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Neutrosophic Linear Fractional Programming Problems

Abdel-Nasser Hussian[1] ▪ Mai Mohamed[2] ▪ Mohamed Abdel-Baset[*2] ▪ Yongquan Zhou[3] ▪ Florentin Smarandache[4]

2 Department of Operations Research, Faculty of Computers and Informatics, Zagazig University, Sharqiyah, Egypt. E-mail: En_mai2011@yahoo.com *E-mail: analyst_mohamed@yahoo.com
3 College of Information Science and Engineering, Guangxi University for Nationalities, China
Email: yongquanzhou@126.com
4 Math & Science Department, University of New Mexico, Gallup, NM 87301, USA.
E-mail: smarand@unm.edu

Abstract

In this chapter, a solution procedure is proposed to solve neutrosophic linear fractional programming (NLFP) problem where cost of the objective function, the resources and the technological coefficients are triangular neutrosophic numbers. Here, the NLFP problem is transformed into an equivalent crisp multi-objective linear fractional programming (MOLFP) problem. By using proposed approach, the transformed MOLFP problem is reduced to a single objective linear programming (LP) problem which can be solved easily by suitable LP problem algorithm.

Keywords

Linear fractional programming; Triangular neutrosophic numbers.

1 Introduction

Linear fractional programming (LFP) is a generalization of linear programming (LP) whereas the objective function in a linear program is a linear function; the objective function in a linear-fractional program is a ratio of two linear functions. Linear fractional programming is used to achieve the highest ratio of profit/cost, inventory/sales, actual cost/standard cost, output/employee, etc. Decision maker may not be able to specify the coefficients (some or all) of LFP problem due to incomplete and imprecise information which tend to be presented in real life situations. Also, aspiration level of objective function and parameters of problem, hesitate decision maker. These situations can be modeled efficiently through neutrosophic environment. Neutrosophy is the study of neutralities as an extension of dialectics. Neutrosophic is the derivative of neutrosophy and it includes neutrosophic set, neutrosophic probability, neutrosophic statistics and neutrosophic logic. Neutrosophic theory means neutrosophy applied in many fields of sciences, in order to solve problems related to indeterminacy. Although intuitionistic fuzzy sets can only handle incomplete information not indeterminate, the neutrosophic set can handle both incomplete and indeterminate information. [1] Neutrosophic sets characterized by three independent degrees namely truth-membership degree (*T*), indeterminacy-membership degree(*I*), and falsity-membership degree (*F*), where *T*,*I*,*F* are standard or non-standard subsets of $]^{-}0, 1^{+}[$. The decision makers in neutrosophic set want to increase the degree of truth-membership and decrease the degree of indeterminacy and falsity membership. The structure of the chapter is as follows: the next section is a preliminary discussion; the third section describes the LFP problem with Charnes and cooper's transformation; the fourth section presents multi-objective linear fractional programming problem; the fifth section presents neutrosophic linear fractional programming problem with solution procedure; the sixth section provides a numerical example to put on view how the approach can be applied; finally, the seventh section provides the conclusion.

2 Preliminaries

In this section, the basic definitions involving neutrosophic set, single valued neutrosophic sets, neutrosophic numbers, triangular neutrosophic numbers and operations on triangular neutrosophic numbers are outlined.

Definition 1. [2]

Let X be a space of points (objects) and $x\in X$. A neutrosophic set A in X is defined by a truth-membership function (x), an indeterminacy-membership function (x) and a falsity-membership function $F(x)$. (x), (x) and $F(x)$ are real standard or real nonstandard subsets of $]^{-}0, 1^{+}[$. That is $TA(x)$:$X\rightarrow]^{-}0, 1^{+}[$, $IA(x)$:$X\rightarrow]^{-}0, 1^{+}[$ and F$A(x)$:$X\rightarrow]^{-}0, 1^{+}[$. There is no restriction on the sum of (x), (x) and $F(x)$, so $0-$ $\leq$ sup $T_A(x)$+ sup $I_A(x)$+ sup $F_A(x)$ $\leq 3+$.

Definition 2. [2]

Let X be a universe of discourse. A single valued neutrosophic set A over X is an object having the form $A=\{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x\in X\}$, where $T_A(x)$:$X\rightarrow[0,1]$, $I_A(x)$:$X\rightarrow[0,1]$ and $F_A(x)$:$X\rightarrow[0,1]$ with $0\leq T_A(x)+ I_A(x)+ F_A(x)\leq 3$ for all $x\in X$. The intervals $T_A(x)$, $I_A(x)$ and $F_A(x)$ denote the truth-membership degree, the indeterminacy-membership degree and the falsity membership degree of x to A, respectively. For convenience, a SVN number is denoted by A= (a, b, c), where a, b, c∈ [0, 1] and a+b+c≤3.

Definition 3.

