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1908 BUCHERER EXPERIMENT AND THE LORENTZ FORCE LAW

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ABSTRACT. The Bucherer experiment of 1908 was not experimental proof of a relativistic mass varying with speed, but proof that electromagnetism and the Lorentz force law fail under relativistic speed conditions. Our conclusions come from a novel re-examination of the experiment based on three different interpretations of Newton's second law as applied to the experiment and to analyze the implications for each of:

(1) force$\propto$dp/dt
(2) force=relativistic_mass$\times$acceleration
(3) the classical $f = ma$

The new interpretation now shows a constant charge-mass ratio for all relativistic speed; both charge and mass would be speed invariant. New relativistic force laws had to be proposed to be consistent with the new experimental findings; the Lorentz force law is now: $\mathbf{F} = q(\sqrt{1 - v^2/c^2}(1 + v^2/c^2)\mathbf{E} + \sqrt{1 - v^4/c^4}\mathbf{v} \times \mathbf{B})$; the Coulomb's law is:$\mathbf{F} = \sqrt{1 - v^2/c^2}(1 + v^2/c^2)(\frac{1}{4\pi\epsilon_0})\frac{q_1 q_2 \hat{\mathbf{r}}}{r^2}$. The Coulomb's law has an additional scalar factor dependent on the relative velocity between the charges; for small speed, the form is: $\mathbf{F} = (1 + \frac{1}{2}\frac{v^2}{c^2})(\frac{1}{4\pi\epsilon_0})\frac{q_1 q_2 \hat{\mathbf{r}}}{r^2}$. This enables the formula for the force between parallel current-carrying conductors:$F_{dl} = \frac{\mu_0}{2\pi R} I_1 I_2 dl$,to be derived free of the concept of the magnetic field. A real possibility exists for a formulation of a revolutionary Newtonian electric theory free of magnetism and the Biot-Savart law.Also,the Bucherer experiment could have been an experimental verification of the relativistic Lorentz force law if the predicted speeds of the electrons had been verified through direct time-of-flight measurements.

1. INTRODUCTION

[Version 4.2.1] It is now common to find that whenever special relativity is discussed, it is accompanied by the assertion that it is one of the best tested and verified physics theory to date. The Kaufmann(1901)[3],

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Bucherer(1908)[4], Neumann[5] and Rogers et.al(1940)[6] experiments have always been represented as conclusive experimental verification of a relativistic mass varying with speed, thereby indirectly also repudiating Newtonian mechanics and verifying special relativity. It will be shown that despite the excellent agreement the relativistic mass of special relativity has with the Bucherer experiment, the experiment is not a verification of special relativity; instead, it is a clear experimental repudiation of the Lorentz force law of electrodynamics at relativistic speeds.

2. THE BUCHERER EXPERIMENT, 1908

The purpose of this paper is not to critique the Bucherer experiment in its details, but only its theoretical basis. Professor A.K.T. Assis gives a simplified description of the Bucherer experiment in one of his papers[2], but with a clear description of the theory behind the experiment. We reproduce it here.

The Bucherer apparatus may be considered as a capacitor with a linear dimension L much greater then the separation of of the two oppositely charge plates with surface charge distribution of $\pm\sigma$. The x-axis is perpendicular to the plates from $-\sigma$ to $+\sigma$. Classical electrodynamics shows that there is a uniform electric field $\mathbf{E}_x = -(\sigma/\epsilon_0)\hat{\mathbf{x}}$ between the capacitor plates. The axes origin is a radium β-particle(electron) source at the center of the capacitor between the plates. The y-axis is the path an electron would leave the capacitor after traversing the distance L leaving the capacitor with a velocity $\mathbf{v}_y$. A uniform magnetic field $\mathbf{B}_z$ in the z-axis direction is superimposed on the capacitor. Only those electrons in the y-direction could leave the capacitor when the electric deflection and the magnetic deflection in the x-direction are in balance. Furthermore, the initial electrons has to have no velocity component in the x-direction else they would collide with the capacitor plates. For electrons that leaves the capacitor moving along the y-axis, the only forces acting on the electrons are from the electric and magnetic deflections that act only in the x-direction; the velocity $\mathbf{v}_y$ is the natural electron ejection velocity. The force acting on the electrons is:

$$\mathbf{F_x} = -e(\mathbf{E}_x + \mathbf{v_y} \times \mathbf{B_z}) \tag{1}$$

Equating the force with zero, we have:

$$v_y = \sigma/\epsilon_0 B_z \tag{2}$$

The Bucherer apparatus is also a velocity selector as changing the magnitude of the voltage across the capacitor and the magnetic field would allow electrons of varying speed to leave the capacitors. Five runs of the experiment were made giving data points for speed from about 0.3c to 0.7c. After the electrons leave the capacitor it would only be under the deflection of the magnetic field and it would travel in a circular path with a constant speed as in (2) until it strikes a photographic plate at some known distance away. From the coordinate of the points the electrons make on the photographic plate and the other dimensions, the radius r of the circular path could be computed. Applying the Lorentz magnetic force as the centripetal force for circular motion, we have:

$$|e(\mathbf{v} \times \mathbf{B})| = ma = mv^2/r \tag{3}$$

a being the centripetal acceleration and v is the constant speed equal to the speed in (2). Combining equations (2) and (3) gives:

$$e/m = \sigma/r\epsilon_0 B^2 \tag{4}$$

The RHS of (4) could be evaluated as the terms are known physical constants or measured variables of the experiment. The ratio e/m for the data points was found to vary with velocity, decreasing with velocity increase. As the electron charge was accepted to be constant, the varying charge-mass ratio of the electron was interpreted to mean that mass increases with velocity. The ratio e/m was found to have a strong correlation with $e/(m_0/\sqrt{1 - v^2/c^2})$. This showed that the experiment was in agreement with the Lorentz-Einstein model where the electromagnetic mass was:

$$m_r = \frac{m_0}{\sqrt{1 - v^2/c^2}} \tag{5}$$

m_0 being the invariant rest mass of the electron. The textbook of Professor Robert Resnik [1] gives a table of the data for the experiment.The Bucherer experiment was viewed as evidence that inertia mass of matter has an electromagnetic origin and that it varies with velocity, not invariant.

3. Interpretation of the Bucherer Experiment

The result of the experiment did have profound implications. Prima facie, it repudiated invariant mass and verified the relativistic mass of special relativity. It was neither. The physicists then had electron models that predicted mass increasing with speed. For whatever reasons, they were unwilling to forego their models and consider alternative interpretations of the experiment. Some were quick to accept the Bucherer experiment as a conclusive repudiation of the invariant mass. None cast any suspicion on the equation (3) which was the basis of experiments such as that of Bucherer's. Electrons were deflected in a circular path and the Lorentz magnetic force of $e(\mathbf{v} \times \mathbf{B})$ was the only force acting on the electrons. It was the application of Newton's second law that gave rise to the equation.

We would re-examine the Bucherer experiment in a manner which has not been considered in the past. The novelty of this treatment is to attempt three different interpretations of Newton's second law to be used in equation (3) of the Bucherer experiment and to analyze the implications of each.

