IX

Using Neutrosophic Sets to Obtain PERT Three-Times Estimates in Project Management

Mai Mohamed[1*] ▪ Mohamed Abdel-Basset[1] ▪ Abdel-Nasser Hussien[2] ▪ Florentin Smarandache[3]

1 Department of Operations Research, Faculty of Computers and Informatics, Zagazig University, Sharqiyah, Egypt. E-mail: en_mai2011@yahoo.com E-mail: analyst_mohamed@yahoo.com
2 Department of Information System, Faculty of Computers and Informatics, Zagazig University, Sharqiyah, Egypt. E-mail: nasserhr@gmail.com
3 Math & Science Department, University of New Mexico, Gallup, NM 87301, USA.
E-mail: smarand@unm.edu

Abstract

Neutrosophic sets have been introduced as a generalization of crisp sets, fuzzy sets, and intuitionistic fuzzy sets to represent uncertain, inconsistent and incomplete information about real world problems. Elements of neutrosophic set are characterized by a truth-membership, falsity-membership and indeterminacy membership functions. For the first time, this chapter attempts to introduce the mathematical representation of Program Evaluation and Review Technique (PERT) in neutrosophic environment. Here the elements of three-times estimates of PERT are considered as neutrosophic elements. Score and accuracy functions are used to obtain crisp model of problem. The proposed method has been demonstrated by a suitable numerical example.

Keywords

Neutrosophic Sets, Project, Project Management, Gantt chart, CPM, PERT, Three-Time Estimate.

1 Introduction

A project is a one-time job that has a definite starting and ending dates, a clearly specified objective, a scope of work to be performed and a predefined budget. Each part of the project has an effect on overall project execution time, so project completion on time depends on rightly scheduled plan. The main problem here is wrongly calculated activity durations due to lack of knowledge and experience. Lewis [1] defines project management as "the planning, scheduling and controlling of project activities to achieve project objectives-performance, cost and time for a given scope of work". The most popularly used techniques for project management are Gantt chart, Program Evaluation and Review Technique (PERT) and Critical Path Method (CPM). Gantt chart is an early technique of planning and controlling projects. Gantt charts are simple to construct, easy to understand and change. They can show plan and actual progress. However, it does not show interrelationships of activities. To overcome the limitation of Gantt chart, two project planning techniques-PERT and CPM were developed in 1950s. Both use a network and graphical model of a project, showing the activities, their interrelationships and starting and ending dates. In case of CPM, activity time can be estimated accurately and it does not vary much. In recent years, by depending on the fuzzy set theory for managing projects there were different PERT methods. However, the existing methods of fuzzy PERT have some drawbacks [2]:

– Cannot find a critical path in a fuzzy project network.

– The increasing of the possible critical paths, which is the higher risk path.

– Can't determine indeterminacy, which exist in real life situations.

In case of PERT, time estimates vary significantly [3][4]. Here three-time estimates which are optimistic(a), pessimistic(b) and most likely(m) are used. In practice, a question often arises as to how obtain good estimates of a, b, and m. The person who responsible for determining values of a, b, and m often face real problem due to uncertain, inconsistent, and incomplete information about real world. It is obvious that neutrosophic set theory is more appropriate than fuzzy set in modeling uncertainty that is associated with parameters such as activity duration time and resource availability in PERT. By using neutrosophic set theory in PERT technique, we can also overcome the drawbacks of fuzzy PERT methods. This chapter is organized as follows: In section 2, the basic concepts neutrosophic sets are briefly reviewed. In section 3, the mathematical model of neutrosophic PERT and the proposed algorithm is presented. In section 4, a suitable numerical example is illustrated. Finally, section 5 concludes the chapter with future work.

2 Preliminaries

In this section, the basic definitions involving neutrosophic set, single valued neutrosophic sets, trapezoidal neutrosophic numbers and operations on trapezoidal neutrosophic numbers are outlined.

Definition 1. [5] Let X be a space of points (objects) and $x\in X$. A neutrosophic set A in X is defined by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or real nonstandard subsets of $]^-0, 1^+[$. That is $T_A(x):X\to]^-0, 1^+[$, $I_A(x):X\to]^-0, 1^+[$ and $F_A(x):X\to]^-0, 1^+[$. There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0- \leq \sup T_A(x)+ \sup I_A(x) + \sup F_A(x) \leq 3+$.

