Planck Mass Measured Totally Independent of Big G Utilizing McCulloch-Heisenberg Newtonian Equivalent Gravity

Espen Gaarder Haug

Norwegian University of Life Sciences – Chr. Magnus Falsens Road, 1430 Ås, Norway

PACS 04.60.-m – Quantum gravity

Abstract – In 2014, McCulloch showed, in a new and interesting way, how to derive a gravity theory from Heisenberg's uncertainty principle that is equivalent to Newtonian gravity. McCulloch utilizes the Planck mass in his derivation and obtains a gravitational constant of $\frac{\hbar c}{m_p^2}$. This is a composite constant, which is equivalent in value to Newton's gravitational constant. However, McCulloch has pointed out that his approach requires an assumption on the value of G, and that this involves some circular reasoning. This is in line with the view that the Planck mass is a derived constant from Newton's gravitational constant, while big G is a universal fundamental constant. Here we will show that we can go straight from the McCulloch derivation to measuring the Planck mass without any knowledge of the gravitational constant. From this perspective, there are no circular problems with his method. This means that we can measure the Planck mass without Newton's gravitational constant, and shows that the McCulloch derivation is a theory of quantum gravity that stands on its own. And further we show that we can easily measure the Schwarzschild radius of a mass without knowing its mass, or Newton's gravitational constant, or the Planck constant. The very essence of gravity is linked to the Planck length and the speed of light, but here we will claim that we do not need to know the Planck length itself. Our conclusion is that Newton's gravitational constant is a universal constant, but it is a composite constant of the form $G = \frac{l_p^2 c^3}{\hbar}$ where the Planck length and the speed of light are the keys to gravity. This could be an important step towards the development of a full theory of quantum gravity.
Key words: Heisenberg, Planck mass, McCulloch gravity, Newton, gravitational constant, Cavendish apparatus.

McCulloch-Heisenberg Newton Equivalent Gravity. – In 2014, McCulloch [1] derived an equivalent gravity to that of Newton [2] directly from the Heisenberg uncertainty principle and gets the following equation for the gravity force (See Appendix A.1 for a short review of his derivation.)

$$F = \frac{\hbar c}{m_p^2}\frac{Mm}{r^2} \tag{1}$$

Where $\frac{\hbar c}{m_p^2} \approx 6.67384 \times 10^{-11} m^3 \cdot kg^{-1} \cdot s^{-2}$. That is basically identical to the empirically-measured gravitational constant value, even if there is large uncertainty in Newton's gravitational constant [3–7]; this is something we will return to later. Still, we cannot know its value without knowing the speed of light, the Planck constant, and the Planck mass. The speed of light is known and can be measured with no knowledge of gravity (and is exact per definition), and the Planck constant can be found from the Watt balance (Kibble balance) [8–10]. However, the Planck mass is unknown and it is generally assumed that we must know G in order to calculate the Planck mass. This point is mentioned by McCulloch himself in a follow-up paper [11]. In that paper he states

> *In the above gravitational derivation, the correct value for the gravitational constant G can only be obtained when it is assumed that the gravitational interaction occurs between whole multiples of the Planck mass, but this last part of the derivation involves some circular reasoning since the Planck mass is defined using the value for G.*

This is fully in line with modern physics' view that the only way to find the Planck mass is to derive it from big G. The Planck mass, the Planck temperature (energy), the Planck length, and the Planck time were introduced in 1899 by Max Planck [12, 13] himself. Planck derived these units, which he called natural units, from what he considered to be the most fundamental universal constants: Newton's gravitational constant, the speed of light, and the Planck constant. Based on this, we need to know Newton's gravitational constant to find the Planck mass from Planck's formula, $m_p = \sqrt{\frac{\hbar c}{G}}$. However, as we will see here, we can build on McCulloch's derivation, complete a few more derivations, and easily design a simple experiment to measure the Planck mass independent of Newton's gravitational constant, or knowledge of any other theories of gravity.