(1) $Force \propto \dfrac{dp}{dt}$. This is an attempt to go back to the original statement as in the *Principia*. Momentum would be the relativistic definition :

$$Force \propto \frac{d}{dt}\left(\frac{mv}{\sqrt{1-v^2/c^2}}\right) \tag{6}$$

This interpretation of Newton's second law as in relativistic mechanics fails - it leads to a force that is fictitious.

A proportionality relation has meaning only when both sides of the relation have defined values. The RHS is defined and has dimension of $[M][L][T^{-2}]$. The LHS is not defined as there is not yet a definition for force; the definition of force as is customary in Newtonian mechanics cannot be assumed here as this interpretation of Newton's second law effectually defines a new mechanics. Such a relativistic mechanics of special relativity has to come out with a definition of force if this new mechanics is to be valid. As there is no defined unit for relativistic force, such a relativistic mechanics may only be a fictitious formulation. Though the relativistic mass as implied in (6) does satisfy the mass as required in the Bucherer experiment, such a force as defined in this case fails. This case is dismissed.

(2) $Force = relativistic\_mass \times acceleration$. Relativistic mass may be defined as $\phi(v)m$ where $\phi(v)$ is a scalar function dependent on velocity v, m being the invariant mass. This interpretation takes the form of a definition of a force as the relation here is an identity, not a proportionality as in the first case. As the dimension of the RHS is $[M][L][T^{-2}]$, the same dimension of force as with classical Newtonian mechanics, we first assume that the force here is defined and has a real unit the same as that for Newtonian mechanics. This definition of force would be consistent with the actual Bucherer experiment as it would accommodate a mass that varies with speed if it is found to be the case in the result. If we take $\phi(v)$ to be $\dfrac{1}{\sqrt{1-v^2/c^2}}$, the relativistic mass would be that of special relativity. As we have seen, such a mass agrees with the result of the Bucherer experiment. The case here seems to give a formulation of relativistic mechanics that has a real unit of force and leads to a valid mechanics which agrees with the Bucherer experiment.

We now use this definition of force in the work-energy theorem to get the formula for kinetic energy.

$$K = W = \int_0^v (\frac{m}{\sqrt{1 - v^2/c^2}}) \frac{dv}{dt} dx = \int_0^v \frac{mv}{\sqrt{1 - v^2/c^2}} dv$$

$$K = mc^2(1 - \sqrt{1 - v^2/c^2}) \tag{7}$$

The formula (7) is not the same as the kinetic energy formula of special relativity which is:

$$K = mc^2(\frac{1}{\sqrt{1 - v^2/c^2}} - 1)$$

If the definition of force in this case formulates a valid relativistic mechanics, it is not the relativistic mechanics of special relativity. In fact, the definition of force in this case is also invalid as a variable mass dependent on speed could not be use to define a consistent standard unit of force. This case too is dismissed.

(3) $Force = mass \times acceleration$, mass being invariant. This is the definition of force in classical Newtonian mechanics. It interprets Newton's second law as an axiom of truth defining a force. This interpretation has been the only one since the time of Newton and there never was any other. Here, mass is an invariant as an axiom of Newton's laws of motion. This force definition is used in the circular motion force equation (3) of the Bucherer experiment and it leads to a result which shows a mass that is not invariant, but increases with speed - there is a contradiction between the physics theory and the experimental result. If the physics underlying the experiment is correct, such a contradiction should not occur. The force law behind the Bucherer experiment is based on an invariant mass and yet, the result shows a mass that increases with speed. This contradiction indicates that the physics on which the experiment is based are not all correct - it includes physics that are incorrect.

The physics behind the Bucherer experiment are Newton's force law, electromagnetism including the Lorentz force law. One of them is invalid giving rise to the contradiction.The classical Newton's force law is one of the best tested laws in physics since the time of Newton, rigorously tested for three centuries without any instant of failure where it is applied - it cannot be incorrect. The conclusion cannot be other than that electromagnetism and the Lorentz force law contain fundamental errors in some manner. As others have noted [7], the Lorentz law has to date not been directly tested under relativistic speeds; that it is the cause of the contradiction is very probable. To assume that only the Lorentz force law alone is invalid and the rest of electromagnetism is all clean and correct is illogical. As Lorentz force law itself involves both fields $\mathbf{E}$ and $\mathbf{B}$, its failure may well have its origin in the very formulation of electromagnetism itself.

The first two cases above have to be ignored. Only the third case need to be considered and the conclusion could only be the failure of the Lorentz force law and the theory of electromagnetism.

The Bucherer experiment was experimental proof that electromagnetism and the Lorentz force law fail under relativistic speed conditions.

4. A Mass Definition Is Not Testable

In the "*Introduction to Special Relativity*"[1], the well known author Robert Resnick shows the Bucherer experiment as "*proof*" that the idea of an invariant mass was contradicted by experiment - mass was verified to vary and even fits the γ-factor for the relativistic mass of special relativity. The invariance of mass in Newtonian mechanics is a definition - defined as an absolute "*quantity of matter*" in the *Principia*. Even the relativistic mass of special relativity is founded on this same mass, but as a "*rest mass*" m_0 with a γ-factor added, $\gamma = \dfrac{1}{\sqrt{1-v^2/c^2}}$:

$$m_r = \frac{m_0}{\sqrt{1-v^2/c^2}} \tag{8}$$

The formula (8) is just a new definition for mass (indirectly through relativistic momentum) giving rise to a new formulation of mechanics of special relativity.

Experiments in the scientific paradigm is meant only to verify or test predictions of a theory, not any of its defined concepts. As an example, the invariance of mass in Newtonian mechanics is not testable, but the prediction that planets orbits the sun in elliptical orbits is verifiable. So the claim of Professor Robert Resnick is logically untenable. Neither the invariant Newtonian mass nor the relativistic mass of special relativity is testable.

5. A New Coulomb's Law and Lorentz Force Law

We will examine here what forms the new Lorentz force law should take in order that the Bucherer experiment would not lead to a contradiction of a variable charge-mass ratio. We examine the form as :

$$\mathbf{F} = q(\phi_e\mathbf{E} + \phi_b\mathbf{v}\times\mathbf{B})$$

where ϕ_e, ϕ_b are two scalar functions of velocity whose forms would be determined. Following the same derivation as done earlier,the constant velocity in the Bucherer experiment for the circular motion of the electron in a uniform magnetic field would be:

$$v = \phi_e \sigma / \phi_b \epsilon_0 B \tag{9}$$

For the equation of circular motion, we now have:

$$|e\phi_b(\mathbf{v} \times \mathbf{B})| = ma = mv^2/r \tag{10}$$

Combining equations (9) and (10), we have a new equation for the charge-mass ratio:

$$e/m = (\phi_e/\phi_b^2)\sigma/\epsilon_0 r B^2 \tag{11}$$

We multiply the nominator and denominator of the RHS of (11) with ϕ, $\phi = \sqrt{1 - v^2/c^2}$, we have:

$$e/m = (\phi_e \phi/\phi_b^2)(\sigma/\phi\epsilon_0 r B^2) \tag{12}$$

As shown in Section 2 above, $\sigma/\phi\epsilon_0 r B^2$ has a strong correlation with e/m_0 for the data set of the original experiment. So, in order that the new Lorentz force law leads to a non-contradictory constant e/m ratio strongly correlated to e/m_0 the condition to be satisfied is: $\phi_e\phi/\phi_b^2 = 1$ or,

$$\phi_e \phi = \phi_b^2 \tag{13}$$

A necessary condition for the form of the new Lorentz force law would be:

$$\phi_e = \phi^{2n-1} f(v)^2; \quad \phi_b = \phi^n f(v) \tag{14}$$

for any integer n, $f(v)$ be any arbitrary scalar function of velocity.

