation) were interpreted by some famous physicists including Feynman as particles moving backwards in time, see for example [18].

Here we will outline a series of inconsistencies related to the choice of a non-optimal definition of momentum.

- Modern physics does not have rest-mass momentum, but does have rest-mass energy, kinetic energy, and total energy. The lack of rest-mass momentum appears to be inconsistent.
- Modern physics uses different formulas for momentum for photons and for matter with rest-mass. For matter, we have $p = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}}$, while for photons, we have $p = \frac{h}{\lambda}$. For photons, we also have $E = pc$. In a series of papers, it seems to be used incorrectly: for matter with rest-mass, one uses $E = pc$ to go from momentum to energy, but this is inconsistent with the relativistic momentum of matter. And $E = pc = mvc$ is not energy.
- When using standard momentum to calculate a matter wave, we get the de Broglie wavelength. Contrary to what modern physics claims, this wave has never been observed (at least not directly). The wave nature of matter has been detected, and a wave related to matter has been measured very accurately. However, this is in relation to the Compton wavelength and not the de Broglie wavelength. Still, one could claim the de Broglie wavelength exists indirectly, as it is a mathematical derivative of the Compton wavelength, namely $\lambda_B = \lambda_c \frac{c}{v}$. The de Broglie wave makes no sense for a rest-mass, as it is then infinite. The idea that an electron at rest should be everywhere in the universe, or have a probability to be anywhere in the universe, for example, simply makes no logical sense. And yet there is a series of different interpretations developed around this concept, even inside the standard paradigm.
- The relativistic energy momentum relation is unnecessarily complex, and, we would say, even mystical as an approach to scientific phenomenon. What is energy and momentum squared? There are no such things physically. The standard relativistic energy momentum relation is problematic because the momentum is ill-specified in the first place, so to get the math to fit observations (energy) one needs an unnecessarily complex formula $E = p^2 c^2 + (mc^2)^2$. This also means the momentum of a particle with mass is an unnecessarily complex function of energy, namely $p = \frac{\sqrt{E^2 - m^2 c^4}}{c}$. At the same time, for a photon it is simply $p = E/c$ (simply by putting $m = 0$ in the relativistic energy momentum formula). By using our redefined momentum, we get a much simpler and logical relativistic energy momentum relation, namely $E_t = p_k c + mc^2$. This also means momentum is always the energy divided by the speed of light, which removes challenges with such things as negative energies and negative probabilities. Be aware they give the same energy.
- The relativistic energy momentum relation leads to possibility of negative momentum, negative energy, negative mass, and a series of famous physicists have even speculated on negative probabilities. This has led to a considerable amount of wild speculation in modern physics that will all disappear with a sound momentum definition.
- Standard physics momentum has led to a non-physical wavelength (the de Broglie wavelength) and impossible mathematical artifacts, such as superluminal and even infinite phase velocity of $\frac{c^2}{v}$. These are derivatives linked to real properties, but they add complexity when what they represent is not fully understood.

Table 1 summarizes how our newly defined momentum brings logic and simplicity back into physics.

