# Einstein/Newton duality: An ontological-phase topological field theory 

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#### Abstract

Newton claimed instantaneous G-influence; Einstein insisted no influence propagated faster than $c$. Quantum Mechanics (QM) the so-called basement of reality, posits a Quantum Gravity, for which no a priori science exists. We propose a paradigm shift with duality between a semi-quantum Standard Model (SM) limit and Large-Scale Additional Dimensionality (LSXD) in a modified M-Theoretic Unified Field (UF) brane arena as the regime of integration described by an Ontological-Phase Topological Field Theory (OPTFT) requiring fundamental changes in the concept of dimensionality and matter. OPTFT is developed to formally describe $3^{\text {rd }}$ regime Unified Field Mechanics (UFM) (classical-Quantum-UFM) to relate Newton-Einstein duality by added degrees of freedom in a semiquantum limit enabling topological Dirac-Majorana doublet fusion supervening the uncertainty principle.


## 1. Introduction - Imminent $3^{\text {rd }}$ Regime Paradigm Shift

An Ontological-Phase Topological Field Theory (OPTFT) requires fundamental changes in the concept of dimensionality and matter. From suggestions by Rowlands, two processes emerge to create XDs (Additional Dimensionality) [1]: 1) duality, with Ds fundamentally different, and 2) anticommutativity, with Ds fundamentally the same [2]. Yang-Mills (YM) Kaluza-Klein (KK) correspondence drives Physics beyond the SM. Horizontal and vertical subspaces in the tangent bundle of $M,(M=M \times G)$ defined by YM connections are orthogonal with respect to a KK metric suggesting orthogonal extension to XD beyond the 4D limit of the SM. CERN LHC research seeks KK XD beyond the SM. Current thinking posits XD as $\hbar$-scale since they are unobserved; however, this is not the only interpretation. A LSXD alternative hidden by subtractive interferometry is proposed [3-5]. Albeit, our OPTFT iteration of M-Theory is based on radical extensions of the original hadronic string theory because of inherent key elements: virtual tachyon/tardyon interactions and a variable concept of string tension, $T_{S}=T_{0}+\Delta \hbar[3,6]$. A and B-type topological string theories, and a related Topological M-Theory with mirror symmetry, while interesting, since they allow sufficient XD by Calabi-Yau mirror symmetry, essential for developing UFM; a distinction between these theories causes our model to diverge, as compactification must be continuous. A key parameter is topological charge in brane dynamics which by definition makes correspondence to a de-BroglieBohm super-quantum potential synonymous with an ontological Force of Coherence, an inherent aspect of UF dynamics [3-5]. Thus, UFM predicts no phenomenal graviton (perceived artifact of incompleteness of Gauge Theory, i.e. Gauge Theory is approximate suggesting new physics).
The difference between 4D quantum field phenomenology and LSXD topological field ontology is the energyless exchange process. Information (Shannon related) is transferred ontologically by the
dynamics of topological switching in M-Theoretic branes carrying topological charge [3-5]. Completing Geometrodynamics inherently includes Newton/Einstein duality [5]; evidenced by interpreting quasar luminosity as $G$-shock waves [3] countering Big Bang interpretations of large redshift, Z based on Doppler recession. Instead, redshift results from periodic photon mass-anisotropy by coupling to a covariant polarized Dirac vacuum [3]. A further conundrum exists by defining a Manifold of Uncertainty (MOU) of finite dimensional radius, allowing a wave-particle-like duality with a quantal-like virtual graviton in the semi-quantum limit - an intermediary between field phenomenology and topological ontology. This has increasing importance for the new field of Relativistic Information Processing (RIP) which introduces G-effects in parallel transport of brane topological switching [3-5]. From the broad context above our central theme is the introduction of a topological formalism for a new set of UF transformations beyond the Galilean, Lorentz-Poincairé. An empirical protocol falsifying the model is developed [3-5]; which if successful has far reaching consequences for experimentally validating XD, M-Theory and leads to obsolescence of usual $\mathrm{TeV} \rightarrow$ PeV supercollider particle sprays by providing a new form of table-top low-energy UFM XD brane collision (LSXD topological fusion) cross section alternatives for viewing putative SUSY partners in a trans-D slice rather than standard $\mathrm{TeV} / \mathrm{PeV}$ cross section collision physics [7].
OPTFT, is a paradigm shift beyond limits of the Standard Model (SM). Required tenets include:

- Redefinition of Matter: The current 0D fermionic singularity becomes cyclic M-Theoretic mirrorsymmetric LSXD dual quaternion/octonion dynamic brane topology.
- Complex (LCU) Least Cosmological Unit Tessellating Space/Spacetime: LCU simplistic similitude of Unit Cell building Crystal Structure.
- Semi-Quantum Limit: Uncertainty a Manifold of Finite Radius: Where Duality of SM \& LSXD Newton - Einstein Enters.
- Described by New Topological Transformation: Beyond the Galilean, Lorentz-Poincairé named Noetic Transform because ultimately it must include physics of the Observer...


## 2. Quasar Luminosity as Gravitational Shock Waves - Newton Einstein G Duality

Conflicts within the SM call into question the fundamental interpretation of the Doppler component of the putative Hubble Expansion Law and the nature of events in associated with conventional coordinates of the line element as attached to the physical basis of the observer. Also of paramount importance is that Einstein's Classical Geometrodynamics is not a complete theory of gravity, as stated by Einstein himself. We postulate nonlinear effects associated with the propagation of light in an intense G-field produce shock waves creating light-booms along boundary conditions at cosmological distances approaching the limit of observation that if correct would explain anomalous Quasi-Stellar Object (QSO) luminosity. These G-shock waves are considered observationally manifest in the spectrum of QSOs and Supernova as a continuous front of light booms produced by superluminal boosts associated with continuous coordinate transformations relative to a distant observer, suggesting that QSOs are a form of Seifert spiral galaxy with Active Galactic Nuclei (AGN) in the vicinity of the putative observational limit of the Hubble radius, $H_{R}$, creating an issue of fundamental basis of Geometrodynamics. Newton's formulation of the G-force law requires each particle to respond instantaneously to every other massive particle regardless of the distance between them which he proved; but the proof is only valid in Euclidian space. Today this would be described by the Poisson equation,

$$
\left(\frac{\partial^{2}}{\partial x^{2}}+\frac{\partial^{2}}{\partial y^{2}}+\frac{\partial^{2}}{\partial z^{2}}\right) \phi(x, y, z,)=f(x, y, z)
$$

according to which, when the mass distribution of a system changes, its G-field instantaneously adjusts. Therefore, theory requires the speed of $G$ to be infinite (instantaneous). Einstein's Geometrodynamics, $G_{\mu \nu}+\Lambda g_{\mu \nu}=\frac{8 \pi G}{c^{4}} T_{\mu \nu}$ a classical extension of Newtonian-G is thus incomplete.

Physical theory incorporates an upper limit on the propagation speed of an interaction, maintaining that instantaneous action-at-a-distance is impossible. However, quantum entanglement between separated quanta enables instantaneous EPR correlations which led to the puzzle as to whether causality or locality is abandoned in transit to $3{ }^{\text {rd }}$ regime natural science.

