Neutrosophic Rare α-Continuity

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ABSTRACT

In this paper, we introduce the concepts of neutrosophic rare α-continuous, neutrosophic rarely continuous, neutrosophic rarely pre-continuous, neutrosophic rarely semi-continuous are introduced and studied in light of the concept of rare set in neutrosophic setting.

KEYWORDS: Neutrosophic rare set; neutrosophic rarely α-continuous; neutrosophic rarely pre-continuous; neutrosophic almost α-continuous; neutrosophic weekly α-continuous; neutrosophic rarely semi-continuous.

1 INTRODUCTION AND PRELIMINARIES

The study of fuzzy sets was initiated by Zadeh (1965). Thereafter the paper of Chang (1968) paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. Currently Fuzzy Topology has been observed to be very beneficial in fixing many realistic problems. Several mathematicians have tried almost all the pivotal concepts of General Topology for extension to the fuzzy settings. In 1981, Azad gave fuzzy version of the concepts given by Levine 1961; 1963 and thus initiated the study of weak forms of several notions in fuzzy topological spaces. Popa (1979) introduced the notion of rare continuity as a generalization of weak continuity (Levine, 1961) which has been further investigated by Long and Herrington (1982) and Jafari (1995; 1997). Noiri (1987) introduced and investigated weakly α-continuity as a generalization of weak continuity. He also introduced and investigated almost α-continuity (Noiri, 1988). The concepts of Rarely α-continuity was introduced by Jafari (2005). The concepts of fuzzy rare α-continuity and intuitionistic fuzzy rare α-continuity were introduced by Dhavaseelan and Jafari (n.d.-b, n.d.-c). After the advent of the concepts of neutrosophy and neutrosophic set introduced by Smarandachethe (1999; 2002), the concepts of neutrosophic crisp set and neutrosophic crisp topological spaces were introduced by Salama and Alblowi (2012).

The purpose of the present paper is to introduce and study the concepts of neutrosophic rare α-continuous functions, neutrosophic rarely continuous functions, neutrosophic rarely pre-continuous functions and neutrosophic rarely semi-continuous functions in light of the concept of rare set in a neutrosophic setting.

Definition 1.1. Let X be a nonempty fixed set. A neutrosophic set [briefly NS] A is an object having the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$, where $\mu_A(x), \sigma_A(x)$ and $\gamma_A(x)$ which represents the degree of membership function ($\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$) and the degree of nonmembership ($\gamma_A(x)$), respectively, of each element $x \in X$ to the set A.

Remark 1.1. (1) A neutrosophic set $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ can be identified to an ordered triple $\langle \mu_A, \sigma_A, \gamma_A\rangle$ in $]0^-, 1^+[$ on X.

(2) For the sake of simplicity, we shall use the symbol $A = \langle \mu_A, \sigma_A, \gamma_A\rangle$ for the neutrosophic set $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$.

Definition 1.2. Let X be a nonempty set and the neutrosophic sets A and B in the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$, $B = \{\langle x, \mu_B(x), \sigma_B(x), \gamma_B(x)\rangle : x \in X\}$. Then

(a) $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x)$, $\sigma_A(x) \leq \sigma_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$;

(b) $A = B$ iff $A \subseteq B$ and $B \subseteq A$;

(c) $\bar{A} = \{\langle x, \gamma_A(x), \sigma_A(x), \mu_A(x)\rangle : x \in X\}$; [complement of A]

(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \vee \gamma_B(x)\rangle : x \in X\}$;

(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \wedge \gamma_B(x)\rangle : x \in X\}$;

(f) $[\,]A = \{\langle x, \mu_A(x), \sigma_A(x), 1 - \mu_A(x)\rangle : x \in X\}$;

(g) $\langle\rangle A = \{\langle x, 1 - \gamma_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$.

Definition 1.3. Let $\{A_i : i \in J\}$ be an arbitrary family of neutrosophic sets in X. Then

(a) $\bigcap A_i = \{\langle x, \wedge\mu_{A_i}(x), \wedge\sigma_{A_i}(x), \vee\gamma_{A_i}(x)\rangle : x \in X\}$;

(b) $\bigcup A_i = \{\langle x, \vee\mu_{A_i}(x), \vee\sigma_{A_i}(x), \wedge\gamma_{A_i}(x)\rangle : x \in X\}$.