The Planck Mass Measured Directly from McCulloch's Derivation and a Cavendish Apparatus. – Newton did not measure the gravitational constant himself; this was first done indirectly by Cavendish [14] in 1798. Using a Cavendish apparatus, we can measure the Planck mass without any knowledge of Newton's gravitational constant, or any knowledge of Newtonian gravity. A Cavendish apparatus consist of two small balls and two larger balls, all made of lead, for example. The torque (moment of force) is given by

$$\kappa\theta \tag{2}$$

where κ is the torsion coefficient of the suspending wire and θ is the deflection angle of the balance. We then have the following well-known relationship

$$\kappa\theta = LF \tag{3}$$

where L is the length between the two small balls in the apparatus. Further, F can be set equal to the gravitational force given by McCulloch's Heisenberg-derived formula

$$F = \frac{\hbar c}{m_p^2}\frac{Mm}{r^2} \tag{4}$$

This means we must have

$$\kappa\theta = L\frac{\hbar c}{m_p^2}\frac{Mm}{r^2} \tag{5}$$

We also have that the natural resonant oscillation period of a torsion balance is given by

$$T = 2\pi\sqrt{\frac{I}{\kappa}} \tag{6}$$

Further, the moment of inertia I of the balance is given by

$$I = m\left(\frac{L}{2}\right)^2 + m\left(\frac{L}{2}\right)^2 = 2m\left(\frac{L}{2}\right)^2 = \frac{mL^2}{2} \tag{7}$$

this means we have

$$T = 2\pi\sqrt{\frac{mL^2}{2\kappa}} \tag{8}$$

and when solved with respect to κ, this gives

$$\begin{aligned} \frac{T^2}{2^2\pi^2} &= \frac{mL^2}{2\kappa} \\ \kappa &= \frac{mL^2}{2\frac{T^2}{2^2\pi^2}} \\ \kappa &= \frac{mL^2 2\pi^2}{T^2} \end{aligned} \tag{9}$$

Next in equation 5 we are replacing κ with this expression, and solving with respect to the Planck mass

$$\begin{aligned} \frac{mL^2 2\pi^2}{T^2}\theta &= L\frac{\hbar c}{m_p^2}\frac{Mm}{r^2} \\ \frac{L^2 2\pi^2 r^2}{\hbar c L M T^2}\theta &= \frac{1}{m_p^2} \\ m_p^2 &= \frac{\hbar c M T^2}{L 2\pi^2 r^2\theta} \\ m_p &= \sqrt{\frac{\hbar c M T^2}{L 2\pi^2 r^2\theta}} \end{aligned} \tag{10}$$

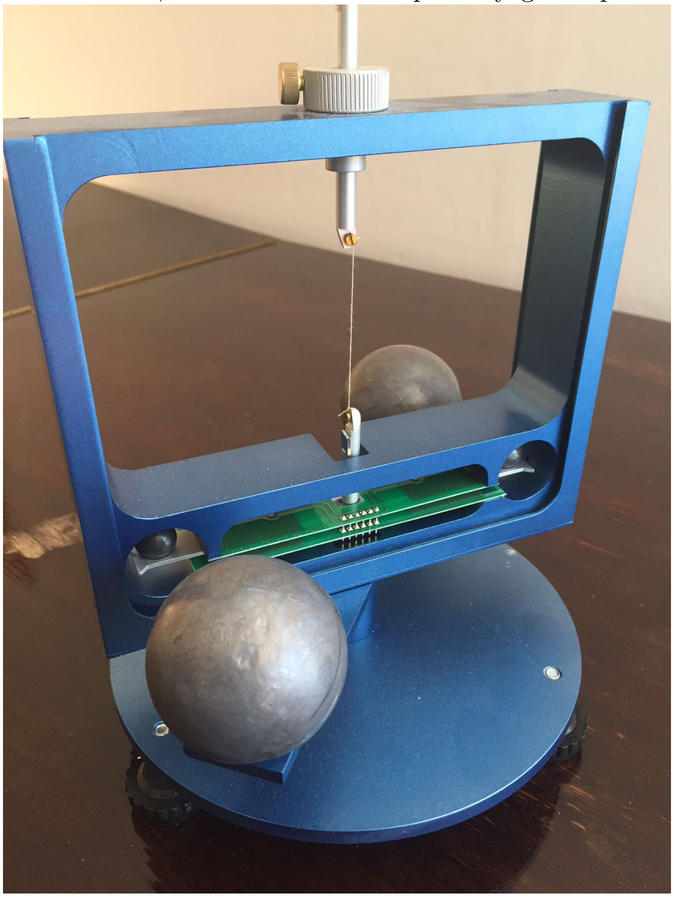
The mass M is the mass of each of the two large lead balls in the Cavendish apparatus, not the mass of the Earth. All we need in order to find the mass of the large balls is an accurate weight. The Planck constant can be found from the Watt balance. The angle θ and the oscillation time period T are what we measure with the Cavendish apparatus. The length L is the distance between the small lead balls and r is the distance between the large lead ball's center to the center of the small lead ball, when the arm is in equilibrium position (mid position).