### 2.1. Note on Cosmological Principle and G-Duality

In summarizing the Cosmological Principle (universe homogeneous and isotropic) [8] events are idealized spacetime instants defined by arbitrary time and position coordinates $t, x, y, z$, written collectively as $x^{i}$ with $i, 0$ to 3 . The standard line element is $d s^{2}=\sum_{i j} g_{i j} d x^{i} d x^{j}=g_{i j} d x^{i} d x^{j}$, where metric tensor $g_{i j}(x)=g_{j i}(x)$ is symmetric [8]. In local Minkowski form all first derivatives of $g_{i j}$ vanish at the event taking the form $d s^{2}=d t^{2}-d x^{2}-d y^{2}-d z^{2}$. The Cosmological Principle generally suggests that clocks of all observers are synchronized throughout all space because of inherent homogeneity and isotropy. Because of this synchronization of clocks for the same world time $t$, for commoving observers the line element becomes, $d s^{2}=d t^{2}+g_{\alpha \beta} d x^{\alpha} d x^{\beta}=d t^{2}-d l^{2}$, where $d l^{2}$ represents spatial separation of events at the same world time, $t$. This spatial component of event $d l^{2}$ can be represented as an Einstein 3-sphere (compatible with dual 6D Calabi-Yau 3-tori) $d l^{2}=d x^{2}+d y^{2}+d z^{2}+d w^{2}$ which is represented by the set of points $(x, y, z, w)$ at a fixed distance $R$ from origin: $R^{2}=x^{2}+y^{2}+z^{2}+w^{2}$ where $w^{2}=R^{2}-r^{2}$ and $r^{2}=x^{2}+y^{2}+z^{2}$, so finally we may write the line element of the Einstein 3 -sphere as $d l^{2}=d x^{2}+d y^{2}+d z^{2}+r^{2} d r^{2} / R^{2}-r^{2}$ [8]. By imbedding an Einstein 3-sphere in a flat HD space, specifically as a subspace of a new complex 12D superspace $[3,4,9]$, new theoretical interpretations of standard cosmological principles are feasible. This is the line element most compatible with the oscillatory spacetime boundary parameters required by our model of nonlinear dual G-shock waves in QSO luminosity [3].
According to MTW [10] junction conditions may act as generators of G-shocks; the dynamics of spacetime geometry for a 3 -surface, $\Sigma$ which includes intrinsic Riemann scalar curvature invariants, $R$, also includes an extrinsic curvature tensor, $K_{i j}$. When imbedded in an enveloping 4-geometry hypersurface it can change (shrinkage and deformation) in vector, $n$ parallel transported as junction conditions applicable to the G-field (spacetime curvature) and the stress-energy generating it. A discontinuity in $K_{i j}$ across a null surface without stress-energy producing it is a geometric manifestation of a G-shock-wave generated by a different embedding in spacetime above $\Sigma$ than below $\Sigma[3,10]$.
Dray and 't Hooft [11] found conditions for introducing G-shock waves in a class of vacuum solutions to Einstein's equations by coordinate shift. Their model generalizes G-shock waves for a massless particle moving in flat Minkowski space formulated as two Schwarzschild black holes of equal masses glued together at the horizon. For a spherical shell of unequal masses moving along $u=u_{0} \neq 0$; their solution [12] represents two Schwarzschild black holes glued together at $u=u_{0}$. By infinitely boosting the Dray-'t Hooft solutions various forms of G-shock waves have been found [13-14]. Sfetsos [15] extends these results to the case with matter fields and a non-vanishing cosmological constant. Using the d-D spacetime metric:

$$
\begin{equation*}
d s^{2}=2 A(u, v) d u d v+g(u, v) h_{i j}(x) d x^{i} d x^{j}, \text { with }(i, j=1,2, \ldots, d-2) \tag{1}
\end{equation*}
$$

he uses a string based dilatonic black hole G-solution [16] from the perspective of a conformal background field theory of coset $S L(2, \mathbf{R}) / \mathbf{R} \otimes \mathbf{R}^{2}$ to achieve a differential shift factor

$$
\begin{equation*}
\left(\frac{d^{2}}{d \rho^{2}}+\frac{1}{\rho} \frac{d}{d \rho}-\varepsilon\right) f(\rho)=-16 \varepsilon \frac{1}{\rho} \delta(\rho) \tag{2}
\end{equation*}
$$

where $\rho^{2}=x^{2}+y^{2}$ and for a black hole singularity case with $\varepsilon=1$,(2) is a modified Bessel equation [15]. Spitkovsky [17] simulates a relativistic Fermi emission shock process that could provide an alternative to, or component process for our G-shock work. His simulations on relativistic collisionless shocks propagating in initially unmagnetized electron-positron pair plasmas showed natural production of accelerated particles as part of shock evolution. He studied the mechanism that populates the suprathermal tail for particles gaining the most energy. The simulation showed the main acceleration occurs near the shock, where for each reflection these particles gain energy, $\Delta E \sim E$ as is expected in relativistic shocks [18].
Newton's $G$ required instantaneous action at a distance or the conservation of angular momentum would be violated; but for Einstein's GR an instantaneous influence would violate causality and SR and so must be mediated by a field. This is the dual nature of gravity that we have put as the basis for our model. We tried to show that it is possible with further study to relate shock phenomena to Gwaves especially for narrow axis massive cosmological objects such as AGN QSOs that readily lend themselves to light-boom effects that could therefore be used to explain QSO luminosity as further evidence of the insurmountable shortcomings of Big Bang cosmology. Our model works best contrasting both modes of the intrinsic dual nature of $G$ because nonlinear jumps in flow occur with discontinuity. From the $2^{\text {nd }}$ Law of Thermo-dynamics entropy is only increases when particles cross a shock. The duality of the propagation of the G-influence is evident in Birkhoff's theorem [3,9] where a spherically symmetric G-field is produced by a massive object such as a QSO at the origin; if there were another concentration of mass-energy elsewhere, this would disturb spherical symmetry. This effect could occur if interference occurs between the usual modes of the G-influence by shock parameters.

