THE UNIVERSE AND LARGE NUMBERS : ARITHMETICS OF PHYSICAL CONSTANTS

Alberto Coe

albamv8@gmail.com

Abstract

It's about the large numbers associated to the field of cosmology and the field of physical constants under an arithmetical point of view . In this paper we have performed an equation that gives an account of some of the previously mentioned concepts . The summarized equation reads : $\frac{30[(A-B)+C]}{C} = \frac{1}{\alpha}$. Inside this equation there are implicitly : the number of photons in the observable universe , the current energy associated to the cosmic microwave backgroud , a parameter that defines a space's expansion factor , gravitational potential energy of the matter , the number of atoms in the obsevable universe and the energy associated to the electrons of the observable universe . Finally , the arithmetic combination among such topics give a surprising result : the inverse of fine structure constant .

Keywords : Cosmology , large numbers , cosmic microwave background ,physical constants , arithmetics.

Introduction

For a long time large numbers have intrigued physicists . Outstanding physicists [1] like Dirac , Weyl , Eddington and other speculated about the so-called ''large numbers hypothesis ''

The original motivation to write this paper was to study the issue of ''large numbers '' from a merely arithmetic point of view : an arithmetical analysis of the energy of matter , the energy of light and the space's expansion and large numbers involved in such subjects.

It's worth to say that this is a preliminary incursion in the field of large numbers . It should be obligatorily a short-haul incursion , we mean , susceptible of future contributions because the subject is very broad .Therefore in this paper we'll analyze relationship between cosmic microwave background and gravitational potential energy and some physical constants and cosmological parameters . It seems to offer an opportunity for the use of a simple arithmetic method : the ratio between different experimental values and the numerical equivalences between different ratios . We must remark on the fact that a numerical analysis of physical constants only makes sense using dimensionless numbers .

Method

First of all notice that all physical constants used in this paper are properly referenced in the appendix .That keep us from continuous references for each physical constant .

Let's start defining the first dimensionless number that works as a true large number and that will be useful to analyze relationship among different large numbers . Such definition involves two length dimensions : Bohr radius and Planck's length.Besides ,our definition includes a mathematical constant : Euler's number (2.7182818…) involved in the mathematics of the exponential growth [2].

Planck's length is a base unit in the system of Planck units [3], also known as natural units .Most of physicists states that Planck length defines the smallest scale's length where space breaks down [4].

Bohr radius is a physical constant that refers to the unit of length in atomic units.In the hydrogen's atom an electron orbit a proton .The ground state of one electron in the atom of hydrogen defines the smallest possible orbit [5]. This orbital radius is almost equal to the Bohr radius , that gives the maximum probability density of the electron in its ground state energy .

Therefore we have two small length dimensions ; on one hand Planck's length is associated to the physics of the space itself , and on the other hand Bohr radius is associated to the scale of the most simple atom , the atom of hydrogen .Since both physical constants are given in meters , the ratio of Bohr radius over Planck's length gives a dimensionless number.From a theoretical point of view we must consider the gap between both length dimensions , i.e. the orders of magnitude that separate Planck length from Bohr radius . we must also introduce a parameter that express conceptually the growth of a system , represented by Euler number . In addition we must multiply both Bohr radius and Planck length by two different expansion factors , by means of wich the initial universe expands until it reaches the current size :

Planck length -> Bohr radius -> (x expansion factors) -> universe radius

The arithmetics of the above ideas could be writed using two physical constants and a mathematical constant . As for the physical constants they represents length dimensions

a_0 refers to Bohr radius = $5.291772 \, x \, 10^{-11} \, m$

L_P refers to Planck's length = $1.6162 \, x \, 10^{-35} \, m$

Mathematical constant is represented by Euler's number $e = 2.7182818 \ldots$

The dimensionless parameter obtained is

$$(N_{...}) = \frac{1}{e}\frac{a_0}{L_P} = 1.2045 \text{ x } 10^{24} \tag{1}$$

Breaking down large numbers

Let's put in action the arithmetics that describes the number of photons , protons and electrons , the energy associated to the cosmic microwave background, the gravitational potential energy of the observable universe , the expansion of the space and the energy of an electron .