Definition 2. [5] Let X be a universe of discourse. A single valued neutrosophic set A over X is an object having the form $A=\{\langle x, T_A(x), I_A(x), F_A(x)\rangle:x\in X\}$, where $T_A(x):X\to[0,1]$, $I_A(x):X\to[0,1]$ and $F_A(x):X\to[0,1]$ with $0\leq T_A(x)+ I_A(x)+ F_A(x)\leq 3$ for all $x\in X$. The intervals $T_A(x)$, $I_A(x)$ and $F_A(x)$ denote the truth-membership degree, the indeterminacy-membership degree and the falsity membership degree of x to A, respectively. For convenience, a SVN number is denoted by $A= (a,b,c)$, where $a,b,c\in[0,1]$ and $a+b+c\leq 3$.

Definition 3. [6] Let $\alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}} \in [0,1]$ and $a_1, a_2, a_3, a_4 \in \boldsymbol{R}$ such that $a_1 \leq a_2 \leq a_3 \leq a_4$. Then a single valued trapezoidal neutrosophic number, $\tilde{a}=\langle(a_1, a_2, a_3, a_4); \alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}}\rangle$ is a special neutrosophic set on the real line set $\boldsymbol{R}$, whose truth-membership, indeterminacy-membership, and falsity-membership functions are given as follows [8]:

$$T_{\tilde{a}}(x) = \begin{cases} \alpha_{\tilde{a}}\left(\frac{x-a_1}{a_2-a_1}\right) & if\ a_1 \leq x \leq a_2 \\ \alpha_{\tilde{a}} & if\ a_2 \leq x \leq a_3 \\ \alpha_{\tilde{a}}\left(\frac{a_4-x}{a_4-a_3}\right) & if\ a_3 < x \leq a_4 \\ 0 & otherwise \end{cases} \quad (1)$$

$$I_{\tilde{a}}(x) = \begin{cases} \frac{(a_2-x+\theta_{\tilde{a}}(x-a_1))}{(a_2-a_1)} & if\ a_1 \leq x \leq a_2 \\ \theta_{\tilde{a}} & if\ a_2 \leq x \leq a_3 \\ \frac{(x-a_3+\theta_{\tilde{a}}(a_4-x))}{(a_4-a_3)} & if\ a_3 < x \leq a_4 \\ 1 & otherwise \end{cases} \quad (2)$$

$$F_{\tilde{a}}(x) = \begin{cases} \frac{\left(a_2 - x + \beta_{\tilde{a}}(x - a_1)\right)}{(a_2 - a_1)} & if\ a_1 \le x \le a_2 \\ \beta_{\tilde{a}} & if\ \ a_2 \le x \le a_3 \\ \frac{(x - a_3 + \beta_{\tilde{a}}\,(a_4 - x))}{(a_4 - a_3)} & if\ a_3 < x \le a_4 \\ 1 & otherwise \end{cases} \quad (3)$$

where $\alpha_{\tilde{a}}$,$\theta_{\tilde{a}}$, $\beta_{\tilde{a}}$ denote the maximum truth-membership degree, minimum indeterminacy-membership degree and minimum falsity-membership degree respectively. A single valued trapezoidal neutrosophic number $\tilde{a}$=$\langle(a_1, a_2, a_3, a_4); \alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}}\rangle$ may express an ill-defined quantity about a,which is approximately equal to $[a_2, a_3]$.

Definition 4. [7] Let $\tilde{a}$=$\langle(a_1, a_2, a_3, a_4); \alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}}\rangle$ and $\tilde{b}$ = $\langle(b_1, b_2, b_3, b_4); \alpha_{\tilde{b}}, \theta_{\tilde{b}}, \beta_{\tilde{b}}\rangle$ be two single valued trapezoidal neutrosophic numbers and $\gamma \neq 0$ be any real number [9]. Then,

