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Absorption of Spin by a Conducting Medium

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Abstract: Two current concepts of the electrodynamics spin are considered in comparison. Then the original concept, going back to Poynting, was used to demonstrate the transfer of energy and spin from a plane circularly polarized electromagnetic wave into a conducting medium. A spin tensor is used. The given calculations show that spin is the same natural property of a plane electromagnetic wave, as energy and momentum. The expression for the torque density acting on a conducting medium is given. This expression is an analogy of the Lorentz force density.

Keywords: Classical Spin, Circular Polarization, Spin Tensor

1. Introduction 1. Spin Density Is Proportional to the Energy Density

It was suggested as early as 1899 by Sadowsky [1] and as 1909 by Poynting [2] that any usual circularly polarized light carries angular momentum volume *density,* and the angular momentum density is proportional to the energy volume density. That is the angular momentum is present in any point of the light.

J. H. Poynting: If we put E for the energy in unit volume and G for the torque per unit area, we have $G = E\lambda / 2\pi$ [2, p. 565].

This sentence points that any absorption of a circularly polarized light results in a mechanical torque density acting on the absorber. We have researched this effect and have found that this torque density induces specific mechanical stresses in the absorber [3].

Now, according to the Lagrange formalism, this angular momentum volume density is *spin density*. The spin of electromagnetic waves is described by a canonical spin tensor [4 – 7].

$$\underset{c}{\Upsilon}^{\lambda\mu\nu} = -2A^{[\lambda}\delta^{\mu]}_{\alpha}\frac{\partial \mathsf{L}}{\partial(\partial_\nu A_\alpha)}, \tag{1}$$

where $\mathsf{L} = -F_{\mu\nu}F^{\mu\nu}/4$ is the canonical Lagrangian, A^λ is the magnetic vector potential of the electromagnetic field, and $F_{\mu\nu}$ is the electromagnetic field tensor. So, any infinitesimal 3-volume dV_ν contains spin

$$dS^{\lambda\mu} = \underset{c}{\Upsilon}^{\lambda\mu\nu}dV_\nu. \tag{2}$$

The spin tensor (1) is appeared in the company of the canonical energy-momentum tensor $\underset{c}{T}^{\mu\nu}$ and of the orbital angular momentum tensor, which is simply a moment of the energy-momentum tensor $\underset{c}{L}^{\lambda\mu\nu} = 2x^{[\lambda}\underset{c}{T}^{\mu]\nu}$. So, the total angular momentum tensor $\underset{c}{J}^{\lambda\mu\nu} = \underset{c}{L}^{\lambda\mu\nu} + \underset{c}{\Upsilon}^{\lambda\mu\nu}$ equals the sum of orbital and spin angular momentums. The canonical energy-momentum, total angular momentum, and spin tensors are, respectively,

$$\underset{c}{T}^{\mu\nu} = -\partial^\mu A_\alpha F^{\nu\alpha} + g^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}/4, \tag{3}$$

$$\underset{c}{J}^{\lambda\mu\nu} = 2x^{[\lambda}\underset{c}{T}^{\mu]\nu} + \underset{c}{\Upsilon}^{\lambda\mu\nu},\ \underset{c}{\Upsilon}^{\lambda\mu\nu} = -2A^{[\lambda}F^{\mu]\nu}. \tag{4}$$

So, the classical electromagnetic field theory provides meaningful descriptions of the spin and the orbital angular momentum separately. Note, spin tensor $\Upsilon^{\lambda\mu\nu}$ is not a moment of energy-momentum tensor $2x^{[\lambda}T^{\mu]\nu}$, and spin density Υ is not (a part of) a moment of linear momentum density $\mathbf{r}\times(\mathbf{E}\times\mathbf{H})/c^2$.

