IV

A Novel Methodology Developing an Integrated ANP: A Neutrosophic Model for Supplier Selection

Abduallah Gamal[1*] ▪ Mahmoud Ismail[2] ▪ Florentin Smarandache[3]

[1]Department of Operations Research, Faculty of Computers and Informatics, Zagazig University,
[2]Department of Operations Research, Faculty of Computers and Informatics, Zagazig University,
Email: abduallahgamal@gmail.com
[3]Math&Science Department, University of New Mexico Gallup, NM87301, USA.
Email: smarand@unm.edu

Abstract

In this research, the main objectives are to study the Analytic Network Process (ANP) technique in neutrosophic environment, to develop a new method for formulating the problem of Multi-Criteria Decision-Making (MCDM) in network structure, and to present a way of checking and calculating consistency consensus degree of decision makers. We have used neutrosophic set theory in ANP to overcome the situation when the decision makers might have restricted knowledge or different opinions, and to specify deterministic valuation values to comparison judgments. We formulated each pairwise comparison judgment as a trapezoidal neutrosophic number. The decision makers specify the weight criteria in the problem and compare between each criteria the effect of each criteria against other criteria. In decision-making process, each decision maker should make $\frac{n \times (n-1)}{2}$ relations for n alternatives to obtain a consistent trapezoidal neutrosophic preference relation. In this research, decision makers use judgments to enhance the performance of ANP. We introduced a real life example: how to select personal cars according to opinions of decision makers. Through solution of a numerical example, we formulate an ANP problem in neutrosophic environment.

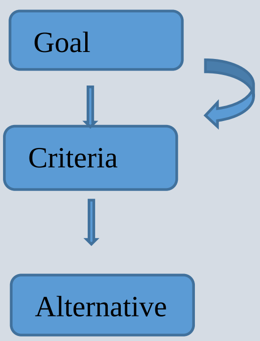
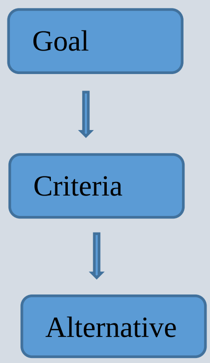
Keywords

Analytic Network Process, Neutrosophic Set, Multi-Criteria Decision Analysis (MCDM).

1 Introduction

The Analytic Network Process (ANP) is a new theory that extends the Analytic Hierarchy Process (AHP) to cases of dependency and feedback, and generalizes the supermatrix approach introduced by Saaty (1980) for the AHP [1]. This research focuses on ANP method, which is a generalization of AHP. Analytical Hierarchy Process (AHP) [2] is a multi-criteria decision making method where, given the criteria and alternative solutions of a specific model, a graph structure is created, and the decision maker is asked to pair-wisely compare the components, in order to determine their priorities. On the other hand, ANP supports feedback and interaction by having inner and outer dependencies among the models' components [2]. We deal with the problem, analyze it, and specify alternatives and the critical factors that change the decision. ANP is considered one of the most adequate technique for dealing with multi criteria decision-making using network hierarchy [19]. We present a comparison of ANP vs. AHP in *Table 1*: how each technique deals with a problem, the results of each technique, advantages and disadvantages.

Table1. Comparison of ANP vs. AHP.

Property	**ANP (Analytic Network Process)**	**AHP (Analytic Hierarch Process)**
Structure	 Network	 Hierarchy

Why are the results different?	The user learns through feedback comparisons that his/her priority for cost is not nearly as high as originally thought when asked the question abstractly, while prestige gets more weight.	The user going top down makes comparisons, when asked, without referring to actual alternatives, and overestimates the importance of cost.
Advantages	a) Using feedback and interdependence between criteria. b) Deal with complex problem without structure.	a) Straightforward and convenient. b) Simplicity by using pairwise comparisons.
Disadvantages	a) Conflict between decision makers. b) Inconsistencies. c) Hole of large scale 1 to 9. d) Large comparisons matrix.	a) Decision maker's capacity. b) Inconsistencies. c) Hole of large scale 1 to 9. d) Large comparisons matrix.

Analytic network process (ANP) consists of criteria and alternatives by decomposing them into sub-problems, specifying the weight of each criterion and comparing each criterion against other criterion, in a range between 0 and 1. We employ ANP in decision problems, and we make pairwise comparison matrices between alternatives and criteria. In any traditional methods, decision makers face a difficult problem to make $\frac{n \times (n-1)}{2}$ consistent judgments for each alternative.

In this article, we deal with this problem by making decision maker using (n-1) judgments. The analysis of ANP requires applying a scale system for pairwise comparisons matrix, and this scale plays an important role in transforming qualitative analysis to quantitative analysis [4].

Most of previous researchers use the scale 1-9 of analytic network process and hierarchy. In this research, we introduced a new scale from 0 to 1, instead of the scale 1-9. This scale 1-9 creates large hole between ranking results, and we overcome this drawback by using the scale [0, 1] [5], determined by some serious mathematical shortages of Saaty's scale, such as:

- Large hole between ranking results and human judgments;
- Conflicting between ruling matrix and human intellect.

The neutrosophic set is a generalization of the intuitionistic fuzzy set. While fuzzy sets use true and false for express relationship, neutrosophic sets use true membership, false membership and indeterminacy membership [6]. ANP employs network structure, dependence and feedback [7]. MCDM is a formal and structured decision making methodology for dealing with complex problems [8]. ANP was also integrated as a SWOT method [9]. An overview of integrated ANP with intuitionistic fuzzy can be found in Rouyendegh, [10].

Our research is organized as it follows: Section 2 gives an insight towards some basic definitions of neutrosophic sets and ANP. Section 3 explains the proposed methodology of neutrosophic ANP group decision making model. Section 4 introduces a numerical example.

2 Preliminaries

In this section, we give definitions involving neutrosophic set, single valued neutrosophic sets, trapezoidal neutrosophic numbers, and operations on trapezoidal neutrosophic numbers.

2.1 Definition [26-27]

Let X be a space of points and $x \in X$. A neutrosophic set A in X is defined by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$, $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or real nonstandard subsets of $]{-0}, 1{+}[$. That is $T_A(x){:}X \rightarrow]{-0}, 1{+}[$, $I_A(x){:}X \rightarrow]{-0}, 1{+}[$ and $F_A(x){:}X \rightarrow]{-0}, 1{+}[$. There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0- \leq \sup(x) + \sup x + \sup x \leq 3+$.

2.2 Definition [13, 14, 26]

Let X be a universe of discourse. A single valued neutrosophic set A over X is an object taking the form $A = \{\langle x, T_A(x), I_A(x), F_A(x), \rangle{:}x \in X\}$, where $T_A(x){:}X \rightarrow [0,1]$, $I_A(x){:}X \rightarrow [0,1]$ and $F_A(x){:}X \rightarrow [0,1]$ with $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$ for all $x \in X$. The intervals $T_A(x)$, $I_A(x)$ and $F_A(x)$ represent the truth-membership degree, the indeterminacy-membership degree and the falsity membership degree of x to A, respectively. For convenience, a SVN number is represented by $A = (a, b, c)$, where $a, b, c \in [0, 1]$ and $a+b+c \leq 3$.

