Gravity without Newton's Gravitational Constant and No Knowledge of Mass Size

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Abstract: In this paper we show that the Schwarzschild radius can be extracted easily from any gravitationally-linked phenomena without having knowledge of the Newton gravitational constant or the mass size of the gravitational object. Further, the Schwarzschild radius can be used to predict any gravity phenomena accurately, again without knowledge of the Newton gravitational constant and also without knowledge of the size of the mass, although this may seem surprising at first.

Hidden within the Schwarzschild radius are the Planck mass and the Planck length, which we will assert, that in addition to the speed of light, (the speed of gravity) contain the secret essence related to gravity. We do not claim to have all the answers, but this seems to indicate that gravity is quantized, even at a cosmological scale, and this quantization is directly linked to the Planck units. This also supports our view that the Newton gravitational constant is a universal composite constant of the form $G = \frac{l_p^2 c^3}{\hbar}$, rather than relying on the Planck units and a function of G. This does not mean that Newton's gravitational constant is not a universal constant, but that it is instead a composite universal constant that depends on the Planck length, the speed of light, and the Planck constant.

This is, to our knowledge, the first paper that shows how a long series of major gravity predictions and measurements can be completed without any knowledge of the mass of the object, or Newton's gravitational constant. As a minimum we think it provides an interesting new angle for evaluating existing gravity theories, and it may even give us a small hint on how to combine quantum gravity with Newton and Einstein gravity.

Key words: Schwarzschild radius, Planck mass, Planck length, measurement, gravitational constant, Heisenberg.

I. INTORDUCTION

We will start this paper on what some will think is a slightly unscientific tone, but we think it is well-suited for a short introduction to what we will show later on. In subsequent sections we will expand on the key points with more rigorous derivations and scientific principles.

Thus, let us assume that an alien came to Earth and gave you the following formula

$$r_e = \frac{g^2 R^2}{c^2} \tag{1}$$

The alien explained that g is the gravitational acceleration, R is the radius from the center of the planet (gravitational mass) to the surface, and c is the well-known speed of light. The gravitational acceleration is easy to measure without any knowledge of gravity; it is about 9.8 m/s^2. The radius of the Earth is not that easy to measure, but we know it is about 6,371,000 meters. As for the speed of light, we can measure it with a low cost kit and or just take the standard accepted speed, which is actually defined as exactly 299792458 m/s. The main point is that one needs no knowledge of gravity to measure each of these input factors.

Next we plug these values into the formula above and get

$$r_e = \frac{9.8^2 \times 6371000^2}{299792458^2} \approx 0.00442588 \tag{2}$$

Some will recognize that this is very similar to half of the value of the Schwarzschild radius of the Earth; this is not a coincidence, as that is indeed exactly what it is, something we will return to soon. Next, we can use this value of r_e and plug into any of the formulas below to calculate almost any major gravity predictions. We can predict the orbital velocity of a satellite or the moon, for example, by the formula

$$v_o = c\sqrt{\frac{r_e}{R_o}} \tag{3}$$

where R_o is the radius from the center of the Earth to the object for which we want to predict orbital velocity. Further, the time dilation between two clocks at different altitudes around a planet is given by

$$\frac{t_h}{t_L} = \frac{\sqrt{1 - 2\frac{r_e}{R_h}}}{\sqrt{1 - 2\frac{r_e}{R_L}}} \tag{4}$$

where R_h is the radius further from the center of the earth than R_L. We can test this by placing one atomic clock at the ocean altitude and one at the top of a 2,000-meter mountain top. We naturally need to synchronize the clocks before performing this task. The clocks will be consistent with our gravity prediction. Again, all we need is r_e, which we can easily extract from the gravitational acceleration on the surface of the Earth, as already shown.

Next we can predict the red-shift; it is given by using the following formula

$$\lim_{r\to+\infty} z(r) = \frac{r_e}{R} \tag{5}$$

This is just one example of how we can perform a series of gravitational predictions that can be confirmed by experiment without any knowledge of Newton's gravitational constant or the mass size of any object. What we have relied on instead is r_e, that can simply be obtained from the gravitational acceleration at the surface of the Earth, the speed of light, and the radius of the earth.