Entity	Standard physics	New theory
Total momentum mass	$p = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}$	$p_t = \frac{mc}{\sqrt{1-\frac{v^2}{c^2}}}$
Kinetic momentum	$p = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}$	$p_k = \frac{mc}{\sqrt{1-\frac{v^2}{c^2}}} - mc$
Kinetic momentum $v \ll c$	$p \approx mv$	$p_k \approx \frac{1}{2}m\frac{v^2}{c}$
Rest-mass momentum	None	$p_r = mc$
Momentum photon	$p = \frac{h}{\lambda} = mc$	$p_t = \frac{mc}{\sqrt{1-\frac{v^2}{c^2}}} = mc$ since $v = 0$ photon-photon collision
From momentum to energy	For photons multiply by c , or else complicated	Just multiply by c for photons and standard mass.
From energy to momentum	For photons divide by c , or else complicated	Just divide by c for photons and standard mass.
Matter wave-1 de Broglie	$\lambda_B = \frac{h}{mv}$ Never observed!	de Broglie is mathematical construct
Matter wave-2 Compton wave The new momentum used	$\lambda_c = \frac{h}{mc}$ Observed. Not understood	$\lambda_c = \frac{h}{mc}$ The only matter wave. Understood
Mass from Compton	$m = \frac{h}{\lambda_c} \frac{1}{c}$	$m = \frac{h}{\lambda_c} \frac{1}{c}$
Mass from de Broglie	$m = \frac{h}{\lambda_B} \frac{1}{v}$ Impossible for rest-mass	$m = \frac{h}{\lambda_B} \frac{1}{v}$ Impossible for rest-mass (artifact)
de Broglie from Compton	$\lambda_B = \lambda_c \frac{c}{v}$	$\lambda_B = \lambda_c \frac{c}{v}$
Compton from de Broglie	$\lambda_c = \lambda_B \frac{v}{c}$	$\lambda_c = \lambda_B \frac{v}{c}$
Phase velocity	$v_p = \frac{E}{p} = \frac{c^2}{v}$ Not understood, cannot carry energy	$v_p = \frac{E}{p} = c$ Understood, can carry energy
Energy momentum relation	$E^2 = p^2c^2 + (mc^2)^2$	$E = p_kc + mc^2$
Energy momentum relation	$E^2 = p^2c^2 + (mc^2)^2$	$E = p_t c$ same as above
Momentum from energy	$p = \frac{\sqrt{E^2 - m^2c^4}}{c} = \frac{\sqrt{\left(\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}\right)^2 - m^2c^4}}{c}$	$p_k = \frac{E - mc^2}{c} = \frac{\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2}{c} = \frac{E_k}{c}$
Momentum from energy	$p = \frac{\sqrt{E^2 - m^2c^4}}{c} = \frac{E}{c} = \frac{h}{\lambda} = \frac{h}{\lambda}$ Partly “trickery” derivation, but correct	$p_k = \frac{E - mc^2}{c} = \frac{\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2}{c} = 0$ photon-photon collision $v = 0$ $p_t = mc^2 = \frac{h}{\lambda}$ Consistent and correct
Negative: energy, momentum, and mass	Cannot be excluded	Totally excluded
Negative probability	Suggested as solution	Absurd and not needed
Max velocity matter	$v < c$	$v \leq c\sqrt{1 - \frac{I_p^2}{\lambda^2}}$
Trans-Planckian crisis	Yes	No

Table 1: Summarizes how our newly defined momentum brings logic and simplicity back into physics.

5 New Relativistic Quantum Mechanics Equation

The standard relativistic energy momentum relationship (rooted in an ill-specified momentum) is given by

$$E^2 = \mathbf{p}^2c^2 + (mc^2)^2 \quad (19)$$

Where \mathbf{p} is the three-momentum (a vector in three dimensions). By turning E and \mathbf{p} into operators and doing substitutions, we get the well-known Klein–Gordon equation

$$-\hbar^2 \frac{\partial^2 \Psi}{\partial t^2} = (-\hbar^2 c^2 \nabla^2 + m^2 c^4) \Psi \quad (20)$$

where Ψ is the position-space wave function. The Klein–Gordon equation is often better known in the form (dividing by \hbar^2 and c^2 on both sides):

$$\frac{1}{c^2} \frac{\partial^2}{\partial t^2} \Psi - \nabla^2 \Psi + \frac{m^2 c^2}{\hbar^2} \Psi = 0 \quad (21)$$

The Klein–Gordon equation has strange properties, such as energy squared, which is one of several reasons that Schrödinger did not like it that much. If we use our new momentum definition and its corresponding relativistic energy momentum relation instead,

$$\begin{aligned} E &= \mathbf{p}_k c + mc^2 \\ E &= \left(\frac{mc}{\sqrt{1 - \frac{\mathbf{v}^2}{c^2}}} - mc \right) c + mc^2 \\ E &= \left(\frac{mc}{\sqrt{1 - \frac{\mathbf{v}^2}{c^2}}} \right) c \\ E &= \mathbf{p}_t c \end{aligned} \quad (22)$$

where \mathbf{v} is the particle’s three-velocity. Now we can substitute E and \mathbf{p}_t with corresponding energy and momentum operators and get a new relativistic quantum mechanical wave equation

$$-\hbar \frac{\partial \Psi}{\partial t} = -\hbar \nabla \cdot (\Psi \mathbf{c}) \quad (23)$$

where $\mathbf{c} = (c_x, c_y, c_z)$ would be the light velocity field. Dividing both sides by \hbar , we can rewrite this as

$$-\frac{\partial \Psi}{\partial t} = -\nabla \cdot (\Psi \mathbf{c}) \quad (24)$$

The light velocity field should satisfy

$$\nabla \cdot \mathbf{c} = 0 \quad (25)$$

that is, the light velocity field is a solenoidal, which means we can rewrite our wave equation as

$$\frac{\partial \Psi}{\partial t} - \mathbf{c} \cdot \nabla \Psi = 0 \quad (26)$$

interestingly the equation has the same structural form as the advection equation, but here for quantum wave mechanics.

