A general definition of division in a field

Hiroshi Okumura
hokmr@yandex.com

Abstract. A historical definition of division by zero is reconsidered.

Let F be a field. The definition of division $/$ in F may be stated as $a/b = ab^{-1}$ in the case $b \neq 0$, and a/b is not defined if $b = 0$. But we can easily define $/$ including the case $b = 0$, i.e., $a/b = ab^{-1}$ if $b \neq 0$, and $a/b = ba$ if $b = 0$. Notice that the definition is made in a natural way. Essentially the same definition was made by the Indian mathematician and astronomer Brahmagupta (598-665+α).

Our definition is justified by Takahasi's theorem [2].

Theorem 1. *Let F be a filed with characteristic different from 2. Let f be a mapping $f : F \times F \to F$ satisfying*

(i) $f(a, b) = ab^{-1}$ *if* $b \neq 0$,

(ii) $f(a, b)f(c, d) = f(ac, bd)$.

Then we have $f(a, 0) = 0$ for any $a \in F$.

Since our generalized definition have not been considered until today, it works as something like an axiom when we consider over a field. Therefore we have to consider whether the definition gives as fruitful results or not. The definition should get canceled, if it do not give any interesting results. Experimenting works with this aspect have been made for more than four years under the leadership of Saburou Saitoh, and a listing with more than eight hundreds results is made, some of which can be seen in [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17].

References

[1] H. Akca, S. Pinelas and S. Saitoh, Incompleteness of the theory of differential equations and open problems, International Journal of Applied Mathematics and Statistics, Int. J. Appl. Math. Stat. Vol. **57**; Issue No. 4; Year 2018, ISSN 0973-1377 (Print), ISSN 0973-7545 (Online).

[2] M. Kuroda, H. Michiwaki, S. Saitoh, M. Yamane, New meanings of the division by zero and interpretations on $100/0 = 0$ and on $0/0 = 0$, Int. J. Appl. Math., **27**(2) (2014) 191–198.

[3] T. Matsuura, H. Michiwaki and S. Saitoh, $\log 0 = \log \infty = 0$ and applications. Differential and Difference Equations with Applications. Springer Proceedings in Mathematics & Statistics. **230** (2018), 293–305.

[4] T. Matsuura and S. Saitoh, Matrices and division by zero $z/0 = 0$, Advances in Linear Algebra & Matrix Theory, **6**(2016), 51-58, Published Online June 2016 in SciRes. `http://www.scirp.org/journal/alamt`, `http://dx.doi.org/10.4236/alamt.2016.62007`.

[5] H. Michiwaki, S. Saitoh and M.Yamada, Reality of the division by zero $z/0 = 0$. IJAPM International J. of Applied Physics and Math. **6**(2015), 1–8. `http://www.ijapm.org/show-63-504-1.html`.

[6] H. Michiwaki, H. Okumura and S. Saitoh, Division by Zero $z/0 = 0$ in Euclidean Spaces, International Journal of Mathematics and Computation, **2**8(2017); Issue 1, 1-16.

[7] Y. Kanai, H. Okumura, A three tangent congruent circle problem, Sangaku J. Math., **1**(2018) 16-20.

[8] H. Okumura, Wasan geometry with the division by 0, Int. J. Geom., **8**(1)(2018), 17-20.

[9] H. Okumura, Is it really impossible to divide by zero?, Biostatistics and Biometrics, **7**(1)(2018), 1-2.

[10] H. Okumura, Solution to 2017-1 Problem 4 with division by zero, Sangaku J. Math., **2**(2018) 27-30.

[11] H. Okumura and S. Saitoh, The Descartes circles theorem and division by zero calculus. `https://arxiv.org/abs/1711.04961(2017.11.14)`.

[12] H. Okumura and S. Saitoh, Harmonic mean and division by zero, Forum Geom., **18**(2018), 155-159.

[13] H. Okumura and S. Saitoh, Remarks for the twin circles of Archimedes in a skewed arbelos by Okumura and Watanabe, Forum Geom., **18**(2018), 97-100.

[14] H. Okumura and S. Saitoh, Applications of the division by zero calculus to Wasan geometry, Glob. J. Adv. Res. Class. Mod. Geom., **7**(2)(2018), 44-49.

[15] H. Okumura, S. Saitoh and T. Matsuura, Relations of 0 and ∞, Journal of Technology and Social Science (JTSS), **1**(2017), 70-77.

[16] S. Pinelas and S. Saitoh, Division by zero calculus and differential equations. Differential and Difference Equations with Applications. Springer Proceedings in Mathematics & Statistics. **230** (2018), 399–418.

[17] S. Saitoh, Mysterious Properties of the Point at Infinity, arXiv:1712.09467 [math.GM](2017.12.17).