2.3 Definition [14, 15, 16]

Suppose $\alpha_{\tilde{a}}$, $\theta_{\tilde{a}}$, $\beta_{\tilde{a}}$ ϵ [0,1] and a_1 , a_2 , a_3 , a_4 ϵ R, where $a_1 \le a_2 \le a_3 \le a_4$. Then, a single valued trapezoidal neutrosophic number $\tilde{a}$=$\langle(a_1 , a_2 , a_3 , a_4); \alpha_{\tilde{a}} , \theta_{\tilde{a}} , \beta_{\tilde{a}}\rangle$ is a special neutrosophic set on the real line set R, whose truth-membership, indeterminacy-membership and falsity-membership functions are defined as:

$$T_{\tilde{a}}(x) = \begin{cases} \alpha_{\tilde{a}}\left(\frac{x-a_1}{a_2-a_1}\right) & (a_1 \le x \le a_2) \\ \alpha_{\tilde{a}} & (a_2 \le x \le a_3) \\ \alpha_{\tilde{a}}\left(\frac{a_4-x}{a_4-a_3}\right) & (a_3 \le x \le a_4) \\ 0 & otherwise \end{cases} \quad (1)$$

$$I_{\tilde{a}}(x) = \begin{cases} \frac{(a_2-x+\theta_{\tilde{a}}(x-a_1))}{(a_2-a_1)} & (a_1 \le x \le a_2) \\ \alpha_{\tilde{a}} & (a_2 \le x \le a_3) \\ \frac{(x-a_3+\theta_{\tilde{a}}(a4-x))}{(a_4-a_3)} & (a_3 \le x \le a_4) \\ 1 & otherwise \end{cases} , \quad (2)$$

$$F_{\tilde{a}}(x) = \begin{cases} \frac{(a_2-x+\beta_{\tilde{a}}(x-a_1))}{(a_2-a_1)} & (a_1 \le x \le a_2) \\ \alpha_{\tilde{a}} & (a_2 \le x \le a_3) \\ \frac{(x-a_3+\beta_{\tilde{a}}(a4-x))}{(a_4-a_3)} & (a_3 \le x \le a_4) \\ 1 & otherwise \end{cases} , \quad (3)$$

where $\alpha_{\tilde{a}}$, $\theta_{\tilde{a}}$ and $\beta_{\tilde{a}}$ represent the maximum truth-membership degree, the minimum indeterminacy-membership degree and the minimum falsity-membership degree, respectively. A single valued trapezoidal neutrosophic number $\tilde{a}$=$\langle(a_1 , a_2 , a_3 , a_4); \alpha_{\tilde{a}} , \theta_{\tilde{a}} , \beta_{\tilde{a}}\rangle$ may express an ill-defined quantity of the range, which is approximately equal to the interval $[a_2 , a_3]$.

2.4 Definition [15, 14]

Let $\tilde{a}$=$\langle(a_1 , a_2 , a_3 , a_4); \alpha_{\tilde{a}} , \theta_{\tilde{a}} , \beta_{\tilde{a}}\rangle$ and $\tilde{b}$=$\langle(b_1 , b_2 , b_3 , b_4); \alpha_{\tilde{b}} , \theta_{\tilde{b}} , \beta_{\tilde{b}}\rangle$ be two single valued trapezoidal neutrosophic numbers, and $\Upsilon \neq 0$ be any real number. Then:

- Addition of two trapezoidal neutrosophic numbers:

$$\tilde{a} + \tilde{b} = \langle(a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}\rangle$$

- Subtraction of two trapezoidal neutrosophic numbers:

$$\tilde{a} - \tilde{b} = \langle(a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}\rangle$$

- Inverse of trapezoidal neutrosophic number:

$$\tilde{a}^{-1} = \langle\left(\frac{1}{a_4}, \frac{1}{a_3}, \frac{1}{a_2}, \frac{1}{a_1}\right); \alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}}\rangle \quad \text{where } (\tilde{a} \neq 0)$$

- Multiplication of trapezoidal neutrosophic number by constant value:

$$\Upsilon\tilde{a} = \begin{cases} \langle(\Upsilon a_1, \Upsilon a_2, \Upsilon a_3, \Upsilon a_4); \alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}}\rangle & \text{if } (\Upsilon > 0) \\ \langle(\Upsilon a_4, \Upsilon a_3, \Upsilon a_2, \Upsilon a_1); \alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}}\rangle & \text{if } (\Upsilon < 0) \end{cases}$$

- Division of two trapezoidal neutrosophic numbers:

$$\frac{\tilde{a}}{\tilde{b}} = \begin{cases} \langle\left(\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}\rangle & \text{if } (a_4 > 0, b_4 > 0) \\ \langle\left(\frac{a_4}{b_4}, \frac{a_3}{b_3}, \frac{a_2}{b_2}, \frac{a_1}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}\rangle & \text{if } (a_4 < 0, b_4 > 0) \\ \langle\left(\frac{a_4}{b_1}, \frac{a_3}{b_2}, \frac{a_2}{b_3}, \frac{a_1}{b_4}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}\rangle & \text{if } (a_4 < 0, b_4 < 0) \end{cases}$$

- Multiplication of trapezoidal neutrosophic numbers:

$$\tilde{a}\tilde{b} = \begin{cases} \langle(a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}\rangle & \text{if } (a_4 > 0, b_4 > 0) \\ \langle(a_1 b_4, a_2 b_3, a_3 b_2, a_4 b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}\rangle & \text{if } (a_4 < 0, b_4 > 0) \\ \langle(a_4 b_4, a_3 b_3, a_2 b_2, a_1 b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}\rangle & \text{if } (a_4 < 0, b_4 < 0) \end{cases}$$

3 Methodology

In this study, we present the steps of the proposed model, we identify criteria, evaluate them, and decision makers also evaluate their judgments using neutrosophic trapezoidal numbers.

In previous articles, we noticed that the scale (1-9) has many drawbacks illustrated by [5]. We present a new scale from 0 to 1 to avoid this drawbacks. We use (n-1) judgments to obtain consistent trapezoidal neutrosophic preference relations instead of $\frac{n \times (n-1)}{2}$, in order to decrease the workload. ANP is used for ranking and selecting the alternatives.

The model of ANP in neutrosophic environment quantifies four criteria to combine them for decision making into one global variable. To do this, we first present the concept of ANP and determine the weight of each criterion based on opinions of decision makers.

Then, each alternative is evaluated with other criteria, considering the effects of relationships among criteria. The ANP technique is composed of four steps in the traditional way [17].

The steps of our ANP neutrosophic model can be introduced as:

Step - 1 constructing the model and problem structuring:

1. Selection of decision makers (DMs).

Form the problem in a network; the first level represents the goal and the second level represents criteria and sub-criteria and interdependence and feedback between criteria, and the third level represents the alternatives. An example of a network structure:

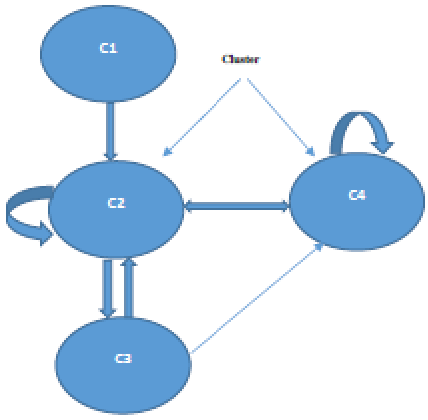


Figure 1. ANP model.

Another example of a network ANP structure [17]:

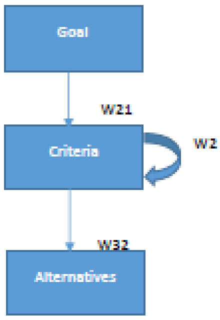


Fig. 2. A Network Structure.

2. Prepare the consensus degree as it follows:

   CD = $\frac{NE}{N} \times 100\%$, where NE is the number of decision makers that have the same opinion and N is the total numbers of experts. Consensus degree should be greater than 50% [16].