Since our main purpose is to construct the tools for developing neutrosophic topological spaces, we must introduce the neutrosophic sets 0_N and 1_N in X as follows:

Definition 1.4. $0_N = \{\langle x, 0, 0, 1\rangle : x \in X\}$ and $1_N = \{\langle x, 1, 1, 0\rangle : x \in X\}$.

Definition 1.5. (Dhavaseelan & Jafari, n.d.-a) A neutrosophic topology (briefly NT) on a nonempty set X is a family T of neutrosophic sets in X satisfying the following axioms:

(i) $0_N, 1_N \in T$,

(ii) $G_1 \cap G_2 \in T$ for any $G_1, G_2 \in T$,

(iii) $\cup G_i \in T$ for arbitrary family $\{G_i \mid i \in \Lambda\} \subseteq T$.

In this case the ordered pair (X, T) or simply X is called a neutrosophic topological space (briefly NTS) and each neutrosophic set in T is called a neutrosophic open set (briefly NOS). The complement $\overline{A}$ of a NOS A in X is called a neutrosophic closed set (briefly NCS) in X.

Definition 1.6. (Dhavaseelan & Jafari, n.d.-a) Let A be a neutrosophic set in a neutrosophic topological space X. Then

$Nint(A) = \bigcup\{G \mid G$ is a neutrosophic open set in X and $G \subseteq A\}$ is called the neutrosophic interior of A;

$Ncl(A) = \bigcap\{G \mid G$ is a neutrosophic closed set in X and $G \supseteq A\}$ is called the neutrosophic closure of A.

Definition 1.7. (Dhavaseelan & Jafari, n.d.-a) Let X be a nonempty set. If r, t, s be real standard or non standard subsets of $]0^-, 1^+[$, then the neutrosophic set $x_{r,t,s}$ is called a neutrosophic point(briefly NP)in X given by

$$x_{r,t,s}(x_p) = \begin{cases} (r, t, s), & \text{if } x = x_p \\ (0, 0, 1), & \text{if } x \neq x_p \end{cases}$$

for $x_p \in X$ is called the support of $x_{r,t,s}$, where r denotes the degree of membership value , t the degree of indeterminacy and s the degree of non-membership value of $x_{r,t,s}$.

Definition 1.8. (Dhavaseelan & Jafari, n.d.-b) An intuitionistic fuzzy set R is called intuitionistic fuzzy rare set if $IFint(R) = 0_\sim$.

Definition 1.9. (Dhavaseelan & Jafari, n.d.-b) An intuitionistic fuzzy set R is called intuitionistic fuzzy nowhere dense set if $IFint(IFcl(R)) = 0_\sim$.

2 MAIN RESULTS

Definition 2.1. A neutrosophic set A in a neutrosophic topological space (X, T) is called

1) a neutrosophic semiopen set (briefly NSOS) if $A \subseteq Ncl(Nint(A))$.

2) a neutrosophic α open set (briefly $N\alpha OS$) if $A \subseteq Nint(Ncl(Nint(A)))$.

3) a neutrosophic preopen set (briefly NPOS) if $A \subseteq Nint(Ncl(A))$.

4) a neutrosophic regular open set (briefly NROS) if $A = Nint(Ncl(A))$.

5) a neutrosophic semipreopen or β open set (briefly $N\beta OS$) if $A \subseteq Ncl(Nint(Ncl(A)))$.

A neutrosophic set A is called a neutrosophic semiclosed set, neutrosophic α-closed set, neutrosophic preclosed set, neutrosophic regular closed set and neutrosophic β-closed set (briefly NSCS, NαCS, NPCS, NRCS and NβCS, resp.), if the complement of A is a neutrosophic semiopen set, neutrosophic α-open set, neutrosophic preopen set, neutrosophic regular open set, and neutrosophic β-open set, respectively.

Definition 2.2. Let a neutrosophic set A of a neutrosophic topological space (X, T). Then neutrosophic α-closure of A (briefly $Ncl_\alpha(A)$) is defined as $Ncl_\alpha(A) = \bigcap\{K|$ K is a neutrosophic α closed set in X and $A \subseteq K\}$.

