Proceedings of the Annual Symposium of the Institute of Solid Mechanics
and Session of the Commission of Acoustics, SISOM 2016 Bucharest 12-13 May

MCDM METHOD FOR N-WISE CRITERIA COMPARISONS AND INCONSISTENT PROBLEMS

Azeddine Elhassouny[1] and Florentin Smarandache[2]

[1]A. Elhassouny, Team RIITM, ENSIAS, University Mohammad V Rabat
Morocco, elhassounyphd@gmail.com

[2]F. Smarandache, University of New Mexico 200 College Road, Gallup, NM 87301, USA
smarand@unm.edu

The purpose of this paper is to present an e[xtension and alternative of the hybrid method based on Saaty's Analytical Hierarchy Process and Technique for Order Preference by Similarity to Ideal Solution method (AHP-TOPSIS), that based on the AHP and its use of pairwise comparisons, to a new method called α -D MCDM-TOPSIS(α -Discounting Method for multicriteria decision making-TOPSIS). The new method overcomes limits of AHP which work only for pairwise comparisons of criteria to any-wise (n-wise) comparisons, with crisp coefficients or with interval-valued coefficients.

α-D MCDM-TOPSIS is verified by some examples to demonstrate how it allows for consistency, inconsistent, weak, inconsistent, and strong inconsistent problems.

Keywords: α-D MCDM-TOPSIS, N-wise criteria comparisons, AHP, TOPSIS, Consistency, Inconsistency.

1. INTRODUCTION

The economic, social and technological problems have been widely resolvedin recent years andmulticriteria decision making methods have played a keyrole [8]. However, the quantity of data, the complexity of the modern world and the recent technological advances have made obviously MCDM methods more challenging than ever, hence the necessity of methods able giving quality solution.

Among the complete, simple and the most often MCDM methods used to improve the reliability of the decision making [10, 11, 15] process is the combined method AHP-TOPSIS [2-4, 8,12-14, 16].

In literature, AHP-TOPSIS is a useful and most applied MCDM method to resolve difficult decision making problems and to select the best one of the alternatives. Its applications are several, [8] developed a support for management and planning of flight mission at NASAbased on AHP-TOPSIS, using AHP-TOPSIS, [14] developed a study how the traffic congestion of urban road are evaluated, [3] established a TOPSIS-AHP solution, tried in the mobile phone industry domains, to choose logistics service provider, [12] summarizing a e-SCM performance with AHP-TOPSIS, for management of supply chain [13] proposed a Topsis-AHP simulation model, [2] developed an AHP and TOPSIS Method to evaluate faculty performance in engineering education, the sharing capacity assessment knowledge of supply chain is evaluated using AHP-TOPSIS method in the [4].

The paper is organized as follows. In the next section, the literature survey for consistency is given. Section 3 and 4 will focus on AHP-TOPSIS and the proposed α -D MCDM-TOPSIS model respectively, in a step by- step fashion. Afterwards, the proposed method is tested on the consistent, weak inconsistent and strong inconsistent examples (section 5). AHP method used one to rank the preferences is considered in section 6. In this section, we discuss developments via the use of an example to compare all methods. Finally, conclusions and perspectives are shown.

2. a-D MCDM METHOD

The general idea of the α-D MCDM is to transforman MCDM inconsistent problem (in that AHP does not work) to an MCDM consistent problem, by discounting each coefficient by the same percentages

Let us assume that $C = \{C_1, C_2, \cdots, C_n\}$, with $n \geq 2$,. are a set of Criteria.

Construct a linear homogeneous system of equations,

Each criterion $C_i$can be expressed as linear homogeneous equation, or as non-linear equation, with crisp coefficients or with interval-valued coefficients of other criteria$C_1, \cdots C_j \cdots C_n$.

$$C_i = f(C_1, \cdots C_j \cdots C_n)$$

Consequently a comparaisons matrix associated tothis linear homogeneous system is constracted.

To determine the weights w_i of the criteria, we solve the previous system.

The α-D MCDM method procedure cited above that introduced by Smarandache is not designed to rank preferences P_i based on C_i criteria, as AHP method do, but to detrmine only weights of criteria in any types of problems (consistent, inconsistent)

AHP as cited above is a complete method designed to calculates the weights of criteria C_i and to rank the preferences P_i.In addition, when the AHP is used with TOPSIS, or other MCDM method, we just benefit from the part of weight calculation criteria and we used TOPSIS to rank preferences or other MCDM methods.

The same, for α-D MCDM, in the first time, is just used to calculate the weight of criteria, that will be used later by TOPSIS to rank preferences and, in the second time, we extended α-D MCDM to a complete method to rank the preferences.