Today there even exists a small, ready-to-use, low budget (a few thousand dollars) Cavendish apparatus, where the angle of the arm (and the time) are measured very accurately by fine electronics and plugged directly into a computer with a USB cable; see Figure 1. Using this low budget apparatus, we can measure the Planck mass with about 5% inaccuracy on our kitchen table without any knowledge of Newton's gravitational constant.

As soon as we know the Planck mass, we have the the complete composite gravitational constant and the McCulloch formula can then be applied to any standard gravitational predictions, such as finding the mass of the Earth, or predicting the orbital velocity of planets and satellites.

Haug [15] has, in a similar way, shown how the Planck length can be found independent of big G, but his derivation did not start out with the McCulloch-Heisenberg Newton equivalent gravity theory, so we think the derivation and discussion in this paper offer important additional insight. See also Appendix A.2, on how we can extend the derivation above using the McCulloch-Heisenberg gravity to find the Planck length and Planck time "directly" from a Cavendish apparatus.

Figure 1 Low budget modern Cavendish apparatus combining old mechanics with modern electronics. It is remarkable that with such an instrument we can measure the Planck mass with only about 5% error from the kitchen table, or here from the top of my grand piano.



Why Newton's Gravitational Constant Likely Is a Universal Composite Constant. – In our analysis, the first strong indication that the gravitational constant is a composite constant is given by its output units, which are $m^3 \cdot kg^{-1} \cdot s^{-2}$. It would be very strange if something concerning the fundamental nature of reality would be meters cubed, divided by kg and seconds squared. The Planck mass, on the other hand, is somewhat easier to relate to, even if it is somewhat of a mystery at a deeper level. The speed of light is also something we can relate to logically; it is the distance light travels in vacuum during a pre-specified time interval. The Planck constant is more complex (and outside the scope of this paper), but it is related to the view that energy seems to come in quanta. In sum though, the Planck mass, the speed of light, and the Planck constant seem to be more intuitive than the gravitational constant.

In 2016, Haug [16] suggested that the gravitational constant of the form $G = \frac{l_p^2 c^3}{\hbar}$, which is basically the same as the McCulloch 2014 constant $\frac{\hbar c}{m_p^2}$. Because the Planck mass can be written as $m_p = \frac{\hbar}{l_p}\frac{1}{c}$ we have

$$G = \frac{\hbar c}{m_p^2} = \frac{\hbar c}{\left(\frac{\hbar}{l_p}\frac{1}{c}\right)^2} = \frac{l_p^2 c^3}{\hbar} \tag{11}$$

Haug has shown that assuming the gravitational constant is a composite will help make all of the Planck units more intuitive. For example, the Planck time is given by $t_p = \sqrt{\frac{G\hbar}{c^5}}$; when rewritten based on the idea that the gravitational constant is a composite, this simply gives the (known) $t_p = \frac{l_p}{c}$. The latter form is also known from before, but the view that the Newtonian gravitational constant is a composite renders the form $t_p = \sqrt{\frac{G\hbar}{c^5}}$ unnecessary. We might then ask, what is the intuition about c^5 and G? The answer may not be so clear. On the other hand, the intuition behind $\frac{l_p}{c}$ is very simple; it is simply a very short distance divided by the speed of light, so given a very short time interval, we can see that it is coming directly out from the formula. Haug's gravitational constant composite formula has the same challenge in that one might think we may end up with a circular problem, again, because modern physics typically assumes that we need to know big G before we can find the Planck units. However, as we have shown in this paper this is not the case.

