Title: A Niels Bohr approach to a Pre-Quantum Theory of Quantum Gravity

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Abstract

As a first step towards a theory of quantum gravity, a Niels Bohr pre-quantum mechanics theory of quantum gravity is proposed. It predicts the existence of gravitons with an energy of 3.47×10^{11} GeV, close to the rarely observed particles coming from a region of the Ursa Major constellation, with the largest energy of 3.2×10^{11} GeV, well above the Greisen-Zatsepin-Kuzmin (GZK) limit, suggesting that these particles are high-energy gravitons.

It was noted that the proposed provisionary theory can explain Heisenberg's uncertainty relation as a result of a mathematical theorem by Bruns, where the nonlinearity of the equations of motion, such as Newton's and Einstein's equations, leads to divergent dense manifold for infinitely closed initial conditions as their solutions.

In addition, the theory can also explain the gravitational origin of the quantum potential in the Madelung-transformed Schrödinger equation. It explains the ratio of dark energy to dark matter, approximately 3:1, as the Madelung constant of a gravitationally interacting fluid made up of positive and negative mass pole-dipole particles filling the vacuum of space.

1. Introduction

Prior to the final discover of the laws of quantum mechanics by Heisenberg, Niels Bohr succeeded in explaining the hydrogen atom, with two assumptions:

1. The motion of the electron around the proton is ruled by Newton’s classical mechanics.
2. To satisfy Planck’s energy quantization condition, the angular momentum of the electron must be equal to an even number of ħ.

In addition, to obtain the quantized orbits for the electron, the frequency of emitted light from jumping to a lower energy orbit was then also explained.

In this communication, an attempt is made to explain the physics of an electron as the simplest elementary particle, like how the hydrogen atom is the simplest atom, by a pre-quantum theory of quantum gravity.

2. Niels Bohr's Pre-Quantum Gravity Theory of the Electron

Following Schrödinger's Zitterbewegung (quivering motion) theory of the Dirac electron [1], it was shown by Hönl, Papapetrou [2], and Bopp [3] that a simple "pole-dipole" particle can describe this "Zitterbewegung." Similar to how the hydrogen atom is composed of a particle with a positive electric charge (the proton) and of an electron with an equal but opposite charge, obtaining its mass from the electric field interaction energy, a pole-dipole is made of two masses, one positive m^+, and the other m^-, but with $|m^-|=|m^+|$, obtaining its interaction energy from the gravitational field set up in between m^+ and m^-. Because the signs of m^+ and m^- are opposite, this gravitational interaction energy is positive, and for energies less than the Planck energy of $\sim 10^{19}$ GeV it can be computed from Newton's law of gravity.

While the equivalence principle of the general theory of relativity outlaws the existence of negative mass particles, which would have to move on "antigeodesics," it does not outlaw pole-dipole particles, with a positive mass pole. There, only the center of mass is moving on a geodesic. For a pole-dipole we thus have for the mass of an electron:

$$m = G\frac{|m^{\pm}|^2}{c^2 r} \text{ (Newton's law)} \tag{1}$$

where G is Newton's constant and r is the separation distance between m^+ and m^-. Supplementing (1) with

$$2|m^{\pm}|rc = \hbar \text{ (Bohr's angular momentum quantization principle)} \tag{2}$$

one obtains from (1) and (2)

$$|m^{\pm}| = \sqrt[3]{\frac{\hbar mc}{2G}} = 6 \times 10^{-13} \text{ g} \tag{3}$$

$$r = 3 \times 10^{-26} \text{ cm} \tag{4}$$

and hence

$$|m^{\pm}|c^2 = 3.31 \times 10^{11} \text{ GeV} \tag{5}$$

This means that if an electron broke up into an m^+ and m^- particle, it would release a graviton of this energy. Comparing this with the highest cosmic ray energies observed, $\simeq 3.12 \times 10^{11}$ GeV and well above the Greisen-Zatsepin-Kuzmin (GZK) limit of 5×10^{10} GeV, suggests that these events are caused by gravitons, which are not subject to the GZK mechanism. The fact that they have been observed in a region of Ursa Major suggests they are emitted from a Kerr black hole located in this area of space. The large gravitational fields in the ergosphere of a Kerr black hole would be capable of splitting electrons and releasing such high-energy gravitons, and through the resonance absorption by electrons in the earth's atmosphere, these gravitons could lead to the $\sim 10^{11}$ GeV cosmic rays observed by our detectors.