In the first time, we will used α-D MCDM for just calculate weight of criteria C_i and not to rank P_i preferences. In this case, when we will calculate the weights of criteria C_i.

We should have $C_i = f(\{C\} \setminus C_i)$

Then criteria C_i is a linear equation of C_j such $C_i = \sum_{j=1\, j\neq i}^{n} x_{ij} C_j$

So the comparisons criteria matrix has the number of criteria by rows and columns (rows number n = number of criteria and columns number also m = number of equations). In the result, we havea square matrix ($n = m$), consequently we can calculate the determinant of this matrix. At this point, we have an $n \times n$ linear homogeneous system and its associated matrix.

$$\begin{cases} x_{1,1}w_1 + x_{1,2}w_2 + \cdots + x_{1,n}w_n = 0 \\ \qquad\qquad\vdots \\ x_{n,1}w_1 + x_{m,2}w_2 + \cdots + x_{n,n}w_n = 0 \end{cases}$$

$$X = \begin{pmatrix} x_{1,1} & \cdots & x_{1,n} \\ \vdots & \ddots & \vdots \\ x_{n,1} & \cdots & x_{n,n} \end{pmatrix}$$

The difference between AHP and α-D MCDM is the ability of the latter to work with consistent and inconsistent problems, and if the problem is inconsistent α-D MCDM method transform it to a consistent problem while AHP is unable and works only for consistent problem.

In the follwing, relationship beteween determinant of matrix and consistency and parameterization of system by α_i in order to get a consistent problem.

Properties 1:

* If $det(X) = 0$, the system has a solutions (i.e MCDM problem is consistent,).

* If $det(X) \neq 0$, the system has a only the null solution solutions (i.e MCDM problem is inconsistent).

• If problem is inconsistence, then construct parameterized matrix denoted $X(\alpha)$ by parameterize the right-hand in order to get $\det(X(\alpha)) = 0$ and use Fairness principe (set equal parameters to all criteria $\alpha = \alpha_1 = \alpha_2 = \cdots = \alpha_k > 0$). To get priority vector, resove the new system obtained and set 1 to secondary variable and normalize the vector by deviding on sum of all components.

3. AHP-TOPSIS METHOD

In the real word decisions problems (case 1., section 3.) we have a multiples preferences and diverse criteria. The MCDM problem can summarized as follow :

- calculate weights w_i of criteria C_i .
- Rank preferences (alternatives) A_i .

Let us assume there are n criteria and theirs pairwise relative importance x_{ij} .

TOPSIS assumes that we have n alternatives (preferences) $A_i(i = 1, 2, \cdots, m)$ and n attributes/criteria $C_j(j = 1, 2, \cdots, n)$ and comparison matrix a_{ij} of preference i with respect to criterion j .

The AHP-TOPSIS method is described in the following steps :

Step 2.1. Construct decision matrix denoted by $A = (a_{ij})_{m\times n}$

Table 1: Decision matrix

	C_1	C_2	$\cdots$	C_n
	w_1	w_2	$\cdots$	w_n
A_1	a_{11}	a_{12}	$\cdots$	a_{1n}
A_2	a_{21}	a_{22}	$\cdots$	a_{2n}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
A_m	a_{m1}	a_{m2}	$\cdots$	a_{mn}

Step 2.2. Determine weights (w_i) of each criterion using AHP Method

Where $\sum_{j=1}^{n} w_j = 1,\ j = 1, 2, \cdots, n$

Step 2.2.1. Build a pairwise comparison matrix of criteria

The pairwise comparison of criterion i with respect criterion j gives a square matrix $(X)_{n\times n} = (x_{ij})$ where x_{ij} represents the relative importance of criterion i over the criterion j . In the matrix, $x_{ij} = 1$ when $i = j$ and $x_{ij} = 1/x_{ji}$. So we get an $n\times n$ pair-wise comparison matrix$(X)_{n\times n}$.

Step 2.2.2. Find the relative normalized weight (w_j) of each criterion defined by follow formula.

$$w_j = \frac{\prod_{j=1}^{n}(x_{ij})^{1/n}}{\sum\prod_{j=1}^{n}(x_{ij})^{1/n}}$$

Then get w_i weight of thei^{th} criterion.

Step 2.2.3. Calculate matrix X_3 and X_4 such that $X_3 = X_1 \times X_2$ and where $X_4 = X_3/X_2$ $X2 = [w_1, w_2, \cdots, w_j]^T$

Step 2.2.4. Findthe largest eigen value of pairwise comparison matrix.

For simplfied the calcul, the largest eigen value of pairwise comparison matrix is the average of X_4.