Let $\tilde{J}$ be a neutrosophic number in the set of real numbers R, then its truth-membership function is defined as

$$T_{\tilde{J}}(J)=\begin{cases}\dfrac{J-a_1}{a_2-a_1}, & a_1 \le J \le a_2,\\ \dfrac{a_2-J}{a_3-a_2}, & a_2 \le J \le a_3,\\ 0, & otherwise.\end{cases} \quad (1)$$

Its indeterminacy-membership function is defined as

$$I_{\tilde{J}}(J)=\begin{cases}\dfrac{J-b_1}{b_2-b_1}, & b_1 \le J \le b_2,\\ \dfrac{b_2-J}{b_3-b_2}, & b_2 \le J \le b_3,\\ 0, & otherwise.\end{cases} \quad (2)$$

And its falsity-membership function is defined as

$$F_{\tilde{J}}(J)=\begin{cases}\dfrac{J-c_1}{c_2-c_1}, & c_1 \le J \le c_2,\\ \dfrac{c_2-J}{c_3-c_2}, & c_2 \le J \le c_3,\\ 1, & otherwise.\end{cases} \quad (3)$$

Definition 4. [3,9]

A triangular neutrosophic number $a^{\sim n} =< (a_1, b_1, c_1); \propto_a^{\sim n}, \theta_a^{\sim n}, \beta_a^{\sim n}) >$ is a special neutrosophic set on the real number set R, where $\propto_a^{\sim n}, \theta_a^{\sim n}, \beta_a^{\sim n} \in [0,1]$

The truth-membership, indeterminacy- membership and falsity- membership functions are defined as follows:

$$T_{a^{\sim n}}(x) = \begin{cases} \dfrac{(x-a_1)\propto_a^{\sim n}}{(b_1-a_1)} & if\ a_1 \le x \le b_1 \\ \propto_a^{\sim n} & if\ \ x = b_1 \\ \dfrac{(c_1-x)\propto_a^{\sim n}}{(c_1-b_1)} & if\ b_1 < x \le c_1 \\ 0 & otherwise \end{cases} \tag{4}$$

$$I_{a^{\sim n}}(x) = \begin{cases} \dfrac{(b_1-x+\theta_a^{\sim n}((x-a_1))}{(b_1-a_1)} & if\ a_1 \le x \le b_1 \\ \theta_a^{\sim n} & if\ \ x = b_1 \\ \dfrac{(x-b_1+\theta_a^{\sim n}(c_1-x))}{(c_1-b_1)} & if\ b_1 < x \le c_1 \\ 1 & otherwise \end{cases} \tag{5}$$

$$F_{a^{\sim n}}(x) = \begin{cases} \dfrac{(b_1-x+\beta_a^{\sim n}((x-a_1))}{(b_1-a_1)} & if\ a_1 \le x \le b_1 \\ \beta_a^{\sim n} & if\ \ x = b_1 \\ \dfrac{(x-b_1+\beta_a^{\sim n}(c_1-x))}{(c_1-b_1)} & if\ b_1 < x \le c_1 \\ 1 & otherwise \end{cases} \tag{6}$$

If $a_1 \geq 0$ and at least $c_1 > 0$ then:

$$a^{\sim n} = < (a_1, b_1, c_1); \propto_a^{\sim n}, \theta_a^{\sim n}, \beta_a^{\sim n}) >$$

is called a positive triangular neutrosophic number, denoted by $a^{\sim n} > 0$. Likewise, if $c_1 \leq 0$ and at least $a_1 < 0$, then:

$$a^{\sim n} = < (a_1, b_1, c_1); \propto_a^{\sim n}, \theta_a^{\sim n}, \beta_a^{\sim n}) >$$

is called a negative triangular neutrosophic number, denoted by $a^{\sim n} < 0$.

Definition 5. [3,10]

Let

$$a^{\sim n} = < (a_1, b_1, c_1); \propto_a^{\sim n}, \theta_a^{\sim n}, \beta_a^{\sim n}) >,$$

and

$$b^{\sim n} = < (a_2, b_2, c_2); \propto_b^{\sim n}, \theta_b^{\sim n}, \beta_b^{\sim n}) >$$

be two single valued triangular neutrosophic numbers and $\gamma \neq 0$ be any real number, then:

$$a^{\sim n} + b^{\sim n} = < (a_1 + a_2\,, b_1 + b_2, c_1 + c_2);\ \propto_a^{\sim n} \wedge \propto_b^{\sim n}, \theta_a^{\sim n} \vee$$

$\theta_b^{\sim n}, \beta_a^{\sim n} \vee \beta_b^{\sim n} >$,

$a^{\sim n}$ - $b^{\sim n} =< (a_1 - c_2, b_1 - b_2, c_1 - a_2); \propto_a^{\sim n} \wedge \propto_b^{\sim n}, \theta_a^{\sim n} \vee \theta_b^{\sim n}, \beta_a^{\sim n} \vee \beta_b^{\sim n} >$,

$$a^{\sim n} b^{\sim n} = \begin{cases} < (a_1 a_2, b_1 b_2, c_1 c_2); \propto_a^{\sim n} \wedge \propto_b^{\sim n}, \theta_a^{\sim n} \vee \theta_b^{\sim n}, \beta_a^{\sim n} \vee \beta_b^{\sim n} > & (c_1 > 0, c_2 > 0) \\ < (a_1 c_2, b_1 b_2, c_1 a_2); \propto_a^{\sim n} \wedge \propto_b^{\sim n}, \theta_a^{\sim n} \vee \theta_b^{\sim n}, \beta_a^{\sim n} \vee \beta_b^{\sim n} > & (c_1 < 0, c_2 > 0) \\ < (c_1 c_2, b_1 b_2, a_1 a_2); \propto_a^{\sim n} \wedge \propto_b^{\sim n}, \theta_a^{\sim n} \vee \theta_b^{\sim n}, \beta_a^{\sim n} \vee \beta_b^{\sim n} > & (c_1 < 0, c_2 < 0) \end{cases}$$