Let's write the equation whose equality we want to demonstrate

$$\frac{\mathbf{30}[(\boldsymbol{A}-\boldsymbol{B})+\boldsymbol{C}]}{\mathbf{2}\boldsymbol{C}} = \frac{\mathbf{1}}{\boldsymbol{\alpha}} \tag{2}$$

Symbol $\boldsymbol{A}$ includes :

$$A = (\, N_\gamma \,\hbar\, \nu_0)(N_{...})\frac{f_B}{f_P} \tag{3}$$

It's assumed a photon /baryon ratio $\sim 10^{10}$ [6]. If the observable universe has around 10^{24} stars like the sun [7] and a typical star has about 10^{57} atoms [8] then there are about 10^{81} atoms in the observable universe . therefore $N_\gamma \sim 10^{91}$ is the approximate number of photons .

In the observable universe , radiation is dominated by the cosmic microwave background photons .

the energy of a photon in the microwave range of frequency is given by the Planck-Einstein equation[9]

$$E = \hbar\, \nu_0 \tag{4}$$

where $\hbar$ is the reduced Planck constant : $1.054572\; x\; 10^{-34}\; J\; s$

$\nu_0 \sim 1.6012\; x\; 10^{11}\; s^{-1}$

refers to the spectral radiance peak of the cosmic microwave backgroud (CMB) that occurs at this frequency mode of vibration[10] . $(N_{...})$ refers to a dimensionless parameter already defined above . Parameters f_B and f_P symbolize two variables working as space's expansion factors over time :

from Bohr radius to universe radius : $f_B \sim 10^{37}$

from Planck length to universe radius : $f_P \sim 10^{61}$

If the *current* radius [11] of the observable universe is roughly $r_U \sim 10^{26}\, m$

$(a_0\ x\ f_B) \sim r_U$ and $(l_P\, x\ f_P) \sim r_U$

Once arithmetics are made we obtain

$$\boldsymbol{A} = 2.034\, x\, 10^{68}\ J \qquad (5)$$

Inside $\boldsymbol{B}$ there are

$$\boldsymbol{B} = U(N_b)^2 = -\,G_N\,\frac{(N_b)^2\, p_m\, e_m}{r_U} \qquad (6)$$

Let's describe , one by one , all items involved

$$U = -\frac{3}{5}\,G_N\,\frac{M^2}{r} \qquad (7)$$

represents the gravitational potential energy for a constant density sphere of mass M and radius r just like has been applyied by Kutner [12] . In the case at hand M^2 refers to the mass of one proton multiplied by the mass of one electron , since it's assumed initial conditions of the universe when particles like protons and electrons (besides photons) were in a thermodynamical equilibrium .

therefore

$$M^2 = (\,p_m)(\,e_m) \qquad (8)$$

p_m symbolizes the mass of the proton : $1.673\, x\, 10^{-27}\, kg$,

e_m refers to electron's mass : $9.11\, x\, 10^{-31}\, kg$

G_N refers to the Newtonian constant of gravitation , $6.674\, x\, 10^{-11}\, m^3\, kg^{-1} s^{-2}$

Number of atoms in the observable universe $N_b \sim 10^{81}$

Radius of the observable universe $r_U \sim 10^{26}\, m$

The arithmetics gives

$$\boldsymbol{B} = -\, G_N \,\frac{(N_b)^2\, p_m\, e_m}{r_U} = -6.1017\ x\ 10^{68}\ J \tag{9}$$

Let's analyze symbol **C** , wich stands for

$$C = (N_e\, \hbar\, \nu_e\,)\ z \tag{10}$$

$N_e\ \sim 10^{81}$ refers to the number of electrons in the observable universe , assuming that number of protons = number of electrons

$\nu_e \sim 7.76\ x\ 10^{20}\ \ s^{-1}$ refers to the frequency mode of vibration of the electron derived from

$$\nu_e = \frac{e_m\, c^2}{\hbar} \tag{11}$$

As for the parameter z , is a dimensionless parameter intrinsically related to parameters