A *perfect* plane monochromatic circularly polarized electromagnetic wave travelling in z-direction and with infinite extension in the xy-directions is represented e.g. by the equations:

$$\mathbf{E}_1 = E_1 \exp(ikz - i\omega t)(\mathbf{x} + i\mathbf{y}) \ \text{[V/m]}, \ \mathbf{H}_1 = -i\varepsilon_0 c\mathbf{E}_1 \ \text{[A/m]}, \ ck = \omega \,. \tag{5}$$

According to the definition (4), the spin volume density in such a wave is given by the component of the spin tensor

$$\underset{c}{\Upsilon}^{ijt} = -2A^{[i}F^{j]t} = -2A^{[i}E^{j]} = \mathbf{E}\times\mathbf{A} \tag{6}$$

For example, Soper [5] writes:

“To describe a circularly polarized plane wave traveling in the z-direction, we can choose a potential

$$A^x = a\cos[\omega(z-t)], \ A^y = -a\sin[\omega(z-t)]$$

The corresponding electric field is $E^k = -\partial_t A^k$.

$$E^x = -a\omega\sin[\omega(z-t)], \ E^y = -a\omega\cos[\omega(z-t)]$$

Thus the spin density carried by this wave is

$$\mathbf{s} = a^2\omega\hat{\mathbf{z}}$$

where $\hat{\mathbf{z}}$ is a unit vector pointing in the z-direction.”

Soper means

$$\underset{c}{\Upsilon}^{xyt} = E^x A^y - E^y A^x = a^2\omega \tag{7}$$

in agreement with (6).

Note that perfect plane electromagnetic waves were used by Einstein [8, § 7]:

"In the system K, very far from the origin of co-ordinates, let there be a source of electrodynamic waves, which in a part of space containing the origin of co-ordinates may be represented to a sufficient degree of approximation by the equations" of type (5).

The canonical spin tensor (4) was successfully used in [9] in order to confirm the fulfillment of the conservation laws with respect to spin when a plane circularly polarized electromagnetic wave reflects from a moving mirror, and in [10] where the absorption of spin of a plane wave is described. These calculations prove the functionality of the spin tensor and show that spin is the same natural property of a perfect plane electromagnetic wave, as energy and momentum; and spin density is proportional to energy density.

The classical experiments [11 – 14] also confirm that the spin density of plane waves is proportional to energy density. In these experiments, the angular momentum of the light was transferred to a half-wave plate, which rotated. So, work was performed *in any point of the plate*. This (positive or negative) amount of work reappeared as an alteration in the energy of the photons, i.e., in the frequency of the light, which resulted in moving fringes *in any point* of the interference pattern in a suitable interference experiment.

Some textbooks and articles point that infinite plane circularly polarized electromagnetic wave carries angular momentum:

F. S. Crawford, Jr.: "A circularly polarized travelling plane wave carries angular momentum" [15, p. 365].

R. Feynman: "… the photons of light that are right circularly polarized carry an angular momentum of one unit along the z-axis …light which is right circularly polarized carries an energy and angular momentum" [16].

K. Bliokh and F. Nori: “… the plane wave carries the spin AM density S defined as the *local* expectation value of the operator $\hat{\mathbf{S}}$” [17, p. 4].

A spin tensor is used when describing plane waves in [18, 19] and in other papers of the author.

2. Introduction 2. Spin Density Is Not Proportional to Gradient of the Energy Density

However, since 1939, another concept of electrodynamics spin is in use. The point is, Belinfante & Rosenfeld added specific terms,

$$\partial_\alpha(A^\mu F^{\nu\alpha}) \ \text{and} \ 2\partial_\alpha(x^{[\lambda}A^{\mu]}F^{\nu\alpha}), \tag{8}$$

to the canonical energy-momentum tensor $\underset{c}{T}^{\mu\nu}$ and to the total angular momentum tensor $\underset{c}{J}^{\lambda\mu\nu}$, respectively [20, 21], [5, Sec. 9.4]. This procedure yields an energy-momentum tensor $\underset{st}{T}^{\mu\nu}$ (which differs from the Maxwell tensor $T^{\mu\nu}$), and an total angular momentum tensor $\underset{st}{J}^{\lambda\mu\nu}$. We named these tensors (9) and (10) "standard" [3]:

$$\begin{aligned}
\underset{st}{T}^{\mu\nu} &= \underset{c}{T}^{\mu\nu} + \partial_\alpha(A^\mu F^{\nu\alpha}) = -\partial^\mu A_\alpha F^{\nu\alpha} + g^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}/4 + \partial_\alpha(A^\mu F^{\nu\alpha}) \\
&= -\partial^\mu A_\alpha F^{\nu\alpha} + g^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}/4 + \partial_\alpha A^\mu F^{\nu\alpha} + A^\mu\partial_\alpha F^{\nu\alpha} \\
&= g^{\mu\lambda}(-\partial_\lambda A_\alpha F^{\nu\alpha} + \partial_\alpha A_\lambda F^{\nu\alpha}) + g^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}/4 + A^\mu\partial_\alpha F^{\nu\alpha} \\
&= -g^{\mu\lambda}F_{\lambda\alpha}F^{\nu\alpha} + g^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}/4 + A^\mu\partial_\alpha F^{\nu\alpha} = T^{\mu\nu} + A^\mu\partial_\alpha F^{\nu\alpha},
\end{aligned} \tag{9}$$

