From cosmological constant to cosmological matrix

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Abstract

The starting point of modern theoretical cosmology were the Einstein equations with the cosmological constant Λ which was introduced by Einstein . The Einstein equations with the cosmological matrix $\Lambda_{\alpha\beta}$ is introduced here.

1 Introduction

The cosmological constant Λ is interpreted at this time as the value of the energy density of the vacuum of space. It was originally introduced by Einstein as an addition to his theory of general relativity to achieve a static universe, which was the accepted view at the time (Einstein, 1917).

Einstein abandoned the concept after Hubble's discovery that all galaxies outside the Local Group (the Milky Way Galaxy) are moving away from each other, forming expanding universe (Hubble, 1929).

Most cosmology researchers assumed the cosmological constant to be zero. The cosmological constant is now considered as the simplest possible form of dark energy since it is constant in both space and time, and this leads to the current standard model of cosmology known as the $\Lambda-$model, which is good fit to many cosmological observations.

The cosmological constant has the same effect as an intrinsic energy density of the vacuum, ϱ_{vac}. In this context, it is commonly moved onto the right-hand side of the Einstein equation, and defined with a proportionality factor, or $\Lambda \sim \varrho_{vac}$. The true dimension of Λ is a length^{-2}. At present time $\Lambda = 1.11 \times 10^{-52} \times \mathrm{m}^{-2}$.

The goal of this article is to introduce the o called cosmological matrix which is more adequate to describe many aspects of universe when intserting into the Einstein equations. So, let us remember the derivation of the Einstein equations.

2 Fok derivation of Einstein equations

The Einstein field equations (EFE) are the space-time geometry equations for the determine of the metric tensor of space-time for a given arrangement of stress-energy in the spacetime. They are the non-linear partial differential equations and the solutions of the EFE are the components of the metric tensor.

The inertial trajectories of particles are geodesics in the resulting geometry calculated using the geodesic equation.

EFE obeying local energy-momentum conservation, they reduce to Newtons law of gravitation where the gravitational field is weak and velocities are much less than the speed of light.

Let us follow the rigorous derivation of the Einstein gravity equations by Fok (Fok, 1961). The similar derivation was performed by Chandrasekhar (1972), Kenyon (1996), Landau et al. (1962), Rindler (2003) and others. Source theory derivation of Einstein equations was performed by Schwinger (1970).

It is well known that the gravity mass M_G of some body is equal to the its intertial mass M_I, where gravity mass is a measure of a massive body to create the gravity field (or, gravity force) and the inertial mass of a massive body is a measure of the ability of the resistence of the body when it is accelerated. At present time we know, that if components of elementary particles have the same gravity and inertial masses, the body composed with such elementary particles has the identical gravity and inertial mass. There is no need to perform experimental verification. So, particle physics brilliantly confirms the identity of the inertial and gravity masses.

According to the Newton theory the gravity potential is given by the equation

$$U(r) = -\kappa \frac{M}{r}, \tag{1}$$

where r is a distance from the center of mass of a body, κ is the gravitational constant.

The potential U is as it is well known the solution of the Poisson equation:

$$\Delta U(r) = -4\pi\kappa\varrho, \tag{2}$$

where ϱ is the density of the distributed masses.

The problem is what is the geometrical formulation of gravity equation (2) following from the space-time element ds, which has the specific form in case of the special theory of relativity.

Let us postulate that the motion of a body moving in the g-field is determined by the variational principle

$$\delta \int ds = \delta \int g_{\alpha\beta} dx^\alpha dx^\beta = 0. \tag{3}$$

In order to get the Newton equation of motion, we are forced to perform the following identity:

$$g_{00} = c^2 - 2U = -4\pi\kappa\varrho, \tag{4}$$

The second mathematical requirement, which has also the physical meaning is the covariance of the derived equation. It means that the necessary mathematical operation are the following replacing of original symbols:

$$U \to g_{\mu\nu} \tag{5}$$

with

$$\Delta U \to Tensor\ equation \tag{6}$$

$$\varrho \to T_{\mu\nu}, \tag{7}$$

where $T_{\mu\nu}$ is the tensor of energy and momentum.

In order to get the tensor generalization of eq. (2) it is necessary to construct new tensor $R_{\mu\nu}$, which is linear combination of the more complicated tensor $R_{\alpha\beta,\mu\nu}$, or

$$R_{\mu\nu} = g^{\alpha\beta} R_{\mu\alpha,\beta\nu} \tag{8}$$

and the scalar quantity R, which is defined by equation

$$R = g^{\lambda\mu} R_{\lambda\mu} \tag{9}$$

and construct the combination tensor $G_{\lambda\mu}$ of the form

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R, \tag{10}$$

which has the mathematical property, that the divergence of this tensor is zero, or,

$$\partial^{\lambda} G_{\lambda\mu} = 0 \qquad (11).$$

With regard to the fact that also the energy-momentum tensor $T_{\mu\nu}$ has the zero divergence, we can identify eq. (10) with the tensor $T_{\mu\nu}$, or

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi\kappa}{c^2} T_{\mu\nu}, \tag{12}$$

where the appeared constant in the last equation is introduce to get the classical limit of the equation.