Furthermore, according to the Perron-Frobenius theorem, principal eigenvalue λ_{max} always exists for the Saaty's matrix and it holds $\lambda_{max} \geq n$; for fully consistent matrix $\lambda_{max} = n$.

Consistency check is then performed to ensure that the evaluation of the pair-wise comparison matrix is reasonable and acceptable.

Step 2.2.5. Determine the consistency ratio CR

After calculation consistency ratio (RC) using equation (eq.), and in order to verify the consistency of the matrix that is considered to be consistent if CR is less than threshold and not otherwise,according to Saaty and search.

At this point, we have the weights of criteria and if the consistency is cheked , we will using TOPSIS to rank preferences.

Step 2.3. Normalize decision matrix

The normalize decision matrix is obtained, which is given here with r_{ij}

$$r_{ij} = a_{ij} / \left(\sum_{i=1}^{m} a_{ij}^2 \right)^{0.5} ; \; j = 1, 2, \cdots, n; i = 1, 2 \cdots, m$$

Step 2.4. Calculate the weighted decision matrix

Weighting each column of obtained matrix by its associated weight.

$$v_{ij} = w_j r_{ij}; \; j = 1, 2, \cdots, n; i = 1, 2 \cdots, m$$

Step 2.5. Determine the positive ideal solution (PIS) and negative ideal solution (NIS)

$$A^+ = (v_1^+, v_2^+, \cdots, v_n^+) = \left\{ \left(max_i \left\{ v_{ij} \mid j \in B \right\} \right), \left(min_i \left\{ v_{ij} \mid j \in C \right\} \right) \right\}$$

$$A^- = (v_1^-, v_2^-, \cdots, v_n^-) = \left\{ \left(min_i \left\{ v_{ij} \mid j \in B \right\} \right), \left(max_i \left\{ v_{ij} \mid j \in C \right\} \right) \right\}$$

The benefit and cost solutions are represent B and C respectively

Step 2.6. Calculate thedistance measure for each alternative from the PIS and NIS

Thedistance measure for each alternative from the PIS is

$$S_i^+ = \left\{ \sum_{j=1}^{n} (v_{ij} - v_j^+)^2 \right\}^{0.5} ; i = 1, 2 \cdots, m$$

Also, the distance measure for each alternative from the NIS is

$$S_i^- = \left\{ \sum_{j=1}^{n} (v_{ij} - v_j^-)^2 \right\}^{0.5} ; i = 1, 2 \cdots, m$$

Step 2.7. Determine the values of relative closeness mesure

For each alternative we calculate the relative closeness mesure as follow :

$$T_i = \frac{S_i^-}{(S_i^+ + S_i^-)} ; i = 1, 2 \cdots, m$$

Ranking alternatives set according to the ordre of relative closeness mesure values T_i.

4. α-D MCDM-TOPSIS METHOD

The MCDM problem description is same of problem used in AHP-TOPSIS method (section 4.), but in this case we have n-wise comparisons matrix of criteria.

Let us assume that $C = \{C_1, C_2, \cdots, C_n\}$, with $n \geq 2$, and $\{A_1, A_2, \cdots, A_m\}$, with $m \geq 1$, are a set of criteria and the set of preferences, respectively.

Let us assume each criteria C_i is a linear homogeneous equation of the other criteria $C_1, C_2, \cdots, C_n$: $C_i = f(\{C\} \setminus C_i)$

The α -D MCDM-TOPSIS method is described in the following steps :

Step 3.1. Construct decision matrix denoted by $A = (a_{ij})_{m\times n}$

Table 2: Decision matrix

	C_1	C_2	$\cdots$	C_n
	w_1	w_2	$\cdots$	w_n
A_1	a_{11}	a_{12}	$\cdots$	a_{1n}
A_2	a_{21}	a_{22}	$\cdots$	a_{2n}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
A_m	a_{m1}	a_{m2}	$\cdots$	a_{mn}

Step 3.2. Determine weights (w_i) of each criterion using α-D MCDM Method

Step 3.2.1. Using α**-D MCDM** to determine the importance weight (w_i) of the criteria

Where $\sum_{j=1}^{n} w_j = 1,\ j = 1, 2, \cdots, n$

Step 3.2.2. Build asystem of equations and its associated matrix

To construct linear system of equations, each criteria C_i be expressed as a linear equation of C_j such as. $C_i = \sum_{j=1\, j\neq i}^{n} x_{ij} C_j$

Consequently, we have a system of n linear equations (one equation of each criterion) with n variables (variablew_i is weight of criterion).