So, on the “expanded” form, we have

$$\frac{\partial \Psi}{\partial t} - c_x \frac{\partial \Psi}{\partial x} - c_y \frac{\partial \Psi}{\partial y} - c_z \frac{\partial \Psi}{\partial z} = 0 \quad (27)$$

The equation above is only for a single particle. In the more general case, we have

$$i\hbar \frac{\partial}{\partial t} |\Psi\rangle = \hat{H}_H |\Psi\rangle \quad (28)$$

where \hat{H}_H basically is the Hamilton operator, but with one big difference compared to the Schrödinger solution: In our model, one cannot use the standard momentum to get to the kinetic energy in the way Schrödinger does, which is why we have marked our Hamilton operator with a different notation (with H as subscript).

Schrödinger had his way of setting the kinetic energy operator equal to $\hat{T} = \frac{\hat{\mathbf{p}} \cdot \hat{\mathbf{p}}}{2m}$, where $\hat{\mathbf{p}} = -i\hbar \nabla$, so he gets the correct energy, but does so through a non-optimal momentum that adds unnecessary complexity in the practical form of his equation. In our theory, we simply have $\hat{H}_H = \hat{T} + \hat{V} = \mathbf{c} \cdot \hat{\mathbf{p}}_t$, where $\hat{\mathbf{p}}_t = -i\hbar \nabla$, so we get no squaring of the momentum as Schrödinger does, which is why our single particle wave equation looks more elegant. Actually, we could say the standard momentum used is a derivative of our new momentum definition, $p = p_t \frac{v}{c}$.

Just as we claim the Compton wavelength is the real matter wave, so too will a momentum directly linked to the Compton wavelength be the more fundamental momentum definition. Schrödinger relies on a momentum derivative, and the fact that it is a very old momentum does not help. We are focusing on a simpler theory. Equation 28 looks identical to the general Schrödinger equation [19], but in our view Schrödinger used a non-optimal momentum (or we would even say ill-specified momentum, rooted in old physics) to get to his practical

use of his formula. This means the wave function in that case is very different than ours, with one approach rooted in the de Broglie matter wave (a derivative) and the other approach rooted in the Compton wave, which is the fundamental physical matter wave.

We encourage others to evaluate this new relativistic quantum mechanics equation to see if there are any mistakes and to check if it is consistent with what we observe. Be aware that it is linked to the Compton wavelength and not to the de Broglie wavelength. It is also interesting to know what type of plane wave solution this relativistic wave equation leads to.

Our new relativistic quantum equation has quite a different plane wave solution than the Klein–Gordon and Schrödinger equations, but at first glance it looks exactly the same:

$$\psi = e^{i(kx - \omega t)} \quad (29)$$

But in our theory we should have $k = \frac{2\pi}{\lambda_c}$, where λ_c is the relativistic Compton wavelength and not the de Broglie wavelength, as in standard wave mechanics. Due to this, we have

$$k = \frac{p_t}{\hbar} = \frac{\frac{mc}{\sqrt{1 - \frac{v^2}{c^2}}}}{\hbar} = \frac{2\pi}{\lambda_c} \quad (30)$$

So we can also write the plane wave solution as

$$e^{i\left(\frac{p_t}{\hbar}x - \frac{E}{\hbar}t\right)} \quad (31)$$

where p_t is the total relativistic momentum as defined earlier. So, our quantum wave function is rooted in the Compton wavelength instead of the de Broglie wavelength. For the formality of it, we look at the momentum and energy operators and see that they are correctly specified

$$\frac{\partial\psi}{\partial x} = \frac{ip_t}{\hbar} e^{i\left(\frac{p_t}{\hbar}x - \frac{E}{\hbar}t\right)} \quad (32)$$

This means the momentum operator likely must be

$$\hat{p}_t = -i\hbar\nabla \quad (33)$$

and for energy we have

$$\frac{\partial\psi}{\partial t} = \frac{-iE}{\hbar} e^{i\left(\frac{p_t}{\hbar}x - \frac{E}{\hbar}t\right)} \quad (34)$$

and this gives us a time operator of

$$\hat{E} = -i\hbar\frac{\partial}{\partial t} \quad (35)$$

The momentum and energy operator are the same as under standard quantum mechanics. The only difference between the non-relativistic and relativistic wave equations is that in a non-relativistic equation we can use

$$k = \frac{p_t}{\hbar} = \frac{mc}{\hbar} = \frac{2\pi}{\lambda_c} \quad (36)$$

instead of the relativistic form $p_t = \frac{mc}{\sqrt{1 - \frac{v^2}{c^2}}}$. And this is again because the first term of a Taylor series expansion is $p_t \approx mc$ when $v \ll c$.