1. $\tilde{a} + \tilde{b} = \langle(a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4);\ \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}},\ \theta_{\tilde{a}} \vee \theta_{\tilde{b}},\ \beta_{\tilde{a}} \vee \beta_{\tilde{b}}\rangle$
2. $\tilde{a} - \tilde{b} = \langle(a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1);\ \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}},\ \theta_{\tilde{a}} \vee \theta_{\tilde{b}},\ \beta_{\tilde{a}} \vee \beta_{\tilde{b}}\rangle$
3. $\tilde{a}\tilde{b} = \begin{cases} \langle(a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4);\ \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}},\ \theta_{\tilde{a}} \vee \theta_{\tilde{b}},\ \beta_{\tilde{a}} \vee \beta_{\tilde{b}}\rangle\ if\ (a_4 > 0, b_4 > 0) \\ \langle(a_1 b_4, a_2 b_3, a_3 b_2, a_4 b_1);\ \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}},\ \theta_{\tilde{a}} \vee \theta_{\tilde{b}},\ \beta_{\tilde{a}} \vee \beta_{\tilde{b}}\rangle\ if\ (a_4 < 0, b_4 > 0) \\ \langle(a_4 b_4, a_3 b_3, a_2 b_2, a_1 b_1);\ \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}},\ \theta_{\tilde{a}} \vee \theta_{\tilde{b}},\ \beta_{\tilde{a}} \vee \beta_{\tilde{b}}\rangle\ if\ (a_4 < 0, b_4 < 0) \end{cases}$
4. $\frac{\tilde{a}}{\tilde{b}} = \begin{cases} \langle\left(\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1}\right);\ \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}},\ \theta_{\tilde{a}} \vee \theta_{\tilde{b}},\ \beta_{\tilde{a}} \vee \beta_{\tilde{b}}\rangle\ if (a_4 > 0, b_4 > 0) \\ \langle\left(\frac{a_4}{b_4}, \frac{a_3}{b_3}, \frac{a_2}{b_2}, \frac{a_1}{b_1}\right);\ \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}},\ \theta_{\tilde{a}} \vee \theta_{\tilde{b}},\ \beta_{\tilde{a}} \vee \beta_{\tilde{b}}\rangle\ if (a_4 < 0, b_4 > 0) \\ \langle\left(\frac{a_4}{b_1}, \frac{a_3}{b_2}, \frac{a_2}{b_3}, \frac{a_1}{b_4}\right);\ \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}},\ \theta_{\tilde{a}} \vee \theta_{\tilde{b}},\ \beta_{\tilde{a}} \vee \beta_{\tilde{b}}\rangle\ if (a_4 < 0, b_4 < 0) \end{cases}$
5. $\gamma\tilde{a} = \begin{cases} \langle(\gamma a_1, \gamma a_2, \gamma a_3, \gamma a_4);\ \alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}}\rangle\ if (\gamma > 0) \\ \langle(\gamma a_4, \gamma a_3, \gamma a_2, \gamma a_1);\ \alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}}\rangle\ if\ (\gamma < 0) \end{cases}$
6. $\tilde{a}^{-1} = \langle\left(\frac{1}{a_4}, \frac{1}{a_3}, \frac{1}{a_2}, \frac{1}{a_1}\right);\ \alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}}\rangle$, where $(\tilde{a} \neq 0)$.

3 PERT in Neutrosophic Environment and the Proposed Model

Like CPM, PERT uses network model. However, PERT has been traditionally used in new projects which have large uncertainty in respect of design, technology and construction. To take care of associated uncertainties, we adopt neutrosophic environment for PERT activity duration.

The three-time estimates for activity duration are:

1. Optimistic time ($\tilde{a}$): it is the minimum time needed to complete the activity if everything goes well.
2. Pessimistic time($\tilde{b}$) : it is the maximum time needed to complete the activity if one encounters problems at every turn.
3. Most likely time, i.e. Mode ($\widetilde{m}$): it is the time required to complete the activity in normal circumstances.

Where $\tilde{a}$,$\tilde{b}, \widetilde{m}$ are trapezoidal neutrosophic numbers.

Based on three time estimates ($\tilde{a}$, $\tilde{b}, \widetilde{m}$), expected time and standard deviation of each activity should be calculated , and to do this we should first obtain crisp values of three time estimates.