This does not mean big G is wrong; it is just likely to be a composite universal constant rather than a fundamental constant.

We find that many gravitational formulas may be seen in a new perspective when rewritten based on the idea that Newton's gravitational constant is a composite constant; we summarize a selection of such gravitational formulas in Table 1.

[a] Actually, Newton's gravitational force has never been observed directly, only indirectly though the predictions that come from mathematically rearranging this formula to develop other predictions, such as orbital velocity.
[b] To my knowledge the escape velocity has not been tested empirically.
[c] At least not directly.
[d] At least not directly.
[e] At least not directly.
[f] Needs further investigation and confirmation; see [17] for more details.
[g] Needs further investigation and confirmation; see [17] for more details.

[7] Needs further investigation and confirmation; see [17] for more details.

Table 1: The table of a series gravity formulas when using the standard Newton gravitational constant and the alternative when arguing that Newton's gravitational constant is a composite constant. Note that N is the number of Planck masses in the gravitational object, this can be found for example by measuring indirectly the Planck mass first.

	Standard form/way	**Planck form**	**Observed**
Gravitational constant	$G \approx 6.67 \times 10^{-11}$	$G = \frac{\hbar c}{m_p^2} = \frac{l_p^2 c^3}{\hbar} \approx 6.67 \times 10^{-11}$	Yes
Cavendish: Gravitational constant	$G = \frac{L2\pi^2 r^2 \theta}{MT^2}$		Yes
Cavendish: Planck mass	Only derived from G	$m_p = \sqrt{\frac{\hbar c M T^2}{L2\pi^2 r^2 \theta}}$	Easy to do
Cavendish: Planck lenght	Only derived from G	$l_p = \sqrt{\frac{\hbar L 2\pi^2 r^2 \theta}{MT^2 c^3}}$	Easy to do
Cavendish: Planck time	Only derived from G	$t_p = \sqrt{\frac{\hbar L 2\pi^2 r^2 \theta}{MT^2 c^5}}$	Easy to do
Cavendish: Schwarzschild radius	Normally dependent on G	$r_s = \frac{L4\pi^2 r^2 \theta}{c^2 T^2}$	Easy to do
Newton gravity force	$F = G\frac{mM}{r^2}$	$F = n_1 n_2 \frac{\hbar c}{r^2}$	"Yes"[1]
Gravitational acceleration field	$g = \frac{GM}{r^2}$	$g = N\frac{l_p}{r^2}c^2$	Yes
Mass from acceleration field	$M = \frac{gr^2}{G}$	$M = \frac{gr^2 \hbar}{l_p^2 c^3}$	Yes
Orbital velocity	$v_o = \sqrt{G\frac{M}{r}}$	$v_o = c\sqrt{N\frac{l_p}{r}}$	Yes
Escape velocity	$v_e = \sqrt{2G\frac{M}{r}}$	$v_e = c\sqrt{N2\frac{l_p}{r}}$	No (?)[2]
Time dilation	$t_2 = t_1\sqrt{1 - \frac{2GM}{rc^2}}$	$t_2 = t_1\sqrt{1 - N2\frac{l_p}{r}}$	Yes
Newton gravitational bending of light	$\delta = \frac{2GM}{rc^2}$	$\delta = N2\frac{l_p}{r}$	Twice of that
GR gravitational bending of light	$\delta = \frac{4GM}{rc^2}$	$\delta = N4\frac{l_p}{r}$	Yes
Gravitational red-shift	$\lim_{r\to+\infty} z(r) = \frac{GM}{r^2}$	$\lim_{r\to+\infty} z(r) = N\frac{l_p}{r}$	Yes
Schwarzschild radius	$r_s = \frac{2GM}{rc^2}$	$r_s = N2l_p$	No
Einstein field equation	$R_{\mu v} - \frac{1}{2}g_{\mu v}R = \frac{8\pi G}{c^4}T_{\mu v}$	$R_{\mu v} - \frac{1}{2}g_{\mu v}R = \frac{8\pi l_p}{m_p c^2}T_{\mu v}$	"Yes"
Einstein constant	$\kappa = \frac{8\pi G}{c^2}$	$\kappa = \frac{8\pi l_p}{m_p}$	Yes
Einstein cosmological constant	$\Lambda = \kappa \rho_{vac}$	$\Lambda = \frac{8\pi l_p}{m_p}\rho_{vac}$	Yes
Hawking temperature	$\frac{c^3}{8\pi GM}\frac{\hbar}{k_b}$	$T = \frac{1}{N8\pi}\frac{m_p c^2}{k_b}$	No[3]
Hawking dissipation time	$t_{ev} = \frac{15360\pi G^2 M^3}{\hbar c^4}$	$T = 15360\pi\frac{l_p}{c}$	No[4]
Bekenstein-Hawking luminosity	$P = \frac{\hbar c^6}{15360\pi G^2 M^2}$	$P = \frac{1}{N^2 15360\pi}\frac{\hbar c^2}{l_p^2}$	No[5]
McCulloch orbital mass	$M = \frac{v^4 \Theta}{2Gc^2(1+Z)}$	$M = \frac{1}{2(1+Z)}m_p\frac{\Theta}{l_p}\frac{v^4}{c^4}$	(?)[6]
McCulloch galaxy velocity	$v^4 = \frac{2GMc^2(1+Z)}{\Theta}$	$v = c\left(2N\frac{l_p}{\Theta}(1+Z)\right)^{\frac{1}{4}}$	(?)[7]