There is another condition that has to be satisfied. If the new form has the term ϕ or $\sqrt{1 - v^2/c^2}$, then $v <= c$ otherwise there will be indeterminacy from the square-root of a negative value. The magnetic force of $\mathbf{v} \times \mathbf{B}$ only changes the direction of the velocity ot a moving charge, but not its speed; only the electric field would be able to change the speed. In order that the speed of a charge particle to not keep on increasing without limit and to exceed the speed of light c, then the electric force of $\mathbf{E}$ on a charge must approach zero as v approaches c. This is a further restriction of the form of the new Lorentz force law.

There are three general cases for the conditions 14:

(1) Case $f = \phi$: $\phi_e = \phi^{2n+1}$; $\quad \phi_b = \phi^{n+1}$ where $n >= 0$.
(2) Case $f = 1/\phi$: This case is equivalent to case (1).
(3) Case $f \neq \phi$: $\phi_e = \phi^{2n-1} f^2$; $\quad \phi_b = \phi^n f$ where $n >= 1$

This shows that there are infinite forms the Lorentz force could take which would have the Bucherer experiment to not lead to any contradictions. The only way to determine what the actual form would be is through experimental verification of the velocity in (9) by a direct time-of-flight measurements of the ejected electron speeds. Furthermore, examination of empirical observations may help to deduce what its actual form should take. If the velocities of electrons of the

Bucherer experiment had been verified through direct time-of-flight measurements, then the Bucherer experiment would have been a verification of the relativistic form of the Lorentz force law.

> *The Bucherer experiment could have been an experiment to verify the Lorentz force law for relativistic speed if the predicted speeds of the electrons had been verified through direct time-of-flight measurements.*

As yet, no direct time-of-flight measurement has ever been made.

We will here only consider the case (3) above with n = 1 where the new *relativistic* Lorentz force law would have the general form:

$$\mathbf{F} = q(\phi(v)f(v)^2\mathbf{E} + \phi(v)f(v)\mathbf{v} \times \mathbf{B}) \tag{15}$$

A new Lorentz force law would need a corresponding revision of the Coulomb's law to take an extra factor $\psi(v) = \phi(v)f(v)^2$ which is a scalar function dependent on v; v is now the relative velocity between the two interacting charges. The new relativistic Coulomb's law is :

$$\mathbf{F} = \frac{\psi(v)}{4\pi\epsilon_0}\frac{q_1 q_2 \hat{\mathbf{r}}}{r^2}; \tag{16}$$

$\psi(v) = \sqrt{1 - v^2/c^2} f(v)^2$; $f(v)$ being a scalar function of v whose actual form need to be determined. With this new Coulomb's law, the force on a moving test charge due to an electric field for a stationary charge configuration would be: $\mathbf{F} = q\psi(v)\mathbf{E}$. This is consistent with the relativistic Lorentz force law as verified by the Bucherer experiment.There is empirical phenomenon on which we could rely on to determine what the probable form $f(v)$ is to take. It is shown in the section below that if $f(v) = \sqrt{1 + v^2/c^2}$, the exact well-known force equation (17) between two long parallel current-carrying conductors may be derived using only Coulomb forces only without any need of magnetism. The fact that this parallel-force equation has been well tested may be taken to mean the form $f(v) = \sqrt{1 + v^2/c^2}$ may most likely be correct.The new force laws that are now also applicable for relativistic speeds are:

Coulomb's Law:

$$\mathbf{F} = \sqrt{1 - v^2/c^2}(1 + v^2/c^2)(\frac{1}{4\pi\epsilon_0})\frac{q_1 q_2 \hat{\mathbf{r}}}{r^2}$$

Lorentz Force Law:

$$\mathbf{F} = q(\sqrt{1 - v^2/c^2}(1 + v^2/c^2)\mathbf{E} + \sqrt{1 - v^4/c^4}\mathbf{v} \times \mathbf{B})$$

With the new Lorentz force law, there would not be any contradiction of the Bucherer experiment with the underlying physics on which the experiment is based on. The electric charge would be an invariant. Mass would be speed invariant, consistent with its definition in Newton's *Principia*. The relativistic Lorentz force law would explain why protons within particle accelerators cannot exceed the speed of

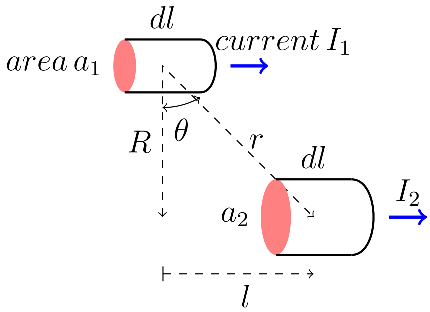

FIGURE 1. Elements $q1, q2$ in two horizontal parallel conductors one above the other,distance R apart, angular offset of angle θ with corresponding uniform charge densities, electron drift velocities ρ_1, v_1, ρ_2, v_2.

light. This is due to the electric force having a necessary factor of $\psi(v)$ which approaches zero as velocity of the protons near that of light speed.

Contemporary physics has incorporated special relativity into electromagnetism. It shows how $\mathbf{F} = q\mathbf{E}$ to be correct for any electrostatic field and for any charge, whether at rest or moving at any speed, including relativistic speed. This derivation relies on the transformation of relativistic force between inertial reference frames based on the Lorentz transformation.[8, 5.8] Our analysis earlier has shown special relativity to be invalidated; the relativistic force based on $\mathbf{F} = \dfrac{d}{dt}(\gamma m\mathbf{v})$ is fictitious. Therefore, $\mathbf{F} = q\mathbf{E}$ at best may only be an approximation for a charge moving at small speed as with the speed of electrons in currents in conductors.

5.1. **Force Between Parallel Current-carrying Conductors.** From classical electromagnetism, two long parallel current-carrying conductors will have forces acting between them; the formula for the force acting on a small element of a conductor of length l by the other long conductor is:

$$F = \frac{\mu_0}{2\pi R}(I_1 I_2 l) \tag{17}$$

It will be shown here that the exact same equation (17) may be derived based only on the relativistic Coulomb forces between the $+q$ and $-q$ charges within the two conductors. This is in contrast to current electromagnetism which derives the equation through the mediation of the Lorentz magnetic force $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$ where $\mathbf{B}$ is based on the Biot-Savart law for the magnetic field.