To obtain crisp values of three-time estimates, we should use score functions and accuracy functions as follows:

Let $\tilde{a}$=$< (a_1, a_2, a_3, a_4$); $\alpha_{\tilde{a}}$,$\theta_{\tilde{a}}$, $\beta_{\tilde{a}} >$ be a single valued trapezoidal neutrosophic number; then

1. score function $S(\tilde{a}) = \left(\frac{1}{16}\right)[a_1 + a_2 + a_3 + a_4]\times\left[\alpha_{\tilde{a}} + (1 - \theta_{\tilde{a}}) + \left(1 - \beta_{\tilde{a}}\right)\right]$; (4)
2. accuracy function $A(\tilde{a}) = \left(\frac{1}{16}\right)[a_1 + a_2 + a_3 + a_4]\times\left[\alpha_{\tilde{a}} + (1 - \theta_{\tilde{a}}) + \left(1 + \beta_{\tilde{a}}\right)\right]$. (5)

After obtaining crisp values of each time estimate by using score function, the expected time and standard deviation of each activity calculated as follows;

$$T_{ij} = \frac{a+4m+b}{6} \quad (6)$$

and

$$\sigma_{ij} = \frac{b-a}{6} \quad (7)$$

where a, m, b are crisp values of optimistic, most likely and pessimistic time respectively,

T_{ij}= Expected time of ij activity and

$\sigma_{ij} =$ Standard deviation of ij activity.

Once the expected time and standard deviation of each activity are calculated, PERT network is treated like CPM network for the purpose of calculation of network parameters like earliest/latest occurrence time of activity, critical path and floats.

Let a network N= $\langle E, A, T \rangle$, being a project model, is given. E is asset of events (nodes) and A$\subset E\times E$ is a set of activities. The set E=$\{1,2, \dots, n\}$ is labeled in such a way that the following condition holds: $(i, j) \in A$ and $i < j$. The activity times in the network are determined by $T_{ij.}$

Notations of network solution and its calculations as follows:

T_i^e=Earliest occurrence time of predecessor event *i*,

T_i^l= Latest occurrence time of predecessor event *i*,

T_j^e=Earliest occurrence time of successor event *j*,

T_j^l= Latest occurrence time of successor event *j*,

$T_{ij}^e/$Start= Earliest start time of an activity ij,

$T_{ij}^e/$Finisht=Earliest finish time of an activityij,

$T_{ij}^l/$Start=Latest start time of an T_i^l activity ij,

$T_{ij}^l/$Finisht= Latest finish time of an activity ij,

T_{ij} = Duration time of activity ij,

Earliest and Latest occurrence time of an event:

T_j^e=maximum $(T_j^e + T_{ij})$, calculate all T_j^e for jth event, select maximum value.

T_i^l=minimum $(T_j^l - T_{ij})$, calculate all T_i^l for ith event, select minimum value.

$T_{ij}^e/$Start=T_i^e,

$T_{ij}^e/$Finisht=$T_i^e + T_{ij}$,

$T_{ij}^l/$Finisht=T_j^l,

$T_{ij}^l/$Start=$T_j^l - T_{ij}$,

Critical path is the longest path in the network. At critical path, T_i^e=T_i^l, for all i.

Slack or Float is cushion available on event/ activity by which it can be delayed without affecting the project completion time.

Slack for ith event = $T_i^l - T_i^e$, for events on critical path, slack is zero.

The expected time of critical path (μ) and its variance (σ^2)calculated as follows;

$\mu = \sum T_{ij,}$ for all ij on critical path.

$\sigma^2 = \sum \sigma_{ij}^2$, for all ij on critical path.

From the previous steps we can conclude the proposed algorithm as follows:

1. To deal with uncertain, inconsistent and incomplete information about activity time, we considered three-time estimates of PERT technique as a single valued trapezoidal neutrosophic numbers.
2. Calculate membership functions of each single valued trapezoidal neutrosophic number, using equation 1, 2 and 3.
3. Obtain crisp model of PERT three-time estimates using score function equation as we illustrated previously.
4. Use crisp values of three time estimates to calculate expected time and standard deviation of each activity.
5. Draw PERT network diagram.
6. Determine floats and critical path, which is the longest path in network as we illustrated previously with details.
7. Calculate expected time and variance of critical path.
8. Determine expected project completion time.

4 Illustrative Example

Let us consider neutrosophic PERT and try to obtain crisp model from it. Since you are given the following data for a project:

Table 1. Input data for neutrosophic PERT.