Definition 2.3. (Jun & Song, 2005) Let a neutrosophic set A of a neutrosophic topological space (X, T). Then neutrosophic α interior of A (briefly $Nint_\alpha(A)$) is defined as $Nint_\alpha(A) = \bigcup\{K|\ K$ is a neutrosophic α open set in X and $K \subseteq A\}$.

Definition 2.4. A neutrosophic set R is called neutrosophic rare set if $Nint(R) = 0_N$.

Definition 2.5. A neutrosophic set R is called neutrosophic nowhere dense set if $Nint(Ncl(R)) = 0_N$.

Definition 2.6. Let (X, T) and (Y, S) be two neutrosophic topological spaces. A function $f : (X, T) \to (Y, S)$ is called

(i) neutrosophic α-continuous if for each neutrosophic point $x_{r,t,s}$ in X and each neutrosophic open set G in Y containing $f(x_{r,t,s})$, there exists a neutrosophic α open set U in X such that $f(U) \leq G$.

(ii) neutrosophic almost α-continuous if for each neutrosophic point $x_{r,t,s}$ in X and each neutrosophic open set G containing $f(x_{r,t,s})$, there exists a neutrosophic α open set U such that $f(U) \leq Nint(Ncl(G))$.

(iii) neutrosophic weakly α-continuous if for each neutrosophic point $x_{r,t,s}$ in X and each neutrosophic open set G containing $f(x_{r,t,s})$, there exists a neutrosophic α open set U such that $f(U) \leq Ncl(G)$.

Definition 2.7. Let (X, T) and (Y, S) be two neutrosophic topological spaces. A function $f : (X, T) \to (Y, S)$ is called

(i) neutrosophic rarely α-continuous if for each neutrosophic point $x_{r,t,s}$ in X and each neutrosophic open set G in (Y, S) containing $f(x_{r,t,s})$, there exist a neutrosophic rare set R with $G \cap Ncl(R) = 0_N$ and neutrosophic α open set U in (X, T) such that $f(U) \leq G \cup R$.

(ii) neutrosophic rarely continuous if for each neutrosophic point $x_{r,t,s}$ in X and each neutrosophic open set G in (Y, S) containing $f(x_{r,t,s})$, there exist a neutrosophic rare set R with $G \cap Ncl(R) = 0_N$ and neutrosophic open set U in (X, T) such that $f(U) \leq G \cup R$.

(iii) neutrosophic rarely precontinuous if for each neutrosophic point $x_{r,t,s}$ in X and each neutrosophic open set G in (Y, S) containing $f(x_{r,t,s})$, there exist a neutrosophic rare set R with $G \cap Ncl(R) = 0_N$ and neutrosophic preopen set U in (X, T) such that $f(U) \leq G \cup R$.

(iv) neutrosophic rarely semi-continuous if for each neutrosophic point $x_{r,t,s}$ in X and each neutrosophic open set G in (Y, S) containing $f(x_{r,t,s})$, there exist a neutrosophic rare set R with $G \cap Ncl(R) = 0_N$ and neutrosophic semiopen set U in (X, T) such that $f(U) \leq G \cup R$.

Example 2.1. Let $X = \{a, b, c\}$. Define the neutrosophic sets A, B and C as follows:
$A = \langle x, (\frac{a}{0}, \frac{b}{0}, \frac{c}{1}), (\frac{a}{0}, \frac{b}{0}, \frac{c}{1}), (\frac{a}{1}, \frac{b}{1}, \frac{c}{0})\rangle$, $B = \langle x, (\frac{a}{1}, \frac{b}{0}, \frac{c}{0}), (\frac{a}{1}, \frac{b}{0}, \frac{c}{0}), (\frac{a}{0}, \frac{b}{1}, \frac{c}{1})\rangle$ and
$C = \langle x, (\frac{a}{0}, \frac{b}{1}, \frac{c}{0}), (\frac{a}{0}, \frac{b}{1}, \frac{c}{0}), (\frac{a}{1}, \frac{b}{0}, \frac{c}{1})\rangle$. Then $T = \{0_N, 1_N, C\}$ and $S = \{0_N, 1_N, A, B, A \cup B\}$ are neutrosophic topologies on X. Let (X, T) and (X, S) be neutrosophic topological spaces. Define $f : (X, T) \to (X, S)$ as a identity function. Clearly f is neutrosophic rarely α-continuous.