6 What Does the de Broglie Wavelength Truly Represent?

When $v \ll c$, the de Broglie wave can also be written as

$$\lambda_B = \frac{h}{mv} = \frac{h}{\frac{h}{\lambda_c} \frac{1}{c} v} = \frac{\lambda_c}{v} c \quad (37)$$

where λ_c is the Compton wavelength of the particle in question. How should we interpret this? First of all, this is the Compton wavelength divided by the velocity of the particle multiplied by the speed of light. $\frac{\lambda_c}{v}$ is the time it takes for the particle to travel its own reduced Compton wavelength. This is a true wavelength that is measurable with a high degree of accuracy, at least for an electron. Under atomism, an indivisible particle with diameter equal to the Planck length is traveling back and forth over the Compton wavelength at the speed of light. Each time it has traveled the Compton wavelength it collides with another indivisible massless particle; this collision is what actually constitutes mass. Thus, the collision creates a Planck mass, but it only lasts for one Planck second. So, the rest-mass of the electron must be given by

$$\frac{c}{\lambda_c} m_p t_p \approx 9.10938 \times 10^{-31} \text{ kg} \quad (38)$$

In this view, $\frac{\lambda}{v}c$, which is the reduced de Broglie wavelength, is simply the distance an indivisible particle has traveled back and forth over the Compton wavelength during the time interval that a whole elementary particle, like an electron, has traveled its own wavelength.

An interesting case is when $v = \lambda$ per time unit chosen, then the de Broglie wavelength has a distance equal to the distance the light has traveled within that time unit. So if the observational time window is one second and $v = \frac{\lambda}{t} = \frac{\lambda}{1}$ m/s, then the de Broglie length is 299,792,458 meters, which is equal to how far light travels in one second. Does this mean that a very slow moving electron is spread out over this distance? Not at all; the idea that the particle is spreading out over distance the slower it moves is illogical and something that we would dispute. What is much more logical is that some building blocks of the electron are traveling back and forth over the Compton wavelength at the speed of light, as we have laid out in a book [20] and series of papers [6, 21]. So, the de Broglie wavelength represents something “important”, but it is not a wave; instead, it represents how far the building blocks of an electron (and other particles) have traveled during the time period the elementary particle (not the indivisible particle, which is even more fundamental) travels its own Compton wavelength.

Further, assume that velocity v is only one Planck length per second. What is the de Broglie wavelength for an electron then? It is

$$\frac{h}{m_e v} = \lambda_e \frac{c}{v} \approx 4.5 \times 10^{31} \text{ m}$$

This is an enormous distance. But again, it should not be interpreted to mean that the electron is spread out over this distance, or that this is the range where it can be found with a given probability. It should be interpreted as representing how far light (the indivisible massless particle) travels back and forth inside the Compton wavelength during the time period it takes an electron (or other fundamental particle) to travel its own Compton length when it is moving at a speed of v .

What about the case when $v = 0$, is the de Broglie wavelength infinite? Then

$$\frac{h}{m \times 0} = \infty \quad (39)$$

This has led to a series of strange and very speculative interpretations, which are common among physicists to this day

The de Broglie wave has infinite extent in space. – A.I. Lvovsky [22] Professor Alexander Lvovsky, Oxford University. p. 100

De Broglie had an extremely strong and concrete physical justification for the infinite wavelength of matter waves, corresponding to the body at rest.Therefore, the infinite wavelength of matter waves, for zero velocity of body, becomes essentially evident. — [23]

The interpretation given by Max Born is likely closer to reality

Physically, there is no meaning in regarding this wave as a simple harmonic wave of infinite extent; we must, on the contrary, regard it as a wave packet consisting of a small group of indefinitely close wave-numbers, that is, of great extent in space. – Max Born

A much better explanation is to consider how long an indivisible particle must travel inside the Compton wavelength at the speed of light, before the elementary particle travels its own Compton length, if it travels at speed zero. Naturally, it needs infinite time in the latter case, as the elementary particle does not move at all and will obviously never travel its own Compton wavelength as long as it is standing still. This means that something moving back and forth over the Compton wavelength at the speed of light will have moved an infinite distance (back and forth only) before the particle has traveled its own Compton wavelength. Based on this model, it is simple and logical to see why the de Broglie wavelength is infinite for a particle at rest. However, the de Broglie wavelength is not truly a wave; it pertains to the indivisible particle and the Compton wave of an elementary particle.

