Relativistic Newtonian Gravitation That Gives the Correct Prediction of Mercury Precession and Needs Less Matter for Galaxy Rotation Observations

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Abstract

In the past, there was an attempt to modify Newton's gravitational theory, in a simple way, to consider relativistic effects. The approach was "abandoned" mainly because it predicted only half of Mercury's precession. Here we will revisit this method and see how a small logical extension can lead to a relativistic Newtonian theory that predicts the perihelion precession of Mercury correctly. In addition, the theory requires much less mass to explain galaxy rotation than standard theories do, and is also interesting for this reason.

Key words: Newton gravity, relativistic adjustment, precession of Mercury, orbital velocity galaxies.

1 Introduction

In 1981 and 1986, Bagge [1] and Phillips [2] each suggested an ad-hoc modification of Newton by simply replacing the smaller mass in the formula with a relativistic mass

$$F = G\frac{M\frac{m}{\sqrt{1-\frac{v^2}{c^2}}}}{r^2} \tag{1}$$

The velocity v is the relative velocity between the two gravitational objects: the velocity of Mercury relative to the Sun, for example. Phillips initially claimed that his derivation, based on this, led to a prediction of the perihelion precession of Mercury equal to that of Einstein's general relativity theory [3]. However, according to criticism from Ghosal in 1987, this approach leads to a perihelion precession of Mercury that is too low. The method has also been criticized by Chow [4] for the same reason. Peters [5] claims that Philipps made a mistake in his Mercury perihelion derivation and that, in reality, his prediction only gives half of the prediction as GR (the GR prediction has been observed). Philipps openly admitted this and discussed his mistakes in detail [6]. He was clear that his theory underestimated the perihelion precession of Mercury, but noted that further adjustments to the theory could potentially be done in the future. Biswas [7] published an interesting paper titled "Special Relativistic Newtonian Gravity" where he claimed

> *The resulting theory is significantly different from the general theory of relativity. However, all known experimental results (precession of planetary orbits, bending of the path of light near the Sun, and gravitational spectral shift) are still explained by this theory.*

However, Peters [8] then pointed out that Biswas had also made a mistake in his derivation, something Biswas agreed to in correspondence with Peters. Ghosal and Cakraborty [9] agree on the criticism of Biswas, but claim his idea was still interesting. Here we will follow up on this discussion and show that there is a simple and logical way to extend this approach in a fruitful manner.

2 Modified Relativistic Newtonian Gravity that Gives the Correct Prediction of the Precession of Mercury

In the relativistic extension of Newton given by [1, 2]

$$F = G\frac{M\frac{m}{\sqrt{1-\frac{v^2}{c^2}}}}{r^2} \tag{2}$$

The velocity v must be interpreted as the velocity between the large and small masses. This extension is, in our view, only valid when the gravity phenomenon is observed from the frame of the large gravitational object, such as predicting the orbital velocity of the moon relative to the Earth, for example. In this case under consideration, however, the small relativistic mass will fall out and we get the same predictions as in standard Newtonian gravity. When it comes to gravity phenomena between two masses as observed from a third frame, we claim it is logical to complete additional ad-hoc modifications to the formula above. When observing the Sun's gravity influence on Mercury, for example, we must also consider the Sun's velocity relative to us as we observe it from Earth. We suggest the following modification

$$F = G\frac{\frac{m}{\sqrt{1-\frac{v_m^2}{c^2}}}\frac{M}{\sqrt{1-\frac{v_M^2}{c^2}}}}{r^2\left(1-\frac{v_M^2}{c^2}\right)} \tag{3}$$

where v_m and v_M are the velocities of the large and small masses as observed from the observer frame, that is to say, in our case, from Earth. As can be seen in our formula, we are suggesting that r (center to center between the two gravitational masses) should be the length contracted depending on the velocity of the two objects relative to the observer; this is best approximated by the velocity of the large gravitational object relative to the observer frame. For example, assume a galaxy with distance r between the galactic center and one of the stars in the arm of the galaxy, as observed from the galactic center. We claim that this distance likely will appear to be contracted, as observed from Earth and as measured with Einstein-Poincaré synchronized clocks. Its contracted length will follow standard Lorentz length contraction, in our formulation, and will be $r\sqrt{1-\frac{v_M^2}{c^2}}$. That is to say, for fast-moving galaxies we have two effects that lead to stronger gravity than predicted by the Newtonian theory. The first effect is that the relativistic mass is relevant for gravity (and this mass is larger than the rest-mass), and the second effect is that the distance center to center between the gravity objects must appear to be contracted, as observed from the laboratory (typically the Earth).