## 3. The Phasor (Phase Vector) Complex Probability Amplitude

Ontological-Phase Topological Field Theory (OPTFT) introduces fundamental $3^{\text {rd }}$ regime postulates: 1) A semi-quantum mirror symmetric Calabi-Yau finite radius manifold of uncertainty, 2) with a 4D Minkowski-Riemann subspace, and 3) cyclical duality of phenomenological (quantal) field mediation and an ontological charge (energyless) topological switching unified field. As initial simplistic modeling of Ontological-Phase we adapt the phasor or phase vector concept as a precursor to ontological topological phase. In general, a phasor is a complex number for a sinusoidal ( $\pi$ rotation) function with amplitude $A$, angular frequency $\omega$ and initial phase $\theta$, with all time invariant. The complex constant is the phasor [4]. Euler's formula can represent sinusoids as the sum of two complex-valued functions: $A \cdot \cos (\omega t+\theta)=A \cdot\left(e^{i(\omega t+\theta)}+e^{-i(\omega t+\theta)} / 2\right)$, or as the real part of function: $A \cdot \cos (\omega t+\theta)=\operatorname{Re}\left\{A \cdot e^{i(\omega t+\theta)}\right\}=\operatorname{Re}\left\{A e^{i \theta} \cdot e^{i \omega t}\right\}$. The function, $A \cdot e^{i(\omega t+\theta)}$ is the analytic representation of $A \cdot \cos (\omega t+\theta)$. Multiplication of the phasor, $A e^{i \theta} e^{i \omega t}$ by a complex constant, $B e^{i \phi}$, produces another phasor changing the amplitude and phase of the underlying sinusoid: $\operatorname{Re}\left\{\left(A e^{i \theta} \cdot B e^{i \phi}\right) \cdot e^{i \omega t}\right\}=\operatorname{Re}\left\{\left(A B e^{i(\theta+\phi)}\right) \cdot e^{i \omega t}\right\}=A B \cos (\omega t+(\theta+\phi))$. If function, $A \cdot e^{i(\omega t+\theta)}$ is depicted in a complex plane, the vector formed by imaginary and real parts rotates around the origin. $A$ is magnitude, $i$ is the imaginary unit, $i^{2}=-1$; one cycle is completed every $\frac{2 \pi}{\omega}$ seconds, and $\theta$ is the angle formed with a real axis at $t=n \frac{2 \pi}{\omega}$, for integer values of $n$ [4].


Figure 1. a) Phasor diagram of three waves in perfect destructive interference. b) Left-Right phase argument, preparation for phase transition channels in Dirac-Majorana duality modes.

This type of addition (Figure 1a) occurs when sinusoids interfere constructively or destructively. Three identical sinusoids with a specific phase difference may perfectly cancel. To illustrate, we place three equal length vectors matching up head to tail to form an equilateral triangle with a $120^{\circ}(2 \pi / 3$ radians) angle between each phasor of $1 / 3$ wavelength, $\lambda / 3$, so the phase difference between each wave is $120^{\circ}, \cos (\omega t)+\cos \left(\omega t+\frac{2 \pi}{3}\right)+\cos \left(\omega t-\frac{2 \pi}{3}\right)=0$. In the three waves example, the phase difference between $1^{\text {st }}$ and last waves is $240^{\circ}$, In the many waves limit, phasors must form a circle for destructive interference, so that the $1^{\text {st }}$ phasor is nearly parallel with the last. Thus, for many sources, destructive interference happens when the $1^{\text {st }}$ and last wave differ by $360^{\circ}$, a full wavelength, $\lambda$ [4]. For any complex number in polar form, such as $r e^{i \theta}$, the phase factor is the complex exponential factor, $e^{i \theta}$. As such, phase factor relates more generally to term phasor, which may have any magnitude (i.e., need not be part of circle group). A phase factor is a unit complex number of absolute value 1 commonly used in quantum mechanics $(\mathrm{QM})$. The variable $\theta$ is referred to as the phase. Multiplying the equation for a plane wave $A^{e i(k \cdot r-\omega t)}$ by a phase factor shifts the phase of the wave by $\theta: e^{i \theta} A e^{i(k \cdot r-\omega t)}=A e^{i(k \cdot r-\omega t+\theta)}$. In QM, a phase factor is a complex coefficient $e^{i \theta}$ that multiplies a ket $|\psi\rangle$ or bra $\langle\phi|$, not, in itself, having any physical meaning in standard QM , since introducing a phase factor does not change the expectation values of a Hermitian operator. That is, the values of $\langle\phi| A|\phi\rangle$ and $\langle\phi| e^{-i \theta} A e^{i \theta}|\phi\rangle$ are the same [4].