$$\begin{cases} x_{1,1}w_1 + x_{1,2}w_2 + \cdots + x_{1,n}w_n = 0 \\ \vdots \\ x_{n,1}w_1 + x_{m,2}w_2 + \cdots + x_{n,n}w_n = 0 \end{cases}$$

In mathematic, each linear system can associeted to a matrix, in this case,denoted by $X = (x_{ij})$, $1 \le i \le n$ and $1 \le j \le n$ where

$$X = \begin{pmatrix} x_{1,1} & \cdots & x_{1,n} \\ \vdots & \ddots & \vdots \\ x_{n,1} & \cdots & x_{n,n} \end{pmatrix}$$

Step 3.2.3. Solve system of equation using whose associeted matrix

Solve the system of equation, the differentes cases are discussed in proporites 1 in that we compute the determinant of X (find strictly positive solution $w_i > 0$).

Solving this homogeneous linear system, in differents cases above the general solution that we set as a solution vector $S = [s_1, s_2, \cdots, s_n]$, and set 1 to secondary variable we get $W = [w_1, w_2, \cdots, w_n]$

Deviding each vector element on sum of all components of vector to get normalized vector where

$$w_j = \frac{s_j}{\sum_{k=1}^{n} s_k}; i = 1, 2\cdots, n$$

Step 3.2.4. Build a pairwise comparison matrix of criteria

The pairwise comparison of criterion i with respect criterion j gives a square matrix $(X)_{n\times n} = (x_{ij})$ where x_{ij} represents the relative importance of criterion i over the criterion j. In the matrix, $x_{ij} = 1$ when

$i = j$ and $x_{ij} = 1/x_{ji}$. So we get an $n \times n$ pair-wise comparison matrix$(X)_{n\times n}$.

Step 3.2.5. Find the relative normalized weight (w_j) of each criterion defined by follow formula.

$$w_j = \frac{\prod_{j=1}^{n}(x_{ij})^{1/n}}{\sum\prod_{j=1}^{n}(x_{ij})^{1/n}}$$

Then get w_i weight of thei^{th} criterion.

Step 2.2.6. Calculate matrix X_3 and X_4 such that $X_3 = X_1 \times X_2$ and where $X_4 = X_3/X_2$

$$X2 = [w_1, w_2, \cdots, w_j]^T$$

Step 3.2.7. Findthe largest eigen value of pairwise comparison matrix.

For simplfied the calcul, the largest eigen value of pairwise comparison matrix is the average of X_4. Furthermore, according to the Perron-Frobenius theorem, principal eigenvalue λ_{max} always exists for the Saaty's matrix and it holds $\lambda_{max} \geq n$; for fully consistent matrix $\lambda_{max} = n$.

Consistency check is then performed to ensure that the evaluation of the pair-wise comparison matrix is reasonable and acceptable.

Step 3.2.8. Determine the consistency ratio (CR).

After calculation consistency ratio (RC) using equation (eq.), and in order to verify the consistency of the matrix that is considered to be consistent if CR is less than threshold and not otherwise,according to Saaty and search.

At this point, we have the weights of criteria and if the consistency is cheked , we will using TOPSIS to rank preferences.

Step 3.3. Normalize decision matrix

The normalize decision matrix is obtained, which is given here with r_{ij}

$$r_{ij} = a_{ij} \Big/ \left(\sum_{i=1}^{m} a_{ij}^2\right)^{0.5} \; ; j = 1, 2, \cdots, n; i = 1, 2 \cdots, m$$

Step 3.4. Calculate the weighted decision matrix

Weighting each column of obtained matrix by its associated weight.

$$v_{ij} = w_j r_{ij}; \; j = 1, 2, \cdots, n; i = 1, 2 \cdots, m$$

Step 3.5. Determine the positive ideal solution (PIS) and negative ideal solution (NIS)

$$A^+ = (v_1^+, v_2^+, \cdots, v_n^+) = \left\{\left(max_i \left\{v_{ij} \mid j \in B\right\}\right), \left(min_i \left\{v_{ij} \mid j \in C\right\}\right)\right\}$$

$$A^- = (v_1^-, v_2^-, \cdots, v_n^-) = \left\{\left(min_i \left\{v_{ij} \mid j \in B\right\}\right), \left(max_i \left\{v_{ij} \mid j \in C\right\}\right)\right\}$$ The benefit and cost solutions are represent B and C respectively

Step 3.6. Calculate thedistance measure for each alternative from the PIS and NIS

The distance measure for each alternative from the PIS is $S_i^+ = \left\{\sum_{j=1}^{n}(v_{ij} - v_j^+)^2\right\}^{0.5} \; ; i = 1, 2 \cdots, m$

Also, the distance measure for each alternative from the NIS is $S_i^- = \left\{\sum_{j=1}^{n}(v_{ij} - v_j^-)^2\right\}^{0.5} \; ; i = 1, 2 \cdots, m$

Step 3.7. Determine the values of relative closeness mesure

For each alternative we calculate the relative closeness mesure as follow :

$$T_i = \frac{S_i^-}{(S_i^+ + S_i^-)}; i = 1, 2 \cdots, m$$

Ranking alternatives set according to the ordre of relative closeness mesure values T_i.