3. Emission of Watt-less Gravitational Waves from a Dirac Spinor

According to Schrödinger [1], a Dirac electron executes a luminal helical motion, with the radius of the helix equal to the Compton wavelength of the electron, superimposed by a "Zitterbewegung" with an oscillatory displacement given by (2) and equal to $r \simeq 10^{-26}$ cm. This situation resembles a double star, except that one of its components has a negative mass. As for a double star, where the center of mass is on a geodesic, the same must be true here, leading to the emission of short wavelength gravitational by the oscillation of m^+ against m^-, or vice versa, with a wavelength on the order of 10^{-26} cm modulated the Compton frequency $mc^2/\hbar$, due to the helical motion of the pole-dipole Dirac particle

To prevent the Dirac particle from disintegrating due to this emission of gravitational waves, there must be a superposition of positive energy – positive space curvature wave - and a likewise negative energy – negative space curvature wave. The source of the positive space curvature wave is the energy-momentum tensor of the positive mass m^+ and negative mass m^-. There, the Dirac particle would be accompanied by a Watt-less gravitational wave, giving a plausible explanation for de Broglie's pilot wave hypothesis. In addition, it would explain the particle-wave duality of quantum mechanics, which Feynman believed could never be explained.

To compute the energy loss (and energy gain) by the emission of positive (and negative) energy gravitational radiation, we use Einstein's quadruple formula [4] for the energy loss of a double star of masses m_1 and m_2, separated by the distance r and orbital frequency ω:

$$-\frac{d\varepsilon}{dt} = \frac{32G}{5c^5}\left(\frac{m_1 m_2}{m_1 + m_2}\right)^2 r^4 \omega^6 \tag{6}$$

Setting $m_1 >> m_2 = m$, and $m_1 + m_2 = m_1$, (approximately true since m_1 is almost motion-less), one has for the energy loss (gain),

$$-\frac{d\varepsilon}{dt} = \frac{32G}{5c^5} m^2 r^4 \omega^6 \tag{7}$$

We set m = $|m^{\pm}|/2$ as the reduced mass, and furthermore multiply (7) by ½ to average over a sinus wave. For the positive energy loss of m^+ and the negative energy loss of m^-which is equal to the positive energy gain of m^- we have,

$$\mp\frac{d\varepsilon}{dt} = \frac{4G}{5c^5} |m^{\pm}|^2 r^4 \omega^6 \tag{8}$$

Setting ω = c/r, we obtain

$$\mp\frac{d\varepsilon}{dt} = \frac{4}{5}\frac{G|m^{\pm}|^2 c}{r^2} \tag{9}$$

Integrating over the time of one revolution we multiply (9) by r/c and obtain,

$$\varepsilon^{\pm} = \mp\frac{4}{5}\frac{G|m^{\pm}|^2}{r} = \mp\frac{4}{5}mc^2 \tag{10}$$

hence,

$$\varepsilon^{+} + \varepsilon^{-} = 0 \tag{11}$$

We now connect this result to the nonrelativistic Schrödinger equation, which led to the Copenhagen interpretation of quantum mechanics and the famous Bohr-Einstein debate, which for decades appeared had been won by Niels Bohr, but may be ultimately won by Einstein, ironically with his gravitational waves.

For a nonrelativistic particle of mass m and velocity v, and $(1/2)mv^2 \ll mc^2$, the Schrödinger equation is:

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + \mathrm{U}\psi \tag{12}$$

with U the potential of an external applied force. Making for (12) the Madelung transformation [8],

$$\begin{aligned}\Psi &= \sqrt{n}e^{iS}\\ \Psi^* &= \sqrt{n}e^{-iS}\end{aligned} \tag{13}$$

where $n = \Psi^*\Psi$ and S the Hamilton action function, one obtains two coupled equations:

$$\begin{gathered}\hbar\frac{\partial S}{\partial t} + \frac{\hbar^2}{2m}(\nabla S)^2 + U + Q = 0\\ \frac{\partial n}{\partial t} + \frac{\hbar}{m}\nabla(nS) = 0\end{gathered} \tag{14}$$

where,

$$Q = \frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{n}}{\sqrt{n}} \tag{15}$$

is called the quantum potential. Setting as in the Hamilton-Jacobi theory of classical mechanics $\boldsymbol{v} = (\hbar/\mathrm{m})\nabla\,\mathrm{S}$, one obtains from (14),

$$\begin{gathered}\frac{\partial\boldsymbol{v}}{\partial t} + \boldsymbol{v}\cdot\frac{\partial\boldsymbol{v}}{\partial\boldsymbol{r}} = -\frac{1}{m_p}\frac{\partial}{\partial\boldsymbol{r}}[\mathrm{U}+\mathrm{Q}]\\ \frac{\partial n}{\partial t} + \mathrm{div}(\mathrm{n}\boldsymbol{v}) = 0\end{gathered} \tag{16}$$

the Euler and continuity equation for a friction-less fluid with ordinary U and quantum potential Q. Setting Q = 0 and making inverse Madelung transformation [9], one obtains the wave equation of classical mechanics:

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m_p}\nabla^2\psi + \left[\mathrm{U} + \overline{\mathrm{Q}}\right]\psi \tag{17}$$

where

$$\overline{Q} = \frac{\hbar^2}{2m}\frac{\nabla^2|\psi\,|}{|\psi\,|} \tag{18}$$

is the inverse quantum potential, whereby the Madelung transformation of (17) would yield (16) with Q = 0. Unlike the Schrödinger equation (12), equation (15) is nonlinear. This simple fact shows it is the quantum potential which makes the Schrödinger equation a linear wave equation. To estimate the value of the quantum potential (15) we set $\nabla^2 \sim 1/{r_c}^2$, where $r_c = \hbar/2mc$, 1/2 of the Compton wave length of a particle with mass m. One finds that,

$$Q \simeq mc^2 \tag{19}$$

Comparing (11) with (19), we have,

$$|Q| \simeq |\varepsilon| \tag{20}$$

which explains why the quantum potential has its cause in the Watt-less emission of gravitational waves by the Dirac equation. If experimentally confirmed, the insight expressed by (20) would be of fundamental importance in all attempts to formulate a correct theory of quantum gravity.

According to Born, the quantum-mechanical probability density of a wave function ψ is given by

$$\boldsymbol{P} = \psi^*\psi \tag{21}$$

For a normalized plane wave, one has the solution of the Schrödinger equation:

$$\psi = e^{i(kx-\omega t)} \tag{22}$$

where the real and imaginary parts are

$$\begin{aligned} \psi_R &= \cos(\mathrm{kx} - \omega\mathrm{t}) \\ \psi_I &= \sin(\mathrm{kx} - \omega\mathrm{t}) \end{aligned} \tag{23}$$

and hence,

$$\boldsymbol{P} = \psi^*\psi = \psi_R^2 + \psi_I^2 = 1 \tag{24}$$

For a Watt-less gravitational wave, one has there a super-position of real and imaginary waves of equal amplitude, also given by (23). The energy flux density E from both with their sum oscillating between positive and negative space curvature is:

$$E = \psi_R^2 - \psi_I^2 = \cos[2(\mathrm{kx} - \omega\mathrm{t})] \tag{25}$$

where $|E| < 1$, and the time average $\bar{E} = 0$.

This then is a credible explanation of the de Broglie pilot wave theory, in opposition to the widely-accepted Copenhagen interpretation. It was by Einstein called “zu billig” (too cheap), but if it should turn out that the de Broglie pilot wave is a gravitational wave in disguise, he would have hardly expressed such a sentiment, and it rather would have served as an incentive to explore if all the other waves propagating with the velocity of light, like electromagnetic waves, are gravitational waves in disguise.

4. Quantum Gravity at the Planck Scale:

The progress in elementary particle physics was made by going from “smaller” to “larger” energies: By Planck’s law $E = h\nu$ to the Planck energy $E_p = \sqrt{\hbar c^5/G}$, at the same time shrinking the length to the gravitational radius of the Planck energy $r_p = \sqrt{\hbar G/c^3} \cong 10^{-33} cm$, one has the two relations:

$$Gm_p^2 = \hbar c \qquad \text{Energy}$$

(26)

$$m_p r_p c = \hbar \qquad \text{Angular momentum}$$

The energy equation is the gravitational coupling constant of mass to the maximum space curvature $K = 1/r_p$. Likewise, the smallest angular momentum for a Dirac particle is $J = (1/2)h$. For a Planck mass dipole $[m_p^+ + m_p^-] = 0$, the sum is zero but the gravitational interaction energy over a distance of separation set equal to r_p is positive and given by:

$$m_o c^2 = -\frac{G(m_p)(-m_p)}{r_p} = m_p c^2 \qquad (27)$$

with an angular momentum $J = (1/2)\hbar$. For this reason, the metric surrounding a Planck mass dipole should be described by a rotating Kerr black hole with an angular momentum equal to $J = (1/2)\hbar$.