Relative Standard Uncertainty. – Assume we have measured the Planck mass (with a standard uncertainty of 1%) on the kitchen table with Cavendish apparatus plugged into our computer. The relative uncertainty in the gravitational constant must then be

$$\frac{\partial G}{\partial m_p} \times \frac{\frac{m_p}{100}}{G} = \frac{2\hbar c}{m_p^3} \times \frac{m_p}{G \times 100} = \frac{1}{50} = 2\% \tag{12}$$

That is to say, the standard uncertainty in the Newton gravitational constant will always be twice that of the standard uncertainty in the Planck mass measurements. This is in line with what is reported by NIST (2014) CODATA, which reports a relative standard uncertainty for the gravitational constant of 4.7×10^{-5} and 2.3×10^{-5} for the Planck mass.

Measuring the Schwarzschild Radius without Knowledge of the Mass of the Object or Newton's Gravitational Constant. – In the section above we had to know the weight of the lead balls to find the Planck mass and we had to know the Planck constant, but without any knowledge of Newtonian gravity or Newton's gravitational constant. This alone we think is remarkable, as it indicates that the Planck mass is very essential. Next, we will show something even more remarkable. From the Schwarzschild metric [18, 19] solution of the Einstein field equation [20], we get a radius today known as the Schwarzschild radius that mathematically is given by modern physics as

$$r_s = \frac{2GM}{c^2} \tag{13}$$

In this case we need to know the Newton gravitational constant, the mass of the object, and the speed of light to measure the Schwarzschild radius. We will show that we remarkably do not need to know the mass or the gravitational constant to find the Schwarzschild radius of an object.

From the derivations above based on McCulloch Heisenberg equivalent Newton gravity in combination with a Cavendish apparatus, we had that

$$\begin{aligned} \kappa\theta &= LF \\ \frac{mL^2 2\pi^2}{T^2}\theta &= L\frac{\hbar c}{m_p^2}\frac{Mm}{r^2} \end{aligned} \tag{14}$$

Dividing by the small mass on each side we have

$$\frac{L2\pi^2}{T^2}\theta = \frac{\hbar c}{m_p^2}\frac{M}{r^2} \tag{15}$$

The large mass M we can again write as a fraction of the mass m_p, that is $M = \frac{M}{m_p}m_p = Nm_p$, where we assume the size of M as well as N and m_p are unknowns.