where $T^{\mu\nu} = -g^{\mu\lambda}F_{\lambda\alpha}F^{\nu\alpha} + g^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}/4$ is the Maxwell tensor,

$$\underset{st}{J}^{\lambda\mu\nu} = \underset{c}{J}^{\lambda\mu\nu} + 2\partial_\alpha(x^{[\lambda}A^{\mu]}F^{\nu\alpha}) = 2x^{[\lambda}\underset{c}{T}^{\mu]\nu} - 2A^{[\lambda}F^{\mu]\nu} + 2\partial_\alpha(x^{[\lambda}A^{\mu]}F^{\nu\alpha})$$

$$= 2x^{[\lambda}\underset{c}{T}^{\mu]\nu} - 2A^{[\lambda}F^{\mu]\nu} + 2\delta_\alpha^{[\lambda}A^{\mu]}F^{\nu\alpha} + 2x^{[\lambda}\partial_\alpha(A^{\mu]}F^{\nu\alpha}) = 2x^{[\lambda}\underset{st}{T}^{\mu]\nu}. \tag{10}$$

But this procedure eliminates the spin tensor ($\underset{st}{\Upsilon}^{\lambda\mu\nu} = 0$). Really, the standard total angular momentum tensor (10) is equal to moment of the standard energy-momentum tensor: $\underset{st}{J}^{\lambda\mu\nu} = 2x^{[\lambda}\underset{st}{T}^{\mu]\nu}$ without a spin term, cf. (4)

So, the corresponding ("standard") spin term is absent. As a result, in the absence of electrodynamics spin tensor, it has been declared that the electrodynamics spin **S** is contained inside the moment of linear momentum together with the orbital momentum **L** [22, p. 7]

$$\int \varepsilon_0 \mathbf{r}\times(\mathbf{E}\times\mathbf{B})dV = \mathbf{L} + \mathbf{S},$$

and that a plane wave has no angular momentum at all.

Heitler W: "A plane wave travelling in z-direction and with infinite extension in the xy-directions can have no angular momentum about the z-axis, because $(\mathbf{E}\times\mathbf{B})$ is in the z-direction and $(\mathbf{r}\times(\mathbf{E}\times\mathbf{B}))_z = 0$ " [23].

According to the nowadays conception, electrodynamics spin density is proportional to *gradient of energy density*, not to the energy density itself:

Allen L., Padgett M. J.: "… the local spin angular momentum density per photon is proportional to the radial intensity gradient of a light beam:

$$j_z = -r\hbar\sigma/(2u^2)\partial(u^2)/\partial r$$

where $\sigma = \pm 1$ for right- and left-handed circularly polarized light respectively, u^2 is the beam intensity, and r is the distance from the axis. For a plane wave there is no gradient and the spin density is zero." [24]

Simmonds J. W., Guttmann M. J.: "The electric and magnetic fields can have a nonzero z-component only within the skin region of this wave. Having z-components within this region implies the possibility of a nonzero z-component of angular momentum within this region. So, the skin region is the only in which the z-component of angular momentum does not vanish" [25, p. 227]

Thus, according to the widespread opinion, an electromagnetic wave without an azimutal phase gradient has an angular momentum only where, there is an intensity gradient, and this angular momentum is spin. Particularly, angular momentum of a circularly polarized beam is a spin, not an orbital angular momentum, and this spin is localized on the surface of the beam. This opinion carries the spin far to the edge of a real wave in compliance with the Humblet transformation [26].

We have criticized [3] the Humblet transformation. We have noted [27] that this concept, "Spin is only in the skin region," threatens us with a considerable nonlocality of the electrodynamics because the concept implies that the energy and momentum of photons are absorbed everywhere in the absorber, but the spin is absorbed in the remote boundary of the wave.

Note, the Belinfante-Rosenfeld procedure does not yield the Maxwell tensor and even does not symmetrize the canonical tensor (3); the term $A^\mu\partial_\alpha F^{\nu\alpha}$ in (9) vanishes only when interactions are absent, but this case has no sense.