5. NUMERICAL EXAMPLES

We examined a numerical example in which a synthetic evaluation desire to rank four alternatives A_1, A_2, A_3 and A_4 with respect to three benefit attribute C_1, C_2 and C_3.

Table 3: Decision matrix

	C_1	C_2	C_3
	w_1	w_2	w_3
A_1	7	9	9
A_2	8	7	8
A_3	9	6	8
A_4	6	7	8

In the examples below we used α-D MCDM and AHP (if it works) to calculate the weights of the criteria w_1, w_2 and w_3. After we used TOPSIS to rank the four alternatives, the decision matrix (Table 6) below is used for the three following examples.

a. Consistent example 1

We use the α-D MCDM

Let the Set of Criteria be $\{C_1, C_2, C_3\}$ with $w_1 = w(C_1) = x$ and $w_3 = w(C_3) = z$.

Let consider the system of equations associated to MCDM problem and its associated matrix.

$$\begin{cases} x = & 4y \\ y = & 3z \\ z = & \dfrac{x}{12} \end{cases} \qquad X1 = \begin{pmatrix} 1 & 4 & 0 \\ 0 & 1 & 3 \\ \dfrac{1}{12} & 0 & 1 \end{pmatrix}$$

We calcutalte $\det(X)$ (in this cas equal= 0) then MCDM problem is consisten we solve the system we get follow solution $S = \begin{bmatrix} 12z & 3z & z \end{bmatrix}$

Seting 1 to secondary varaible the general solution becomes $S = \begin{bmatrix} 12 & 3 & 1 \end{bmatrix}$

and normalizing vector is (deviding by sum=12+3+1) the weights vector is $W = \begin{bmatrix} \frac{12}{16} & \frac{3}{16} & \frac{1}{16} \end{bmatrix}$

Using AHP, we get the same result

The pairwise comparison matrix of criteria is : $X1 = \begin{pmatrix} 1 & 4 & 12 \\ \frac{1}{4} & 1 & 3 \\ \frac{1}{12} & \frac{1}{3} & 1 \end{pmatrix}$

whose maximum eigenvalue is $\lambda_{max} = 3$ and its corresponding normalized eigenvector (Perron-Frobenius vector) is $W = \begin{bmatrix} \frac{12}{16} & \frac{3}{16} & \frac{1}{16} \end{bmatrix}$

We use TOPSIS to rank the four alternatives

Table 4: Calculate (a_{ij}^2) for each column and divide each column by $(\sum_{i=1}^{n} a_{ij}^2)^{1/2}$ to get r_{ij}

a_{ij}^2	C_1	C_2	C_3
w_i	*12/16*	*3/16*	*1/16*
A_1	*49*	*81*	*81*
A_2	*64*	*49*	*64*
A_3	*81*	*36*	*64*
A_4	*36*	*49*	*64*
$\sum_{i=1}^{n} a_{ij}^2$	*230*	*215*	*273*

Table 5: Multiply each column by w_j to get v_{ij}

v_{ij}	C_1	C_2	C_3
w_i	*12/16*	*3/16*	*1/16*
A_1	*0.3462*	*0.1151*	*0.0340*
A_2	*0.3956*	*0.0895*	*0.0303*
A_3	*0.4451*	*0.0767*	*0.0303*
A_4	*0.2967*	*0.0895*	*0.0303*
v_{max}	*0.4451*	*0.1151*	*0.0340*
v_{min}	*0.2967*	*0.0767*	*0.0303*

Table 6: The separation mesure values and the final rankings for decision matrix (Table 4) using AHP-TOPSIS and α -D MCDM-TOPSIS

	S_i^+	S_i^-	T_i	*Rank*
A_1	*0.0989*	*0.0627*	*0.3880*	*3*
A_2	*0.0558*	*0.0997*	*0.6412*	*2*
A_3	*0.0385*	*0.1484*	*0.7938*	*1*
A_4	*0.1506*	*0.0128*	*0.0783*	*4*

Table 10 presents the rank of alternatives (A_1, A_2, A_3, A_4) and separation measure values of each alternative from the PIS and from NIS in wish the weighted values are calculated by AHP or α -D MCDM. The both methods AHP and α -D MCDM with fairnessp rinciple give the same weights as proven above methods together give same result in consistent problem.

b. Weak inconsistent example 2 where AHP does not work

Consider another example investigated by [7] for which AHP does not work (i.e AHP-TOPSIS doses not work too), **then we use the α -D MCDM to calculate the weights values and ranking the four alternatives by TOPSIS (see Table 14)**.

