Gravitation and Cosmology without Divergences*

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The orthodox general theory of relativity (GR) predicts metric divergences, typically when the normalized gravitational potential $\phi = -½$, giving rise to event horizons. But all standard experimental tests are in the regime where $|\phi|<<1/2$, suggesting that alternative large-potential extrapolations should be considered. A simple alternative formulation of GR is presented which focuses on gravitational time dilation and length contraction by a factor of $(1-\phi)$, which is always > 1 since $\phi < 0$. This formulation duplicates the predictions of the orthodox theory for weak potentials with $|\phi| << ½$, but avoids all divergences for larger values. Furthermore, this suggests "dim stars" rather than black holes, and provides an intuitive picture of large gravitational red shifts in cosmic expansion.

Introduction

General relativity has been around for more than 100 years, but it is not experimentally proven to the degree generally believed. At the heart of general relativity are gravitational time dilation and length contraction. For a test mass m at a distance r from a massive star M, the gravitational potential energy is $\Phi = -GMm/r$. The normalized gravitational potential is $\phi = \Phi/mc^2 = -GM/rc^2$, which is dimensionless. More generally, $\phi<0$, with $|\phi|<<1$ in experimentally accessible configurations. For example, at the surface of the sun, $\phi = -2 \times 10^{-6}$. The standard GR factors for time dilation and length contraction are given in Table I. The factors in a simple alternative formulation (Kadin 2016) are also given, and are clearly identical in the limit $|\phi|<<1$.

The standard tests are unable to distinguish between these two alternatives. But the orthodox factors go to zero or infinity for $|\phi| = ½$. This seems physically unreasonable; nothing much

Table I. Factors for gravitational time dilation and length contraction in orthodox and alternative theories.

	Orthodox theory	**Alternative theory**
Time dilation	$(1+2\phi)^{-0.5}$	$(1-\phi)$
Length contraction	$(1+2\phi)^{0.5}$	$1/(1-\phi)$

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should happen for $|\phi| = ½$. The divergence and sign change may indicate a mathematical artifact, arising from extrapolation of equations beyond their regime of validity. The alternative factors are simple and never diverge. While they are not unique in being non-divergent, it is interesting to explore what they predict for the large $|\phi|$ limit.

These days, almost everyone believes that there are black holes associated with certain specific massive stars, as well as supermassive black holes in the centers of galaxies. I suggest that black holes (and the associated event horizons) are mathematical artifacts of an incorrectly extrapolated theory. However, this alternative extrapolation predicts what might be called "dim stars", gravitationally compressed objects that permit only a small fraction of light (and other particles) to escape in a narrow cone along the radial direction. In the orthodox theory, there is nothing denser than a neutron star, which in any case would be irretrievably lost inside the event horizon of a black hole. On the contrary, I would suggest a dense quark-lepton plasma, perhaps similar to the state in the early phases after the Big Bang. There are certainly gravitationally condensed objects in massive stars and galactic centers, but their identification as black holes is dependent on models with multiple fitting parameters. This non-divergent alternative also gives important insights into the expansion of the early universe and the cosmic microwave background (CMB) radiation.

Quantum Waves, Time and Space

Light waves taken collectively can have any frequency and corresponding wavelength; the only characteristic quantity is the speed of light c: $\omega = kc$. In contrast, fundamental quantum waves do have characteristic frequency and wavelength, with a relativistic dispersion relation given by

$$(\omega\tau)^2 = 1 + (k\Lambda)^2, \tag{1}$$

where $\tau = \hbar/mc^2$ and $\Lambda = \hbar/mc$, so that $\Lambda/\tau = c$. One can select m to be the electron mass, but any other fundamental mass will scale in the same way. The characteristic frequency for an electron (its frequency at rest) is $2\pi/\tau \sim 10^{20}$ Hz, corresponding to a rest energy of 511 keV. I have argued that τ and Λ define local time and space, and that these are more fundamental than the mathematical concept of spacetime (Kadin 2014, 2016, 2017).

Now assume that τ is subject to gravitational time dilation, and Λ is subject to gravitational length contraction:

$$\tau = \tau_0 g; \qquad \Lambda = \Lambda_0/g; \qquad c = c_0/g^2 \tag{2}$$

where $g = 1-\phi$ is a positive factor that is always >1, and τ_0, Λ_0, and c_0 correspond to potential $\phi=0$. Since τ and Λ define local time and space, a local measurement will always measure a constant speed of light, but measurements of distant motion will show variations. Eq. (2) corresponds to $mc^2 = m_0c_0^2/g \approx m_0c_0^2+\Phi$, so that the potential shifts the rest energy of a particle.