Step - 2 Pairwise comparison matrices to determine weighting

1. Identify the alternatives of a problem A = {A1, A2, A3, …, Am}.
2. Identify the criteria and sub-criteria, and the interdependency between them:
   C = {C1, C2, C3, …, Cm}.
3. Determine the weighting matrix of criteria that is defined by decision makers (DMs) for each criterion (W_1).
4. Determine the relationship interdependencies among the criteria and the weights, the effect of each criterion against another in the range from 0 to 1.
5. Determine the interdependency matrix from multiplication of weighting matrix in step 3 and interdependency matrix in step 4.
6. Decision makers make pairwise comparisons matrix between alternatives compared to each criterion, and focus only on (n-1) consensus judgments instead of using $\frac{n \times (n-1)}{2}$ [16].

$$\tilde{R}=\begin{bmatrix} (l_{11}, m_{11l}, m_{11u}, u_{11}) & (l_{11}, m_{11l}, m_{11u}, u_{11}) & \cdots & (l_{1n}, m_{1nl}, m_{1nu}, u_{1n}) \\ (l_{21}, m_{21l}, m_{21u}, u_{21}) & (l_{22}, m_{22l}, m_{22u}, u_{22}) & \cdots & (l_{2n}, m_{2nl}, m_{2nu}, u_{2n}) \\ \cdots & \cdots & \cdots & \cdots \\ (l_{n1}, m_{n1l}, m_{n1u}, u_{n1}) & (l_{n2}, m_{n2l}, m_{n2u}, u_{n2}) & \cdots & (l_{nn}, m_{nnl}, m_{nnu}, u_{nn}) \end{bmatrix}$$

To make the comparisons matrix accepted, we should check the consistency of the matrix.

Definition 5 The consistency of a trapezoidal neutrosophic reciprocal preference relations $\tilde{R}$ = ($\check{r}_{ij}$) n × n can be expressed as:

$\check{r}_{ij}$ = $\check{r}_{ik}$ + $\check{r}_{kj}$ – (0.5, 0.5, 0.5, 0.5) where i, j, k = 1, 2 … n. can also be written as l_{ij} = l_{ik} + l_{kj} – (0.5, 0.5, 0.5, 0.5), m_{ijl} = m_{ikl} + m_{mjl} – (0.5, 0.5, 0.5, 0.5), m_{iju} = m_{iku} + m_{kju} - (0.5, 0.5, 0.5, 0.5), u_{ij} = m_{ik} + m_{kj} – (0.5, 0.5, 0.5, 0.5) , where i, j, k = 1, 2 … n and for $\check{r}_{ik}$ = 1- $\check{r}_{kj}$ {Abdel-Basset, 2017 [16]}.

Definition 6 In order to check whether a trapezoidal neutrosophic reciprocal preference relation $\tilde{R}$ is additive approximation - consistency or not [16].

$$\check{r}_{ij} = \frac{\check{r}_{ij}+c_x}{1+2c_x} \tag{5}$$

$$\check{r}_{ij} = \frac{-\check{r}_{ij}+c_x}{1+2c_x} \tag{6}$$

$$u_{ij} - m_{ij} = \Delta \tag{7}$$

We transform the neutrosophic matrix to pairwise comparison deterministic matrix by adding (α, θ, β), and we use the following equation to calculate the accuracy and score

$$S\,(\tilde{a}_{ij}) = \frac{1}{16}\,[a_1 + b_1 + c_1 + d_1] \times (2 + \alpha_{\tilde{a}} - \theta_{\tilde{a}} - \beta_{\tilde{a}}) \tag{8}$$

and

$$A\,(\tilde{a}_{ij}) = \frac{1}{16}\,[a_1 + b_1 + c_1 + d_1] \times (2 + \alpha_{\tilde{a}} - \theta_{\tilde{a}} + \beta_{\tilde{a}}) \tag{9}$$

We obtain the deterministic matrix by using S ($\tilde{a}_{ij}$).

From the deterministic matrix, we obtain the weighting matrix by dividing each entry by the sum of the column.

Step - 3 Formulation of supermatrix

The supermatrix concept is similar to the Markov chain process [18].

1. Determine scale and weighting data for the n alternatives against n criteria $\mathrm{w}_{21}, \mathrm{w}_{22}, \mathrm{w}_{23}, \dots, \mathrm{w}_{2\mathrm{n}}$.
2. Determine the interdependence weighting matrix of criteria comparing it against another criteria in range from 0 to 1, defined as:

$$W_3 = \begin{array}{c} \\ \mathrm{C}_1 \\ \mathrm{C}_2 \\ \mathrm{C}_3 \\ \mathrm{C}_\mathrm{n} \end{array} \begin{array}{c} \begin{array}{cccc} \mathrm{C}_1 & \mathrm{C}_2 & \mathrm{C}_3 & \mathrm{C}_\mathrm{n} \end{array} \\ \begin{bmatrix} (0-1) & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & (0-1) \end{bmatrix} \end{array} \tag{10}$$

3. We obtain the weighting criteria $W_c = W_3 \times W_1$.
4. Determine the interdependence matrix $\tilde{A}_{criteria}$ among the alternatives with respect to each criterion.

$$\tilde{A}_{criteria}$$

$$\begin{bmatrix} (0.5, 0.5, 0.5, 0.5) & (l_{11}, m_{11l}, m_{11u}, u_{11}) & \dots & (l_{1n}, m_{1nl}, m_{1nu}, u_{1n}) \\ (l_{21}, m_{21l}, m_{21u}, u_{21}) & (0.5, 0.5, 0.5, 0.5) & \dots & (l_{2n}, m_{2nl}, m_{2nu}, u_{2n}) \\ \dots & \dots & (0.5, 0.5, 0.5, 0.5) & \dots \\ (l_{n1}, m_{n1l}, m_{n1u}, u_{n1}) & (l_{n2}, m_{n2l}, m_{n2u}, u_{n2}) & \dots & (0.5, 0.5, 0.5, 0.5) \end{bmatrix}$$

Step - 4 Selection of the best alternatives

1. Determine the priorities matrix of the alternatives with respect to each of the n criteria W_{An} where n is the number of criteria.

   Then, $W_{A1} = W_{\tilde{A}_{C1}} \times W_{21}$

   $W_{A2} = W_{\tilde{A}_{C1}} \times W_{22}$

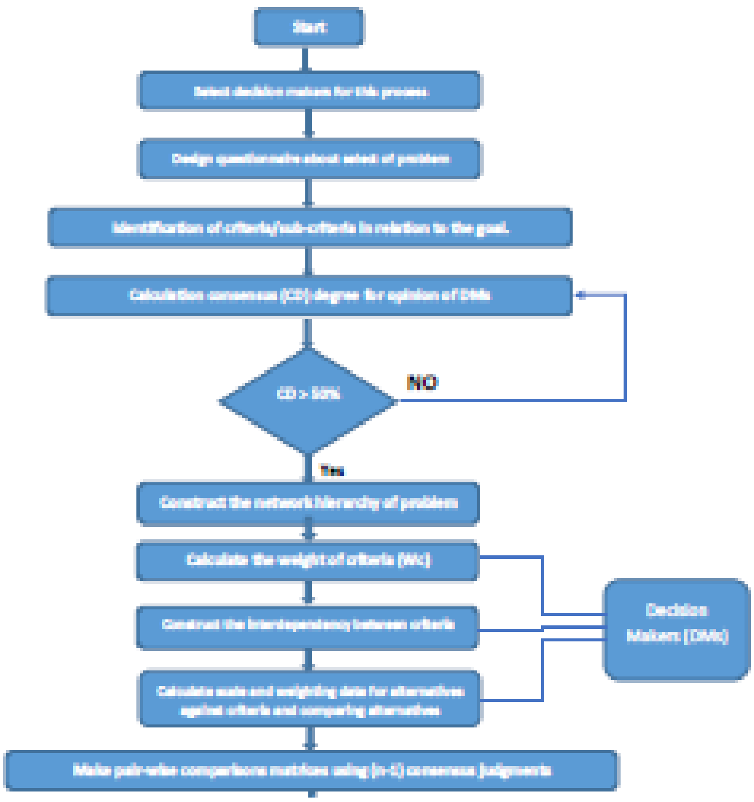
   $W_{A3} = W_{\tilde{A}_{C1}} \times W_{23}$

   $W_{An} = W_{\tilde{A}_{Cn}} \times W_{2n}$

   Then, $W_A = [\, W_{A1}, W_{A2}, W_{A3}, \ldots, W_{An}]$.