In 1859, LeVerrier pointed out that the perihelion of Mercury evidently precesses at a slightly faster rate than predicted by Newtonian mechanics. The Lagrangian is given by

$$L = T - V \tag{4}$$

This gives

$$L = \frac{mc^2}{\sqrt{1-\frac{v_m^2}{c^2}}} + G\frac{M\frac{m}{\sqrt{1-\frac{v_m^2}{c^2}}}}{r\left(1-\frac{v_M^2}{c^2}\right)} \tag{5}$$

When $v_M << c$, we can use a Taylor series expansion and get

$$\begin{aligned} L &= \frac{mc^2}{\sqrt{1-\frac{v_m^2}{c^2}}} + G\frac{M\frac{m}{\sqrt{1-\frac{v_m^2}{c^2}}}}{r} + \frac{v_M^2}{c^2}G\frac{M\frac{m}{\sqrt{1-\frac{v_m^2}{c^2}}}}{r} \\ L &= \frac{mc^2}{\sqrt{1-\frac{v_m^2}{c^2}}} + \frac{GMm}{r\sqrt{1-\frac{v_m^2}{c^2}}} + \frac{v_M^2}{c^2}\frac{GMm}{r\sqrt{1-\frac{v_m^2}{c^2}}} \end{aligned} \tag{6}$$

And to simplify further, we can set $k = GMm$ and this gives

$$L = \frac{mc^2}{\sqrt{1-\frac{v_m^2}{c^2}}} + \frac{k}{r\sqrt{1-\frac{v_m^2}{c^2}}} + \frac{v_M^2}{c^2}\frac{k}{r\sqrt{1-\frac{v_m^2}{c^2}}} \tag{7}$$

Next assume that $v_m << c$ and $v_m \approx v_M$, we can then use a Taylor series expansion and we get

$$L = mc^2 + \frac{1}{2}mv_m^2 + \frac{k}{r} + \frac{3}{2}\frac{v_M^2}{c^2}\frac{k}{r} + O(c^{-4}) \tag{8}$$

given extensive calculations, this seems to lead to the same prediction as GR for Mercury precession, that is

$$\delta = \frac{6\pi m}{c^2 a(1-e^2)} \tag{9}$$

How realistic it is to set $v_m \approx v_M$ we are uncertain about, and also we have not gone though the calculations and see what predictions we get if v_m being significant different from v_M.

3 Orbital Velocity

An important aspect of our gravity theory is that it needs less mass than Newton or Einstein gravity to fit the actual observations. How much of the so-called "missing" ordinary matter and dark matter this theory can explain will require further evaluation; this is something we plan to do in the future, and we encourage others to do so as well. In particular, when v_M are significantly close to the speed of light, then Newton gravity (and GR) will need much more mass to explain the gravity phenomena than this theory does, simply because it indirectly uses rest-mass where we think relativistic mass is the relevant form. In addition, modern physics uses a non-contracted distance r between the gravitational objects, rather than the contracted distance suggested here.

For example, the orbital velocity in our theory should be (see appendix for derivation)

$$v_o \approx \sqrt{\frac{GM}{r}} \frac{1}{\sqrt{1-\frac{v_M^2}{c^2}}} \tag{10}$$

This means the orbital velocity of the moon relative to the Earth will be as before, $v_o = \sqrt{\frac{GM}{r}}$. This is because the velocity of the large mass M relative to the observer (from the large mass, the Earth) is zero in this case. However, for gravity phenomena not related to the Earth, we will typically have a v_M significantly different from zero. We can then solve this with respect to the large gravity mass M we are studying – a distant galaxy, for example, and we get

$$M \approx \frac{v_o^2 r \left(1-\frac{v_M^2}{c^2}\right)}{G} \tag{11}$$

This formula should be compared carefully to observations to see how much better this theory fits with them than the standard Newton theory is known to do. We do not claim that this is the solution to the galaxy rotation problem, but simply suggest that our theory should be investigated further, particularly in the light of the fact that all experiments conducted thus far have failed to detect dark matter from cosmic rays – despite ongoing experimental research for the last 30 years. This should give strong motivation for investigating alternative theories more carefully.

4 Summary

In the past, several ad-hoc modifications of Newton's gravity theory have been proposed and discussed. These approaches have been criticized for predicting only half of the perihelion of the precession of Mercury. Taking that work as a start, however, we have suggested some logical extensions to this theory. If we are looking at relativistic effects, they should be evaluated from the observer frame. In this case, when the gravity phenomenon is not observed from the large gravity mass itself, but rather from an outside frame such as the Earth, then we also must take into account the velocity of the Sun relative to the Earth. After completing an ad-hoc adjustment accordingly, we find the same prediction of precession of Mercury as general relativity theory predicts. In addition, this adjustment requires less mass to explain galaxy rotations. Although we have not tested these results further, we think this is interesting enough to require further investigation, and hope this paper will highlight the way for future research.

Appendix

The orbital velocity is found in the standard way by setting the gravitational force and the centripetal force equation equal to each other and solving with respect to $v$xt

$$F = ma = \frac{mv^2}{r^2} \tag{12}$$

We assume the small mass (the star orbiting the galaxy) is moving at a speed much slower than the galaxy is moving relative to us, that is $v << v_M$, then we have

$$
\begin{aligned}
G\frac{m\frac{M}{\sqrt{1-\frac{v_M^2}{c^2}}}}{r^2\left(1-\frac{v_M^2}{c^2}\right)} - \frac{mv^2}{r\sqrt{1-\frac{v_M^2}{c^2}}} &= 0 \\
G\frac{\frac{M}{\sqrt{1-\frac{v_M^2}{c^2}}}}{r^2\left(1-\frac{v_M^2}{c^2}\right)} - \frac{v^2}{r\sqrt{1-\frac{v_M^2}{c^2}}} &= 0 \\
v &= \sqrt{\frac{GM}{r}\frac{1}{\sqrt{1-\frac{v_M^2}{c^2}}}} \qquad (13)
\end{aligned}
$$

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