In this paper, we confirm the Poynting's and Sadowsky's concept by a new calculation.

3. Absorption of Energy and Spin

Let

$$\mathbf{E} = \exp[i(\breve{k}z - t)](\mathbf{x} + i\mathbf{y}), \quad \mathbf{B} = \exp[i(\breve{k}z - t)]\breve{k}(-i\mathbf{x} + \mathbf{y}), \quad \breve{k} = k_1 + ik_2 \tag{11}$$

is a damping plane circularly polarised electromagnetic wave, which is propageted in a conducting medium for $z > 0$. We set $\varepsilon_0 = \mu_0 = c = \omega = 1$, and we indicate complex numbers by the *breve* mark: $\breve{k}$. Then

$$\mathbf{E}_1 = \exp[i(z - t)](1 + \breve{k})(\mathbf{x} + i\mathbf{y})/2, \quad \mathbf{B}_1 = \exp[i(z - t)](1 + \breve{k})(-i\mathbf{x} + \mathbf{y})/2, \tag{12}$$

$$\mathbf{E}_2 = \exp[i(-z - t)](1 - \breve{k})(\mathbf{x} + i\mathbf{y})/2, \quad \mathbf{B}_2 = \exp[i(-z - t)](1 - \breve{k})(i\mathbf{x} - \mathbf{y})/2 \tag{13}$$

are the incident and reflected waves for $z < 0$, respectively.

Vector potential waves can be written by the formula $\mathbf{A} = -\int \mathbf{E}dt = -i\mathbf{E}$

$$\mathbf{A} = \exp(i\breve{k}z - it)(-i\mathbf{x} + \mathbf{y}), \tag{14}$$

$$\mathbf{A}_1 = \exp(iz - it)(1 + \breve{k})(-i\mathbf{x} + \mathbf{y})/2, \tag{15}$$

$$\mathbf{A}_2 = \exp(-iz - it)(1-\breve{k})(-i\mathbf{x}+\mathbf{y})/2\,. \tag{16}$$

The use of the Maxwell energy-momentum tensor gives the Poynting vectors of the waves:

$$T^{tz} = \Re\{-\bar{F}_{ti}F^{zi}\}/2 = \Re\{\bar{E}^x B^y - \bar{E}^y B^x\}/2 = \Re\{\bar{e}_k \breve{k} e_k - (-i\bar{e}_k)(-i\breve{k}e_k)\}/2 = k_1 \exp(-2k_2 z)\,, \tag{17}$$

$$T_1^{tz} = \Re\{\bar{E}_1^x B_1^y - \bar{E}_1^y B_1^x\}/2 = \Re\{(1+\bar{k})\bar{e}(1+\breve{k})e - (1+\bar{k})(-i\bar{e})(1+\breve{k})(-ie)\}/8 = (1+2k_1+k^2)/4\,, \tag{18}$$

$$T_2^{tz} = \Re\{\bar{E}_2^x B_2^y - \bar{E}_2^y B_2^x\}/2 = \Re\{(1-\bar{k})\bar{e}(1-\breve{k})e - (1-\bar{k})(-i\bar{e})(1-\breve{k})(-ie)\}/8 = -(1-2k_1+k^2)/4\,, \tag{19}$$

$$T_1^{tz} + T_2^{tz} = T^{tz}\Big|_{z=0}\,. \tag{20}$$

Here we denoted to shorten the record: $e_k \equiv \exp(i\breve{k}z - it)$, $e \equiv \exp(iz - it)$, or $e \equiv \exp(-iz - it)$ and $k^2 = \bar{k}\breve{k}$

The use of the canonical spin tensor (1.4) gives the spin fluxes in the waves:

$$\underset{c}{\mathrm{Y}}^{xyz} = \Re\{-\bar{A}^x F^{yz} + \bar{A}^y F^{xz}\}/2 = \Re\{\bar{A}^x B^x + \bar{A}^y B^y\}/2 = \Re\{i\bar{e}_k \breve{k}(-ie_k) + \bar{e}_k \breve{k} e_k\}/2 = k_1 \exp(-2k_2 z)\,, \tag{21}$$

$$\underset{c}{\mathrm{Y}}_1^{xyz} = \Re\{\bar{A}_1^x B_1^x + \bar{A}_1^y B_1^y\}/2 = \Re\{(1+\bar{k})i\bar{e}(1+\breve{k})(-ie) + (1+\bar{k})\bar{e}(1+\breve{k})e\}/8 = (1+2k_1+k^2)/4\,, \tag{22}$$