The approximative solution of the last equation is as follows

$$ds^2 = (c^2 - 2U)dt^2 - \left(1 + \frac{2U}{c^2}(dx^2 + dy^2 + dz^2)\right) \qquad (13).$$

The space-time element is able to explaine the shift of the frequency of light in gravitational field and the deflection of light in the gravitaional field of masiive body with mass M.

So, we have seen that the basic mathematical form of the Einstein general relativity is the Riemann manifold specified by the metric with the physical meaning. The crucial principle is the equality of the inertial and gravitational masses. The principle of equivalence is not the crucial principle of general relativity and it means that the easy logical consequence of this fact is that the Copernicus system is not equivalent to the Ptolemy system.

3 From cosmological constant to cosmological matrix

Let us write the Einstein equations (12) in the following form

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = -\frac{8\pi\kappa}{c^2}T_{\lambda\mu}, \tag{14}$$

where Λ is the new cosmological constant is introduced ad libitum, which enables to find new form of the cosmological model and their solutions in the mathematical form.

The introduction of the comological matrix is as follows is based on the the analogy with the term (9)

$$Rg_{\mu\nu} = (R_{\alpha\beta}g^{\alpha\beta})g_{\mu\nu} \tag{15}$$

Now, if we perform transformation

$$R_{\alpha\beta} \to \Lambda_{\alpha\beta}, \tag{16}$$

then we get

$$Rg_{\mu\nu} \to \Lambda_{\alpha\beta}g^{\alpha\beta}g_{\mu\nu} \tag{17}$$

We suppose here some mathematical freedom in postulating the cosmological matrix to get the Einstein equations with the cosmological matrix $\Lambda_{\alpha\beta}$ in the form:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + (\Lambda_{\alpha\beta}g^{\alpha\beta})g_{\mu\nu} = -\frac{8\pi\kappa}{c^2}T_{\mu\nu}, \tag{18}$$

where

$$\Lambda_{\alpha\beta} = \begin{pmatrix} \Lambda_{00} & \Lambda_{01} & \Lambda_{02} & \Lambda_{03} \\ \Lambda_{10} & \Lambda_{11} & \Lambda_{12} & \Lambda_{13} \\ \Lambda_{20} & \Lambda_{21} & \Lambda_{22} & \Lambda_{23} \\ \Lambda_{30} & \Lambda_{31} & \Lambda_{32} & \Lambda_{33} \end{pmatrix}. \tag{19}$$

The generalization of cosmology and new deal of cosmology is then based on the gravity equations (18).

4 From Schwarzschild space to the space with cosmological matrix

It is well known that the Schwarzschild solution of the Einstein equation is of the spherical form (Rindler, 2003):

$$ds^2 = \left(1 - \frac{2m}{r}\right) dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2), \qquad (20)$$

where m, r, θ is mass of a gravitating body, radius, sperical angle and the azimutal angle.

In case if the Einstein equations with the cosmological constant Λ, we get the so called modified Schwarzschild space-time with the corresponding solution as it follows (Rindler, 2003):

$$ds^2 =$$

$$\left(1 - \frac{2m}{r} - \frac{1}{3}\Lambda r^2\right) dt^2 - \left(1 - \frac{2m}{r} - \frac{1}{3}\Lambda r^2\right) dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2), \quad (21)$$

representing Schwarzschild-de Sitter space because for small r it approximates Schwarzschild space-time and for large r de Sitter space-time if Λ is positive.

The solution of the Einstein equations with the cosmological matrix (13) can be obtained by the usual mathematical methods of the relativistic physics.

5 Discussion

Einstein included the cosmological constant (”biggest blunder” of his life) as a term in his field equations for general relativity because he was dissatisfied that his equations did not allow, apparently, for a static universe. Einstein predicted the expansion of the universe in theory, before it was demonstrated in observation of the cosmological red shift.

However, soon after Einstein developed his static theory, observations by Edwin Hubble indicated that the universe appears to be expanding which this was consistent with a original solution that had been found by the mathematician Friedmann, working on the Einstein equations of general relativity.

So, adding the cosmological constant to Einstein's equations does not lead to a static universe at equilibrium because the equilibrium is unstable.

However, the cosmological constant remained a subject of theoretical and empirical interest. The cosmological data in the past decades strongly suggests that our universe has a positive cosmological constant. The explanation of this small but positive value is an brilliant theoretical goal.

We present here the generalization of Einstein's gravitational theory, by the cosmological matrix and it is not excluded that we define here the new deal of mathematical physics of differential equations and the new deal of cosmology.

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