$$\gamma a^{\sim n} = \begin{cases} < (\gamma a_1, \gamma b_1, \gamma c_1); \propto_a^{\sim n}, \theta_a^{\sim n}, \beta_a^{\sim n} > & (\gamma > 0) \\ < (\gamma c_1, \gamma b_1, \gamma a_1); \propto_a^{\sim n}, \theta_a^{\sim n}, \beta_a^{\sim n} > & (\gamma < 0) \end{cases}$$

$$a^{\sim n -1} =< \left(\frac{1}{c_1}, \frac{1}{b_1}, \frac{1}{a_1}\right); \propto_a^{\sim n}, \theta_a^{\sim n}, \beta_a^{\sim n} > (a^{\sim n} \neq 0)$$

$$a^{\sim n}/b^{\sim n} = \begin{cases} < (a_1/c_2, b_1/b_2, c_1/a_2); \propto_a^{\sim n} \wedge \propto_b^{\sim n}, \theta_a^{\sim n} \vee \theta_b^{\sim n}, \beta_a^{\sim n} \vee \beta_b^{\sim n} > & (c_1 > 0, c_2 > 0) \\ < (c_1/c_2, b_1/b_2, a_1/a_2); \propto_a^{\sim n} \wedge \propto_b^{\sim n}, \theta_a^{\sim n} \vee \theta_b^{\sim n}, \beta_a^{\sim n} \vee \beta_b^{\sim n} > & (c_1 < 0, c_2 > 0) \\ < (c_1/a_2, b_1/b_2, a_1/c_2); \propto_a^{\sim n} \wedge \propto_b^{\sim n}, \theta_a^{\sim n} \vee \theta_b^{\sim n}, \beta_a^{\sim n} \vee \beta_b^{\sim n} > & (c_1 < 0, c_2 < 0) \end{cases}$$

3 Linear Fractional Programming Problem (LFPP)

In this section, the general form of LFP problem is discussed. Also, Charnes and Cooper's [4] linear transformation is summarized.

The linear fractional programming (LFP) problem can be written as:

$$\text{Max } Z(x) = \frac{\sum c_j x_j + p}{\sum d_j x_j + q} = \frac{c^T x + p}{d^T x + q} = \frac{N(x)}{D(x)}, \qquad (7)$$

Subject to

$x \in s = \{x \in R^n : Ax = b, x \geq 0\}$,

where $j = 1,2, \ldots, n, A \in R^{m \times n}$, $b \in R^m$, $c_j, d_j \in R^n$, and $p, q \in R$. For some values of x, $D(x)$ may be equal to zero. To avoid such cases, we require that either $\{Ax \leq b, x \geq 0 \Rightarrow D(x) > 0\}$ or $\{Ax \leq b, x \geq 0 \Rightarrow D(x) < 0\}$. For convenience here, we consider the first case, i.e.

$$\{AX = b, x \geq 0, D(x) > 0\} \qquad (8)$$

Using Charnes and Cooper's linear transformation the previous LFP problem is equivalent to the following linear programming (LP) problem:

Max $c^T y + pt$,

Subject to

$$d^T y + qt = 1, \tag{9}$$

$$Ay - bt = 0,$$

$$t \geq 0, y \geq 0, y \in R^n, t \in R.$$

Consider the fractional programming problem

$$\text{Max } Z(x) = \frac{N(x)}{D(x)}, \tag{10}$$

Subject to

$$Ax \leq b, x \geq 0,$$

$$x \in \Delta = \{x: Ax \leq b, x \geq 0, D(x) > 0$$

$$x \in \Delta = \{x: Ax \leq b, x \geq 0 \Rightarrow D(x) > 0\}$$

By the transformation $t = \frac{1}{D(x)}$, $y = tx$ we obtained the following:

$$\text{Max } tN\left(\frac{y}{t}\right),$$

Subject to

$$A\left(\frac{y}{t}\right) - b \leq 0,$$

$$tD\left(\frac{y}{t}\right) = 1,$$

$$t > 0, y \geq 0. \tag{11}$$

By replacing the equality constraint $tD\left(\frac{y}{t}\right) = 1$ by an inequality constraint $tD\left(\frac{y}{t}\right) \leq 1$

We obtained the following:

$$\text{Max } tN\left(\frac{y}{t}\right),$$

Subject to

$$A\left(\frac{y}{t}\right) - b \leq 0,$$

$$tD\left(\frac{y}{t}\right) \leq 1,$$

$$t > 0, y \geq 0. \qquad (12)$$

If in equation 10, $N(x)$ is concave, $D(x)$ is concave and positive on Δ, and $N(x)$ is negative for each $x \in \Delta$, then $Max_{x\in\Delta} \frac{N(x)}{D(x)} \Leftrightarrow Min_{x\in\Delta} \frac{-N(x)}{D(x)} \Leftrightarrow Max_{x\in\Delta} \frac{D(x)}{-N(x)}$, where $-N(x)$ is convex and positive. Now linear fractional program (10) transformed to the following LP problem:

Max $tD\left(\frac{y}{t}\right)$,

Subject to

$$A\left(\frac{y}{t}\right) - b \leq 0,$$

$$-tN\left(\frac{y}{t}\right) \leq 1,$$

$$t > 0, y \geq 0. \qquad (13)$$

4 Multi-objective Linear Fractional Programming Problem

In this section, the general form of MOLFP problem is discussed and the procedure for converting MOLFP problem into MOLP problem is illustrated.