Proposition 2.1. Let (X, T) and (Y, S) be any two neutrosophic topological spaces. For a function $f : (X, T) \to (Y, S)$ the following statements are equivalents:

(i) The function f is neutrosophic rarely α-continuous at $x_{r,t,s}$ in (X, T).

(ii) For each neutrosophic open set G containing $f(x_{r,t,s})$, there exists a neutrosophic α open set U in (X, T) such that $Nint(f(U) \cap \overline{G}) = 0_N$.

(iii) For each neutrosophic open set G containing $f(x_{r,t,s})$, there exists a neutrosophic α open set U in (X, T) such that $Nint(f(U)) \leq Ncl(G)$.

(iv) For each neutrosophic open set G in (Y, S) containing $f(x_{r,t,s})$, there exists a neutrosophic rare set R with $G \cap Ncl(R) = 0_N$ such that $x_{r,t,s} \in Nint_\alpha(f^{-1}(G \cup R))$.

(v) For each neutrosophic open set G in (Y, S) containing $f(x_{r,t,s})$, there exists a neutrosophic rare set R with $Ncl(G) \cap R = 0_N$ such that $x_{r,t,s} \in Nint_\alpha(f^{-1}(Ncl(G) \cup R))$

(vi) For each neutrosophic regular open set G in (Y, S) containing $f(x_{r,t,s})$, there exists a neutrosophic rare set R with $Ncl(G) \cap R = 0_N$ such that $x_{r,t,s} \in Nint_\alpha(f^{-1}(G \cup R))$

Proof. $(i) \Rightarrow (ii)$ Let G be a neutrosophic open set in (Y, S) containing $f(x_{r,t,s})$. By $f(x_{r,t,s}) \in G \leq Nint(Ncl(G))$ and $Nint(Ncl(G))$ containing $f(x_{r,t,s})$, there exists a neutrosophic rare set R with $Nint(Ncl(G)) \cap Ncl(R) = 0_N$ and a neutrosophic α open set U in (X, T) containing $x_{r,t,s}$ such that $f(U) \leq Nint(Ncl(G)) \cup R$. We have $Nint(f(U) \cap \overline{G})Nint(\overline{G}) \leq Nint(Ncl(G) \cup R) \cap \overline{(Ncl(G))} \leq Ncl(G) \cup Nint(R) \cap \overline{(Ncl(G))} = 0_N$.

$(ii) \Rightarrow (iii)$ Obvious.

$(iii) \Rightarrow (i)$ Let G be a neutrosophic open set in (Y, S) containing $f(x_{r,t,s})$. Then by (iii), there exists a neutrosophic α-open set U containing $x_{r,t,s}$ such that $Nint(f(U) \leq Ncl(G)$. We have $f(U) = (f(U) \cap \overline{(Nint(f(U)))}) \cup Nint(f(U)) < (f(U) \cap \overline{(Nint(f(U)))}) \cup Ncl(G) = (f(U) \cap \overline{(Nint(f(U)))}) \cup G \cup (Ncl(G) \cap \overline{G}) = (f(U) \cap \overline{(Nint(f(U)))} \cap \overline{G}) \cup G \cup (Ncl(G) \cap \overline{G})$. Set $R_1 = f(U) \cap \overline{(Nint(f(U)))} \cap \overline{G}$ and $R_2 = Ncl(G) \cap \overline{G}$. Then R_1 and R_2 are neutrosophic rare sets. More $R = R_1 \cup R_2$ is a neutrosophic set such that $Ncl(R) \cap G = 0_N$ and $f(U) \leq G \cup R$. This show that f is neutrosophic rarely α-continuous.

$(i) \Rightarrow (iv)$ Suppose that G be a neutrosophic open set in (Y, S) containing $f(x_{r,t,s})$. Then there exists a neutrosophic rare set R with $G \cap Ncl(R) = 0_N$ and U be a neutrosophic α-open set in (X, T) containing $x_{r,t,s}$ such that $f(U) \leq G \cup R$. It follows that $x_{r,t,s} \in U \leq f^{-1}(G \cup R)$. This implies that $x_{r,t,s} \in Nint_\alpha(f^{-1}(G \cup R))$.