In addition, from the de Broglie theory, we have phase velocity; it is given by

$$v_p = \frac{E}{p} = \frac{c^2}{v} \quad (40)$$

Since the speed of a particle always must be below the speed of light, this means the phase velocity always is above the speed of light, and it is even infinite if the velocity of the particle is zero. However, the superluminal phase velocity is claimed not to violate special relativity, because phase propagation carries no energy. The question then is, What is the phase velocity if it carries no energy? Is it something imaginary, just math? Yes, it is more or less just a mathematical artifact, but let us look more closely at what it represents. It is said that the phase velocity is equal to the product of the frequency multiplied by the wavelength. In our Compton clock model, an electron has an internal Compton frequency of $\frac{c}{\lambda} \approx 1.24 \times 10^{20}$. If we calculate the de Broglie wavelength and multiply by this Compton frequency, we get the phase velocity

$$\frac{c}{\lambda} \frac{h}{mv} = \frac{c}{\lambda} \frac{h}{\frac{h}{\lambda} \frac{1}{c} v} = \frac{c^2}{v} \quad (41)$$

So, the phase velocity is the Compton frequency times the de Broglie wavelength. But why the combination of them? – This is our question. We have to keep in mind what the de Broglie wavelength actually represents: again, it is how far light (an indivisible particle) travels back and forth over the Compton wavelength of the electron (or any other elementary particle) during the time it takes for the particle to travel its own Compton wavelength. The so-called phase wave for particles is, from a deeper perspective, a strange mix of aspects that a particle has. As the phase velocity contains information about the non-optimal defined momentum that can be used to find the correct energy, we can naturally use it in derivation, but the phase velocity interpreted on its own is almost absurd. It is nothing physical, but it is linked to the Compton frequency multiplied by the de Broglie wavelength. It would be much more meaningful to simply multiply the Compton frequency with the Compton length, then we get the speed of light, which simply shows that there is something that the electron is built from that moves at the speed of light. In our model, it is an indivisible massless particle.

On the other hand, the de Broglie frequency multiplied by the de Broglie wavelength is also the speed of light.

$$\frac{c}{\lambda_B} \frac{h}{mv} = \frac{c}{\lambda_B} \frac{h}{\frac{h}{\lambda_c} \frac{1}{c} v} = c \quad (42)$$

We will conclude that the de Broglie wavelength not really is a wavelength. It is, however, related to interesting and deeper internal aspects of elementary particles. Recent developments in modern atomism fit well and have good explanations for this; further information about atomism is offered in many of my other papers and my book.

7 Historical Perspective

So, if we are right that the Compton wavelength is truly essential for matter and the derivative de Broglie wavelength is less so, then why has this not been explored before? In fact, the Compton wavelength has played an increasingly important role over time, especially with the finding that matter is related to the Compton frequency. Bear in mind that Einstein had a copy of de Broglie's PhD thesis (even before it was accepted) on matter waves and thought it was brilliant work. We are not claiming otherwise, as de Broglie was possibly the first to claim that matter had both a particle- and a wave-like nature. In addition, his paper on this was published in a very prestigious journal, namely *Nature*. Compton published his Compton wavelength in *Physical Review*, which was perhaps less prestigious at that time. Still, both men were awarded the Nobel Prize in physics, Compton in 1927 and de Broglie in 1929; clearly their work was acknowledged as being very important in a short period of time.

More important for the advance of QM was that de Broglie was much firmer in his hypothesis on matter waves, while Compton's work seems to be of a more speculative but experimental nature: yes, if we do this scattering experiment involving electrons, we observe some waves that we also can calculate. Still, it is more to it. The de Broglie wavelength, as discussed previously, represents something taking place internally within elementary particles, even if it is less of a wave than the Compton wave. It is easier to understand this from the use of the Compton wavelength, while the de Broglie wavelength is a more complex way to obtain the information, perhaps unnecessarily so. The existing quantum theory, rooted in the de Broglie wavelength, is complicated, which opens it up for many convoluted interpretations. In this paper, we have presented a simpler alternative, but naturally the analysis will benefit from closer examination and comparison with other frameworks.

Further, our new maximum velocity formula plays an important part in getting a consistent theory. The maximum velocity formula for matter is what binds light and matter together. It shows how light is matter at the very collision point between photons, and how light has two velocities (zero and c) and not one. This is a dramatic new insight, but as I have shown in series of papers, it simplifies existing physics and help us get rid of all infinity challenges and so forth.

8 New Slightly Modified Uncertainty Principle

The Heisenberg [24] uncertainty principle in its momentum position form is given by

$$\Delta p \Delta x \geq \hbar \quad (43)$$

With our redefined momentum this would be

$$\Delta p_t \Delta x \geq \hbar \quad (44)$$

But a momentum consists quantum-wise of many parts:

$$p_t = \frac{mc}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{\frac{h}{\lambda_c} \frac{1}{c} c}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{\frac{h}{\lambda_c}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (45)$$