### 3.1. Berry Phase - Precursor to Ontological Phase

However, differences in phase factors between two interacting quantum states can be measurable under certain conditions such as in Berry phase, which has important consequences [4]. The argument for a complex number $z=x+i y$, denoted $\arg z$, is defined as:

- Geometrically, in the complex plane, as the angle $\phi$ from the positive real axis to the vector representing $z$. The numeric value given by the angle in radians is positive if measured counterclockwise (Figure 1b).
- Algebraically, the argument is defined as any real quantity, $\phi$ such that $z=r(\cos \phi+i \sin \phi)=r e^{i \phi}$ for some positive real $r$ (Euler's formula). The quantity $r$ is the modulus of $z$, as $|z|$ : $r=\sqrt{x^{2}+y^{2}}$.
Use of the terms amplitude for the modulus and phase for the argument are often used equivalently; by both definitions, the argument of any (non-zero) complex number has many possible values: firstly, as a geometrical angle, whole circle rotations do not change the point, so angles differing by an integer multiple of $2 \pi$ radians are the same. Similarly, from the periodicity of $\sin$ and $\cos$, the $2^{\text {nd }}$ definition also has this property. An N-particle system can be represented in non-relativistic QM by a wave function, $\psi\left(x_{1}, x_{2}, \ldots x_{n}\right)$, where each $x_{i}$ is a point in 3D space. A classical phase-space contains a
real-valued function in 6 N Ds (each particle contributes 3-spatial coordinates and 3-momenta. Quantum phase-space involves a complex-valued function on a 3 N dimensional space. Position and momenta are represented by non-commuting operators, and $\psi$ lives in the maths structure of a Hilbert space. Aside from these differences, the analogy holds. In physics, this addition occurs with constructively or destructively interfering sinusoids. The static vector concept provides useful insight into questions like: What phase difference is required for three identical sinusoids to perfectly cancel (again Figure 1a)? Waves are characterized by amplitude and phase, and both may vary as a function of those parameters. According to Berry [19], if parameter of the Hamiltonian of quantum system undergoes adiabatic changes, cyclically returning to original values, the wave function can acquire geometrical and dynamical phase. This additional Berry phase is $\neq 0$ when the trajectory in parameter space is near a point of degenerate states. Berry assumed the Hamiltonian is Hermitian (linear) in deviations of parameters from a point. He considered such points to be monopole-like when calculating geometrical phase. Thus, such points generate a field coinciding in monopole-like form, and the flux of Berry's field through a contour gives the geometrical phase of the system. Berry phase occurs in Aharonov-Bohm effects, where the adiabatic parameter is the magnetic field enclosed by two cyclical interference paths forming a loop and conical intersections (adiabatic parameters are molecular coordinates) of two potential energy surfaces, a set of geometrical points where the two potential energy surfaces are degenerate (intersect) and the non-adiabatic couplings between these two states are non-vanishing. Generally, geometric phase occurs whenever at least two wave parameters in the vicinity of a singularity/hole in the topology; two are required because either the set of nonsingular states will not be simply connected (shrink closed curve to point), or there will be nonzero holonomy. A Berry phase difference is acquired over the course of a cycle, when a system is subjected to cyclic adiabatic processes resulting from the geometrical properties of the parameter space of the Hamiltonian $[4,19]$. In addition to QM it can occur whenever there are at least two parameters describing a wave in the vicinity of a singularity or topological hole.
In a quantum system at the $n^{\text {th }}$ eigenstate, if adiabatic (adapts to gradually changing external conditions; but for rapidly varying conditions there is insufficient time, so the spatial probability density remains unchanged) evolution of the Hamiltonian evolves the system such that it remains in the $n^{\text {th }}$ eigenstate, while also obtaining a phase factor. The phase obtained has a contribution from the state's time evolution and another from the variation of the eigenstate with the changing Hamiltonian. The $2^{\text {nd }}$ term is Berry phase which for non-cyclical variations of the Hamiltonian can be made to vanish by a different choice of the phase associated with the eigenstates of the Hamiltonian at each point in the evolution. But if variation is cyclical, Berry phase cannot be cancelled, as it is invariant and becomes an observable property of the system. From the Schrödinger equation, the Berry phase $\gamma$ is: $\gamma[C]=i \prod_{C}\langle n, t|\left(\nabla_{R}|n, t\rangle\right) d R$, where $R$ parameterizes the cyclic adiabatic process. It follows a closed path $C$ in the appropriate parameter space. Geometric phase along the closed path $C$ can also be calculated by integrating the Berry curvature over surface enclosed by $C$ [4]. The Foucault pendulum is a simple example of geometric phase. The pendulum precess when it is taken around a general path $C$. For transport along the equator, the pendulum does not precess. But if $C$ is made up of geodesic segments, precession arises from the angles where the segments of the geodesics meet; the total precession is equal to the net deficit angle, which equals the solid angle enclosed by $C$ modulo $2 \pi$. We can approximate any loop by a sequence of geodesic segments, from which the most general result is that the net precession is equal to the enclosed solid angle. Since there are no inertial forces on the pendulum precess, precession, relative to the direction of motion along the path, is entirely due to the turning of the path. Thus, the orientation of the pendulum undergoes parallel transport [4].
Topological quantum field theories (TQFT) were created to avoid infinities in quantum field theory. In topological field theory, the concern is topological invariants, objects computed from a topological space (smooth manifold) without any metric. Topological invariance is invariance under the diffeomorphism group of the manifold. TQFT flourished through the work of Witten and Atiyah [4].