The relativistic Coulomb's law has a factor $\psi(v) = \sqrt{1 - v^2/c^2}(1 + v^2/c^2)$ and is valid for any speed $v <= c$. The drift speed of current electrons in conductors is in the order of $10^{-5} m/s$, very much smaller than c. For currents in conductors, $\psi(v)$ may be approximated by series expansion: $\sqrt{1-v^2/c^2} = 1 - \dfrac{v^2}{2c^2} + ...$ giving $\psi(v) = 1 + \dfrac{v^2}{2c^2} + ...$ The Coulomb's law would then be:

$$\mathbf{F} = k(1 + \frac{v^2}{2c^2})\frac{q_1 q_2 \hat{\mathbf{r}}}{r^2}$$

Figure (1) shows two typical elements of the same length dl. The aim is to find the force that acts on the single top element q_1 due the the long conductor 2. Element 1 will have a $+q_1$ charge due to the fixed proton lattice ions; it will be balanced by an equal amount $-q_1$ of drift electrons. It is similar for the element 2. If the classical Coulomb's law is used, the forces between the charges between the two elements will exactly balance - the Coulomb forces will not give rise to any net force between the parallel conductors. As we are now using a relativistic Coulomb's law with a scalar factor dependent on the relative speed of the interacting charges - electrons have a drift speed - the Coulomb forces between the charges will not exactly balance as before; it will give rise to a net force between the current elements, either attractive or repulsive.

The method is straightforward. The force between the charges q_1 and q_2 will be along the direction of θ. The total force on element q_1 is found by integrating the forces for the conductor 2 from $-\infty$ to $+\infty$. From symmetry, the force between the elements need only the transverse components as the longitudinal components will cancel out when integrated. The forces between elements q_1 and q_2 are:

(1) f^{++}; repulsion between $+q_1$ and $+q_2$. As the lattice ions are stationary, there is no speed dependency; $f^{++} = \dfrac{kq_1q_2}{r^2}$; $k = \dfrac{1}{4\pi\epsilon_0}$.

(2) f^{--}; repulsion between $-q_1$ and $-q_2$; here, there is no loss of generality by assuming $v_2 >= v_1$. $f^{--} = \dfrac{kq_1q_2}{r^2}(1 + \dfrac{(v_3)^2}{2c^2})$; $v_3 = v_2 - v_1$.

(3) f^{+-}; attraction between $+q_1$ and $-q_2$ and between $-q_1$ and $+q_2$. $f^{+-} = \dfrac{kq_1q_2}{r^2}(2 + \dfrac{(v_2)^2}{2c^2} + \dfrac{(v_1)^2}{2c^2})$

The net attractive force f_{12} is $f^{+-} - f^{++} - f^{--}$:

$$f_{12} = \frac{kq_1q_2}{r^2}(\frac{v_2^2}{2c^2} + \frac{v_1^2}{2c^2} - \frac{(v_2 - v_1)^2}{2c^2}) \tag{18}$$

$$f_{12} = \frac{kq_1q_2}{r^2}(\frac{v_1v_2}{c^2})$$

$$f_{12} = \frac{kq_1v_1}{c^2}(\frac{q_2v_2}{r^2}) = A\frac{q_2v_2}{r^2}; A = \frac{kq_1v_1}{c^2} \tag{19}$$

The following relations apply:

$$q = \rho a dl; \quad I = av\rho \tag{20}$$

Substituting (20) into (19), we have:

$$f_{12} = A\rho_2 a_2 v_2 \frac{dl}{r^2}$$

As $\frac{1}{r} = \frac{cos\theta}{R}$,we have:

$$f_{12} = A\rho_2 a_2 v_2 \frac{dlcos^2\theta}{R^2} \tag{21}$$

Refering to Figure (1), we see that $l = Rtan\theta$ giving $dl = Rsec^2\theta d\theta = \frac{Rd\theta}{cos^2\theta}$. For integration over the whole length of conductor 2, we need only the transverse component of f_{12}, i.e multiplying (21) by $cos\theta$. Substituting for dl:

$$f_{12-transverse} = A\rho_2 a_2 v_2 \frac{cos\theta d\theta}{R}$$

The resultant force on a typical element dl of conductor 1 due to the long conductor 2 is:

$$F_{dl} = \frac{A\rho_2 a_2 v_2}{R} \int_{-\pi/2}^{\pi/2} cos\theta d\theta = \frac{2A\rho_2 a_2 v_2}{R} = \frac{q_1 v_1}{2\pi R\epsilon_0 c^2}(\rho_2 a_2 v_2)$$

As $q_1 v_1 = I_1 dl, \quad \mu_0 = \frac{1}{\epsilon_0 c^2}, \quad I_2 = \rho_2 a_2 v_2,$ we finally have:

$$F_{dl} = \frac{\mu_0}{2\pi R} I_1 I_2 dl \tag{22}$$

The formula is valid for any element of length l of the conductor where l is much smaller than the lengths of the parallel conductors. For parallel currents, the force would be attractive. The formula is also valid for anti-parallel currents where the force would be repulsive. This cable form f (v) is to take. It is shown in the section below that if f (v) = 1 + v 2 /c 2 , the exact well-known force equation (17) between two long parallel current-carryingble form f (v) is to take. It is shown in the section below that if f (v) = 1 + v 2 /c 2 , the exact well-known force equation (17) between two long parallel current-carryingn be seen in equation (18). The term $(v2 - v1)^2$ becomes $(v2 + v1)^2$ and would cause a change in sign in f_{12}. This sign change would cause a sign change in the final equation (22).

A curious observation may be made of equation (22). It is used as the basis to define the SI unit of Ampere for electric currents. This fact may be taken to mean that the equation has been rigorously verified experimentally by all the standard's laboratories around the world to be reliable and consistent; any inconsistency of the equation, if any, should by now be discovered. So far, no inconsistency of the equation has ever been observed. As the equation (22) is also derived based on the new relativistic Lorentz force law and the Coulomb's law, the implication here is that it is a verification of the new force laws for speed much smaller than the light speed.

> *The fact that the formula for the forces between long parallel conductors has been rigorously tested is a verification of the relativistic Lorentz force law and the Coulomb's law for speed much smaller than that of the light speed.*

What has just been demonstrated shows that the incorporation of the motion of electric charges itself into a relativistic Coulomb's law may enable a formulation of a Newtonian electric theory without the need of the concept of the magnetic field, thus making the Biot-Savart law redundant.

