The differential equation of a swarm of smart particles (insects, birds, fish, robots, etc.) can be obtained using two axioms:

1. there is a steady density μ for the fluid dynamics
2. the flow velocity has the density gradient as a module

the Euler's equation (see Fluid mechanics of Landau Lifshitz for some ideas used here):

$$\begin{array}{l} \frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v}) \\ \rho \frac{d\mathbf{v}}{dt} = -\nabla p \Longrightarrow \frac{\partial \mathbf{v}}{\partial t} = -(\mathbf{v} \cdot \nabla)\mathbf{v} - \frac{1}{\rho}\nabla p \end{array}$$

I simplify the equations using a force in each volumes that is proportional to the gradient of the density of smart particles:

$$\mathbf{F}_{dV} = -\alpha \ \nabla|\rho - \mu| \ \rho dV$$

so that the intelligent particles move towards points with optimal density μ (this is the force that the fluid apply to the volume dV, but I consider this force like the force of the swarm of ρdV particles):

$$\alpha = \frac{w^2}{\mu}$$

so that, $\mathbf{F}_{dV}$ is the force in the swarm (like a bird that change direction using a force to obtain the optimal density).

I write the Euler's equation for this force (I consider a constant pressure on an infinitesimal volume dV that it is equal to the density gradient):

$$\begin{array}{l} \partial_t \rho = -\nabla \cdot (\rho \mathbf{v}) \\ \rho \ d_t \mathbf{v} = -\alpha \ \rho \ \nabla|\rho - \mu| \Longrightarrow \partial_t \mathbf{v} = -(\mathbf{v} \cdot \nabla)\mathbf{v} - \alpha \ sgn(\rho - \mu) \ \nabla \rho \end{array}$$

I solve this equations for little perturbation of the density:

$$\rho = \mu + \epsilon$$

so that:

$$\begin{array}{l} \partial_t \epsilon = -\mu \ \nabla \cdot \mathbf{v} \\ \partial_t \mathbf{v} = -\alpha \ sgn(\rho - \mu) \ \nabla \epsilon \end{array}$$

to simplify the equations (like in the sound wave):

$$\mathbf{v} = \nabla \phi$$

so that:

$$\partial_t \epsilon = -\mu\ \Delta\phi$$
$$\partial_t \nabla\phi = -\alpha\ sgn(\rho-\mu)\nabla\epsilon$$
$$\epsilon = -\frac{1}{\alpha\ sgn(\rho-\mu)}\partial_t\phi$$
$$\partial^2_{tt}\phi - \alpha\ \mu\ sgn(\rho-\mu)\Delta\phi = 0$$
$$\partial^2_{tt}\phi - w^2 sgn(\rho-\mu)\ \Delta\phi = 0$$

there are two regions; the internal region that has a sound wave solution with velocity w, and a external region that has exponential decay.

The perturbative solution near the border of the swarm are sound wave of particles with constant velocity w, and out of the swarm the solution are an exponential space decay; a little perturbation because of draft, or predators, lead an attraction at the border.