$$\underset{c}{\mathrm{Y}}_2^{xyz} = \Re\{\bar{A}_2^x B_2^x + \bar{A}_2^y B_2^y\}/2 = \Re\{(1-\bar{k})i\bar{e}(1-\breve{k})(-ie) + (1-\bar{k})\bar{e}(1-\breve{k})e\}/8 = -(1-2k_1+k^2)/4\,, \tag{23}$$

$$\underset{c}{\mathrm{Y}}_1^{xyz} + \underset{c}{\mathrm{Y}}_2^{xyz} = \underset{c}{\mathrm{Y}}^{xyz}\Big|_{z=0}\,. \tag{24}$$

The difference between the energy and spin fluxes in the incident and reflected waves is absorbed in the medium. The equality between the energy and spin fluxes is natural because energy of photon $\hbar\omega$ equals spin of photon $\hbar$ if $\omega = 1$.

Current density in the medium can be written by the formula $\mathbf{j} = \mathrm{curl}\,\mathbf{B} - \partial_t \mathbf{E}$:

$$j^x = -\partial_z B^y - \partial_t E^x = i(1-\breve{k}^2)\exp[i(\breve{k}z - t)], \quad j^y = \partial_z B^x - \partial_t E^y = (\breve{k}^2 - 1)\exp[i(\breve{k}z - t)]\,. \tag{25}$$

Absorbed power density equals $w = -\partial_z T^{tz} = (\mathbf{j}\cdot\mathbf{E})$. Really (see (3.7)),

$$(\mathbf{j}\cdot\mathbf{E}) = \Re\{\bar{j}^x E^x + \bar{j}^y E^y\}/2 = 2k_1 k_2 \exp(-2k_2 z)\,. \tag{26}$$

Now consider the divergence of the spin tensor (4)

$$\begin{aligned} \partial_\nu \underset{c}{\mathrm{Y}}^{\lambda\mu\nu} &= -\partial_\nu (2A^{[\lambda}F^{\mu]\nu}) \\ &= -\partial_\nu A^\lambda \partial^\mu A^\nu + \partial_\nu A^\lambda \partial^\nu A^\mu + \partial_\nu A^\mu \partial^\lambda A^\nu - \partial_\nu A^\mu \partial^\nu A^\lambda - A^\lambda \partial_\nu^\mu A^\nu + A^\lambda \partial_\nu^\nu A^\mu + A^\mu \partial_\nu^\lambda A^\nu - A^\mu \partial_\nu^\nu A^\lambda \end{aligned} \tag{27}$$

The first and third terms are zero because $\partial_x = 0, \partial_y = 0, A^z = A^t = 0$, the fifth and seventh terms are zero because of $\partial_\nu A^\nu = 0$, the second and fourth terms mutually cancel. So, we have

$$\partial_\nu \underset{c}{\mathrm{Y}}^{\lambda\mu\nu} = A^\lambda \partial_\nu^\nu A^\mu - A^\mu \partial_\nu^\nu A^\lambda\,. \tag{28}$$

And, because of $\partial_\nu^\nu A^\mu = j^\mu$,

$$-\partial_z \underset{c}{\mathrm{Y}}^{xyz} = \mathbf{j}\times\mathbf{A} = \Re\{\bar{j}^x A^y - \bar{j}^y A^x\}/2 = 2k_1 k_2 \exp(-2k_2 z) \tag{29}$$

The divergence of the spin tensor is expressed in terms of the *torque volume density* $d\tau^{xy}/dV$ which is an analogue of the Lorentz force density [28 (33.7)]: $-\partial_i T^{ki} = j_i F^{ki} = \mathbf{j}\times\mathbf{B}$.

$$-\partial_z \underset{c}{\mathrm{Y}}^{xyz} = d\tau^{xy}/dV = (\mathbf{j}\times\mathbf{A})^z\,. \tag{30}$$

This result was published in [29].

4. Conclusion

The given calculations show that spin is a natural property of a plane electromagnetic wave, similar to energy. We have confirmed the Poynting statement that the absorption of energy of circularly polarized light is accompanied by the absorption of volume density of torque.

We are eternally grateful to Professor Robert Romer, having courageously published the question: "Does a plane wave really not carry spin?" [30].

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