The MOLFP problem can be written as follows:

Max $z(x) = [z_1(x), z_2(x), \ldots, z_k(x)]$,

Subject to

$$x \in \Delta = \{x: Ax \leq b, x \geq 0\} \qquad (14)$$

With $b \in R^m, A \in R^{m\times n}, and\ z_i(x) = \frac{c_i x + p_i}{d_i x + q_i} = \frac{N_i(x)}{D_i(x}$, $c_i, d_i \in R^n$ and $p_i, q_i \in R, i = 1,2, \ldots, k$.

Let I be the index set such that $I = \{i:\ N_i(x) \geq 0\ for\ x \in \Delta\}$ and

$I^c = \{i:\ N_i(x) < 0\ for\ x \in \Delta\}, where\ I \cup I^c = \{1,2,\ldots,K\}$. Let $D(x)$ be positive on Δ where Δ is non-empty and bounded. For simplicity, let us take the least value of 1/ $(d_i x + q_i)$ and 1/ [-$(c_i x + p_i)$] is t for $i \in I$ and $i \in I^c$, respectively i.e.

$$\frac{1}{(d_i x + q_i)} \geq t \text{ for } i \in I \text{ and } \frac{-1}{(c_i x + p_i)} \geq t \text{ for } i \in I^c \qquad (15)$$

By using the transformation $y = tx\ (t > 0)$, and equation 15, MOLFP problem (14) may be written as follows:

Max $z_i(y,t)$={ $tN_i\left(\frac{y}{t}\right)$, for $i \in I$; $tD_i\left(\frac{y}{t}\right)$, for $i \in I^c$}

Subject to

$tD_i\left(\frac{y}{t}\right) \le 1$, for $i \in I$,

- $tN_i\left(\frac{y}{t}\right) \le 1$, for $i \in I^c$, (16)

$A\left(\frac{y}{t}\right) - b \le 0$,

$t, y \ge 0$.

If $i \in I$, then truth- membership function of each objective function can be written as:

$$T_i\left(tN_i\left(\frac{y}{t}\right)\right) = \begin{cases} 0 & if\ tN_i\left(\frac{y}{t}\right) \le 0, \\ \frac{tN_i\left(\frac{y}{t}\right)}{z_i - a_i} & if 0 \le tN_i\left(\frac{y}{t}\right) \le z_i + a_i\ , \\ 1 & if\ tN_i\left(\frac{y}{t}\right) \ge z_i + a_i \end{cases} \tag{17}$$

If $i \in I^c$, then truth- membership function of each objective function can be written as:

$$T_i\left(tD_i\left(\frac{y}{t}\right)\right) = \begin{cases} 0 & if\ tD_i\left(\frac{y}{t}\right) \le 0, \\ \frac{tD_i\left(\frac{y}{t}\right)}{z_i - a_i} & if 0 \le tD_i\left(\frac{y}{t}\right) \le z_i + a_i\ , \\ 1 & if\ tD_i\left(\frac{y}{t}\right) \ge z_i + a_i \end{cases} \tag{18}$$

If $i \in I$, then falsity- membership function of each objective function can be written as:

$$F_i\left(tN_i\left(\frac{y}{t}\right)\right) = \begin{cases} 1 & if\ tN_i\left(\frac{y}{t}\right) \le 0, \\ 1 - \frac{tN_i\left(\frac{y}{t}\right)}{z_i - c_i} & if 0 \le tN_i\left(\frac{y}{t}\right) \le z_i + c_i\ , \\ 0 & if\ tN_i\left(\frac{y}{t}\right) \ge z_i + c_i \end{cases} \tag{19}$$

If $i \in I^c$, then falsity- membership function of each objective function can be written as:

$$F_i\left(tD_i\left(\frac{y}{t}\right)\right) = \begin{cases} 1 & if\ tD_i\left(\frac{y}{t}\right) \le 0, \\ 1 - \frac{tD_i\left(\frac{y}{t}\right)}{z_i - c_i} & if 0 \le tD_i\left(\frac{y}{t}\right) \le z_i + c_i\ , \\ 0 & if\ tD_i\left(\frac{y}{t}\right) \ge z_i + c_i \end{cases} \quad (20)$$

If $i \in I$, then indeterminacy- membership function of each objective function can be written as

$$I_i\left(tN_i\left(\frac{y}{t}\right)\right) = \begin{cases} 0 & if\ tN_i\left(\frac{y}{t}\right) \le 0, \\ \frac{tN_i\left(\frac{y}{t}\right)}{z_i - d_i} & if 0 \le tN_i\left(\frac{y}{t}\right) \le z_i + d_i\ , \\ 0 & if\ tN_i\left(\frac{y}{t}\right) \ge z_i + d_i \end{cases} \quad (21)$$

If $i \in I^c$, then indeterminacy - membership function of each objective function can be written as:

$$I_i\left(tD_i\left(\frac{y}{t}\right)\right) = \begin{cases} 0 & if\ tD_i\left(\frac{y}{t}\right) \le 0, \\ \frac{tD_i\left(\frac{y}{t}\right)}{z_i - d_i} & if 0 \le tD_i\left(\frac{y}{t}\right) \le z_i + d_i\ , \\ 0 & if\ tD_i\left(\frac{y}{t}\right) \ge z_i + d_i \end{cases} \quad (22)$$

where, a_i , d_i and c_i are acceptance tolerance, indeterminacy tolerance and rejection tolerance.