6. Conclusions

The Bucherer type experiments of the early 20th Century have always been claimed to be experimental proof of relativistic mass and the repudiation of the invariant mass of Newtonian mechanics. Our argument has shown this to be not the case. Firstly, the concept of mass is never experimentally testable; only the predictions of a theory is testable. Secondly, the result of the Bucherer experiment was actually experimental proof that electromagnetism and the current form of the Lorentz force law fail under relativistic speed conditions. A new relativistic Coulombs's Law and Lorentz force law are here proposed that would be consistent with the Bucherer experiment. It is found that the force between parallel current-carrying conductors may be fully explained through just the Coulomb forces based on a revised Coulomb's law. This shows that a potential exists for a revolutionary formulation of an Newtonian electric theory without magnetism, without the need of the concept of a magnetic field thus making the Biot-Savart law redundant.Also,the Bucherer experiment could have been an experimental verification of the relativistic Lorentz force law if the predicted speeds of the electrons had been verified through direct time-of-flight measurements.

ABSTRACT. 1908年的 Bucherer 实验并不是相对论质量随速度变化的实验证明,而是证明电磁学和洛伦兹力律在相对论速度条件下失败。我们的结论来自对牛顿第二定律三种不同解释的实验的新分析，并分析了每种情况的含义:(1)力与动量变化率成正比 $f \propto dp/dt$; (2)力是相对论质量和加速度的乘积: $f = relativistic\_mass \times acceleration$; (3)经典: $f = ma$。 现在的新的解释显示了所有相对论速度的恒定电荷 - 质量比; 电荷和质量都将是速度不变的。 必须提出新的相对论力量法律与新的实验结果一致; 现在的洛伦兹定律是：$\mathbf{F} = q(\sqrt{1-v^2/c^2}(1+v^2/c^2)\mathbf{E} + \sqrt{1-v^4/c^4}\mathbf{v}\times\mathbf{B})$; 库仑定律是：$\mathbf{F} = \sqrt{1-v^2/c^2}(1+v^2/c^2)(\frac{1}{4\pi\epsilon_0})\frac{q_1q_2\hat{\mathbf{r}}}{r^2}$。 库仑定律具有依赖于电荷之间的相对速度的附加标量因子。 对于小速度，形式是： :$\mathbf{F} = (1+\frac{1}{2}\frac{v^2}{c^2})(\frac{1}{4\pi\epsilon_0})\frac{q_1q_2\hat{\mathbf{r}}}{r^2}$。这使得能够导出平行载流导体之间的力的公式:$F_{dl} = \frac{\mu_0}{2\pi R}I_1I_2dl$，而不需要磁场的概念。一种革命性的牛顿电学理论的新形式，而不需要磁性和 Biot-Savart 定律，存在一个真正的可能性。此外,如果通过直接飞行时间测量验证了电子的预测速度,Bucherer 实验可能是新相对论洛伦兹力定律的实验验证。

1. 介绍

[版本4.2.1] 现在常见的是，在讨论狭义相对论时，伴随着它是迄今为止是经过最严格测试的物理理论之一。考夫曼(Kaufmann 1901），Bucherer 1908，诺伊曼(Neumann) 和罗杰斯等人(Rogers et.al 1940)的实验被认为是相对论质量随速度而变化的确定性实证检验，间接推翻牛顿力学,验证狭义相对论。 虽然相对论质量与Bucherer实验是有一个很好的一致性,实验并不是一个特别的相对论证据。相反，这是洛仑兹力法在相对论速度下失败的绝对实验证明。

2. 1908 BUCHERER 实验

本文的目的不是对Bucherer实验的细节进行评论，而只是理论的依据。 教授
A.K.T.Assis [2] 在他的一篇论文中简要介绍了Bucherer实验，并对实验背后的理论进行了清晰的描述。 我们在这里重现。

Bucherer装置可以被认为是具有线性尺寸 L 的电容器，远大于两个相对充电板的分离，其表面电荷分布为 $\pm\sigma$。 x-axis 垂直于从 $-\sigma$ 到 $+\sigma$ 的盘。经典电动力学表明电容器板之间有一个均匀的电场，它们是 $\mathbf{E}_x = -(\sigma/\epsilon_0)\hat{\mathbf{x}}$。 轴原点是板之间电容器中心处的镭 β-粒子(电子）源。 y-axis 是电子在穿过距离 L 离开电容器之后离开电容器的路径，以 $\mathbf{v}_y$ 的速度离开电容器。 z-axis 方向上的均匀磁场 $\mathbf{B}_z$ 叠加在电容上。当 x 方向的电偏转和磁偏转平衡时，只有 y 方向的那些电子可能会留下电容。此外，初始电子必须在 x 方向没有速度分量，否则它们将与电容器板碰撞。对于离开电容器沿着 y-axis 移动的电子，作用在电子上的唯一力是来自仅在 x 方向上作用的电和磁

偏转;速度 $\mathbf{v}_y$ 是自然电子喷射速度。作用于电子的力是:

$$\mathbf{F_x} = -e(\mathbf{E}_x + \mathbf{v_y} \times \mathbf{B_z}) \tag{1}$$

将力等于零，得:

$$v_y = \sigma/\epsilon_0 B_z \tag{2}$$

Bucherer 装置也是速度选择器，用于改变电容器两端的电压的大小，并且磁场将允许变化速度的电子离开电容器。进行了5次实验，数据点的速度从 $0.3c$ 到 $0.7c$。 在电子离开电容器之后，它将仅在磁场的偏转之下，并且将以（2）中的恒定速度在圆形路径中行进，直到在某个已知距离处撞击照相板。 从电子在照相板上形成的点的坐标和其他尺寸，可以计算圆形路径的半径 r。 应用洛伦兹磁力作为圆周运动的向心力，得:

$$|e(\mathbf{v} \times \mathbf{B})| = ma = mv^2/r \tag{3}$$

a 是向心加速度，v 是等于（2) 中速度的恒定速度。 组合方程（(2) 和 (3)给出:

$$e/m = \sigma/r\epsilon_0 B^2 \tag{4}$$

方程 (4） 的右手边可以被评估为术语是实验的已知物理常数或测量变量。 发现数据点的 e/m 的比率随速度而变化，随着速度的增加而减小。 随着电子电荷被认为是恒定的，电子的电荷-质量比变化被解释为质量随着速度而增加。 发现 e/m 的比率与 $e/m_0/\sqrt{1-v^2/c^2}$ 有很强的相关性。 这表明实验与洛伦兹爱因斯坦(Lorentz-Einstein)模型一致，其中电磁质量为:

$$m_r = \frac{m_0}{\sqrt{1-v^2/c^2}} \tag{5}$$

m_0 被电子的静止质量。 Robert Resnik教授的教科书 [1] 给出了实验数据表.Bucherer 实验被认为是物质的惯性质量具有电磁起源，并且随速度变化而不是不变的证据。

3. BUCHERER实验的解释

实验结果确实有深远的影响。 初步的表面,它否定了不变质量,并验证了狭义相对论的相对论质量。它既不是。 当时的物理学家有电子模型预测电子的质量随着速度而增加。 无论什么原因，他们不愿意放弃模型，并考虑对实验的替代解释。 有些人很快接受了 Bucherer 实验，作为对不变质量的决定性否定。对于诸如 Bucherer 的实验基础的方程式 (3), 没有引起任何怀疑。 电子以圆形路径偏转，并且 $e(\mathbf{v} \times \mathbf{B})$ 的洛伦兹磁力是作用于电子的唯一力。 正是牛顿第二定律的应用引起了这个方程。

