The Balan-Killing Manifolds

A.Balan

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Abstract

We define here a differential equation over the metric of a Spin manifold, calling Balan-Killing metrics the solutions of this equation.

1 The Spin manifolds

A Spin manifold [F] admits a spinor bundle by reduction of the tangent bundle with the orthogonal group to the spinor bundle with the group spin. The spinors ψ admit a multiplication by the vectors X, called the Clifford multiplication $X.\psi$.

Definition 1 *A Killing spinor is defined by the equation:*

$$\nabla_X \psi = \mu\, X.\psi$$

with ∇ the Levi-Civita connection and $\mu \in \mathbf{R}$.

2 The Balan-Killing metrics

We introduce here the following differential equation over the metric. Let R^∇ be the spinor curvature.

Definition 2 *We call a Balan-Killing manifold, a manifold M with riemannian metric g such that $\exists \mu$, $\forall X, Y$ vectors and $\forall \psi$ spinor:*

$$R^\nabla(X,Y)\psi = \mu\, Ricc^\nabla(X.Y - Y.X).\psi$$

. is the Clifford multiplication and ∇ is the Levi-Civita connection over the spinors, $Ricc^\nabla$ is the Ricci curvature as an endomorphism of the spinors.

For example, a flat space is a Balan-Killing manifold.

3 The Balan-Killing spinors

Definition 3 *We call a Balan-Killing spinor, a spinor ψ which verifies the equation:*

$$d(h(Z.\psi,\psi)(X,Y) = \mu\, h((X.Y - Y.X).\psi,\psi)$$

h is the hermitian metric over the spinor bundle, d is the differential operator over the forms.

It is a differential equation over the spinors.

References

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