$(N_{...})$ and $\frac{f_B}{f_P}$ already defined above :

$$z = \frac{4 l_P}{a_0}\,\frac{f_P}{f_B} = 1.2216 \tag{12}$$

Therefore

$$C = (N_e\, \hbar\, \nu_e\,) x\ 1.2216 = 1\ x\ 10^{68} J \tag{13}$$

Once described one by one all items , will type the detailed arithmetics

$$\frac{30[(A-B)+C]}{2C} = \frac{1}{\alpha}\ \rightarrow\ \frac{30[(2.034\ x\ 10^{68}\ J)-(-\ 6.10175\ x\ 10^{68}\ J)+\ 10^{68}\ J]}{2\ x\ 10^{68}\ J} =\ 137.036 \tag{14}$$

As we wanted to demonstrate .

Discussion

we have put into action an example of the arithmetics of large numbers at a cosmological level .

It's about energies : energy of the light of the CMB and the gravitational potential energy inherent to a system of roughly 10^{81} particles like protons and electrons . it's also about the expansion of the observable universe as an exponential growth . In the first stages of the universe temperature was very huge .Both radiation (photons)and matter (electrons and protons) were in thermodynamic equilibrium

As universe expands temperature decreases also . one proton and one electron could combine to form hydrogen and light was no longer scattered . This epoch is called recombination .

Fine structure constant characterizes the strength of the electromagnetic interaction between elementary charged particles , the streng of the coupling of an elementary charged particle with the electromagnetic field . Therefore in this paper we have studied how physical subjects , such the cosmic microwave background and the gravitational potential energy associated to the ordinary matter , all together in an equation are intimately related to the fine structure constant .

Conclusion

We have studied briefly the issue of ''large numbers '' from a merely arithmetic point of view : an arithmetical analysis of the energy of matter , the energy of light and the space's expansion and large numbers involved in such subjects. A summarized equation bring together the number of photons in the observable universe , the current energy associated to the cosmic microwave backgroud , a parameter that defines a space's expansion factor , gravitational potential energy of the matter , the number of atoms in the obsevable universe and the energy associated to the electrons of the observable universe

Bibliography

[1] Norriss S. Hetherington 2014 . Encyclopedia of Cosmology … p.355

[2] Martin B. Reed 2011 . Core maths for the Biosciences , p.295

[3] Paul F. Kisak 2015 . Planck units : the fundamental units of our Universe , p.10

[4] Gregory Lessing Garrett 2018 . The scientism delusion , p.280

[5] Robert G. Mortimer 2008 . Physical Chemistry , p.651

[6] Tai L. Chow 2007 . Gravity , black holes and the very early universe : an introduction to general relativity and cosmology , p.208

[7] Don Anderson 2010 . Musings of an old soul , p.151

[8] T.W.Hartquist , J.E.Dyson 2004 . Blowing bubbles in the cosmos : astronomical winds , jets and explosions , p.15

[9] David Warren Ball 2001 . The basics of spectroscopy , p.12

[10] Lucio Cerrito 2017 . Radiation and detectors : introduction to the physics of radiation and detection devices , p.20

[11] Sunil Batra 2016 . Physics vol I , p.2-23

[12] Marc L. Kutner 2003 . Astronomy : a physical perspective , p.227

[13] Mobilereference -2010- Tables of Universal ,Electromagnetic , Atomic and Nuclear & Physico-Chemical Constants .

Appendix . Table of physical constants [13]

Planck length , l_P : $1.6162 \ x \ 10^{-35} \ m$

Bohr radius , a_0 : $5.291772 \ x \ 10^{-11} \ m$

Reduced Planck constant , $\hbar = \frac{h}{2\pi} = 1.054572 \ x \ 10^{-34} \ Js$

Newtonian constant of gravitation , G_N : $6.6735 \ x \ 10^{-11} \ \frac{m^3}{kg \ s^2}$

Proton mass , m_p : $1.67262 \ x \ 10^{-27} \ kg$

Electron mass , m_e : $9.109382 \ x \ 10^{-31} \ kg$