Zimmermann [5] proved that if membership function $\mu_D(y, t)$ of complete solution set(y, t), has a unique maximum value$\mu_D(y^*, t^*)$ then (y^*, t^*) which is an element of complete solution set (y, t) can be derived by solving linear programming with one variableλ.

Using Zimmermann's min operator and membership functions, the model (14) transformed to the crisp model as:

Maxλ,

Subject to

$$T_i\left(tN_i\left(\frac{y}{t}\right)\right) \geq \lambda \text{ , for } i \in I$$

$$T_i\left(tD_i\left(\frac{y}{t}\right)\right) \geq \lambda \text{ , for } i \in I^c$$

$$F_i\left(tN_i\left(\frac{y}{t}\right)\right) \leq \lambda \text{ , for } i \in I$$

$$F_i\left(tD_i\left(\frac{y}{t}\right)\right) \leq \lambda \text{ , for } i \in I^c$$

$$I_i\left(tN_i\left(\frac{y}{t}\right)\right) \leq \lambda \text{ , for } i \in I$$

$$I_i\left(tD_i\left(\frac{y}{t}\right)\right) \leq \lambda \text{ , for } i \in I^c$$

$$tD_i\left(\frac{y}{t}\right) \leq 1, \quad \text{for } i \in I,$$

$$- tN_i\left(\frac{y}{t}\right) \leq 1, \quad \text{for } i \in I^c, \tag{23}$$

$$A\left(\frac{y}{t}\right) - b \leq 0,$$

$$t, y, \lambda \geq 0 .$$

5 Neutrosophic Linear Fractional Programming Problem

In this section, we propose a procedure for solving neutrosophic linear fractional programming problem where the cost of the objective function, the resources, and the technological coefficients are triangular neutrosophic numbers.

Let us consider the NLFP problem:

$$\text{Max } z(x^{\sim n}) = \frac{\sum c_j^{\sim n} x_j + p^{\sim n}}{\sum d_j^{\sim n} x_j + q^{\sim n}}$$

Subject to

$$\sum a_{ij}^{\sim n} x_j \leq b_i^{\sim n}, i = 1,2, \ldots, m, \tag{24}$$

$$x_j \geq 0, j = 1,2, \ldots, n.$$

We assume that $c_j^{\sim n}$, $p^{\sim n}, d_j^{\sim n}, q^{\sim n}, a_{ij}^{\sim n}$ *and* $b_i^{\sim n}$ are triangular neutrosophic numbers for each $i = 1,2, \ldots, m$ and $j = 1,2, \ldots, n.$ therefore, the problem (24) can be written as: (25)

$$\text{Max } z(x^{\sim n}) = \frac{\sum(c_{j1}, c_{j2}, c_{j3};\ \alpha_c, \theta_c, \beta_c)x_j + (p_1, p_2, p_3;\ \alpha_p, \theta_p, \beta_p)}{\sum(d_{j1}, d_{j2}, d_{j3};\ \alpha_d, \theta_d, \beta_d)x_j + (q_1, q_2, q_3;\ \alpha_q, \theta_q, \beta_q)}$$

Subject to

$$\sum(a_{ij1}, a_{ij2}, a_{ij3};\ \alpha_a, \theta_a, \beta_a)x_j \leq (b_{i1}, b_{i2}, b_{i3}; \alpha_b, \theta_b, \beta_b), i = 1,2, \ldots, m,$$

$$x_j \geq 0, j = 1,2, \ldots, n.$$

where $\propto, \theta, \beta \in [0,1]$ and stand for truth-membership, indeterminacy and falsity-membership function of each neutrosophic number.

Here decision maker wants to increase the degree of truth-membership and decrease the degree of indeterminacy and falsity membership. Using the concept of component wise optimization, the problem (25) reduces to an equivalent MOLFP as follows:

$$\text{Max } z_1(x) = \frac{\sum c_{j1}\ x_j + p_1}{\sum d_{j3}\ x_j + q_3},$$

$$\text{Max } z_2(x) = \frac{\sum c_{j2}\ x_j + p_2}{\sum d_{j2}\ x_j + q_2},$$

$$\text{Max } z_3(x) = \frac{\sum c_{j3}\ x_j + p_3}{\sum d_{j1}\ x_j + q_1}, \tag{26}$$

$$\text{Max } z_4(x) = \frac{\sum \alpha_c x_j + \alpha_p}{\sum \beta_d x_j + \beta_q},$$

$$\text{Max } z_5(x) = 1 - \frac{\sum \theta_{c,} x_j + \theta_p}{\sum \theta_{d,} x_j + \theta_q},$$

$$\text{Max } z_6(x) = 1 - \frac{\sum \beta_c x_j + \beta_p}{\sum \alpha_d x_j + \alpha_q},$$