我们将以过去没有做过的方式重新审视 Bucherer 实验。这种分析的新颖性是尝试对牛顿第二定律的三种不同解释，以便在实验的方程 (3) 中使用，并分析其中每个的含义。

(1) $Force \propto \frac{dp}{dt}$。 这是试图回到原来 “Newton‘s Principia” 的声明。动量将是相对论定义:

$$Force \propto \frac{d}{dt}\left(\frac{mv}{\sqrt{1-v^2/c^2}}\right) \tag{6}$$

牛顿第二定律在相对论力学中的解释失败了-它导致了力的虚构。比例关系只有当关系的双方都有定义值时才有意义。右侧被定义，量纲为 $[M][L][T^{-2}]$。 左侧的力尚未定义; 因此左侧未定义。在牛顿力学中习惯的力的定义不能在这里假设，因为牛顿第二定律的这种解释有效地定义了一种新的力学。如果这种新的机制是有效的，狭义相对论的这种力的定义必须先出现在相对论力学中。由于没有相对论力的确定单位，这种相对论力学只能是虚构的。虽然REF中所示的相对论质量满足了Bucherer实验中所要求的质量，但是在这种情况下定义的力就会失效。此案被驳回。

(2) $Force = relativistic\_mass \times acceleration$: 相对论质量可以定义为 $\phi(v)m$，其中 $\phi(v)$ 是取决于速度 v 的标量函数，m 是不变质量。 这种解释是以力的定义的形式，因为这里的关系是个相等值,而不是第一种情况下的比例。由于右手边的量纲为 $[M][L][T^{-2}]$，与经典的牛顿力学相同的力量维度，我们首先假设这里的力被定义，并且具有与牛顿力学相同的真实单位。力的这种定义将与实际的 Bucherer 实验一致，因为它将适应随速度变化的质量，如果在结果中被发现是这样。如果我们将 $\phi(v)$ 作为 $\frac{1}{\sqrt{1-v^2/c^2}}$，质量将是狭义相对论的质量。正如我们所看到的，这样的质量与 Bucherer 实验的结果一致。这里的情况似乎是给出了一个相对论力学的表达式，它具有真实的力量单位，并导致与 Bucherer 实验一致的有效力学。

我们现在在工作能量定理中使用力的定义来得到动能公式。

$$K = W = \int_0^v \left(\frac{m}{\sqrt{1-v^2/c^2}}\right)\frac{dv}{dt}dx = \int_0^v \frac{mv}{\sqrt{1-v^2/c^2}}dv$$

$$K = mc^2(1-\sqrt{1-v^2/c^2}) \tag{7}$$

公式（7）与狭义相对论的动能公式不一样，即:

$$K = mc^2\left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right)$$

如果这种情况下的力量定义形成了一种有效的相对论力学，那就不是狭义相对论的相对论力学。 事实上，在这种情况下，力的定义也是无效的，因为取决于速度的变量质量不能用于定义一致的标准力单位。 此案也被驳回。

(3) $Force = mass \times acceleration$, 质量是不变的。这是古典牛顿力学中力的定义。它将牛顿的第二定律解释为定义力量的真理公理。这个解释一直是牛顿以来唯一的公理解释。在这里，质量是牛顿运动定律的一个不变量。该力定义用于 Bucherer 实验的圆形运动力方程 (3)，它使显示质量不变，但随着速度的增加。这代表了这实验的物理学理论与实验结果之间存在矛盾。如果实验所基于的物理学是正确的，那么这种矛盾就不应该发生。 Bucherer 实验背后的力法基于不变质量，但结果表明随着速度增加的质量。这个矛盾表明，实验所依据的物理学并不完全正确 - 它包括不正确的物理理论。

Bucherer 实验的理论是基于牛顿力法和洛伦兹力法在内的电磁学。其中一个是无效的，导致矛盾。牛顿时代以来，经典的牛顿定律是一个经过最严格检验的物理学规律。经过三个世纪的严格测试，没有一次失败;问题不应该归结于牛顿定律。结论应该是电磁学和洛伦兹力定律包含一些根本误差。正如其他人指出的那样 [7]，洛伦兹法律迄今尚未以相对论速度直接测试; 这是矛盾的原因是非常有可能的。假设只有洛伦兹的力法是无效的而其余的电磁学是清洁的，正确的是不合逻辑的。 洛伦兹力法涉及电场 E 和磁场 B，因此它的失败很可能源于电磁本身的制定。

上述前两种情况必须予以忽视。 只有第三种情况需要考虑，结论只能是洛仑兹力法和电磁理论的失败。

Bucherer实验证明了电磁学和洛伦兹力律在相对论速度条件下失败.

4. 质量定义不可验

在 "*Introduction to Special Relativity*"[1]中，着名作家 Robert Resnick 显示 Bucherer 实验却实证明即不变质量的想法与实验互相矛盾；质量被证明是一个变量，甚至适合狭义相对论的相对论质量的 γ-因子。牛顿力学中质量的不变性是一种定义 - 定义为 "Newton's Principia" 中绝对的 “物质数量” 。狭义相对论的质量也是建立在这个相同的不变质量上，但是随着 m_0 加上 γ-因子, $\gamma = \dfrac{1}{\sqrt{1-v^2/c^2}}$:

$$m_r = \frac{m_0}{\sqrt{1-v^2/c^2}} \tag{8}$$

公式 (8) 只是质量的新定义（间接通过相对论动量），产生了狭义相对论力学的新形式。

科学范式的实验仅用于验证或测试理论的预测，而不是其定义的任何概念。 例如，牛顿力学中质量的不变性是不可测试的，但行星绕着椭圆轨道运行的太阳的预测是可以验证的。 所以Robert Resnick教授的说法在逻辑上是站不住脚的。 牛顿质量的不变量和狭义相对论的相对论质量都不可测。

5. 新库仑法和洛伦兹力法

我们将在这里研究新的洛伦兹力法应该采取什么形式，以便 Bucherer 实验不会导致可变电荷质量比的矛盾。 我们检查表格为：

$$\mathbf{F} = q(\phi_e \mathbf{E} + \phi_b \mathbf{v} \times \mathbf{B})$$

其中 ϕ_e, ϕ_b 是速度的两个标量函数，其形式将被确定。按照前面所做的相同的推导，在 Bucherer 实验中的均匀磁场中电子的圆周运动的恒定速度将是：

$$v = \phi_e \sigma / \phi_b \epsilon_0 B \tag{9}$$

对于圆周运动方程，我们现在有：

$$|e\phi_b(\mathbf{v} \times \mathbf{B})| = ma = mv^2/r \tag{10}$$

结合方程式 (9) 和 (10), 我们有一个新的电荷质量比方程：

$$e/m = (\phi_e/\phi_b^2)\sigma/\epsilon_0 r B^2 \tag{11}$$

我们乘以 (11) 右侧的分子和分母与 ϕ, $\phi = \sqrt{1 - v^2/c^2}$, 得：

$$e/m = (\phi_e \phi/\phi_b^2)(\sigma/\phi \epsilon_0 r B^2) \tag{12}$$

如上述第2节所示，$\sigma/\phi\epsilon_0 r B^2$ 与原始实验的数据集与 e/m_0 有很强的相关性。 因此，为了使新的洛伦兹力法导致与 e/m_0 强相关的非矛盾的常数 e/m 比，要满足的条件是：$\phi_e \phi/\phi_b^2 = 1$ 或者,