2. In the last we rank the priorities of criteria and obtain the best alternatives by multiplication of the W_A matrix by the Weighting criteria matrix W_c, i.e.

$W_A \times W_c$



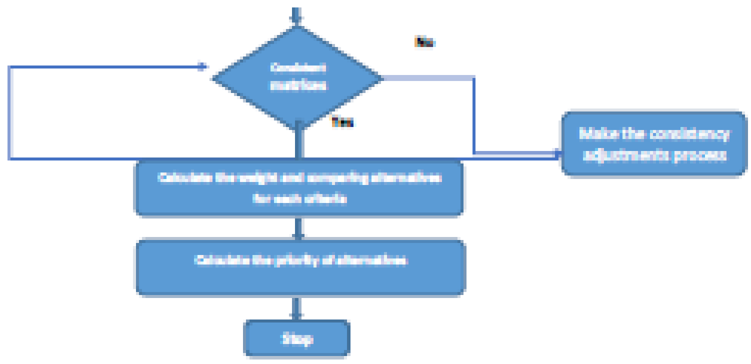


Figure 3. Schematic diagram of ANP with neutrosophic.

4 Numerical Example

In this section, we present an example to illustrate the ANP in neutrosophic environment - selecting the best personal car from four alternatives: Crossover is alternative A1, Sedan is alternative A2, Diesel is alternative A3, Nissan is alternative A4. We have four criteria C_j (j = 1, 2, 3, and 4), as follows: C_1 for price, C_2 for speed, C_3 for color, C_4 for model. The criteria to be considered is the supplier selections, which are determined by the DMs from a decision group. The team is split into four groups, namely DM_1, DM_2, DM_3 and DM_4, formed to select the most suitable alternatives. The criteria to be considered in the supplier's selection are determined by the DMs team from the expert's procurement office.

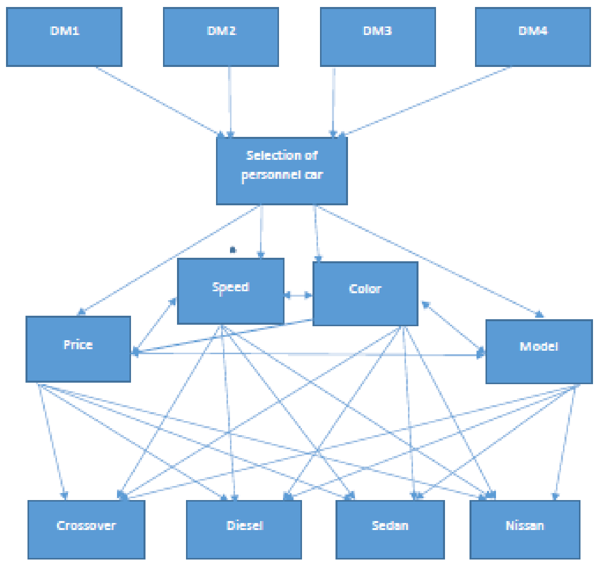


Figure 4. Network structure of the illustrative example.

In this example, we seek to illustrate the improvement and efficiency of ANP, the interdependency among criteria and feedback, and how a new scale from 0 to 1 improves and facilitates the solution and the ranking of the alternatives.

Step - 1: In order to compare the criteria, the decision makers assume that there is no interdependency among criteria. This data reflects relative weighting without considering interdependency among criteria. The weighting matrix of criteria that is defined by decision makers is W_1= (P, S, C, M) = (0.33, 0.40, 0.22, 0.05).

Step - 2: Assuming that there is no interdependency among the four alternatives, (A_1 A_2, A_3, A_4), they are compared against each criterion. Decision makers determine the relationships between each criterion and alternative, establishing the neutrosophic decision matrix between four alternatives (A_1, A_2,A_3, A_4) and four criteria (C_1, C_2 , C_3 , C_4):

$$R = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{bmatrix} (0.3\,,0.5,0.2,0.5) & (0.6,0.7,0.9,0.1) & (0.7\,,0.2,0.4,0.6) & (0.3\,,0.6,0.4,0.7) \\ (0.6\,,0.3,0.4,0.7) & (0.2\,,0.3,0.6,0.9) & (0.6\,,0.7,0.8,0.9) & (0.3\,,0.5,0.2,0.5) \\ (0.3\,,0.5,0.2,0.5) & (0.3\,,0.7,0.4,0.3) & (0.8\,,0.2,0.4,0.6) & (0.2\,,0.5,0.6,0.8) \\ (0.4\,,0.3,0.1,0.6) & (0.1\,,0.4,0.2,0.8) & (0.5\,,0.3,0.2,0.4) & (0.6\,,0.2,0.3,0.4) \end{bmatrix} \end{array}$$

The last matrix appears consistent to definition 6 (5, 6, 7). Then, by ensuring consistency of trapezoidal neutrosophic additive reciprocal preference relations, decision makers (DMs) should determine the maximum truth-membership degree (α), minimum indeterminacy-membership degree (θ), and minimum falsity-membership degree (β) of single valued neutrosophic numbers, as in definition 6 (c). Therefore:

$$R = \begin{array}{c} A1 \\ A2 \\ A3 \\ A3 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{bmatrix} (0.3,0.5,0.2,0.5;\ 0.3,0.4,0.6) & (0.6,0.7,0.9,0.1;\ 0.4,0.3,0.5) & (0.7,0.2,0.4,0.6;\ 0.8,0.4,0.2) & (0.3,0.6,0.4,0.7;\ 0.4,0.5,0.6) \\ (0.6,0.3,0.4,0.7;\ 0.2,0.5,0.8) & (0.2,0.3,0.6,0.9;\ 0.6,0.2,0.5) & (0.6,0.7,0.8,0.9;\ 0.2,0.5,0.7) & (0.3,0.5,0.2,0.5;\ 0.5,0.7,0.8) \\ (0.3,0.5,0.2,0.5;\ 0.4,0.5,0.7) & (0.3,0.7,0.4,0.3;\ 0.2,0.5,0.9) & (0.8,0.2,0.4,0.6;\ 0.4,0.6,0.5) & (0.2,0.5,0.6,0.8;\ 0.4,0.3,0.8) \\ (0.4,0.3,0.1,0.6;\ 0.2,0.3,0.5) & (0.1,0.4,0.2,0.8;\ 0.7,0.3,0.6) & (0.5,0.3,0.2,0.4;\ 0.3,0.4,0.7) & (0.6,0.2,0.3,0.4;\ 0.6,0.3,0.4) \end{bmatrix} \end{array}$$

S $(\tilde{a}_{ij}) = \frac{1}{16}[a_1 + b_1 + c_1 + d_1] \times (2 + \alpha_{\tilde{a}} - \theta_{\tilde{a}} - \beta_{\tilde{a}})$

And

A $(\tilde{a}_{ij}) = \frac{1}{16}[a_1 + b_1 + c_1 + d_1] \times (2 + \alpha_{\tilde{a}} - \theta_{\tilde{a}} + \beta_{\tilde{a}})$

The deterministic matrix can be obtained by S $(\tilde{a}_{ij})$ equation in the following step:

$$R = \begin{array}{c c} & \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{array}{c} A1 \\ A2 \\ A3 \\ A3 \end{array} & \begin{bmatrix} 0.122 & 0.23 & 0.261 & 0.163 \\ 0.113 & 0.238 & 0.188 & 0.10 \\ 0.113 & 0.085 & 0.163 & 0.17 \\ 0.123 & 0.169 & 0.105 & 0.178 \end{bmatrix} \end{array}$$

Scale and weighting data for four alternatives against four criteria is derived by dividing each element by the sum of each column. The comparison matrix of four alternatives and four criteria is the following:

$$\begin{array}{c c} & \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{array}{c} A1 \\ A2 \\ A3 \\ A3 \end{array} & \begin{bmatrix} 0.259 & 0.319 & 0.364 & 0.268 \\ 0.240 & 0.329 & 0.262 & 0.164 \\ 0.240 & 0.118 & 0.227 & 0.278 \\ 0.261 & 0.234 & 0.146 & 0.291 \end{bmatrix} \\ & \begin{array}{cccc} w_{21} & w_{22} & w_{23} & w_{24} \end{array} \end{array}$$

Step - 3: Decision makers take into consideration the interdependency among criteria. When one alternative is selected, more than one criterion should be considered. Therefore, the impact of all the criteria needs to be examined by using pairwise comparisons. By decision makers' group interviews, four sets of weightings have been obtained. The data that the decision makers prepare for the relationships between criteria reflect the relative impact degree of the four criteria with respect to each of four criteria. We make a graph to show the relationship between the interdependency among four criteria, and the mutual effect.