Let the Set of Criteria be $\{C_1, C_2, C_3\}$ with $w_1 = w(C_1) = x$ and $w_3 = w(C_3) = z$.

Let consider the system of equations associated to MCDM problem and its associated matrix.

$$\begin{cases} x = & 2y+3z \\ y = & \dfrac{x}{2} \\ z = & \dfrac{x}{3} \end{cases} \qquad X1 = \begin{pmatrix} 1 & -2 & -3 \\ \dfrac{-1}{2} & 1 & 0 \\ \dfrac{-1}{3} & 0 & 1 \end{pmatrix}$$

The solution of this system is $x = y = z = 0$ sine the sum of weights should be = 1, then this solution is not acceptable.

Parameterizing the right-hand side coefficient of each equations by α_i we get

$$\begin{cases} x = 2\alpha_1 y + 3\alpha_2 z \\ y = \dfrac{\alpha_3 x}{2} \\ z = \dfrac{\alpha_4 x}{3} \end{cases}$$

we solve the system we get follow solution $S = \begin{cases} y = \frac{\alpha_3 x}{2} \\ z = \frac{\alpha_4 x}{3} \end{cases}$ or $S = \begin{bmatrix} x & \dfrac{\alpha_3 x}{2} & \dfrac{\alpha_4 x}{3} \end{bmatrix}$

Seting 1 to secondary varaible the general solution becomes $S = \begin{bmatrix} 1 & \dfrac{\alpha_3}{2} & \dfrac{\alpha_4}{3} \end{bmatrix}$

Applying Fairness Principle: then replace $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha$ in whence $\alpha = \dfrac{\sqrt{2}}{2}$

$$S = \begin{bmatrix} 1 & \dfrac{\sqrt{2}}{4} & \dfrac{\sqrt{2}}{6} \end{bmatrix}$$

and normalizing vector is (deviding by sum) the weights vector is $W = \begin{bmatrix} 0.62923 & 0.22246 & 0.14831 \end{bmatrix}$

TOPSIS is used to rank the four alternative: application of TOPSIS method is in the same manner as in the previous example (the four alternatives (Ai)are ranked in following table 10)

Table 7: Calculate (a_{ij}^2) for each column

a_{ij}^2	C_1	C_2	C_3
	0.62923	*0.22246*	*0.14831*
A_1	*49*	*81*	*81*
A_2	*64*	*49*	*64*
A_3	*81*	*36*	*64*
A_4	*36*	*49*	*64*
$\sum_{i=1}^{n} a_{ij}^2$	*230*	*215*	*273*

Table 8: Divide each column by $(\sum_{i=1}^{n} a_{ij}^2)^{1/2}$ to get r_{ij}

r_{ij}	C_1	C_2	C_3
	0.62923	0.22246	0.14831
A_1	0.4616	0.6138	0.5447
A_2	0.5275	0.4774	0.4842
A_3	0.5934	0.4092	0.4842
A_4	0.3956	0.4774	0.4842
$\sum_{i=1}^{n} a_{ij}^2$	230	215	273

Table 9: Multiply each column by w_j to get v_{ij}

v_{ij}	C_1	C_2	C_3
	0.62923	0.22246	0.14831
A_1	0.2904	0.1365	0.0808
A_2	0.3319	0.1062	0.0718
A_3	0.3734	0.0910	0.0718
A_4	0.2489	0.1062	0.0718
v_{max}	0.3734	0.1365	0.0808
v_{min}	0.2489	0.0910	0.0718

Table 10: The separation mesure values and the final rankings for decision matrix (Table 4) using α -D MCDM-TOPSIS

	S_i^+	S_i^-	T_i	Rank
A_1	0.0830	0.0622	0.4286	3
A_2	0.0522	0.0844	0.6178	2
A_3	0.0464	0.1245	0.7285	1
A_4	0.1284	0.0152	0.1057	4

c. Jean Dezert's strong inconsistent example 3

Smarandache [7] introduced a Jean Dezert's Strong Inconsistent example, Let consider the system of equations associated to MCDM problem and its associated matrix.