Where does the uncertainty in momentum come from? The Planck constant h and the speed of light c are constants, so the uncertainty cannot come from them. The uncertainty in the momentum must come from uncertainty in the Compton wavelength, or in the velocity of the particle. But these two, we will claim, are the same thing, as the Compton wavelength undergoes length contraction and is directly linked to the velocity. So, we can say that uncertainty in the momentum comes from uncertainty in the velocity, which is directly linked to uncertainty in the Compton wavelength, due to then uncertainty in length contraction. Therefore, we think the (modified) Heisenberg uncertainty principle can be seen as

$$\frac{mc}{\sqrt{1 - \frac{(\Delta v)^2}{c^2}}} \Delta x \geq \hbar \quad (46)$$

which can also be written as

$$\frac{h}{\lambda_c \sqrt{1 - \frac{(\Delta v)^2}{c^2}}} \Delta x \geq \hbar \quad (47)$$

By assuming there is a minimum length, and by setting the minimum uncertainty in the position to the Planck length, $\Delta x = l_p$ we get

$$\frac{mc}{\sqrt{1 - \frac{(\Delta v)^2}{c^2}}} l_p \geq \hbar \quad (48)$$

Solved with respect to Δv we get

$$\Delta v \leq c \sqrt{1 - \frac{l_p^2}{\lambda^2}} \quad (49)$$

That is, we get our maximum velocity for matter [3, 4], which is rooted in a minimum length equal to the Planck length, which gives a maximum velocity and therefore also a maximum uncertainty in velocity. We get the same maximum uncertainty in velocity from the energy and time version of the Heisenberg uncertainty principle. If, on the other hand, one uses the standard momentum instead of our newly defined momentum, one gets a structural difference in the maximum velocity when using deriving it from the momentum position principle and when doing it from the energy time principle, see [25]. Numerically it makes no difference for any known particle, as the first part of their series expansions are the same, but still this is not good. It points to yet another inconsistency that is indirectly created by an ill-specified momentum that we can say is rooted in the de Broglie wavelength rather than the Compton wavelength. There is no such inconsistency in our theory rooted in the Compton matter wavelength.

Based on our new theory we will also get that the uncertainty principle breaks down at the Planck scale; it switches from an uncertainty to certainty principle. This is because the maximum velocity for a photon with momentum is zero. This can also be proven in a much more formal way from the wave equation, and holds for the old non-optimal wave equations as well as from our new wave equation, see [6] for how to do this. A photon only has momentum when it is mass, and it only has mass in the one Planck second it spends in collision with another photon. This solves a series of interpretation crises in modern QM. Such things as entanglement can now be likely be explained by hidden variable theories. Bell's theorem [26] and the rejection of Einstein's suggested hidden variable theories are rooted in the idea that the Heisenberg uncertainty principle always holds. It holds all the way to the Planck scale, but then breaks down. The Planck scale exhibits very high energies, but only when the observational window is a Planck second. In larger observational time-windows, one should actually look for a very small particle to be the Planck mass particle (the collision point between photons); in a one second observational time-window, it will only be approximately 10^{-51} kg

We will also mention that we recently have shown in several Preprint papers that the Planck length can be measured totally independent of G and \hbar . The Planck length and the speed of light are the two of the most important universal constants and the Planck length leads to a maximum velocity for matter that also is related to photon collisions, that are the very deeper essence of matter.

We have also recently shown that special relativity is inconsistent with any minimum length, as one always can let an object (or elementary particle) travel at a velocity close enough to the speed of light so that any length becomes shorter than any given minimum length. Clearly there is substantial evidence pointing towards the fact that our two modifications are needed. Namely a redefined momentum rooted in the Compton wavelength rather than the de Broglie wavelength, and also a minimum length that leads to a maximum velocity for all matter. This forms the basis for a new QM that is simpler and more logical than existing theories.

9 Breakdown of the Heisenberg Uncertainty Principle at the Planck Scale

The plane wave function is given by

$$\Psi = e^{i\left(\frac{p_t}{\hbar}x - \frac{E}{\hbar}t\right)} \quad (50)$$

the total momentum p_t is given by

$$p_t = \frac{mc}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{\frac{\hbar}{\lambda} \frac{1}{c} c}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{\hbar}{\lambda \sqrt{1 - \frac{v^2}{c^2}}} \quad (51)$$

Then we can rewrite the wave function as

$$\Psi = e^{i\left(\frac{\frac{\hbar}{\lambda \sqrt{1 - \frac{v^2}{c^2}}}}{\hbar} x - \frac{\frac{\hbar c}{\lambda \sqrt{1 - \frac{v^2}{c^2}}}}{\hbar} t\right)} \quad (52)$$

Next remember we have $v_{max} = c\sqrt{1 - \frac{l_p^2}{\lambda^2}}$, and in the case of a Planck mass particle, we have $v_{max} = c\sqrt{1 - \frac{l_p^2}{l_p^2}} = 0$. Further, as explained earlier in the paper, the Planck mass particle (a photon–photon collision) only lasts for one Planck second, and the Planck mass particle has a fixed “size” (reduced Compton wavelength) equal to the Planck length. This means to observe a Planck mass particle we must have $x = l_p$ and $t = \frac{l_p}{c}$. This gives

$$\Psi = e^{i\left(\frac{1}{l_p} l_p - l_p c \frac{l_p}{c}\right)} = e^{i \times 0} = 1 \quad (53)$$

That is, the Ψ is always equal to one in the special case of the Planck mass particle, see also [6]. This means if we derive the Heisenberg Uncertainty principle from this wave function, in the special case of a Planck mass particle it breaks down.