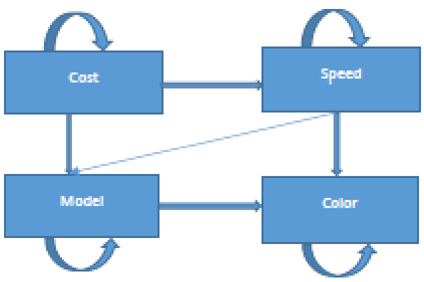


Figure 5. Interdependence among the criteria.

The interdependency weighting matrix of criteria is defined as:

$$\mathrm{w}_3 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{bmatrix} 1 & 0.8 & 0.4 & 0 \\ 0 & 0.2 & 0.5 & 0.6 \\ 0 & 0 & 0.1 & 0.3 \\ 0 & 0 & 0 & 0.1 \end{bmatrix} \end{array}$$

$$\mathrm{w}_c = \mathrm{w}_3 \times \mathrm{w}_1 = \begin{bmatrix} 1 & 0.8 & 0.4 & 0 \\ 0 & 0.2 & 0.5 & 0.6 \\ 0 & 0 & 0.1 & 0.3 \\ 0 & 0 & 0 & 0.1 \end{bmatrix} \times \begin{bmatrix} 0.33 \\ 0.40 \\ 0.22 \\ 0.05 \end{bmatrix} = \begin{bmatrix} 0.738 \\ 0.220 \\ 0.037 \\ 0.005 \end{bmatrix}$$

Thus, it is derived that w_c= (C_1,C_2 ,C_3 ,C_4) = (0.738, 0.220, 0.037, 0.005).

Step - 4: The interdependency among alternatives with respect to each criterion is calculated by respect of consistency ratio that the decision makers determined. In order to satisfy the criteria 1 (C_1), which alternative contributes more to the action of alternative 1 against criteria 1 and how much more? We defined the project interdependency weighting matrix for criteria C_1 as:

a. First criteria (C_1)

DMs compare criteria with other criteria, and determine the weighting of every criteria:

$$\tilde{A}_{C1} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & A_3 & A_4 \end{array} \\ \begin{bmatrix} (0.5\,,0.5,0.5,0.5) & (0.3, 0.2,0.4,0.5) & y & y \\ y & (0.5\,,0.5,0.5,0.5) & (0.1\,,0.2,0.4,0.8) & y \\ y & y & (0.5\,,0.5,0.5,0.5) & (0.2\,,0.3,0.4,0.7) \\ y & y & y & (0.5\,,0.5,0.5,0.5) \end{bmatrix} \end{array}$$

where y indicates preference values that are not determined by decision makers. Then, we can calculate these values and make them consistent with their judgments. Let us complete the previous matrix according to definition 5 as follows:

$\widetilde{R}_{13} = \tilde{r}_{12} + \tilde{r}_{23}$ - (0.5 , 0.5,0.5,0.5) = $(-0.1, -0.1, 0.3, 0.8)$

$\widetilde{R}_{31}$ = 1 - $\widetilde{R}_{13}$ = 1 - $(-0.1, -0.1, 0.3, 0.8)$ = (0.2, 0.7,1.1,1.1)

$\widetilde{R}_{32} = \tilde{r}_{31} + \tilde{r}_{12}$ - (0.5 , 0.5,0.5,0.5) = (0.0,0.4 ,1.0,1.1)

$\widetilde{R}_{21}$ = 1 - $\widetilde{R}_{12}$ = 1 – (0.3, 0.2,0.4,0.5) = (0.5, 0.6, 0.8, 0.7)

$\widetilde{R}_{14} = \tilde{r}_{13} + \tilde{r}_{34}$ - (0.5 , 0.5,0.5,0.5) = $(-0.1, -0.3, 0.2, 1.1)$

$\widetilde{R}_{24} = \tilde{r}_{21} + \tilde{r}_{14}$ - (0.5 , 0.5,0.5,0.5) = $(-0.1, -0.2, 0.5, 1.2)$

$\widetilde{R}_{41}$ = 1 - $\widetilde{R}_{14}$ = 1 – $(-0.1, -0.3, 0.2, 1.0)$ = (1.0, 0.8, 1.3, 1.1)

$\widetilde{R}_{42}$ = 1 - $\widetilde{R}_{24}$ = 1 – $(-0.1, -0.2, 0.5, 1.2)$ = (0.2, 0.5, 1.2, 1.1)

$\widetilde{R}_{43}$ = 1 - $\widetilde{R}_{34}$ = 1 – (0.2, 0.3,0.4,0.7) = (0.3, 0.6, 0.7, 0.8)

The comparison matrix will be as follows:

$$\tilde{A}_{C1} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & A_3 & A_4 \end{array} \\ \begin{bmatrix} (0.5\,,0.5,0.5,0.5) & (0.3,0.2,0.4,0.5) & (-0.1,-0.1,0.3,0.8) & (-0.1,-0.3,0.2,1.1) \\ (0.5,0.6,0.8,0.7) & (0.5\,,0.5,0.5,0.5) & (0.1\,,0.2,0.4,0.8) & (-0.1,-0.2,0.5,1.2) \\ (0.2,0.7,1.1,1.1) & (0.0,0.4\,,1.0,1.1) & (0.5\,,0.5,0.5,0.5) & (0.2\,,0.3,0.4,0.7) \\ (1.0,0.8,1.3,1.1) & (0.2,0.5,1.2,1.1) & (0.3,0.6,0.7,0.8) & (0.5\,,0.5,0.5,0.5) \end{bmatrix} \end{array}$$

According to definition 6, one can see that this relation is not a trapezoidal neutrosophic additive reciprocal preference relation. By using Eq. 5, Eq. 6 and Eq. 7 in definition 6, we obtain the following:

$$\tilde{A}_{C1} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & A_3 & A_4 \end{array} \\ \begin{bmatrix} (0.5\,,0.5,0.5,0.5) & (0.3,0.2,0.4,0.5) & (0.1,0.1,0.3,0.8) & (0.1,0.3,0.2,1.0) \\ (0.5,0.6,0.8,0.7) & (0.5\,,0.5,0.5,0.5) & (0.1\,,0.2,0.4,0.8) & (0.1,0.2,0.5,1.0) \\ (0.2,0.7,1.0,1.0) & (0.0,0.4\,,1.0,1.0) & (0.5\,,0.5,0.5,0.5) & (0.2\,,0.3,0.4,0.7) \\ (1.0,0.8,1.0,1.0) & (0.2,0.5,1.0,1.0) & (0.3,0.6,0.7,0.8) & (0.5\,,0.5,0.5,0.5) \end{bmatrix} \end{array}$$

We check if the matrix is consistent according to definition 6. By ensuring consistency of trapezoidal neutrosophic additive reciprocal preference relations, decision makers (DMs) should determine the maximum truth-membership degree (α), the minimum indeterminacy-membership degree (θ) and the minimum falsity-membership degree (β) of single valued neutrosophic numbers as in definition 6.