$$\begin{aligned} \frac{L2\pi^2}{T^2}\theta &= \frac{\hbar c}{m_p^2}\frac{Nm_p}{r^2} \\ \frac{L2\pi^2}{T^2}\theta &= \frac{\hbar c}{m_p}\frac{N}{r^2} \\ \frac{m_p}{N} &= \frac{\hbar c T^2}{r^2 L2\pi^2\theta} \end{aligned} \tag{16}$$

And we can write any elementary particle mass as

$$m = \frac{\hbar}{\bar{\lambda}}\frac{1}{c} \tag{17}$$

The Planck mass has a reduced Compton wavelength of $\bar{\lambda} = l_p$, but we do not need to know this, nor do we need knowledge of the size of the Planck mass, as soon will become very clear. We now have the reduced Planck constant on both sides of the equation. So we can divide by the reduced Planck constant on both sides and get rid of it

$$\begin{aligned} \frac{\frac{\hbar}{\bar{\lambda}}\frac{1}{c}}{N} &= \frac{\hbar c T^2}{r^2 L2\pi^2\theta} \\ \frac{1}{N\bar{\lambda}} &= \frac{c^2T^2}{L2\pi^2 r^2\theta} \\ 2N\bar{\lambda} &= \frac{L4\pi^2 r^2\theta}{c^2T^2} \end{aligned} \tag{18}$$

The measurement we get out $2N\bar{\lambda}$ is actually the Schwarzschild radius of the large lead ball in the Cavendish apparatus. We encourage other researchers to check this out experimentally, even if it is quite "obvious" after one has studied this for a period of time. That is, to find the Schwarzschild radius of the large lead balls in the Cavendish apparatus we only need to know the distance between the small lead balls L, the radius between the large and small lead balls r, the resonant oscillation period of a torsion balance T, the deflection angle of the balance θ that our built-in microchip sensor reads off, and the well-known speed of light. On a somewhat humorous note, one might say, "Eureka". It is quite remarkable that we can measure the Schwarzschild radius of the large lead balls with a measurement error of roughly about $\pm 5\%$, using a small low budget Cavendish apparatus as shown in this paper. What we find is that the number of an essential mass times its reduced Compton wavelength is what is hidden in the Schwarzschild radius, and we can discover this without any knowledge of Newton or Einstein gravity, but instead we find it based on the Heisenberg uncertainty principle and the idea that there is one unique mass, which is the Planck mass, and the entire analysis can be done even without knowing the mass of the lead ball, the mass of the Earth, the gravitational constant, the Planck mass, or the Planck constant.

In our view, this gives strong evidence that the Planck length and the speed of light are the essential fundamental constants for gravity and not the Newtonian gravitational constant, something that also several of the Planck equivalent gravity formulas in Table 1 clearly indicate. This clearly points towards that the gravitational constant not is a fundamental constant. It is the Planck length, the Planck constant, and the speed of light that are essential, and the gravitational constant is a composite constant of the form $G = \frac{\hbar c}{m_p^2} = \frac{l_p^2 c^3}{\hbar}$. Again, we are not saying that the Newton gravitational constant not is universal; we simply claim it is a universal composite constant, and that it is the Planck length, together with the speed of light, that is truly essential for gravity.

That the Planck length is so essential in gravity should also be of great importance for development of quantum gravity theories. This confirms what has long been expected, that gravity indeed is linked to the Planck length. Here we show that this not is only the case at the subatomic scale, but also for macroscopic measurements. The Schwarzschild radius is quantized and comes in Planck length units. We could be closer to a unified theory between the quantum scale and the cosmic scale than we might anticipate, based on research up until now.

Conclusion. – We have shown that the Planck mass can be measured with a Cavendish apparatus without any prior knowledge of gravity except for the McCulloch gravity derived directly from Heisenberg's uncertainty principle. This no longer posits the Planck mass as simply being a derived constant from big G, but possibly makes it even more important than big G, since the gravitational constant can be written as a composite constant $G = \frac{\hbar c}{m_p^2} = \frac{l_p^2 c^3}{\hbar}$. Further, we have demonstrated that the Schwarzschild radius of an object can be measured with no knowledge of Newton's gravitational constant and also no knowledge of the size of the mass. This indicates that the Planck units are the truly fundamental units and that the gravitational constant likely is a composite constant. This also implies that the Planck units play a very central role in gravity. After years of thinking about the problem, we have come up with this gravity experiment to measure the Planck mass, the Planck length, and Planck time. It is quite remarkable it has taken us about 119 years since Max Planck first suggested the natural Planck units to discover a way to measure them totally independent of any knowledge of Newton's gravitational constant. This could be an important step toward a theory of quantum gravity that unites the quantum scale and the cosmological scale.