10 Is the Zero Mass of Photons Only a Hypothesis ? We Think So: Photons Are the Missing Mass Gap!

Modern physics assumes that photons have zero mass – that they are, in fact, massless. The reasoning behind this hypothesis is likely related to Einstein’s [27, 28] relativistic energy mass formula, which is given by

$$\frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (54)$$

As Einstein himself pointed out, we must have $v < c$, because if not, then we would need an infinite energy to accelerate anything with rest-mass to the speed of light, or, in his own words:

This expression approaches infinity as the velocity v approaches the velocity of light c . The velocity must therefore always remain less than c , however great may be the energies used to produce the acceleration¹

At the same time, we know light moves at the speed of light by definition, so indeed, modern physics seems to have no other choice but to assume that photons have no mass. (However, as we will soon see, there is another choice). This also explains why modern physics use a kind of double accounting when working with matter and light from the relativistic energy momentum relation. It is important to understand that light being seen as massless is mainly an assumption based on the problem that most of the relativistic equations break down for anything moving at the speed of light. So, in this way they need to operate with two set of equations, or use two different assumptions when applying the same formula, such as the relativistic energy momentum formula. Furthermore, as stated previously, Einstein’s relativistic energy formula cannot hold for photons, as it would lead to infinite mass (energy). In the relativistic energy momentum relation, we must assume photons have a mass of zero in order to find their momentum, while we also have to assume that other particles do have mass in order to calculate their momentum. While modern physics has made great progress on understanding the connection between light (pure energy) and matter, through Einstein’s $E = mc^2$, for example, we will claim that something important is still missing between the direct link between photons and mass.

¹This quote is taken from page 53 in the 1931 edition of Einstein’s book *Relativity: The Special and General Theory*. English translation version of Einstein’s book by Robert W. Lawson.

energy level, but it is minuscule compared to a limit that is a little shorter than infinite, but with no real limit on how close to infinite a relativistic mass can be.

- Our maximum velocity for matter means that we can now use the relativistic energy momentum relation, both the old one and our new one to find the mass of a photon; it is the Planck mass. This we can also find directly from Einstein's relativistic energy formula, which also holds for photons under this framework.
- Our maximum velocity formula gives us the hidden link between photons and traditional particles with mass. It is clear from this that elementary particles such as electrons are trapped photons moving back and forth and colliding occasionally. Further, the maximum velocity formula we get from only adding one assumption, namely that there is a minimum length, indicates that we are working with quantum at the deepest level.
- Our maximum velocity formula gives a totally new and revolutionary insight in the missing part of photons. Photons have two velocities, not one; they move with the speed of light when they are not colliding with other photons, but they are standing absolutely still when they are colliding with other photons. Correspondingly, photons also have zero mass (massless) when not colliding and have mass when colliding. This leads to a binary photon model at the deepest level, at the Planck scale
- Our maximum velocity also means that the Heisenberg uncertainty principle must break down at the Planck scale. This opens up room for hidden variable theories again, as Bell's theorem is rooted in the idea that the Heisenberg uncertainty principle always holds. We have also shown how Heisenberg's uncertainty principle contains an inconsistency with respect to the momentum and position principle and the energy and time principle. Further, our theory means that mass can be seen as a Compton frequency clock; this seems to solve the Pauli Objection [].
- All in all, our theory leads to a simpler and more logical framework for understanding these relationships; one that is still consistent with all experiments.

Clearly, we are making some extraordinary claims, so the work requires serious documentation. We encourage readers to study our other published papers as well as our working papers. With so many outstanding questions, inconsistencies, absurd predictions, and anomalies, it is not inappropriate to think outside the box in the field of physics. We welcome constructive feedback and feel that new, well-documented, and carefully articulated approaches are worth serious consideration and discussion.

11 Conclusion

We have suggested a new momentum definition that corresponds to the true matter wave, that is the Compton wavelength, rather than the de Broglie wavelength, which is a mathematical derivative of the true physical wavelength in matter. This gives us a new and simpler relativistic energy momentum relation. This also eliminates the need for speculation on such things as negative energy, negative mass, negative probabilities, and particles moving backwards in time, all of which have been considered in the effort to patch the hole created by the use of standard momentum and its corresponding de Broglie wavelength. We have also suggested a new relativistic quantum mechanics wave equation based on this relationship.