$$\tilde{A}_{C1} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & A_3 & A_4 \end{array} \\ \begin{bmatrix} (0.5\,,0.5,0.5,0.5) & (0.3,0.2,0.4,0.5;0.7\,,0.2,0.5) & (0.1,0.1,0.3,0.8;0.5\,,0.2,0.1) & (0.1,0.3,0.2,1.0;0.5,0.2,0.1) \\ (0.5,0.6,0.8,0.7;0.7\,,0.2,0.5) & (0.5\,,0.5,0.5,0.5) & (0.1\,,0.2,0.4,0.8;0.4,0.5,0.6) & (0.1,0.2,0.5,1.0;0.5,0.1,0.2) \\ (0.2,0.7,1.0,1.0;0.8,0.2\,,0.1) & (0.0,0.4\,,1.0,1.0;0.3,0.1,0.5) & (0.5\,,0.5,0.5,0.5) & (0.2\,,0.3,0.4,0.7;0.7,0.2,0.5) \\ (1.0,0.8,1.0,1.0;0.6,0.2,0.3) & (0.2,0.5,1.0,1.0;0.6,0.2,0.3) & (0.3,0.6,0.7,0.8;0.9,0.4,0.6) & (0.5\,,0.5,0.5,0.5) \end{bmatrix} \end{array}$$

We make sure the matrix is deterministic, or we transform the previous matrix to be a deterministic pairwise comparison matrix, to calculate the weight of each criterion using equation (8, 9) in definition 6.

The deterministic matrix can be obtained by S ($\tilde{a}_{ij}$) equation in the following step:

$$\tilde{A}_{C1} = \begin{bmatrix} 0.5 & 0.175 & 0.179 & 0.22 \\ 0.325 & 0.5 & 0.122 & 0.25 \\ 0.453 & 0.265 & 0.5 & 0.2 \\ 0.38 & 0.354 & 0.285 & 0.5 \end{bmatrix}$$

We present the weight of each alternatives according to each criteria from the deterministic matrix easily by dividing each entry by the sum of the column; we obtain the following matrix as:

$$\tilde{A}_{C1} = \begin{bmatrix} 0.30 & 0.135 & 0.165 & 0.188 \\ 0.196 & 0.386 & 0.112 & 0.214 \\ 0.273 & 0.198 & 0.460 & 0.171 \\ 0.229 & 0.274 & 0.262 & 0.427 \end{bmatrix}$$

b. Second criteria (C_2)

DMs compare criteria with other criteria, and determine the weighting of every criteria:

$$\tilde{A}_{C2} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & A_3 & A_4 \end{array} \\ \begin{bmatrix} (0.5\,,0.5,0.5,0.5) & (0.3,0.6,0.4,0.5) & y & y \\ y & (0.5\,,0.5,0.5,0.5) & (0.5\,,0.2,0.4,0.9) & y \\ y & y & (0.5\,,0.5,0.5,0.5) & (0.5\,,0.3,0.4,0.7) \\ y & y & y & (0.5\,,0.5,0.5,0.5) \end{bmatrix} \end{array}$$

where y indicates preference values that are not determined by decision makers, then we can calculate these values and make them consistent with their judgments.
We complete the previous matrix according to definition 5 as follows:

The comparison matrix will be as follows:

$$\tilde{A}_{C2} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & A_3 & A_4 \end{array} \\ \begin{bmatrix} (0.5\,,0.5,0.5,0.5) & (0.3,0.6,0.4,0.5) & (0.3,0.3,0.3,0.9) & (0.3,0.1,0.2,1.1) \\ (0.5,0.6,0.4,0.7) & (0.5\,,0.5,0.5,0.5) & (0.5\,,0.2,0.4,0.9) & (0.3,0.2,0.1,1.3) \\ (0.1,0.7,0.7,0.7) & (-0.1,0.8\,,0.3,0.5) & (0.5\,,0.5,0.5,0.5) & (0.5\,,0.3,0.4,0.7) \\ (1.0,0.8,0.9,0.7) & (0.3,0.9,0.8,0.7) & (0.3,0.6,0.7,0.5) & (0.5\,,0.5,0.5,0.5) \end{bmatrix} \end{array}$$

According to definition 6, one can see that this relation is not a trapezoidal neutrosophic additive reciprocal preference relation. By using Eq. 5, Eq. 6 and Eq. 7 in definition 6, we obtain the following:

$$\tilde{A}_{C2} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & A_3 & A_4 \end{array} \\ \begin{bmatrix} (0.5\,,0.5,0.5,0.5) & (0.3,0.6,0.4,0.5) & (0.3,0.3,0.3,0.9) & (0.3,0.1,0.2,1.0) \\ (0.5,0.6,0.4,0.7) & (0.5\,,0.5,0.5,0.5) & (0.5\,,0.2,0.4,0.9) & (0.3,0.2,0.1,1.0) \\ (0.1,0.7,0.7,0.7) & (0.1,0.8\,,0.3,0.5) & (0.5\,,0.5,0.5,0.5) & (0.5\,,0.3,0.4,0.7) \\ (1.0,0.8,0.9,0.7) & (0.3,0.9,0.8,0.7) & (0.3,0.6,0.7,0.5) & (0.5\,,0.5,0.5,0.5) \end{bmatrix} \end{array}$$

Let us check that the matrix is consistent according to definition 6. Then, by ensuring consistency of trapezoidal neutrosophic additive reciprocal preference relations, decision makers (DMs) should determine the maximum truth-membership degree (α), the minimum indeterminacy-membership degree (θ) and the minimum falsity-membership degree (β) of single valued neutrosophic numbers, as in definition 6. Then:

$$\tilde{A}_{C2} = \begin{array}{c|cccc} & A_1 & A_2 & A_3 & A_4 \\ \hline A_1 & (0.5, 0.5, 0.5, 0.5) & (0.3, 0.6, 0.4, 0.5; 0.7, 0.3, 0.5) & (0.3, 0.3, 0.3, 0.9; 0.5, 0.2, 0.1) & (0.3, 0.1, 0.2, 1.0; 0.5, 0.2, 0.1) \\ A_2 & (0.5, 0.6, 0.4, 0.7; 0.7, 0.3, 0.5) & (0.5, 0.5, 0.5, 0.5) & (0.5, 0.2, 0.4, 0.9; 0.4, 0.5, 0.6) & (0.3, 0.2, 0.1, 1.0; 0.5, 0.1, 0.4) \\ A_3 & (0.1, 0.7, 0.7, 0.7; 0.8, 0.2, 0.3) & (0.1, 0.8, 0.3, 0.5; 0.4, 0.2, 0.5) & (0.5, 0.5, 0.5, 0.5) & (0.5, 0.3, 0.4, 0.7; 0.6, 0.2, 0.5) \\ A_4 & (1.0, 0.8, 0.9, 0.7; 0.6, 0.4, 0.3) & (0.3, 0.9, 0.8, 0.7; 0.6, 0.2, 0.3) & (0.3, 0.6, 0.7, 0.5; 0.9, 0.4, 0.5) & (0.5, 0.5, 0.5, 0.5) \end{array}$$

Let us be sure the matrix is deterministic, or transform the previous matrix to be deterministic pairwise comparison matrix, to calculate the weight of each criteria using equation (8, 9) in definition 6.