II. THE SCHWARZSCHILD RADIUS IN A NEW PERSPECTIVE

The Schwarzschild radius comes out from the Schwarzschild metric [1, 2] solution of the Einstein field equation [3] and is given by

$$r_s = \frac{2GM}{c^2} \tag{6}$$

where G is the Newton's gravitational constant, M is the mass of the object, and c is the well-known speed of light. In other words, we need to know the mass of the object of interest and the Newton gravitational constant to find its Schwarzschild radius. The escape velocity [4] from a mass M at the radius R from the center of the mass is given by

$$v_e = \sqrt{\frac{2GM}{R}} \tag{7}$$

When we replace the radius in the escape velocity with the Schwarzschild radius $r = r_s = \frac{2GM}{c^2}$ we get

$$v_e = \sqrt{\frac{2GM}{\frac{2GM}{c^2}}} = c \tag{8}$$

So, if an object with mass M is packed inside the Schwarzschild radius, then we have a mass where even light cannot escape from inside the radius. This phenomenon is often known as a black hole, and the Schwarzschild radius is often linked to black holes.

Any object we have observed directly in the sky or on Earth has mass where the radius is extending outside the Schwarzschild radius. In other words, no mass has directly been detected that has all of its mass inside the Schwarzschild radius (even though recent gravitational wave detections may have detected collisions of black holes, that is something we not will discuss further in this paper).

What is important here is that the Newton gravitational constant, not the mass of the cosmological or smaller-sized objects, needs to be known to find the Schwarzschild radius. The Schwarzschild radius can be found directly as described in the section above from the gravitational acceleration of the Earth, or directly from the measured orbital velocity of a satellite such as the moon by simply using the formula

$$r_s = 2r_e = 2\frac{v_o^2 R}{c^2} \tag{9}$$

where R is now the radius from the center of the Earth to the orbital object of interest. Further, v_o is the "easily" observed orbital velocity of the moon, for example. Alternatively, we could use two atomic clocks, measure time dilation between them, and then plug the values into this formula to find the Schwarzschild radius

$$r_e = R\sqrt{1 - \frac{t_0^2}{t_f^2}} \tag{10}$$

In 2016, Haug [5] suggested that the gravitational constant is likely a universal composite constant of the form

$$G = \frac{l_p^2 c^3}{\hbar} \tag{11}$$

Which is basically identical to a similar composite constant suggested by McCulloch [6]. This leads to an evaluation of the Schwarzschild radius at a deeper level by

$$r_s = \frac{GM}{c^2} = \frac{\frac{l_p^2 c^3}{\hbar} M}{c^2} = 2Nl_p \tag{12}$$

where N is the number of Planck masses in the mass M. This is not new in itself, but the idea that we can find the Schwarzschild radius with no knowledge off G or even the mass and use this to predict "all" known gravity phenomena is new, to the best of our knowledge. What seems important for gravity phenomena is the combination measurement of N and l_p, and in this case, we do not need to know N separately (the number of Planck masses) or l_p seperately, but the combination of the two Nl_p will be sufficient.

There are several good reasons clearly pointing towards the Newton gravitational constant is a composite consrant; here is a selection of them:

1. If we never need the gravitational constant for any gravitation observation, not even in calibrating a model, does this imply that it is not central for gravity either? See Table 1 for a long series of gravity calculations and observations that all can be done without any knowledge of the Newton gravitational constant. Further, if we want to separate out the Planck units, we need to know the size of the mass of the gravity object, otherwise that is not

necessary. For all gravity predictions and phenomena in the table we need no knowledge of Newton's gravitational constant, or the size of the mass.

2. The output units of Newton's gravitational constant are given by: $m^3 \cdot kg^{-1} \cdot s^{-2}$. It would seem very strange if something fundamental existed at the deepest level that is meters cubed, divided by kg and seconds squared. It cannot be excluded, but one should first undertake a serious attempt to find a simpler explanation. We will claim this strongly indicates that the Newton gravitational constant must be a composite universal constant consisting of more fundamental constants.

3. By re-formulating G as a composite of the form $G = \frac{l_p^2 c^3}{\hbar}$, a long series of the Planck units are dramatically simplified and become more logical. For example, the Planck time described as $t_p = \sqrt{\frac{G\hbar}{c^5}}$, such formulas give minimal intuition. We may ask, what is the logical meaning of c^5 and what is the deep logic behind the gravitational constant? When replacing G with its composite form, we simply show the Planck time as $t_p = \frac{l_p}{c}$, so the time it takes for light to travel the Planck length – this is naturally known.

4. The Planck mass and the Planck length can be measured totally independent of any knowledge of the Newton gravitational constant, as recently shown by Haug [7, 8]. This means the elements of a Newtonian composite gravitational constant all are known. It seems more logical that at a fundamental level there exists a unique and likely the shortest possible length with any real meaning, namely the Planck length, as well as the speed of light. In addition, we have the Planck constant that is more complex, but in all observable gravity phenomena the gravitational constant even cancels out, and we are left with the Schwarzschild radius (or half of this in many cases) as the essential thing we need to know and can measure easily. Again, this consists of the number of Planck masses times the Planck length in the gravity object of interest.