$$X = \begin{pmatrix} 1 & 9 & \frac{1}{9} \\ \frac{1}{9} & 1 & 9 \\ 9 & \frac{1}{9} & 1 \end{pmatrix} \qquad \begin{cases} x = 9y, x > y \\ x = \frac{1}{9}z, x < z \\ y = 9z, y > z \end{cases}$$

We followthe same process asthe example above we get the general solution

$$W = \begin{bmatrix} \frac{1}{6643} & \frac{81}{6643} & \frac{6561}{6643} \end{bmatrix}$$

We use TOPSIS to rank the four alternatives

Table 11: Calculate (a_{ij}^2) for each column

a_{ij}^2	C_1	C_2	C_3
	0.0002	0.0122	0.9877
A_1	49	81	81
A_2	64	49	64
A_3	81	36	64
A_4	36	49	64
$\sum_{i=1}^{n} a_{ij}^2$	230	215	273

Table 12: Divide each column by $(\sum_{i=1}^{n} a_{ij}^2)^{1/2}$ to get r_{ij}

r_{ij}	C_1	C_2	C_3
	0.0002	0.0122	0.9877
A_1	0.503	0.699	0.623
A_2	0.574	0.543	0.553
A_3	0.646	0.466	0.553
A_4	0.431	0.543	0.553
$\sum_{i=1}^{n} a_{ij}^2$	230	215	273

Table 13: Multiply each column by w_j to get v_{ij}

v_{ij}	C_1	C_2	C_3
	0.0002	0.0122	0.9877
A_1	0.0001	0.0075	0.5380
A_2	0.0001	0.0058	0.4782
A_3	0.0001	0.0050	0.4782
A_4	0.0001	0.0058	0.4782
v_{max}	0.0001	0.0075	0.5380
v_{min}	0.0001	0.0050	0.4782

Table 14: The separation mesure values and the final rankings for decision matrix (Table 4) using α -D MCDM-TOPSIS

	S_i^+	S_i^-	T_i	Rank
A_1	0.0000	0.0598	0.99966	1
A_1	0.0598	0.0008	.01372	2
A_1	0.0598	0.0000	0.00049	4
A_1	0.0598	0.0008	.013715	3

Table 15: Summary of the results of three examples of all methods

Example	Alternative	AHP-TOPSIS		α-DMCDM-TOPSIS	
Consistent example 1	A_1	*0.3880*	*3*	*0.3880*	*3*
	A_2	*0.6412*	*2*	*0.6412*	*2*
	A_3	*0.7938*	*1*	*0.7938*	*1*
	A_4	*0.0783*	*4*	*0.0783*	*4*
Weak Inconsistent Example 2	A_1	*Does not works*		*0.3880*	*3*
	A_2			*0.6412*	*2*
	A_3			*0.7938*	*1*
	A_4			*0.0783*	*4*
Strong Inconsistent Example 3	A_1	*Does not works*		*0.999668*	*1*
	A_2			*0.013719*	*2*
	A_3			*0.000497*	*4*
	A_4			*0.013715*	*3*

For the three examples presented in this paper, the table 25 (that summarized all results of all methods) illustrate that the AHP and AHP-TOPSIS methods works just for the first example in wish the criteria and alternatives are consistent in their pairwise comparisons. Ours proposed methods α -D MCDM-TOPSIS works not only for consistent example 1, in that gives the same results as AHP and AHP-TOPSIS methods, but for weak inconsistent and strong inconsistent examples.

The results have claimed that AHP-TOPSIS and our α -D MCDM-TOPSIS methods preserves the ranking order of the alternatives and overcome the nearness scored values problem. By using AHP-TOPSIS and our alpha-D MCDM-TOPSIS methods, the scored value of the A3 was changed from 0.2789 to 0.7938, the scored value of the A2 was changed from 0.2604 to 0.6412, and the scored value of the A4 was changed from 0.2104 to 0.0783. The bigger differences between the score values of alternatives 0.7155 ((A3) 0.7938-(A4) 0.0783) is also subject to gain additional insight into this phenomenon.

In the two last examples (weak inconsistent and strong inconsistent), one sees that the importance discounting ours approaches (what we call the α -D MCDM-TOPSIS approachs) will suggest, that can uses to solve real-life problems in wish criteria are not only pairwise but n-wise comparisons and the problems are not perfectly consistent. It is however worth to note that the ranking order of the four alternatives obtained by both methods is similar but scored values of are slightly different, in wish the same remarks mentioned above are available between extended α -D MCDM and α -D MCDM-TOPSIS. α -D MCDM-TOPSIS method allows to take into account also any nembers of alternatives and any weights of criteria.