The deterministic matrix can be obtained by S ($\tilde{a}_{ij}$) equation in the following step:

$$\tilde{A}_{C2} = \begin{bmatrix} 0.5 & 0.214 & 0.247 & 0.22 \\ 0.216 & 0.5 & 0.163 & 0.20 \\ 0.316 & 0.181 & 0.5 & 0.226 \\ 0.404 & 0.354 & 0.3 & 0.5 \end{bmatrix}$$

We present the weight of each alternatives according to each criteria from the deterministic matrix by dividing each entry by the sum of the column; we obtain the following matrix:

$$\tilde{A}_{C2} = \begin{bmatrix} 0.50 & 0.215 & 0.244 & 0.192 \\ 0.216 & 0.503 & 0.161 & 0.175 \\ 0.273 & 0.182 & 0.495 & 0.197 \\ 0.229 & 0.356 & 0.259 & 0.436 \end{bmatrix}$$

c. Third criteria (C_3)

DMs compare criteria with other criteria, and determine the weight of every criteria.

$$\tilde{A}_{C3} = \begin{array}{c|cccc} & A_1 & A_2 & A_3 & A_4 \\ \hline A_1 & (0.5, 0.5, 0.5, 0.5) & (0.6, 0.7, 0.9, 0.1) & y & y \\ A_2 & y & (0.5, 0.5, 0.5, 0.5) & (0.6, 0.7, 0.8, 0.9) & y \\ A_3 & y & y & (0.5, 0.5, 0.5, 0.5) & (0.2, 0.5, 0.6, 0.8) \\ A_4 & y & y & y & (0.5, 0.5, 0.5, 0.5) \end{array}$$

where y indicates preference values that are not determined by decision makers; then, we can calculate these values and make them consistent with their judgments.

We complete the previous matrix according to definition 5 as follows:

$$\tilde{A}_{C3} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & A_3 & A_4 \end{array} \\ \begin{bmatrix} (0.5\,,0.5,0.5,0.5) & (0.6,0.7,0.9,0.1) & (0.7,0.9,1.2,1.4) & (0.4,0.7,1.3,1.7) \\ (0.0,0.1,0.3,0.4) & (0.5\,,0.5,0.5,0.5) & (0.6\,,0.7,0.8,0.9) & (0.3,0.5,0.9,1.2) \\ (-0.4,-0.2,0.1,0.3) & (-0.3,0.0\,,0.5,0.8) & (0.5\,,0.5,0.5,0.5) & (0.2\,,0.5,0.6,0.8) \\ (-0.7,-0.3,0.3,0.6) & (-0.6,-0.1,0.7,1.1) & (0.2,0.4,0.5,0.8) & (0.5\,,0.5,0.5,0.5) \end{bmatrix} \end{array}$$

According to definition 6, one can see that the relation is not a trapezoidal neutrosophic additive reciprocal preference relation. By using Eq. 5, Eq. 6 and Eq. 7 in definition 6, we obtain the following:

$$\tilde{A}_{C3} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & A_3 & A_4 \end{array} \\ \begin{bmatrix} (0.5\,,0.5,0.5,0.5) & (0.6,0.7,0.9,0.1) & (0.7,0.9,1.0,1.0) & (0.4,0.7,1.0,1.0) \\ (0.0,0.1,0.3,0.4) & (0.5\,,0.5,0.5,0.5) & (0.6\,,0.7,0.8,0.9) & (0.3,0.5,0.9,1.0) \\ (0.4,0.2,0.1,0.3) & (0.3,0.0\,,0.5,0.8) & (0.5\,,0.5,0.5,0.5) & (0.2\,,0.5,0.6,0.8) \\ (0.7,0.3,0.3,0.6) & (0.6,0.1,0.7,1.0) & (0.2,0.4,0.5,0.8) & (0.5\,,0.5,0.5,0.5) \end{bmatrix} \end{array}$$

Then, let us check that the matrix is consistent according to definition 6. Then, by ensuring consistency of trapezoidal neutrosophic additive reciprocal preference relations, decision makers (DMs) should determine the maximum truth-membership degree (α), the minimum indeterminacy-membership degree (θ) and the minimum falsity-membership degree (β) of the single valued neutrosophic numbers as in definition 6. Then:

$$\tilde{A}_{C3} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & A_3 & A_4 \end{array} \\ \begin{bmatrix} (0.5\,,0.5,0.5,0.5) & (0.6,0.7,0.9,0.1;0.7\,,0.2,0.5) & (0.7,0.9,1.0,1.0;0.5\,,0.2,0.1) & (0.4,0.7,1.0,1.0;0.5,0.2,0.3) \\ (0.0,0.1,0.3,0.4;0.8\,,0.2,0.6) & (0.5\,,0.5,0.5,0.5) & (0.6\,,0.7,0.8,0.9;0.5,0.2,0.1) & (0.3,0.5,0.9,1.0;0.5,0.1,0.2) \\ (0.4,0.2,0.1,0.3;0.5,0.3\,,0.4) & (0.3,0.0\,,0.5,0.8;0.8,0.5,0.3) & (0.5\,,0.5,0.5,0.5) & (0.2\,,0.5,0.6,0.8;0.6,0.4,0.2) \\ (0.7,0.3,0.3,0.6;0.5,0.2,0.1) & (0.6,0.1,0.7,1.0;0.3,0.1,0.5) & (0.2,0.4,0.5,0.8;0.3,0.1,0.5) & (0.5\,,0.5,0.5,0.5) \end{bmatrix} \end{array}$$

Let us be sure the matrix is deterministic, or transform the previous matrix to be deterministic pairwise comparison matrix, to calculate the weight of each criteria using equation (8, 9) in definition 6.

The deterministic matrix can be obtained by S ($\tilde{a}_{ij}$) equation in the following step:

$$\tilde{A}_{C3} = \begin{bmatrix} 0.5 & 0.4 & 0.49 & 0.41 \\ 0.1 & 0.5 & 0.41 & 0.37 \\ 0.18 & 0.24 & 0.5 & 0.56 \\ 0.38 & 0.30 & 0.20 & 0.5 \end{bmatrix}$$

We present the weight of each alternatives according to each criteria from the deterministic matrix by dividing each entry by the sum of the column; we obtain the following matrix:

$$\tilde{A}_{C3} \;=\begin{bmatrix} 0.43 & 0.27 & 0.30 & 0.22 \\ 0.08 & 0.35 & 0.26 & 0.20 \\ 0.15 & 0.16 & 0.31 & 0.30 \\ 0.33 & 0.21 & 0.12 & 0.27 \end{bmatrix}$$

d. Four criteria (C_4)

DMs compare criteria with other criteria, and determine the weighting of every:

$$\tilde{A}_{C4} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & A_3 & A_4 \end{array} \\ \begin{bmatrix} (0.5\,,0.5,0.5,0.5) & (0.4,0.5,0.3,0.7) & y & y \\ y & (0.5\,,0.5,0.5,0.5) & (0.4\,,0.2,0.7,0.5) & y \\ y & y & (0.5\,,0.5,0.5,0.5) & (0.4,0.6,0.5,0.8) \\ y & y & y & (0.5\,,0.5,0.5,0.5) \end{bmatrix} \end{array}$$

Where y indicates the preference values that are not determined by decision makers; then, we can calculate these values and make them consistent with their judgments.

We complete the previous matrix according to definition 5 as follows:

$$\tilde{A}_{C4} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & A_3 & A_4 \end{array} \\ \begin{bmatrix} (0.5\,,0.5,0.5,0.5) & (0.4,0.5,0.3,0.7) & (0.3,0.2,0.5,0.7) & (0.2,0.3,0.5,1.0) \\ (0.3,0.7,0.5,0.6) & (0.5\,,0.5,0.5,0.5) & (0.4\,,0.2,0.7,0.5) & (0.0,0.5,0.5,1.1) \\ (0.3,0.7,0.5,0.6) & (0.2,0.5\,,0.6,0.9) & (0.5\,,0.5,0.5,0.5) & (0.4,0.6,0.5,0.8) \\ (0.3,0.7,0.5,0.6) & (-0.1,0.5,0.5,1.0) & (0.2,0.5,0.4,0.6) & (0.5\,,0.5,0.5,0.5) \end{bmatrix} \end{array}$$