Subject to

$$\sum a_{ij1} x_j \leq b_{i1},$$

$$\sum a_{ij2} x_j \leq b_{i2},$$

$$\sum a_{ij3} x_j \leq b_{i3},$$

$$\sum \alpha_a x_j \leq \alpha_b,$$

$$\sum \theta_{a,} x_j \leq \theta_b,$$

$$\sum \beta_{a,} x_j \leq \beta_b,$$

$$x_j \geq 0, i = 1,2, \dots, m;\ j = 1,2, \dots, n.$$

Let us assume that z_1, z_2, z_3, z_4, z_5 and $z_6 \geq 0$ for the feasible region. Hence, the MOLFP problem can be converted into the following MOLP problem:

$$\text{Max } z_1(y,t) = \sum c_{j1}\ y_j + p_1 t,$$

$$\text{Max } z_2(y,t) = \sum c_{j2}\ y_j + p_2 t,$$

$$\text{Max } z_3(y,t) = \sum c_{j3}\ y_j + p_3 t,$$

$$\text{Max } z_4(y,t) = \sum \alpha_c y_j + \alpha_p t,$$

$$\text{Max } z_5(y,t) = 1 - \left(\sum \theta_c y_j + \theta_p t\right),$$

$$\text{Max } z_6(y,t) = 1 - \left(\sum \beta_c y_j + \beta_p t\right),$$

Subject to

$$\sum d_{j3}\ y_j + q_3 t \leq 1,$$

$$\sum d_{j2}\ y_j + q_2 t \leq 1,$$

$$\sum d_{j1}\ y_j + q_1 t \leq 1,$$

$$\sum \beta_d y_j + \beta_q t \leq 1,$$

$$\sum \theta_{d,} y_j + \theta_q t \leq 1,$$

$$\sum \alpha_d y_j + \alpha_q t \leq 1,$$

$$\sum a_{ij1}\ y_j - b_{i1} t \leq 0,$$

$$\sum a_{ij2}\ y_j - b_{i2} t \leq 0,$$

$$\sum a_{ij3}\, y_j \;- b_{i3} t \leq 0,$$
$$\sum \alpha_a \; y_j \;- \alpha_b \, t \leq 0,$$
$$\sum \theta_a \; y_j \;- \theta_{b,} t \leq 0, \qquad (27)$$
$$\sum \beta_a \; y_j \;- \beta_b \, t \leq 0,$$
$$t, y_j \;\geq 0, i = 1,2, \ldots, m; j = 1,2, \ldots, n.$$

Solving the transformed MOLP problem for each objective function, we obtain $z_1^*, z_2^*, z_3^*, z_{4,}^* z_5^*$ and z_6^* .

Using the membership functions defined in previous section, the above model reduces to:

Max λ,

Subject to

$$\sum c_{j1}\, y_j \;+ p_1\, t - z_1^* \lambda \geq 0,$$
$$\sum c_{j2}\, y_j \;+ p_2\, t - z_2^* \lambda \geq 0,$$
$$\sum c_{j3}\;\, y_j \;+ p_3\, t - z_3^* \lambda \geq 0,$$
$$\sum \alpha_c\, y_j \;+ \alpha_p\; t - z_4^* \lambda \geq 0,$$
$$1 - \left(\sum \theta_c\, y_j \;+ \theta_p\, t\right) - z_5^* \lambda \leq 0,$$
$$1 - \left(\sum \beta_c y_j \;+ \beta_p\, t\right) - z_6^* \lambda \leq 0,$$
$$\sum d_{j3}\;\, y_j \;+ q_3\, t \leq 1,$$
$$\sum d_{j2}\;\, y_j \;+ q_2\, t \leq 1,$$
$$\sum d_{j1}\; y_j \;+ q_1\, t \leq 1,$$

$$\sum \beta_d y_j + \beta_q t \leq 1,$$

$$\sum \theta_{d,} y_j + \theta_q t \leq 1,$$

$$\sum \alpha_d y_j + \alpha_q t \leq 1,$$

$$\sum a_{ij1} y_j - b_{i1} t \leq 0,$$

$$\sum a_{ij2} y_j - b_{i2} t \leq 0,$$

$$\sum a_{ij3} y_j - b_{i3} t \leq 0,$$

$$\sum \alpha_a y_j - \alpha_b t \leq 0,$$

$$\sum \theta_a y_j - \theta_{b,} t \leq 0,$$

$$\sum \beta_a y_j - \beta_{b,} t \leq 0, t, y_j, \lambda \geq 0, i = 1,2, \dots, m; j = 1,2, \dots, n$$

$$t, y_j, \lambda \geq 0, i = 1,2, \dots, m; j = 1,2, \dots, n. \qquad (28)$$

5.1 Algorithm

The proposed approach for solving NLFP problem can be summarized as follows:

Step 1. The NLFP problem is converted into MOLFP problem using componentwise optimization of triangular neutrosophic numbers.

Step 2. The MOLFP problem is transformed into MOLP problem using the method proposed by Charnes and Cooper.

Step 3. Solve each objective function subject to the given set of constraints.