6. CONCLUSION

We have proposed tow multicritere decision making methods, Extended- α -D MCDM and α -D MCDM-TOPSIS models that allows to works for consistent and inconsistent MCDM problem In addition, three examples have demonstrated the α -D MCDM-TOPSIS model is efficient and robust.

Ours approch α -D MCDM-TOPSIS give the same result as AHP-TOPSIS and AHP in consistent MCDM problems and elements of decision matrix are pairwise comparisons, but for weak inconsistent and strong inconsistent MCDM problems in which AHP and AHP-TOPSIS are limited and unable, ours proposed methods Extended- α -D MCDM and α -D MCDM -TOPSIS give a justifiable results.

Furthermore ours proposed approaches α -D MCDM-TOPSIS can uses to solve real-life problems in wish criteria are not only pairwise but n -wise comparisons and the problems are not perfectly consistent.

REFERENCES

1. Alonso, J.A., Lamata, M.: Consistency in the analytic hierarchy process : a new approch. International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems **14**(4), 445–459 (2006)
2. Ghosh, D.N.: Analytic hierarchy process and topsis method to evaluate faculty performance in engineering education. Dipendra Nath Ghosh et al UNIASCIT **1**(2), 63–70 (2011)
3. Jayanta, A., P.Guptaa, S.K.Gargb, M.Khana: Topsis-ahp based approach for selection of reverse logistics service provider: A case study of mobile phone industry. Procedia Engineering 97, Elsevier **97**, 2147–2156 (2014). DOI 10.1016/j.proeng.2014.12.458
4. Maheshwarkar, M., Sohani, N.: Combined ahp-topsis based approach for the evaluation of knowledge sharing capabilities of supply chain partners. Management Science and Engineering **7**(3), 27–32 (2013). DOI 10.3968/j.mse.1913035X20130703.2573
5. Pelaez, J.I., Lamata, M.T.: New measure of consistency for positive reciprocal matrices. An Intematiml Journal Computers and Mathematics with Applications Elsevier **46**, 1839–1845 (2003). DOI lO.l016/SO898-1221(03)00407-3
6. Saaty, T.: Fundamentals of decision making and priority theory with the ahp. RWS Publications, Pittsburgh, PA, U.S.A. (1994)
7. Smarandache, F.: Î±-discounting method for multi-criteria decision `making (α -D MCDM). Fusion 2010 International Conference Scotland (2010)
8. Tavana, M., Hatami-Marbini, A.: A group ahp-topsis framework for human spaceflight mission planning at nasa. Expert Systems with Applications **38**, 13,588–13,603 (2011). DOI 10.1016/j.eswa.2011.04.108
9. Triantaphyllou, E., Mann, S.H.: Using the analytic hierarchy process for decision making in engineering application : dome challenges. Interl Journal of Industrial Engineering: Applications and Practice **2**(1), 35–44 (1995)
10. Victor Vladareanu, Gabriela Tont, Luige Vladareanu, Florentin Smarandache, , “The navigation of mobile robots in non-stationary and non-structured environments”, Inderscience Publishers, Int. J. Advanced Mechatronic Systems, Vol. 5, No. 4, 2013, pg.232- 243, ISSN online: 1756-8420, ISSN print: 1756-8412.
11. F.Smarandache, L.Vladareanu, Applications of Neutrosophic Logic to Robotics - An Introduction, The 2011 IEEE International Conference on Granular Computing Kaohsiung, Taiwan, Nov. 8-10, 2011, pp. 607-612, ISBN 978-1-4577-0370-6, IEEE Catalog Number: CFP11GRC-PRT.
12. Tyagia, M., Kumarb, P., Kumarc, D.: A hybrid approach using ahp-topsis for analyzing e-scm performance. 12th Global Congress On Manufacturing And Management Procedia Engineering Elsevier **97**, 2195–2203 (2014)
13. Wu, M.: Topsis-ahp simulation model and its application to supply chain management. World Journal of Modelling and Simulation **3**(3), 196–201 (2007). DOI ISSN 1 746-7233
14. Yu, J., Wang, L., Gong, X.: Study on the status evaluation of urban road intersections traffic congestion base on ahp-topsis modal. 13th COTA International Conference of Transportation Professionals CICTP 2013 **96**(609–616) (2013)
15. Victor Vladareanua, Radu I. Munteanub, Ali Mumtazc, Florentin Smarandachedand Luige Vladareanua *, “The optimization of intelligent control interfaces using Versatile Intelligent Portable Robot Platform”, Procedia Computer Science 65 (2015): 225 – 232, ELSEVIER 2015
16. Weak consistency: A new approach to consistency in the saaty’s analytic hierarchy process. Mathematica **52**(2), 71–83