According to definition 6, one can see that this relation is not a trapezoidal neutrosophic additive reciprocal preference relation. By using Eq. 5, Eq. 6 and Eq. 7 in definition 6, we obtain the following:

$$\tilde{A}_{C4} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & A_3 & A_4 \end{array} \\ \begin{bmatrix} (0.5\,,0.5,0.5,0.5) & (0.4,0.5,0.3,0.7) & (0.3,0.2,0.5,0.7) & (0.2,0.3,0.5,1.0) \\ (0.3,0.7,0.5,0.6) & (0.5\,,0.5,0.5,0.5) & (0.4\,,0.2,0.7,0.5) & (0.0,0.5,0.5,1.0) \\ (0.3,0.7,0.5,0.6) & (0.2,0.5\,,0.6,0.9) & (0.5\,,0.5,0.5,0.5) & (0.4,0.6,0.5,0.8) \\ (0.3,0.7,0.5,0.6) & (0.1,0.5,0.5,1.0) & (0.2,0.5,0.4,0.6) & (0.5\,,0.5,0.5,0.5) \end{bmatrix} \end{array}$$

Then, we check that the matrix is consistent according to definition 6. By ensuring consistency of trapezoidal neutrosophic additive reciprocal preference relations, decision makers (DMs) should determine the maximum truth-membership degree (α), the minimum indeterminacy-membership degree (θ) and the minimum falsity-membership degree (β) of the single valued neutrosophic numbers, as in definition 6.

$$\tilde{A}_{C4} = \begin{array}{c|cccc} & A_1 & A_2 & A_3 & A_4 \\ \hline A_1 & (0.5, 0.5, 0.5, 0.5) & (0.4, 0.5, 0.3, 0.7; 0.4, 0.3, 0.6) & (0.3, 0.2, 0.5, 0.7; 0.2, 0.3, 0.5) & (0.2, 0.3, 0.5, 1.0; 0.3, 0.1, 0.8) \\ A_2 & (0.3, 0.7, 0.5, 0.6; 0.7, 0.4, 0.5) & (0.5, 0.5, 0.5, 0.5) & (0.4, 0.2, 0.7, 0.5; 0.3, 0.5, 0.6) & (0.0, 0.5, 0.5, 1.0; 0.4, 0.3, 0.2) \\ A_3 & (0.3, 0.5, 0.8, 0.7; 0.7, 0.4, 0.5) & (0.2, 0.5, 0.6, 0.9; 0.7, 0.4, 0.3) & (0.5, 0.5, 0.5, 0.5) & (0.4, 0.6, 0.5, 0.8; 0.7, 0.3, 0.5) \\ A_4 & (0.0, 0.5, 0.7, 0.8; 0.5, 0.2, 0.4) & (0.1, 0.5, 0.5, 1.0; 0.5, 0.3, 0.6) & (0.2, 0.5, 0.4, 0.6; 0.4, 0.6, 0.2) & (0.5, 0.5, 0.5, 0.5) \end{array}$$

Let us be sure the matrix is deterministic, or transform the previous matrix to be deterministic pairwise comparison matrix, to calculate the weight of each criteria using equation (8, 9) in definition 6.

The deterministic matrix can be obtained by S ($\tilde{a}_{ij}$) equation in the following step:

$$\tilde{A}_{C4} = \begin{bmatrix} 0.5 & 0.18 & 0.15 & 0.17 \\ 0.24 & 0.5 & 0.13 & 0.23 \\ 0.29 & 0.27 & 0.5 & 0.27 \\ 0.23 & 0.21 & 0.17 & 0.5 \end{bmatrix}$$

We present the weight of each alternative according to each criteria from the deterministic matrix by dividing each entry by the sum of the column; we obtain the following matrix:

$$\tilde{A}_{C4} = \begin{bmatrix} 0.40 & 0.16 & 0.16 & 0.15 \\ 0.19 & 0.43 & 0.14 & 0.19 \\ 0.23 & 0.23 & 0.5 & 0.23 \\ 0.18 & 0.18 & 0.18 & 0.42 \end{bmatrix}$$

Step 4: The priorities of the alternative W_A with respect to each of the four criteria are given by synthesizing the results from Steps 2 and 4 as follows:

$$W_{A1} = W_{\tilde{A}_{C_1}} \times W_{21} = \begin{bmatrix} 0.199 \\ 0.172 \\ 0.273 \\ 0.299 \end{bmatrix}$$

$$W_{A2} = W_{\tilde{A}_{C_2}} \times W_{22} = \begin{bmatrix} 0.303 \\ 0.294 \\ 0.251 \\ 0.347 \end{bmatrix}$$

$$W_{A3} = W_{\tilde{A}_{C_3}} \times W_{23} = \begin{bmatrix} 0.327 \\ 0.209 \\ 0.210 \\ 0.241 \end{bmatrix}$$

$$W_{A4} = W_{\tilde{A}_{C_4}} \times W_{24} = \begin{bmatrix} 0.222 \\ 0.216 \\ 0.305 \\ 0.250 \end{bmatrix}$$

The matrix W_A is defined by grouping together the above four columns:

W_A =[$W_{A1}, W_{A2}, W_{A3}, W_{A4}$]

Step 5: The overall priorities for the candidate alternatives are finally calculated by multiplying W_A and W_c:

$$= W_A \times W_c = \begin{array}{c} \begin{array}{cccc} W_{A1} & W_{A2} & W_{A3} & W_{A4} \end{array} \\ \begin{bmatrix} 0.199 & 0.303 & 0.327 & 0.222 \\ 0.172 & 0.294 & 0.209 & 0.216 \\ 0.273 & 0.251 & 0.210 & 0.305 \\ 0.299 & 0.347 & 0.241 & 0.250 \end{bmatrix} \end{array} \times \begin{bmatrix} 0.738 \\ 0.220 \\ 0.037 \\ 0.005 \end{bmatrix} = \begin{bmatrix} 0.226 \\ 0.200 \\ 0.265 \\ 0.307 \end{bmatrix}$$

The final results in the ANP Neutrosophic Phase are (A1, A2, A3, A4) = (0.226, 0.200, 0.265, 0.307). These ANP Neutrosophic results are interpreted as follows. The highest weighting of criteria in this problem selection example is A4. Next is A1. These weightings are used as priorities in selecting the best personnel car.

Then, it is obvious that the four alternative has the highest rank, meaning that Nissan is the best car according to this criteria, followed by Crossover, Diesel and, finally, Sedan.

Table 2. Ranking of alternatives.

Car Name	Priority
Crossover	0.22
Diesel	0.20
Nissan	0.26
Sedan	0.30

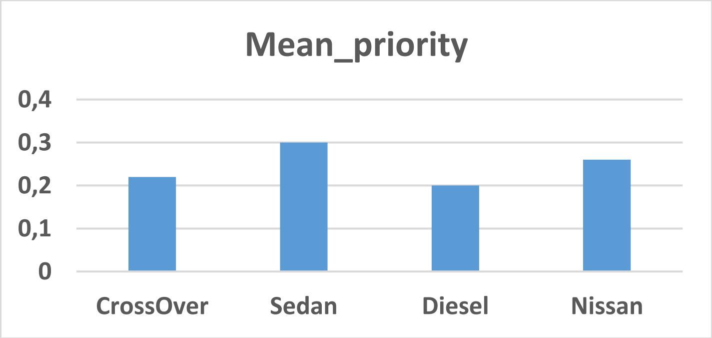


Figure 6. ANP ranking of alternatives.

5 Conclusion

This research employed the ANP technique in neutrosophic environment for solving complex problems, showing the interdependence among criteria, the feedback and the relative weight of decision makers (DMs). We analyzed how to determine the weight for each criterion, and the interdependence among criteria, calculating the weighting of each criterion to each alternative. The proposed model of ANP in neutrosophic environment is based on using of $(n - 1)$ consensus judgments instead of $\frac{n \times (n-1)}{2}$ ones, in order to decrease the workload. We used a new scale from 0 to 1 instead of that from 1 to 9. We also presented a real life example as a case study. In the future, we plan to apply ANP in neutrosophic environment by integrating it with other techniques, such as TOPSIS.

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