Step 4. Define membership functions for each objective function as in section four.

Step 5. Use Zimmermann's operator and membership functions to obtain crisp model.

Step 6. Solve crisp model by using suitable algorithm.

6 Numerical Example

A company manufactures 3 kinds of products I, II and III with profit around 8, 7 and 9 dollars per unit, respectively. However, the cost for each one unit of the products is around 8, 9 and 6 dollars, respectively. Also it is assumed that a fixed cost of around 1.5 dollars is added to the cost function due to expected duration through the process of production. Suppose the materials needed for manufacturing the products I, II and III are about 4, 3 and 5 units per pound, respectively. The supply for this raw material is restricted to about 28 pounds. Man-hours availability for product I is about 5 hours, for product II is about 3 hours, and that for III is about 3 hours in manufacturing per units. Total man-hours availability is around 20 hours daily. Determine how many products of I, II and III should be manufactured in order to maximize the total profit. Also during the whole process, the manager hesitates in prediction of parametric values due to some uncontrollable factors.

Let x_1, x_2 and x_3 units be the amount of I, II and III, respectively to be produced. After prediction of estimated parameters, the above problem can be formulated as the following NLFPP:

$$\text{Max } Z(x^{\sim n}) = \frac{8^{\sim n}x_1 + 7^{\sim n}x_2 + 9^{\sim n}x_3}{8^{\sim n}x_1 + 9^{\sim n}x_2 + 6^{\sim n}x_3 + 1.5^{\sim n}}$$

Subject to

$$\begin{aligned} &4^{\sim n}x_1 + 3^{\sim n}x_2 + 5^{\sim n}x_3 \le 28^{\sim n}, \\ &5^{\sim n}x_1 + 3^{\sim n}x_2 + 3^{\sim n}x_3 \le 20^{\sim n}, \\ &x_1, x_2, x_3 \ge 0. \end{aligned} \tag{29}$$

with

$8^{\sim n} = (7,8,9; 0.5,0.8,0.3)$, $7^{\sim n} = (6,7,8; 0.2,0.6,0.5)$,
$9^{\sim n} = (8,9,10; 0.8, 0.1, 0.4)$,
$6^{\sim n} = (4,6,8; 0.75, 0.25, 0.1)$, $1.5^{\sim n} = (1, 1.5, 2; 0.75, 0.5, 0.25)$,
$4^{\sim n} = (3,4,5; 0.4, 0.6, 0.5)$, $3^{\sim n} = (2,3,4; 1, 0.25, 0.3)$,
$5^{\sim n} = (4,5,6; 0.3, 0.4, 0.8)$, $28^{\sim n} = (25,28,30; 0.4, 0.25, 0.6)$,
$20^{\sim n} = (18,20,22; 0.9, 0.2, 0.6)$.

This problem is equivalent to the following MOLFPP:

$$\text{Max} z_1(x) = \frac{7x_1 + 6x_2 + 8x_3}{9x_1 + 10x_2 + 8x_3 + 2},$$

$$\text{Max} z_2(x) = \frac{8x_1 + 7x_2 + 9x_3}{8x_1 + 9x_2 + 6x_3 + 1.5},$$

$$\text{Max} z_3(x) = \frac{9x_1 + 8x_2 + 10x_3}{7x_1 + 8x_2 + 4x_3 + 1},$$

$$\text{Max} z_4(x) = \frac{0.5x_1+0.2x_2+0.8x_3}{0.3x_1+0.4x_2+0.1x_3+0.25},$$
$$\text{Max} z_5(x) = 1 - \frac{0.8x_1+0.6x_2+0.1x_3}{0.8x_1+0.1x_2+0.25x_3+0.5},$$
$$\text{Max} z_6(x) = 1 - \frac{0.3x_1+0.5x_2+0.4x_3}{0.5x_1+0.8x_2+0.75x_3+0.75}, \quad (30)$$

Subject to

$3x_1 + 2x_2 + 4x_3 \leq 25,$

$4x_1 + 3x_2 + 5x_3 \leq 28,$

$5x_1 + 4x_2 + 6x_3 \leq 30,$

$4x_1 + 2x_2 + 2x_3 \leq 18,$

$5x_1 + 3x_2 + 3x_3 \leq 20,$

$6x_1 + 4x_2 + 4x_3 \leq 22,$

$0.4x_1 + x_2 + 0.3x_3 \leq 0.4,$

$0.6x_1 + 0.25x_2 + 0.4x_3 \leq 0.25,$

$0.5x_1 + 0.3x_2 + 0.8x_3 \leq 0.5,$

$0.3x_1 + x_2 + x_3 \leq 0.9,$

$0.4x_1 + 0.25x_2 + 0.25x_3 \leq 0.2,$

$0.8x_1 + 0.3x_2 + 0.3x_3 \leq 0.6,$

Using the transformation, the problem is equivalent to the following MOLPP:

$$\text{Max } z_1(y,t) = 7y_1 + 6y_2 + 8y_3,$$
$$\text{Max } z_2(y,t) = 8y_1 + 7y_2 + 9y_3,$$
$$\text{Max } z_3(y,t) = 9y_1 + 8y_2 + 10y_3,$$
$$\text{Max } z_4(y,t) = 0.5y_1 + 0.2y_2 + 0.8y_3,$$
$$\text{Max } z_5(y,t) = 0.5y_2 + 0.15y_3 + 0.5,$$
$$\text{Max } z_6(y,t) = 0.2y_1 + 0.3y_2 + 0.35y_3 + 0.75, \quad (31)$$