$(iv) \Rightarrow (v)$ Suppose that G be a neutrosophic open set in (Y, S) containing $f(x_{r,t,s})$. Then there exists a neutrosophic rare set R with $G \cap Ncl(R) = 0_N$ such that $x_{r,t,s} \in Nint_\alpha(f^{-1}(G \cup R))$. Since $G \cap Ncl(R) = 0_N, R \leq \overline{G}$, where $\overline{G} = \overline{(Ncl(G))} \cup (Ncl(G) \cap \overline{G})$. Now, we have $R \leq R \cup \overline{(Ncl(G))} \cup (Ncl(G) \cap \overline{G})$. Now, $R_1 = R \cap \overline{(Ncl(G))}$. It follows that R_1 is a neutrosophic rare set with $Ncl(G) \cap R_1 = 0_N$. Therefore $x_{r,t,s} \in Nint_\alpha(f^{-1}(G \cup R)) \leq Nint_\alpha(f^{-1}(G \cup R_1))$.

$(v) \Rightarrow (vi)$ Assume that G be a neutrosophic regular open set in (Y, S) containing $f(x_{r,t,s})$. Then there exists a neutrosophic rare set R with $Ncl(G) \cap R = 0_N$ such that $x_{r,t,s} \in Nint_\alpha(f^{-1}(Ncl(G) \cup R))$. Now $R_1 = R \cup (Ncl(G) \cup \overline{G})$. It follows that R_1 is a neutrosophic rare set and $(G \cap Ncl(R_1)) = 0_N$. Hence $x_{r,t,s} \in Nint_\alpha(f^{-1}(Ncl(G) \cup R)) = Nint_\alpha(f^{-1}(G \cup (Ncl(G) \cap \overline{G})) \cup R) = Nint_\alpha(f^{-1}(G \cup R_1))$. Therefore $x_{r,t,s} \in Nint_\alpha(f^{-1}(G \cup R_1))$.

$(vi) \Rightarrow (ii)$ Let G be a neutrosophic open set in (Y, S) containing $f(x_{r,t,s})$. By $f(x_{r,t,s}) \in G \leq Nint(Ncl(G))$ and the fact that $Nint(Ncl(G))$ is a neutrosophic regular open in (Y, S), there exists a neutrosophic rare set R and $Nint(Ncl(G)) \cap Ncl(R) = 0_N$, such that $x_{r,t,s} \in Nint_\alpha(f^{-1}(Nint(Ncl(G)) \cup R)$. Let $U = Nint_\alpha(f^{-1}(Nint(Ncl(G)) \cup R)$. Hence U is a neutrosophic α-open set in (X, T) containing $x_{r,t,s}$ and therefore $f(U) \leq Nint(Ncl(G)) \cup R$. Hence, we have $Nint(f(U) \cap \overline{G}) = 0_N$. □

Proposition 2.2. Let (X, T) and (Y, S) be any two neutrosophic topological space. Then a function $f : (X, T) \to (Y, S)$ is a neutrosophic rarely α-continuous if and only if $f^{-1}(G) \leq Nint_\alpha(f^{-1}(G \cup R))$ for every neutrosophic open set G in (Y, S), where R is a neutrosophic rare set with $Ncl(R) \cap G = 0_N$.

Proof. Suppose that G be a neutrosophic rarely α-open set in (Y, S) containing $f(x_{r,t,s})$. Then $G \cap Ncl(R) = 0_{_N}$ and U be a neutrosophic α-open set in (X, T) containing $x_{r,t,s}$, such that $f(U) \leq G \cup R$. It follows that $x_{r,t,s} \in U \leq f^{-1}(G \cup R)$. This implies that $f^{-1}(G) \leq Nint_{\alpha}(f^{-1}(G \cup R))$. □

Definition 2.8. A function $f : (X, T) \to (Y, S)$ is neutrosophic $I\alpha$-continuous at $x_{r,t,s}$ in (X, T) if for each neutrosophic open set G in (Y, S) containing $f(x_{r,t,s})$, there exists a neutrosophic α-open set U containing $x_{r,t,s}$, such that $Nint(f(U)) \leq G$.

If f has this property at each neutrosophic point $x_{r,t,s}$ in (X, T), then we say that f is neutrosophic $I\alpha$-continuous on (X, T).