$$\phi_e \phi = \phi_b^2 \tag{13}$$

洛伦兹新法律形式的必要条件是：

$$\phi_e = \phi^{2n-1} f(v)^2; \quad \phi_b = \phi^n f(v) \tag{14}$$

对于任何整数 n，$f(v)$ 是速度的任意标量函数。

还有一个条件必须满足。 如果新形式具有术语 $\phi = \sqrt{1 - v^2/c^2}$,则 $v <= c$ 否则将存在来自负值的平方根的不确定性。 $\mathbf{v} \times \mathbf{B}$ 的磁力仅改变移动电荷的速度方向，而不是其速度; 只有电场才能改变速度。为了使电荷粒子的速度不会持续增加而不受限制并超过光速 c，则电荷的电力在v接近 c 时必须接近零。 这是对洛伦兹新法律形式的进一步限制。

条件ref有三种一般情况：

(1) 情况 $f = \phi$: $\phi_e = \phi^{2n+1}$; $\quad \phi_b = \phi^{n+1}$ 其中 $n >= 0$。
(2) 情况 $f = 1/\phi$: 这种情况等同于情况（1）。
(3) 情况 $f \neq \phi$: $\phi_e = \phi^{2n-1} f^2$; $\quad \phi_b = \phi^n f$ 其中 $n >= 1$。

这表明洛伦兹力量可以采取无限的形式，这将使得 Bucherer 实验不会导致任何矛盾。 确定实际形式的唯一方法是通过对喷射的电子速度的直接飞行时间测量来对 (9) 中的速度进行实验验证。 此外，对实证观察的考察可能有助于推断实际形式应该采取什么。 如果通过直接飞行时间测量验证了Bucherer实验的电子速度，那么 Bucherer 实验就是验证洛伦兹力定律的相对论形式。

> 如果通过直接飞行时间测量验证了电子的预测速度，
> 则 **Bucherer** 实验便是验证如果洛伦兹力法在相对论
> 速度下有效。

至今还没有直接的飞行时间测量。

我们在这里只考虑上面的情况 (3), $n = 1$ 其中新洛伦兹力法将具有一般形式:

$$\mathbf{F} = q(\phi(v)f(v)^2\mathbf{E} + \phi(v)f(v)\mathbf{v} \times \mathbf{B}) \tag{15}$$

一个新的洛伦兹力法将需要对库仑定律进行相应的修订以增加额外的因素 $\psi(v) = \phi(v)f(v)^2$ 其是取决于 v 的标量函数; v 现在是两个相互作用的电荷之间的相对速度。 新的相对论库仑定律是:

$$\mathbf{F} = \frac{\psi(v)}{4\pi\epsilon_0}\frac{q_1q_2\hat{\mathbf{r}}}{r^2}; \tag{16}$$

$\psi(v) = \sqrt{1 - v^2/c^2}f(v)^2$ 和 $f(v)$ 是 v 的标量函数，其实际形式需要确定。 使用这种新的库仑定律，由于固定电荷配置的电场而导致的移动测试电荷的力将是： $\mathbf{F} = q\psi(v)\mathbf{E}$。 这与通过 Bucherer 实验验证的相对论洛伦兹力定律一致。 有一个经验现象，我们可以依靠它来确定 $f(v)$ 的可能形式是什么。 在下面的部分中显示，如果 $f(v) = \sqrt{1 + v^2/c^2}$, 两个长并联载流导体之间的精确的着名力方程 (17) 可以仅使用没有任何磁学的库仑力来推导出来。这个平行力方程已被很好地测试的事实可以被认为是指 $f(v) = \sqrt{1 + v^2/c^2}$ 形式很可能是正确的。现在适用于相对论速度的新的力量法则是:

库仑定律:

$$\mathbf{F} = \sqrt{1 - v^2/c^2}(1 + v^2/c^2)(\frac{1}{4\pi\epsilon_0})\frac{q_1q_2\hat{\mathbf{r}}}{r^2}$$

洛伦兹力法:

$$\mathbf{F} = q(\sqrt{1 - v^2/c^2}(1 + v^2/c^2)\mathbf{E} + \sqrt{1 - v^4/c^4}\mathbf{v} \times \mathbf{B})$$

利用新的洛伦兹力量法，Bucherer 实验与实验所依据的物理学不存在任何矛盾。 电荷是不变的。 质量将是速度不变的，与其在 “Newton’s Principia” 中的定义一致。 相对论的洛伦兹力法则将解释为什么粒子加速器内的质子不能超过光速。 这是由于电力具有必要的 $\psi(v)$ 因子，随质子接近光速的速度而接近零。

当代物理学已将狭义相对论纳入电磁学。 它显示 $\mathbf{F} = q\mathbf{E}$ 可以应用于任何静电场和任何电荷，无论是静止还是移动，包括相对论速度。 这种推导依赖于基于洛伦兹变换引用电荷的惯性参考系之间相对论力的转换 [8]。 我们之前的分析表明狭义相对论无效; 基于 $\mathbf{F} = \frac{d}{dt}(\gamma m\mathbf{v})$ 的相对论力量是虚构的。 因此，$\mathbf{F} = q\mathbf{E}$ 最好只能是与电流在导体中的电子速度一样以小速度移动的电荷的近似值。

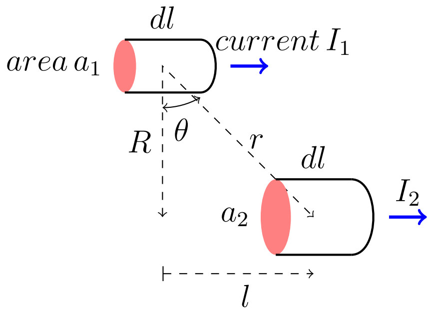

FIGURE 1. 两个水平并联导体中的两个元件 $q1, q2$，距离 R 分开，角度 θ 的角度偏移; 相应的均匀电荷密度，电子漂移速度为 ρ_1, v_1, ρ_2, v_2。

5.1. 并联载流导体之间的力. 从传统的电磁学角度来看，两条长的平行载流导体将具有作用于它们之间的力; 通过另一个长导体作用在长度为l的导体的小元件上的力的公式为:

$$F = \frac{\mu_0}{2\pi R}(I_1 I_2 l) \tag{17}$$

这里将显示，也能基于两个导体内的 $+q$ 和 $-q$ 电荷之间的新库仑力来推导出完全相同的方程 (17)。这与通过洛伦兹磁力 $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$ 的调解得出方程的电流相反，其中 $\mathbf{B}$ 基于磁场的 Biot-Savart 定律。