* * *

Thanks to Victoria Terces for helping me edit this manuscript.

***.** – REFERENCES

[1] M. E. McCulloch. Gravity from the uncertainty principle. *Astrophysics and Space Science*, 349(2), 2014.

[2] I. Newton. *Philosophiae Naturalis Principia Mathematica*. London, 1686.

[3] G. S. Bisnovatyi-Kogan. Checking the variability of the gravitational constant with binary pulsars. *International Journal of Modern Physics D*, 15(07), 2006.

[4] B. Fixler, G. T. Foster, J. M. McGuirk, and M. A. Kasevich. Atom interferometer measurement of the Newtonian constant of gravity. *Science*, 315, 2007.

[5] S. Galli, A. Melchiorri, G. F. Smoot, and O. Zahn. From Cavendish to Planck: Constraining Newton's gravitational constant with CMB temperature and polarization anisotropy. *Physical Review D*, 80, 2009.

[6] G. Rosi, F. Sorrentino, L. Cacciapuoti, M. Prevedelli, and G. M. Tino. Precision measurement of the Newtonian gravitational constant using cold atoms. *Nature*, 510, 2014.

[7] F. Scholkmann. Indications for a diurnal and annual variation in the anisotropy of diffusion patterns: A reanalysis of data presented by J. Dai (2014, Nat. Sci.). *Progress in Physics*, 10, 2014.

[8] B. P. Kibble, J. H. Sanders, and A. H. Wapstra. A measurement of the gyromagnetic ratio of the proton by the strong field method. *Atomic Masses and Fundamental Constants*, 5, 1975.

[9] M. Stock. The Watt balance: Determination of the Planck constant and redefinition of the kilogram. *Philosophical Transactions of the Royal Society*, 369:3936–3953, 2011.

[10] I. A. Robinson and S. Schlamminger. First determination of the Planck constant using the LNE Watt balance. *Metrologia*, 52(2) 2016.

[11] M. E. McCulloch. Quantised inertia from relativity and the uncertainty principle. *EPL (Europhysics Letters)*, 115(6), 2016.

[12] M. Planck. *Naturlische Masseinheiten*. Der Königlich Preussischen Akademie Der Wissenschaften, p . 479, 1899.

[13] M. Planck. *Vorlesungen über die Theorie der Wärmestrahlung*. Leipzig: J.A. Barth, p. 163. See also the English translation "The Theory of Radiation" (1959) Dover, 1906.

[14] H. Cavendish. Experiments to determine the density of the Earth. *Philosophical Transactions of the Royal Society of London, (part II)*, 88, 1798.

[15] E. G. Haug. Can the Planck length be found independent of big G ? *Applied Physics Research*, 9(6), 2017.

[16] E. G. Haug. The gravitational constant and the Planck units. A simplification of the quantum realm. *Physics Essays Vol. 29, No. 4*, 2016.

[17] M. E. McCulloch. Galaxy rotations from quantised inertia and visible matter only. *Astrophysics and Space Science*, 362, 2017.

[18] K. Schwarzschild. Über das gravitationsfeld eines massenpunktes nach der einsteinschen theorie. *Sitzungsberichte der Deutschen Akademie der Wissenschaften zu Berlin, Klasse fur Mathematik, Physik, und Technik*, page 189, 1916.

[19] K. Schwarzschild. über das gravitationsfeld einer kugel aus inkompressibler flussigkeit nach der einsteinschen theorie. *Sitzungsberichte der Deutschen Akademie der Wissenschaften zu Berlin, Klasse fur Mathematik, Physik, und Technik*, page 424, 1916.