5. We can derive a gravitational theory from scratch based on the Heisenberg uncertainty principle that, combined with the analysis given here, generates a long series of gravity equations that give correct predictions without any knowledge of the Newton gravitational constant.

[1] Needs further investigation and confirmation; see [9] for more details.

III. MCCULLOCH-HEISENBERG NEWTON EQUIVALENT GRAVITY

We will also mention a recently-published way of deriving Newtonian equivalent gravity that is potentially linked to the above analysis. In 2014 McCulloch [6] derived Newton's gravitational force from Heisenberg's uncertainty principle. Although the method can be criticized, it provides an interesting perspective on the themes of this paper. Here we will here give a short overview of his derivation and point out several valid questions that we will answer, in part.

Heisenberg's uncertainty principle [10] is given by

$$\Delta p \Delta x \geq \hbar \tag{13}$$

McCulloch goes on to say " Now $E = pc$ so" :

$$\Delta E \Delta x \geq \hbar c \tag{14}$$

This assumption only holds for the Planck momentum $E = pc = m_p cc$, in our opinion. Further, from equation 14, McCulloch suggests that

$$F = \frac{1}{(\Delta x)^2} \sum_i^n \sum_j^N (\hbar c)_{i,j} \tag{15}$$

where $\sum_i^n$ is the number of Planck masses in a smaller mass m we are working with, and $\sum_i^N$ corresponds to the the number of Planck masses in the larger mass we are working with. From this, McCulloch gets the equation

$$F = \frac{\hbar c}{m_p^2} \frac{mM}{(\Delta x)^2} \tag{16}$$

Further, McCulloch replaces Δx with the radius and points out that

$$G = \frac{\hbar c}{m_p^2} \tag{17}$$

The McCulloch gravitational constant is a composite $\frac{\hbar c}{m_p^2} \approx 6.67384 \times 10^{-11} m^3 \cdot kg^{-1} \cdot s^{-2}$. This is equivalent to the empirically observed Newton gravitational constant. It should be observed that there are still large measurement errors in the gravitational constant; see [11–15].

This means his derivation is equivalent to the Newtonian gravity formula

$$F = G \frac{mM}{r^2} \tag{18}$$

Although the idea is promising, several aspects of this derivation and gravity concept should be questioned.

First of all, in a follow up paper McCulloch states, [16].

TABLE I. The table shows that the most common gravitational measurements and predictions can be done without any knowledge of Newton's gravitational constant. Is it not time to ask if the Newton gravitational constant is a composite constant? Only when we want to separate out the Planck units or the gravitational constant do we need to know the mass size of the gravitational object.

What to measure/predict	**Formula**	**How**	**Is it easy to do**	**Knowledge mass size**
Half Schwarzschild radius	$r_e = \frac{g^2 R^2}{c^2}$	Observed g (9.8 m/s^2 Earth)	Yes	No
Half Schwarzschild radius	$r_e = \frac{v_o^2 R_o}{c^2}$	Observed orbital velocity	Yes	No
Half Schwarzschild radius	$r_e = R\sqrt{1-\frac{t_0^2}{t_f^2}}$	From time dilation needs high precision clocks	less so	No
Half Schwarzschild radius	$r_e = z(r)R$	From red-shift	less so	No
Half Schwarzschild radius	$r_e = \frac{\delta R}{4}$	From light-bending	less so	No
Gravitational acceleration field	$g = \frac{r_e}{R^2}c^2$	Find r_e first	Yes	No
Orbital velocity	$v_o = c\sqrt{\frac{r_e}{R}}$	Find r_e first	Yes	No
Escape velocity	$v_e = c\sqrt{2\frac{r_e}{R}}$	Find r_e first	Yes	No
Time dilation	$t_2 = t_1\sqrt{1-2\frac{r_e}{R}}$	Find r_e first	Yes	No
Soldner bending of light	$\delta = 2\frac{r_e}{R}$	Find r_e first	Not possible as wrong	No
GR bending of light	$\delta = 4\frac{r_e}{R}$	Find r_e first	Yes	No
Gravitational red-shift	$\lim_{r\to+\infty} z(r) = \frac{r_e}{R}$	Find r_e first	Yes	No
Bekenstein-Hawking luminosity	$P = \frac{1}{15360\pi}\frac{\hbar c^2}{r_e^2}$	Find r_e first	Yes	No
Planck mass	$m_p = \sqrt{\frac{\hbar^3 2\pi^2 L R^2\theta}{l_p^2 M T^2 c^5}}$	Cavendish apparatus	Yes	Yes
Planck length	$l_p = \sqrt{\frac{\hbar L 2\pi^2 R^2\theta}{M T^2 c^3}}$	Cavendish apparatus	Yes	Yes
Planck time	$t_p = \sqrt{\frac{\hbar L 2\pi^2 R^2\theta}{M T^2 c^5}}$	Cavendish apparatus	Yes	Yes
Gravitational constant	$G = \frac{\hbar c}{m_p^2} = \frac{l_p^2 c^3}{\hbar} \approx 6.67\times 10^{-11}$	Cavendish apparatus	Yes	Yes