Orthodox GR focuses on the constancy of the speed of light, but from an alternative viewpoint it is clear that c varies in a gravitational potential. The curvature of light near a star is quantitatively equivalent to optical refraction with a variable index of refraction $n(\mathbf{r}) = g^2(\mathbf{r})$. In classical optics, any traveling wave maintains constant ω as it propagates through space, as a direct consequence of linearity. The wavelength will change, but the frequency always remains the same. For gravitation, this will also be true, provided that one uses a fixed reference for time and space, rather than the varying local reference frames along the trajectory. One can then calculate the trajectory $\mathbf{r}(t)$ of a wave packet by noting that

$$d\omega/dt = 0 = (\partial\omega/\partial\mathbf{r})\cdot(d\mathbf{r}/dt) + (\partial\omega/\partial\mathbf{k})\cdot(d\mathbf{k}/dt) \quad (3)$$

Since the group velocity of any wave packet is $\partial\omega/\partial\mathbf{k} = d\mathbf{r}/dt$, we also have $d\mathbf{k}/dt = -\partial\omega/\partial\mathbf{r}$. These are essentially the classical Hamiltonian equations of motion, since $E = \hbar\omega$ and $\mathbf{p} = \hbar\mathbf{k}$.

For a massive particle in a gravitational potential, Eq. (1) becomes, in dimensionless units,

$$\omega^2 g^2 = 1 + k^2/g^2, \qquad \text{or} \qquad E^2 g^2 = 1 + p^2/g^2. \quad (4)$$

In general, this will give curving trajectories for either a photon or a massive particle, because $g(\mathbf{r})$ is non-uniform (see Fig. 1). For a central gravitational well $g(r) = 1+GM/rc^2 = 1 + r_1/r$, a radial velocity will remain straight, but a velocity at an angle will bend, and for large g, most trajectories will be unable to escape the well. For photons, this is equivalent to total internal reflection in a medium with a non-uniform index of refraction. This effect is responsible, for example, for the confinement of light in an optical fiber, where the differences in refractive index are small. For large g, confinement is even more effective, for both photons and particles.

These equations make no reference to curved spacetime or a metric, but are completely equivalent to the orthodox formulation, at least for weak gravitational potentials. After a trajectory is calculated based on the uniform $\mathbf{r}$ and t, it can be converted to local values of $\mathbf{r}$ and t, if that is desired. That would recover the constant speed of light, but space would then become non-Euclidian. This orthodox picture is unnecessarily confusing, and obscures the fact that these are essentially classical trajectories.

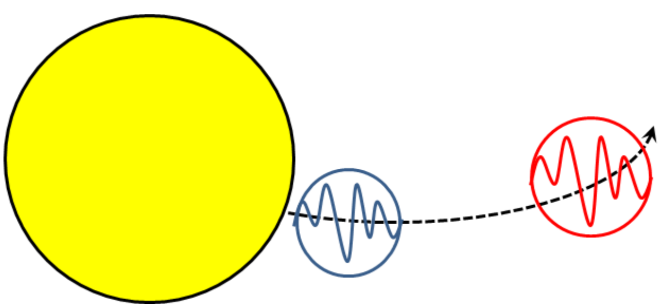

Fig. 1. A blue photon emitted from a gravitational potential well is red-shifted and bends as it moves out of the well, due to an effective index of refraction $n(r) = g^2 = (1+GM/rc^2)^2$.

Gravitational Red Shift

It is useful to compare the explanations of the gravitational red shift in the two complementary pictures. In the wave-based picture, if a blue photon (in local units) is emitted in the gravitational well, neither its frequency f nor its wavelength λ would match a blue photon outside the well, measured with respect to units outside the well. For example, if g=2 in the well, both f and λ would be half their usual values, and $c = f\lambda$ would be 1/4th the usual value, all referenced to clocks and rulers outside the well. But as this photon propagates out, f remains constant, while λ increases by a factor of 4, as the photon speeds up. The resulting photon will be near-infrared, substantially red-shifted.

If one throws a ball upward, one expects it to slow down. It seems odd that an upwards-moving photon would speed up, but this is also true for a highly relativistic particle. Fig. 2 plots the velocity v (in units of c_0) of an upwards-moving photon and a massive particle, as a function of distance r, in units of $r_1 = GM/c^2$, based on the equation

$$v = (1/g^2)[1-1/(g^2E^2)]^{0.5} \tag{5}$$

The top line is for E>>1, for which the particle always moves at the local value of c, equivalent to a photon. The bottom line for E = 1 corresponds to "escape velocity", where the particle starts out relativistic and speeds up, but eventually slows down asymptotically as r approaches infinity. For smaller energy, the particle would reach a maximum distance, then turn around and start falling again. But only particles with velocities within a small angle of the vertical can escape, even at high energy. This escape angle is expected to scale as $1/g^2$, so that the fraction of escaping particles would decrease sharply as $1/g^4$. For large g one would have not a black hole with a sharp event horizon, but rather a very dim star comprising a dense, gravitationally confined sub-nuclear plasma.