Activity	**Immediate Predecessors**	**Time (days)**		
		$\tilde{a}$	$\tilde{m}$	$\tilde{b}$
A	------	$\tilde{1}$	$\tilde{3}$	$\tilde{5}$
B	------	$\tilde{2}$	$\tilde{4}$	$\tilde{6}$
C	A	$\tilde{7}$	$\tilde{9}$	$\widetilde{15}$
D	A	$\tilde{8}$	$\widetilde{10}$	$\widetilde{16}$
E	B	$\tilde{6}$	$\tilde{9}$	$\widetilde{14}$
F	C,D	$\widetilde{11}$	$\widetilde{15}$	$\widetilde{17}$
G	D,E	$\tilde{6}$	$\widetilde{11}$	$\widetilde{19}$
H	F,G	$\tilde{8}$	$\widetilde{12}$	$\widetilde{20}$

In the previous table $\tilde{a}$, $\tilde{m}$ and $\tilde{b}$ are optimistic, most likely and pessimistic time in neutrosophic environment, and considered as a single valued trapezoidal neutrosophic numbers.

Let

$\tilde{1} = \langle(0,2,4,5); 0.8,0.6,0.4\rangle, \tilde{2} = \langle(1,3,5,6);\ \ 0.2,0.3,0.5\rangle,$

$\tilde{3} = \langle(1,2,5,6); 0.2,0.5,0.6\rangle, \tilde{4} = \langle(1,2,5,7); 0.5,0.4,0.9\rangle,$

$\tilde{5} = \langle(2,4,7,10); 0.8,0.2,0.4\rangle, \tilde{6} = \langle(3,7,9,12); 0.7,0.2,0.5\rangle, \tilde{7} = \langle(5,8,9,13); 0.4,0.6,0.8\rangle,\ \ \tilde{8} = \langle(1,6,10,13); 0.9,0.1,0.3\rangle,$

$\tilde{9} = \langle(6,8,10,15); 0.6,0.4,0.7\rangle, \widetilde{10} = \langle(1,6,11,15); 0.7,0.6,0.3\rangle,$
$\widetilde{11} = \langle(5,8,15,20); 0.8,0.2,0.5\rangle, \widetilde{12} = \langle(4,8,17,25); 0.3,0.6,0.4\rangle,$

$\widetilde{14} = \langle(7, 10,19,30); 0.8,0.4,0.7\rangle$, $\widetilde{15} = \langle(8, 10,20,35); 0.5,0.2,0.4\rangle$,
$\widetilde{16} = \langle(5, 15,25,30); 0.7,0.5,0.6\rangle$, $\widetilde{17} = \langle(10, 15,20,25); 0.2,0.4,0.6\rangle$,
$\widetilde{19} = \langle(15, 17,23,25); 0.9,0.7,0.8\rangle$, $\widetilde{20} = \langle(10, 12,27,30); 0.2,0.3,0.5\rangle$.

Step 1: To obtain crisp values of each single valued trapezoidal neutrosophic number, we should calculate score function as follows:

Score function $S(\tilde{1}) = \left(\frac{1}{16}\right)[0 + 2 + 4 + 5]\times[0.8 + (1 - 0.6) + (1 - 0.4)] = 1.24$

Score function $S(\tilde{2}) = \left(\frac{1}{16}\right)[1 + 3 + 5 + 6]\times[0.2 + (1 - 0.3) + (1 - 0.5)] = 1.31$

Score function $S(\tilde{3}) = \left(\frac{1}{16}\right)[1 + 2 + 5 + 6]\times[0.2 + (1 - 0.5) + (1 - 0.6)] = 0.96$

Score function $S(\tilde{4}) = \left(\frac{1}{16}\right)[1 + 2 + 5 + 7]\times[0.5 + (1 - 0.4) + (1 - 0.9)] = 1.12$

Score function $S(\tilde{5}) = \left(\frac{1}{16}\right)[2 + 4 + 7 + 10]\times[0.8 + (1 - 0.2) + (1 - 0.4)] = 3.16$

Score function $S(\tilde{6}) = \left(\frac{1}{16}\right)[3 + 7 + 9 + 12]\times[0.7 + (1 - 0.2) + (1 - 0.5)] = 3.87$

Score function $S(\tilde{7}) = \left(\frac{1}{16}\right)[5 + 8 + 9 + 13]\times[0.4 + (1 - 0.6) + (1 - 0.8)] = 2.19$

Score function $S(\tilde{8}) = \left(\frac{1}{16}\right)[1 + 6 + 10 + 13]\times[0.9 + (1 - 0.1) + (1 - 0.3)] = 4.68$

Score function $S(\tilde{9}) = \left(\frac{1}{16}\right)[6 + 8 + 10 + 15]\times[0.6 + (1 - 0.4) + (1 - 0.7)] = 3.66$