In the above gravitational derivation, the correct value for the gravitational constant G can only be obtained when it is assumed that the gravitational interaction occurs between whole multiples of the Planck mass, but this last part of the derivation involves some circular reasoning since the Planck mass is defined using the value for G.

The Planck mass was first derived by Max Planck in 1899 [17, 18], who assumed that there were three fundamental universal constants, namely the speed of light, the Newton gravitational constant, and the reduced Planck constant. The Planck mass was given as $m_p = \sqrt{\frac{\hbar c}{G}}$. To find the Planck mass, we need to know G, as pointed out by McCulloch and the McCulloch derivation appears to rest on Newtonian theory.

However, as has recently been shown by Haug, this is not necessarily the case, as we can measure the mass m_p directly using a Cavendish apparatus without any prior knowledge of Newton gravity theory or the Newtonian gravitational constant [8]. In that case, we need to know the Planck constant in addition to the speed of light. The Planck constant can be measured independent of any knowledge of the Newton gravitational constant, using the Kibble balance, for example; see [19–21]. The main point is that we do not need to know the Newton gravitational constant to find the Planck mass.

Secondly, how can a principle derived by Heisenberg to understand the uncertainty in quantum world be relevant to this area of physics? Haug has recently re-derived the Heisenberg uncertainty principle with a specific focus on the Planck scale [22]. He has shown that the uncertainty principle likely collapses at the Planck scale and should be replaced with a certainty principle in the special case of the Planck mass:

$$m_p c l_p = \hbar \tag{19}$$

that is, when we are at the Planck scale for a particle with position l_p. We may also ask, how can the Planck mass be relevant when we are working with cosmological objects? In other working papers, we have presented a model of how the Planck mass particle could be the building block of all other particles. This may sound absurd at first, as the Planck mass is so much larger than any known particle. However, recent research has indicated that mass at a deeper level can be seen as a Compton clock. This suggests that the Planck mass is related to the Planck time, and instead of looking for a very large mass (compared to any observed particle), we should be looking for a very small mass, approximately 1.17×10^{-51} kg.

Finally, is it on solid theoretical ground that McCulloch basically transforms the mass momentum relation into an energy position relation? Due to the Pauli objection [23], the energy time version of the uncertainty relation is not considered valid by many physicists, because according to Pauli one cannot find a time operator that is both Hermitian and self-adjoint. McCulloch does not need a time operator, as he does not use time, but instead uses position in relation to energy. However, due to the fact that energies have been proven to come in quanta and there is typically assumed continuous position when deriving the Heisenberg principle, then the position operator will likely also not be Hermitian and self-adjoint with respect to energy. In other words, the McCulloch derivation could run into the Pauli objection. More likely we think that the energy position and the energy time version of the uncertainty principle first introduced by Heisenberg actually are valid. Further, several researchers have recently suggested ways to get around the Pauli objection; see [24].

IV. CONCLUSION

We have shown how a long series of gravity predictions and measurements are totally independent of knowledge of the Newton gravitational constant, or the size of the mass in question. One component is (half) of the Schwarzschild radius, which at a deeper level is the number of Planck masses in the gravitational object multiplied by the Planck length. However, for most gravitational observations and predictions we do not need to break down the Schwarzschild radius into these fundamental components.

We also show, that contrary to possibly beliefs, we do not need any knowledge of the mass of the gravity object or the Newton gravitational constant to find the Schwarzschild radius of a cosmological object, or even a small clump of matter on Earth. This strongly supports our recent view that the Newton gravitational constant is a composite constant of the form $G = \frac{\hbar c}{m_p^2} = \frac{l_p^2 c^3}{\hbar}$. In understanding this we may be a bit closer to understanding the link between the quantum world and the macroscopic world in terms of gravity. We have not shown any new predictions in gravity, but we think our new angle on existing theory is interesting and also relevant from a practical point of view.

If our theory is correct, then any student or researcher can now, based on only on measuring the gravitational acceleration on Earth, perform a long series of accurate gravity predictions without any knowledge of G or the mass of the object.

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