Example 2.2. Let $X = \{a, b, c\}$. Define the neutrosophic sets A and B as follows: $A = \langle x, (\frac{a}{0}, \frac{b}{1}, \frac{c}{0}), (\frac{a}{0}, \frac{b}{1}, \frac{c}{0}), (\frac{a}{1}, \frac{b}{0}, \frac{c}{1}) \rangle$ and $B = \langle x, (\frac{a}{1}, \frac{b}{0}, \frac{c}{0}), (\frac{a}{1}, \frac{b}{0}, \frac{c}{0}), (\frac{a}{0}, \frac{b}{1}, \frac{c}{1}) \rangle$. Then $T = \{0_{_N}, 1_{_N}, A\}$ and $S = \{0_{_N}, 1_{_N}, B\}$ are neutrosophic topologies on X. Let (X, T) and (X, S) be neutrosophic topological spaces. Let $f : (X, T) \to (X, S)$ as defined by $f(a) = f(b) = b$ and $f(c) = c$ is neutrosophic $I\alpha$-continuous.

Proposition 2.3. Let (Y, S) be a neutrosophic regular space. Then the function $f : (X, T) \to (Y, S)$ is neutrosophic $I\alpha$ continuous on X if and only if f is neutrosophic rarely α-continuous on X.

Proof. $\Rightarrow$ It is obvious.
$\Leftarrow$ Let f be neutrosophic rarely α-continuous on (X, T). Suppose that $f(x_{r,t,s}) \in G$, where G is a neutrosophic open set in (Y, S) and a neutrosophic point $x_{r,t,s}$ in X. By the neutrosophic regularity of (Y, S), there exists a neutrosophic open set G_1 in (Y, S) such that G_1 containing $f(x_{r,t,s})$ and $Ncl(G_1) \leq G$. Since f is neutrosophic rarely α-continuous, then there exists a neutrosophic α open set U, such that $Nint(f(U)) \leq Ncl(G_1)$. This implies that $Nint(f(U)) \leq G$ which means that f is neutrosophic $I\alpha$-continuous on X. □

Definition 2.9. A function $f : (X, T) \to (Y, S)$ is called neutrosophic pre-α-open if for every neutrosophic α-open set U in X such that $f(U)$ is a neutrosophic α-open in Y.

Proposition 2.4. If a function $f : (X, T) \to (Y, S)$ is a neutrosophic pre-α-open and neutrosophic rarely α-continuous then f is neutrosophic almost α-continuous.

Proof. suppose that a neutrosophic point $x_{r,t,s}$ in X and a neutrosophic open set G in Y, containing $f(x_{r,t,s})$. Since f is neutrosophic rarely α-continuous at $x_{r,t,s}$, then there exists a neutrosophic α-open set U in X such that $Nint(f(U)) \subset Ncl(G)$. Since f is neutrosophic pre-α-open, we have $f(U)$ in Y. This implies that $f(U) \subset Nint(Ncl(Nint(f(U)))) \subset Nint(Ncl(G))$. Hence f is neutrosophic almost α-continuous. □

For a function $f : X \to Y$, the graph $g : X \to X \times Y$ of f is defined by $g(x) = (x, f(x))$, for each $x \in X$.

Proposition 2.5. Let $f : (X, T) \to (Y, S)$ be any function. If the $g : X \to X \times Y$ of f is neutrosophic rarely α-continuous then f is also neutrosophic rarely α-continuous.

Proof. Suppose that a neutrosophic point $x_{r,t,s}$ in X and a neutrosophic open set W in Y, containing $g(x_{r,t,s})$. It follows that there exists neutrosophic open sets 1_X and V in X and Y respectively, such that $(x_{r,t,s}, f(x_{r,t,s})) \in 1_X \times V \subset W$. Since f is neutrosophic rarely α-continuous, there exists a neutrosophic α-open set G such that $Nint(f(G)) \subset Ncl(V)$. Let $E = 1_X \cap G$. It follows that E be a neutrosophic α-open set in X and we have $Nint(g(E)) \subset Nint(1_X \times f(G)) \subset 1_X \times Ncl(V) \subset Ncl(W)$. Therefore g is neutrosophic rarely α-continuous. □

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