A generalization of the Levi-Civita connection

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Abstract

We define a generalization of the Levi-Civita which depends on an automorphisme of the tangent vector bundle.

1 The Levi-Civita connection

For a riemannian manifold (M, g), the Levi-Civita connection is defined by the two conditions:

$$dg = g(\nabla)$$

$$T(\nabla) = 0$$

The metric is invariant by the Levi-Civita connection and the torsion is zero.

2 The generalization of the Levi-Civita connection

Let be M a manifold with a bilinear form g (non symmetric) and an automorphism ϕ such that:

$$g(\phi(X), Y) = g(\phi(Y), X)$$

The connection ∇ of Levi Civita is defined by the two conditions:
1)

$$Z.g(\phi(X), Y) = g(\nabla_Z \phi(X), Y) + g(\nabla_Z \phi(Y), X)$$

2)

$$\nabla_X \phi(Y) - \nabla_Y \phi(X) - \phi([X, Y]) = 0$$

If $\phi = 1$, then we recover the Levi-Civita connection. 1) is a condition of invariance of the metric and 2) is a torsionless condition.

References

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