[20] Albert Einstein. Näherungsweise integration der feldgleichungen der gravitation. *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften Berlin*, 1916.

[21] W. Heisenberg. Über den anschaulichen inhalt der quantentheoretischen kinematik und mechanik. *Zeitschrift für Physik*, (43):172–198, 1927.

Appendix A.1: Newton's Gravity from Heisenberg's Uncertainty Principle. – In 2014, McCulloch derived Newton's gravitational force from Heisenberg's uncertainty principle; for a more in detailed derivation see the McCulloch papers [1, 11]. Heisenberg's uncertainty principle [21] is given by

$$\Delta p \Delta x \geq \hbar \tag{19}$$

McCulloch goes on to say " Now $E = pc$ so" :

$$\Delta E \Delta x \geq \hbar c \tag{20}$$

This assumption only holds for the Planck momentum $E = pc = m_p cc$. It is implied indirectly in the McCulloch derivation that the Planck mass somehow plays an essential role in gravity. Further, from equation 20, McCulloch goes on to suggest that

$$F = \frac{1}{(\Delta x)^2} \sum_i^n \sum_j^N (\hbar c)_{i,j} \tag{21}$$

where $\sum_i^n$ is the number of Planck masses in a smaller mass m we are working with, and $\sum_i^N$ corresponds to the the number of Planck masses in the larger mass we are working with. From this, McCulloch gets the equation

$$F = \frac{\hbar c}{m_p^2} \frac{mM}{(\Delta x)^2} \tag{22}$$

McCulloch also replaces Δx with the radius, something we think is sound, based on extensive analysis. Further, he correctly points out that

$$G = \frac{\hbar c}{m_p^2} \tag{23}$$

which basically means his derivation is equivalent to the Newtonian gravity formula

$$F = G \frac{mM}{r^2} \tag{24}$$

Appendix A.2: The Planck Time and the Planck Length. – We can also find the Planck time directly from McCulloch-Heisenberg Newton equivalent gravity using a Cavendish experiment by utilizing the derivation below

$$\begin{aligned}
\kappa\theta &= LF \\
\frac{mL^2 2\pi^2}{T^2}\theta &= L\frac{\hbar c}{m_p^2}\frac{Mm}{r^2} \\
\frac{mL^2 2\pi^2}{T^2}\theta &= L\frac{\hbar c}{\left(\frac{\hbar}{l_p}\frac{1}{c}\right)^2}\frac{Mm}{r^2} \\
\frac{mL^2 2\pi^2}{T^2}\theta &= L\frac{t_p c^5}{\hbar}\frac{Mm}{r^2} \\
t_p^2 &= \frac{\hbar L^2 2\pi^2 r^2}{LMT^2 c^5}\theta \\
t_p &= \sqrt{\frac{\hbar L 2\pi^2 r^2 \theta}{MT^2 c^5}}
\end{aligned} \tag{25}$$

Similarly, we can also find the Planck length directly from the McCulloch-Heisenberg Newton equivalent gravity, taking into account that an elementary particle can be written as

$$m = \frac{\hbar}{\bar{\lambda}} \frac{1}{c} \tag{26}$$

In this case, we know the mass is the Planck mass, so the reduced Compton wavelength is related to the Planck length that we can find directly using a Cavendish apparatus

$$\begin{aligned}
\kappa\theta &= LF \\
\frac{mL^2 2\pi^2}{T^2}\theta &= L\frac{\hbar c}{m_p^2}\frac{Mm}{r^2} \\
\frac{mL^2 2\pi^2}{T^2}\theta &= L\frac{\hbar c}{\left(\frac{\hbar}{l_p}\frac{1}{c}\right)^2}\frac{Mm}{r^2} \\
l_p^2 &= \frac{\hbar L^2 2\pi^2 r^2 \theta}{LMT^2 c^3} \\
l_p &= \sqrt{\frac{\hbar L 2\pi^2 r^2 \theta}{MT^2 c^3}}
\end{aligned} \tag{27}$$

In other words, all of the natural Planck units can be found directly from a quantum-derived Newtonian equivalent gravity theory.