To experimentally move from SM Hilbert space to UFM ontological-phase space we must define topological switching [3-5]. We begin looking at the ambiguous Necker cube [4] where the central vertices switch ontologically (energyless) by topological charge.

## 4. Tight Bound States Below the Lowest Bohr Orbit

Recently, Tight Bound States (TBS) due to em-interactions at small distances below the lowest Bohr orbit have been postulated for the Hydrogen atom [20-21]. Summarizing this seminal work: in the usual atomic physics spin-orbit and spin-spin coupling perturbations give rise to only small corrections in classic Bohr energy levels. However, with distances in the $1 / r^{3}$ and $1 / r^{4}$ range these interaction terms, until now overlooked, can be much higher than the Coulomb term at distances $\ll$ than the Bohr radius - predicting new HD physics [20]. In a further development, Corben noticed motion of a point charge in a magnetic dipole field at rest is highly relativistic with orbits of nuclear dimensions. Investigation by [20-21] for extending the Pauli equation to a two-body system defined by the Hamiltonian, $H=\frac{1}{2 m_{1}}\left(P_{1}-e_{1} A\left(r_{1}\right)\right)^{2}+\frac{1}{2 m_{2}}\left(P_{2}-e_{2} A\left(r_{2}\right)\right)^{2}+\frac{1}{4 \pi \varepsilon_{0}} \frac{e_{1} e_{2}}{\left|r_{1}-r_{2}\right|}+V_{d d}$ with, $m_{1}$ mass, $P_{1}$ momentum, $e_{1}$ charge, $r_{1}$ position of the particles $(i=1,2), A$ is electromagnetic vector potential and $V_{d d}$, the dipole-dipole interaction term:

$$
V_{d d}=-\left(\frac{\mu_{0}}{4 \pi}\right) \mu_{1} \mu_{2} \delta\left(r_{1}-r_{2}\right)+\left(\frac{\mu_{0}}{4 \pi}\right)\left[\frac{\mu_{1} \mu_{2}}{\left|r_{1}-r_{2}\right|^{3}}-\frac{3\left[\mu_{1}\left(r_{1}-r_{2}\right)\right] \cdot\left[\mu_{2}\left(r_{1}-r_{2}\right)\right]}{\left|r_{1}-r_{2}\right|^{5}}\right] .
$$