Subject to

$9y_1 + 10y_2 + 8y_3 + 2t \leq 1,$

$8y_1 + 9y_2 + 6y_3 + 1.5t \leq 1,$

$7y_1 + 8y_2 + 4y_3 + t \leq 1,$

$0.3y_1 + 0.4y_2 + 0.1y_3 \ + 0.25t \leq 1$,

$0.8y_1 + 0.1y_2 + 0.25y_3 \ + 0.5t \leq 1$,

$0.5y_1 + 0.8y_2 + 0.75y_3 \ + 0.75t \leq 1$,

$3y_1 + 2y_2 + 4y_3 \ - 25t \leq 0$,

$4y_1 + 3y_2 + 5y_3 \ - 28t \leq 0$,

$5y_1 + 4y_2 + 6y_3 \ - 30t \leq 0$,

$4y_1 + 2y_2 + 2y_3 \ - 18t \leq 0$,

$5y_1 + 3y_2 + 3y_3 \ - 20t \leq 0$,

$6y_1 + 4y_2 + 4y_3 \ - 22t \leq 0$,

$0.4y_1 + y_2 + 0.3y_3 \ - 0.4t \leq 0$,

$0.6y_1 + 0.25y_2 + 0.4y_3 \ - 0.25t \leq 0$,

$0.5y_1 + 0.3y_2 + 0.8y_3 \ - 0.5t \leq 0$,

$0.3y_1 + y_2 + y_3 \ - 0.9t \leq 0$,

$y_1, y_2, y_3 \ \geq 0, \qquad t > 0.$

Solving each objective at a time we get

Z1= 0.7143

Z2=0.8036

Z3=0.8929

Z4=0.0714

Z5=0.833

Z6=0.7813.

$z_1 \ = \quad , z_2 \ = \quad , z_3 = \quad , z_4 = \quad , z_5 \ = \ , z_6 =$

Now the previous problem can be reduced to the following LPP:

Max λ

Subject to

$7y_1 + 6y_2 + 8y_3 \ - z_1 \ \lambda \ \geq 0,$

$8y_1 + 7y_2 + 9y_3 \ - z_2 \ \lambda \ \geq 0,$

$9y_1 + 8y_2 + 10y_3 \ - z_3 \ \lambda \ \geq 0,$

$$
\begin{aligned}
&0.5y_1 + 0.2y_2 + 0.8y_3 \ - z_4 \ \lambda \ \geq 0, \\
&0.5y_2 + 0.15y_3 \ + 0.5 - z_5 \ \lambda \ \leq 0, \\
&0.2y_1 + 0.3y_2 + 0.3y_3 \ + 0.75 - z_6 \ \lambda \ \leq 0, \\
&9y_1 + 10y_2 + 8y_3 \ + 2t \leq 1\,, \\
&8y_1 + 9y_2 + 6y_3 \ + 1.5t \leq 1\,, \\
&7y_1 + 8y_2 + 4y_3 \ + t \leq 1\,, \\
&0.3y_1 + 0.4y_2 + 0.1y_3 \ + 0.25t \leq 1\,, \\
&0.8y_1 + 0.1y_2 + 0.25y_3 \ + 0.5t \leq 1\,, \\
&0.5y_1 + 0.8y_2 + 0.75y_3 \ + 0.75t \leq 1\,, \\
&3y_1 + 2y_2 + 4y_3 \ - 25t \leq 0\,, \\
&4y_1 + 3y_2 + 5y_3 \ - 28t \leq 0\,, \\
&5y_1 + 4y_2 + 6y_3 \ - 30t \leq 0\,, \\
&4y_1 + 2y_2 + 2y_3 \ - 18t \leq 0\,, \\
&5y_1 + 3y_2 + 3y_3 \ - 20t \leq 0\,, \\
&6y_1 + 4y_2 + 4y_3 \ - 22t \leq 0\,, \\
&0.4y_1 + y_2 + 0.3y_3 \ - 0.4t \leq 0\,, \\
&0.6y_1 + 0.25y_2 + 0.4y_3 \ - 0.25t \leq 0\,, \\
&0.5y_1 + 0.3y_2 + 0.8y_3 \ - 0.5t \leq 0\,, \\
&0.3y_1 + y_2 + y_3 \ - 0.9t \leq 0\,, \\
&y_1, y_2, y_3 \ \geq 0, \qquad t > 0.
\end{aligned}
\tag{32}
$$

Solving by LINGO we have

y_1=0

y_2=0

y_3=0.0893

t=0.1429

λ=1

7 Conclusion

In this chapter, a method for solving the NLFP problems where the cost of the objective function, the resources and the technological coefficients are triangular neutrosophic numbers, is proposed. In the method, NLFP problem is transformed to a MOLFP problem and the resultant problem is converted to a LP problem. In future, the proposed approach can be extended for solving multi-objective neutrosophic linear fractional programming problems (MONLFPPs).

Acknowledgements

The authors would like to thank anonymous referees for the constructive suggestions that improved both the quality and clarity of the chapter.

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