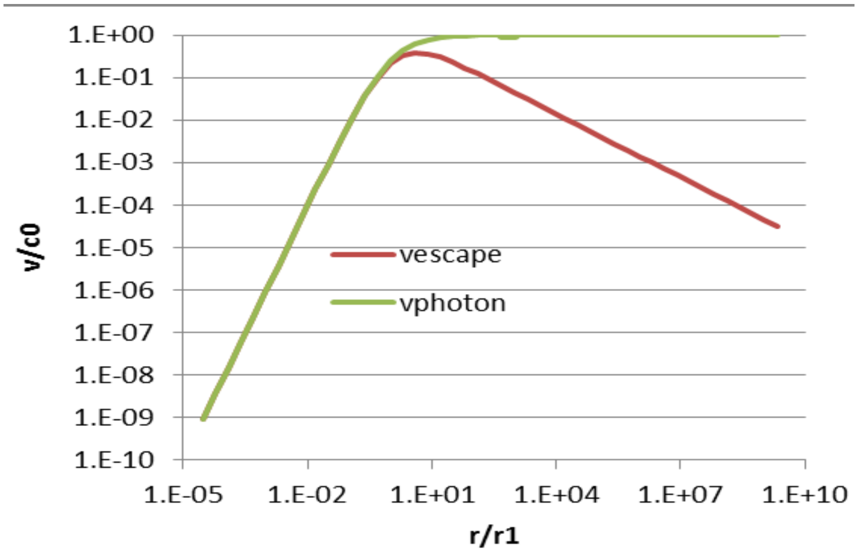

Fig. 2. Plot of v vs. r, in dimensionless units, for photon and massive particle escaping a gravitational potential well, from Eq. (5).

As an example, consider parameters for what has been identified as a supermassive black hole in the center of the Milky Way galaxy, with an estimated four million solar masses. The scaling length is r_1 = 6 million km, and the range on the plot corresponds to r from 60 km to 600 billion km. Consider a photon emitted on the left of the plot with absolute energy 0.3 meV, in a region where g = 30,000, which would appear locally to be an ultraviolet photon with energy ~ 10 eV. But on the right of the plot, it gets red-shifted to a microwave photon, with the same initial absolute energy of 0.3 meV, corresponding to the peak of a black-body thermal distribution of 3 K. Something similar may happen with the cosmic microwave background radiation.

Expansion of the Universe

From the point of view presented here, one can think of the early universe as a gravitationally confined dense gas of relativistic elementary particles moving in random directions, much like the dim star above, but much larger and more massive. The gravitational potential at a location **r** within the universe would be a sum over factors $-GM_i/|\mathbf{r}_i\text{-}\mathbf{r}|c^2$, which is dominated by distant matter. One can therefore crudely estimate $\phi = GM_{tot}/r_{tot}c^2 = r_1/r_{tot}$.

Consider the estimated mass of the observable universe ~ 10^{53} kg, corresponding to $r_1 \sim 10^{26}$ m ~ 10 billion light years. The present effective radius of the universe is $r_{tot} \sim 5 \times 10^{26}$ m ~ 50 billion light years, so that $g = 1+r_1/r_{tot} = 1.2$, not very large. The observation of relatively recent Hubble red shifts may include contributions from both Doppler and gravitational shifts. But the early universe was much smaller than this, giving $g >> 1$, and a Hubble shift dominated by gravitational effects.

The early universe could expand into the surrounding vacuum, but the very high index $n = g^2$ prevents all but a very small fraction of the particles from escaping. Still, that small fraction permits the radius to expand. Comparing to Fig. 2, one would expect that the expansion rate of the universe would be significantly smaller than the velocities shown for a radial particle in the small r regime, since most particle trajectories are not radial. One aspect of cosmological expansion that has received great attention in recent years is the apparent acceleration attributed to dark energy. But perhaps this is an artifact of orthodox models, which might be alternatively explained by the transition from an early sluggish expansion (due to gravitational trapping) to a more recent free expansion (see also Kadin, 2012, 2013).

As r_{tot} increases, g decreases, which in turn decreases the average energy (and temperature) in local units as $1/g \propto r_{tot}$. This local cooling occurs even if the absolute energy of particles remains constant. At some time, the local temperature will cool sufficiently that stable hydrogen atoms can form, enabling the universe to transition from an opaque plasma to a transparent gas that can transmit photons below the (local) ionization energy of hydrogen. Some of those photons can escape, but the overwhelming majority are reflected back. Most of those photons are still traveling around the universe, representing the cosmic background microwave radiation.

Conclusions

The present essay has proposed an unorthodox high-gravity extrapolation without divergence, without black holes and event horizons, but instead with dim stars that trap most photons and particles. Unlike a black hole that hides its internal structure, this alternative suggests a high-density quark-lepton plasma, similar to that in the early universe. While non-divergent gravity may seem physically reasonable, only high-precision gravitational measurements can identify the proper high-field extrapolation. The next generation of experimental, observational, and cosmological tests may help to answer these questions.

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