5. The Cause of Uncertainty in the Nonlinearity of Einstein's Gravitational Field Equation

For any wave mechanics, like for acoustic waves, but also (linear) gravitational waves, made up from a waveform $\psi = Ae^{i(kx-\omega t)}$, the Fourier theorem leads to the equations:

$$\Delta k \Delta x \geq 1$$
$$\Delta \omega \Delta t \geq 1 \tag{28}$$

where Δk and $\Delta \omega$ are the spread in wave number and frequency of a wave package. The problem in the interpretation of quantum mechanics enters by identifying a wave package with a particle of momentum $p = \hbar k$, and energy $E = \hbar \omega$, whereby (28) are the two uncertainty relations:

$$\Delta p \Delta x \geq \hbar$$
$$\Delta E \Delta t \geq \hbar \tag{29}$$

requiring in the Copenhagen interpretation a superluminal (relativity-violating) collapse of the wave function. This is different in the de Broglie-Bohm pilot wave interpretation, where one always has real particles, and the particle is not a wave package. If the pilot wave is a Watt-less gravitational wave, it leads to all the interference phenomena of a wave. But the questions arises, if everything is deterministic, from where then can come any uncertainty? The answer is from the nonlinearity of Einstein's gravitational field equation. That nonlinearity can lead to a different kind uncertainty has been recognized by Heisenberg [7]: "I might mention a most paradoxical result of this mathematical analysis – the theorem by Bruns. He proved that in an even infinitely close neighborhood of a point where the perturbation theory converges, there must always be other points where the perturbation theory diverges. So, one can say that the points where the perturbation theory converges and those where it diverges form a dense manifold. This result suggests that after a very long time one can never know where the orbit finally will go." Heisenberg's comment was made in the context of Newton's classical equations of motion. Like Einstein's gravitational field equation Newton's equation is nonlinear. The nonlinearity implies that the initial conditions of position and velocity for the emission of gravitational waves by the Dirac equation would have to be more accurately known as a Planck length and the likewise accurate particle velocity at this length. This is impossible, since no instrument can be built of parts that small, and because of the theorem by Bruns, even that would not be enough. This means that there always will be an uncertainty, (and where the Gods can interfere).

It is as if nature wants to avoid the nonlinearity of deterministic classical physics by linear quantum mechanics, which in reality is only statistical and not deterministic.

6. Planck Mass Plasma Conjecture

According to Heisenberg [8], a unified theory of elementary particles must be nonlinear to explain the large non-dimensional numbers, like the proton to electron mass ratio, or the fine-structure constant. As an example he mentioned the large Reynolds numbers of nonlinear classical fluid dynamics. And as with classical fluid dynamics, the field equations of Einstein's general theory of relativity are also nonlinear. From the two fundamental equations (26) follows the "Planck Mass Plasma" conjecture [9], that space is densely filled by a medium of gravitationally interacting Planck mass pole-dipole particles, as a bound state of a positive and negative Planck mass particle, with a positive gravitational field mass equal to a Planck mass, as it was shown above in (27). Under this assumption, the vacuum of space would resemble an ionic crystal similar to NaCl, by replacing the electrostatic $1/r^2$ Coulomb force with Newton's $1/r^2$ gravitational force. The Planck mass plasma conjecture then gives a simple explanation for the 3:1 ratio of dark energy to dark matter as the Madelung constant in agreement with the empirical evidence [10]. It also explains the small value of the cosmological constant, which is similar to the small residual electric charge of solid matter.

The Planck mass plasma conjecture may even explain the fine-structure constant. As it was shown by Wilczek [11], at the GUT scale of $\sim 10^{16}$ GeV, this constant is about 1/25 rather than 1/137, which can be explained by the stable configuration of a ring vortex lattice [12] as it was obtained by Schlager for a two-dimensional lattice of line vortices [13].

7. Conclusion

An attempt to narrow the way towards the still unknown theory of quantum gravity, by following the example of Niels Bohr, cannot rely on the rich experimental material available to him. In this attempt we should be forced to be guided by well-established physical principles, and not on mathematical speculations. Following these restrictions, we could make the prediction of ultra-high-energy gravitons, which are not subject to the GZK limit. Another success is the correct explanation of the 3:1 ration of dark energy and dark matter, along with explaining the small value of the cosmological constant. But the most important result is likely the conjectured quantum mechanical uncertainty by the Watt-less emission of very short length, ultra-high-energy gravitational waves.

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