新库仑定律具有因子 $\psi(v) = \sqrt{1 - v^2/c^2}(1+v^2/c^2)$ 并且对于任何速度 $v <= c$ 有效。 导体中当前电子的漂移速度约为 $10^{-5} m/s$，远小于 c。 对于导体中的电流，$\psi(v)$ 可以通过串联扩展近似: $\sqrt{1 - v^2/c^2} = 1 - \frac{v^2}{2c^2} + ...$ 给 $\psi(v) = 1 + \frac{v^2}{2c^2} + ...$。 那么库仑定律就是:

$$\mathbf{F} = k(1 + \frac{v^2}{2c^2})\frac{q_1 q_2 \hat{\mathbf{r}}}{r^2}$$

图 (1)显示了相同长度 dl 的两个典型元素。 目的是找到由于在下面的长导体而作用在单个顶部元件 q_1 上的力。由于固定的质子晶格离子，元件1将具有 $+q_1$ 电荷; 它将被等量的 $-q_1$ 漂移电子的电荷均衡。 元素 2 类似。如果使用古典库仑定律，两个元素之间的电荷之间的力将完全平衡 - 库仑力不会在平行导体之间产生任何净力。因为我们现在正在使用新的库仑定律，所以标量因素取决于交互电荷的相对速度。 由于电子具有漂移速度，所以电荷之间的库仑力不会像以前的平衡。 它将导致现有元素之间的净力，吸引力或排斥。

该方法很简单。 电荷 q_1 与 q_2 之间的力将沿着 θ 的方向。 通过将导体2的力从 $-\infty$ 到 $+\infty$ 的积分来找到元件 q_1 上的总力。 从对称性来看，元件之间的力仅需要横向分量，因为当积分时纵向分量将抵消。 元素 q_1 与 q_2 之间的力是:

(1) f^{++}; $+q_1$ 与 $+q_2$ 之间的作用排斥。 当晶格离子静止时，没有速度依赖性; $f^{++} = \frac{kq_1q_2}{r^2}$; $\quad k = \frac{1}{4\pi\epsilon_0}$.

(2) f^{--}; $-q_1$ 与 $-q_2$ 之间的作用排斥。 在这里,假设 $v_2 >= v_1$ 不会失去一般性。 $f^{--} = \frac{kq_1q_2}{r^2}(1 + \frac{(v_3)^2}{2c^2})$; $v_3 = v_2 - v_1$.

(3) f^{+-}; $+q_1$ 与 $-q_2$ 和 $-q_1$ 与 $+q_2$ 之间的作用吸引。 $f^{+-} = \frac{kq_1q_2}{r^2}(2 + \frac{(v_2)^2}{2c^2} + \frac{(v_1)^2}{2c^2})$

净吸引力为: f_{12} is $f^{+-} - f^{++} - f^{--}$:

$$f_{12} = \frac{kq_1q_2}{r^2}(\frac{v_2^2}{2c^2} + \frac{v_1^2}{2c^2} - \frac{(v_2 - v_1)^2}{2c^2}) \tag{18}$$

$$f_{12} = \frac{kq_1q_2}{r^2}(\frac{v_1v_2}{c^2})$$

$$f_{12} = \frac{kq_1v_1}{c^2}(\frac{q_2v_2}{r^2}) = A\frac{q_2v_2}{r^2}; A = \frac{kq_1v_1}{c^2} \tag{19}$$

以下关系适用:

$$q = \rho adl; \quad I = av\rho \tag{20}$$

将 (20) 代入 (19),得:

$$f_{12} = A\rho_2a_2v_2\frac{dl}{r^2}$$

从 $\frac{1}{r} = \frac{cos\theta}{R}$, 得:

$$f_{12} = A\rho_2a_2v_2\frac{dlcos^2\theta}{R^2} \tag{21}$$

从参考图 (1),可看 $l = Rtan\theta$ 导致 $dl = Rsec^2\theta d\theta = \frac{Rd\theta}{cos^2\theta}$。 为了在导体 2 的整个长度上进行整合,我们只需要 f_{12} 的横向分量,即将 (21) 乘以 $cos\theta$。 代替 dl:

$$f_{12-transverse} = A\rho_2a_2v_2\frac{cos\theta d\theta}{R}$$

由于长导体2导致的导体1的典型元件dl上的合力为:

$$F_{dl} = \frac{A\rho_2a_2v_2}{R}\int_{-\pi/2}^{\pi/2} cos\theta d\theta = \frac{2A\rho_2a_2v_2}{R} = \frac{q_1v_1}{2\pi R\epsilon_0c^2}(\rho_2a_2v_2)$$

从 $q_1v_1 = I_1dl$, $\mu_0 = \frac{1}{\epsilon_0c^2}$, $I_2 = \rho_2a_2v_2$, 得:

$$F_{dl} = \frac{\mu_0}{2\pi R}I_1I_2dl \tag{22}$$

该公式对于导体的长度 l 的任何元件都是有效的,其中 l 远小于平行导体的长度。 对于并联电流,力将是有吸引力的。 该公式对于反并联电流也是有效的,其中力将是排斥的。 这可以在等式 (18) 中看出。 术语 $(v2 - v1)^2$ 变为 $(v2 + v1)^2$,并将导致 f_{12} 中的符号更改。 这个符号变反将导致最终方程式 (22) 中的符号变反。

可以对方程式REF进行好奇的观察。 它用作定义电流的安培的SI单位的基础。 这个事实可能被认为是由世界各地所有标准实验室通过实验得到严格验证的方程式是可靠和一致的; 如果方程式有不一致的话, 现在应该以被发现。 到目前为止，还没有观察到方程的不一致。由于方程式REF也是基于新的洛仑兹力定律和库仑定律得出的，所以这里的意义在于，它是对小速度的新定律的验证。

> 由于两条长电流平行导体之间的力的公式已经被严格测试，所以它成为对于速度远小于光速的新的洛伦兹力定律和库仑定律的验证。

刚刚证明的结果表明，将电荷本身的运动纳入到新的论库仑定律中可以使经典电学理论的形成不需要磁场的概念，从而使得 Biot-Savart 定律变得多余。

6. 结论

二十世纪初的 Bucherer 型实验一直被认为是相对论质量的实验证明和牛顿力学不变质量的否定。我们的论据显示不是这样。 首先，质量的概念是不可测试的；只有理论的预测是可以测试的。其，Bucherer 实验的结果实际上是实验证明，电磁学和洛伦兹力律的当前形式在相对论速度条件下失败。这里提出了一种新的库仑法和洛伦兹强制法，这与 Bucherer 实验一致。发现平行载流导体之间的力可以通过库仑力基于修正的库仑定律来充分解释。这表明，牛顿电学理论的革命性制定的潜力存在，而不需要磁场的概念，从而使得 Biot-Savart 定律成为冗余。此外，Bucherer 实验可能是对新的洛伦兹力的实验验证如果通过直接飞行时间测量验证了电子的预测速度，那么法则就是这样。

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