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12 Appendix A: A More Intuitive Invariant Mass

Our new relativistic energy-momentum relation is given by

$$E = p_k c + m c^2 \quad (59)$$

From this we find that the invariant mass is given by

$$m = \frac{E - p_k c}{c^2} \quad (60)$$

Which simply means the rest-mass is the total energy minus the kinetic energy divided by c^2 . That is simply the rest-mass is equal to the rest-mass energy divided by c^2 . In the modern relativistic momentum energy relation, this also holds true, but here the equation is much less intuitive, as it is rooted in a momentum

derivative – the de Broglie equivalent momentum. From the standard relativistic energy momentum relation, we have

$$\begin{aligned} E^2 &= p^2 c^2 + m^2 c^4 \\ m &= \sqrt{\frac{E^2}{c^2} - p^2} \\ &\quad c^2 \end{aligned} \quad (61)$$

As we can see, this formula is not intuitive. Mathematically, it is correct, as it actually predicts the same invariant mass as our formula, but all the squaring etc. is needed to get the de Broglie wave linked momentum to give the correct energy and thereby the correct mass.

Appendix B: Mass from the Standard Relativistic Energy Momentum Relation versus the New Momentum Relation

In modern physics one sometimes also use the relativistic energy momentum relation to predict mass from particles

$$E^2 = p^2 c^2 + m^2 c^4 \quad (62)$$

Solved with respect to mass, we get the well-known invariant mass formula

$$m = \frac{\sqrt{\frac{E^2}{c^2} - p^2}}{c^2} \quad (63)$$

In modern physics, we must now use a mathematical “trick” to get this to hold for both mass and photons. For standard particles with mass, we must have

$$\begin{aligned} m &= \frac{\sqrt{\frac{E^2}{c^2} - p^2}}{c} \\ m &= \frac{\sqrt{\frac{m^2 c^2}{1 - \frac{v^2}{c^2}} - \left(\frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}}\right)^2}}{c} \\ m &= \frac{\sqrt{\frac{m^2 c^2}{1 - \frac{v^2}{c^2}} - \frac{m^2 v^2}{1 - \frac{v^2}{c^2}}}}{c^2} \\ m &= \frac{\sqrt{\frac{m^2 c^2}{1 - \frac{v^2}{c^2}} + \frac{m^2 c^2 (1 - \frac{v^2}{c^2})}{1 - \frac{v^2}{c^2}} - \frac{m^2 c^2}{1 - \frac{v^2}{c^2}}}}{c} \\ m &= \frac{mc}{c} \end{aligned} \quad (64)$$

However, this expression cannot also be used for photons, as photons in the models are always mass-less. So, when working with photons, modern physics flips into another definition of the momentum. Now we are not allowed to use Einstein’s relativistic momentum formula, as it would give infinite momentum and thereby infinite mass, but instead we are setting the momentum to $\frac{E}{c}$, which is in conflict with Einstein’s relativistic momentum formula. In other words, a kind of double accounting system is needed to make the relativistic energy momentum formula fit both traditional particles with mass and photons that purportedly have no mass.

$$\begin{aligned} m &= \frac{\sqrt{\frac{E^2}{c^2} - p^2}}{c} \\ m &= \frac{\sqrt{\frac{E^2}{c^2} - \left(\frac{E}{c}\right)^2}}{c} \\ m &= 0 \end{aligned} \quad (65)$$

This double accounting system with different rules for photons and other masses is not needed under our new model. Actually, derivation 64 also holds for both standard particles and photons, but then one must realize

the maximum velocity for anything with momentum is $v_{max} = c\sqrt{1 - \frac{l^2}{\lambda^2}}$, which then is zero for a photon. In other words, photons stand still in a photon–photon collision; their momentum is then zero. However, even if the end result is correct, there is another interpretation problem here, as the relativistic energy momentum use a non-optimal momentum that is a derivative of the more fundamental momentum.

In our new momentum relation, we simply have

$$E = p_k c + mc^2 \quad (66)$$

That solved with respect to mass, we get the well-known mass formula

$$m = \frac{E - p_k c}{c^2} \quad (67)$$

and since the kinetic momentum is: $p_k = \frac{mc}{\sqrt{1 - \frac{v^2}{c^2}}} + mc$, we can rewrite the expression above as

$$m = \frac{\frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} - \frac{mc}{\sqrt{1 - \frac{v^2}{c^2}}}c + mcc}{c^2} = \frac{mc^2}{c^2} \quad (68)$$

So this is just simple logic. What is important is that this holds for both photons and other particles. A photon always has a mass equal to the Planck mass, and this is in line with observations and when observed over longer time intervals than the Planck second, we still get the correct mass, as discussed in section 9.