Score function $S(\widetilde{10}) = \left(\frac{1}{16}\right)[1 + 6 + 11 + 15]\times[0.7 + (1 - 0.6) + (1 - 0.3)] = 3.71$

Score function $S(\widetilde{11}) = \left(\frac{1}{16}\right)[5 + 8 + 15 + 20]\times[0.8 + (1 - 0.2) + (1 - 0.5)] = 6.3$

Score function $S(\widetilde{12}) = \left(\frac{1}{16}\right)[4 + 8 + 17 + 25]\times[0.3 + (1 - 0.6) + (1 - 0.4)] = 4.39$

Score function $S(\widetilde{14}) = \left(\frac{1}{16}\right)[7 + 10 + 19 + 30]\times[0.8 + (1 - 0.4) + (1 - 0.7)] = 7.01$

Score function $S(\widetilde{15}) = \left(\frac{1}{16}\right)[8 + 10 + 20 + 35]\times[0.5 + (1 - 0.2) + (1 - 0.4)] = 8.67$

Score function $S(\widetilde{16}) = \left(\frac{1}{16}\right)[5 + 15 + 25 + 30] \times [0.7 + (1 - 0.5) + (1 - 0.6)] = 7.5$

Score function $S(\widetilde{17}) = \left(\frac{1}{16}\right)[10 + 15 + 20 + 25] \times [0.2 + (1 - 0.4) + (1 - 0.6)] = 5.25$

Score function $S(\widetilde{19}) = \left(\frac{1}{16}\right)[15 + 17 + 23 + 25] \times [0.9 + (1 - 0.7) + (1 - 0.8)] = 7$

Score function $S(\widetilde{20}) = \left(\frac{1}{16}\right)[10 + 12 + 27 + 30] \times [0.2 + (1 - 0.3) + (1 - 0.5)] = 6.91$

Step 2: By putting score functions values as crisp values of each time estimate, we can calculate the expected time and variance of each activity as we illustrated with equations in the previous section. The expected time of each activity has been calculated and presented in table 2.

Table2**.** The expected time of each activity in the project.

Activity	Immediate Predecessors	Expected Time(days)
A	------	1
B	------	2
C	A	3
D	A	4
E	B	4
F	C,D	8
G	D,E	6
H	F,G	5

Step 3: Draw the network diagram by using Microsoft Project 2010.

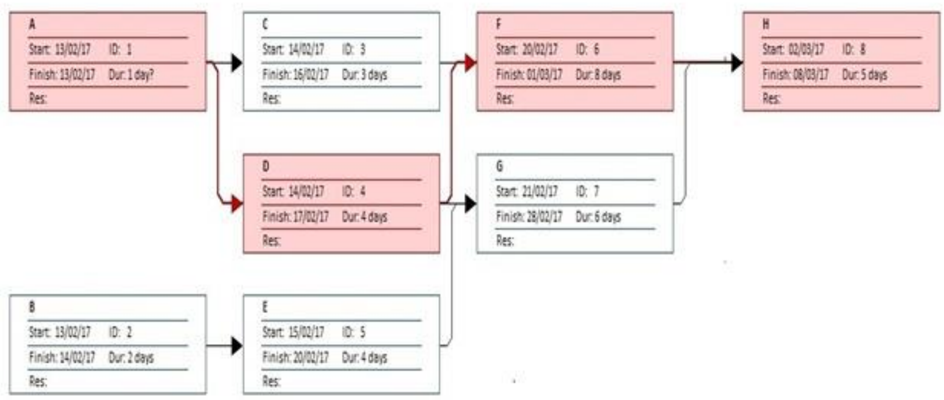

Fig. 1. Network of activities with critical path

From figure 1, we find that the critical path is A-D-F-H and is denoted by red line. The expected project completion time = $t_A + t_D + t_F + t_H$ = 18 days.

5 Conclusion

Neutrosophic set is a generalization of classical set, fuzzy set and intuitionistic fuzzy set because it not only considers the truth-membership and falsity- membership, but also an indeterminacy membership which is very obvious in real life situations. In this chapter, we have considered the three-time estimates of PERT as a single valued trapezoidal neutrosophic numbers and we used score function to obtain crisp values of three-time estimates. In future, the research will be extended to deal with different project management techniques.

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