In a center-of-mass frame with normal magnetic moment, $\mu=\frac{e}{m} s$ Hamiltonian, $H$ above is:

$$
\begin{align*}
& H=\frac{1}{2 m_{1}} p^{2}-\left(\frac{\mu_{0}}{4 \pi}\right) \frac{e_{1} e_{2}}{m_{1} m_{2}} \frac{S L}{r^{3}}+\left(\frac{\mu_{0}}{4 \pi}\right)^{2} \frac{e_{1}^{2} e_{2}^{2} \hbar^{2}}{4 m_{1} m_{2} m} \frac{1}{r^{4}}+\frac{1}{4 \pi \varepsilon_{0}} \frac{e_{1} e_{2}}{r}- \\
& \left(\frac{\mu_{0}}{4 \pi}\right) \frac{e_{1} e_{2}}{m_{1} m_{2}} s_{1} s_{2} \delta(r)+\left(\frac{\mu_{0}}{4 \pi}\right) \frac{e_{1} e_{2}}{m_{1} m_{2}}\left[\frac{s_{1} s_{2}}{r^{3}}-\frac{3\left(s_{1} r\right) \cdot\left(s_{2} r\right)}{r^{5}}\right] . \tag{3}
\end{align*}
$$

The possibility of TBS physics as derived from Hamiltonian (3) is shown in simplified form when limited to spherically symmetric terms by the radial Schrödinger equation [20]: $\frac{d^{2} X}{d r^{2}}+\frac{2 m}{\hbar^{2}}[E-V(r)] X=0$ and contains a form for the effective potential in the inverse power law: $V(r)=\frac{A}{r^{4}}+\frac{B}{r^{3}}+\frac{C}{r^{2}}+\frac{D}{r}$. At large distances this potential is an attractive Coulomb tail with a repulsive core at small distances due to the $\frac{A}{r^{4}}$ term [20]. For proper values of potential $V$ its coefficients could have another potential well in addition to the one at distances of the order of the Bohr radius (location of new physics). Additional theoretical details on the seminal development of TBS by Vigier can be found in [4-5,21]. Idealization of SM elementary particles as 0D points/charge in coordinate context with no known composite subparticles, arose because size is considered irrelevant. Paraphrasing Rowlands: fundamental physics reduces to explaining the structures and interactions of fermions. Fermions appear as singularities not extended objects, with no obvious way of creating such structures within 3D observation space. But, the Dirac equation suggests fermions require a double, rather than single, vector space, confirmed by the double rotation of spin $1 / 2$ objects, and associated zitterbewegung and Berry phase shift. The 2nd 'space' reveals that it is an 'antispace', with the same information as real space but in less accessible form. The two spaces cancel forming a norm 0 (nilpotent) object with the exact mathematical structure required for a fermionic singularity [5]. He further notes that fermions as singularities exist in a multiply-connected space requiring double rotations to return to starting position. Fermions also undergo zitterbewegung continually switching
between real space and complex vacuum space. The double circuit in real space is required because a fermion only exists in this space for half its existence. It is not coincidental that fermion algebra (gamma matrices) requires a commutative combination of two vector spaces for full representation; thus, obviously constructing a singularity requires a dual space [5,22]. The nilpotent space-antispace model extends understanding of a singularity in terms of the SM, but quaternionic algebra is not a penultimate description of nature; Rowlands' model, avant garde to the SM is not sufficiently radical to satisfy the needs of UFM [4-7]; but inspires a basis for correspondence to LSXD UFM OPTFT.

### 4.1. TBS Access Requires Violation of the Quantum Uncertainty Principle

Demonstrating new TBS spectral lines requires experimentally surmounting the quantum uncertainty principle. A new Noetic Transform will tell us whether one or two additional doublings of a Roland's type space anti-space model are required for up to five additional spectral lines which complete the radius of the MOU. Yang-Mills Kaluza-Klein equivalence provides an empirical path extending standard model particle physics. Einstein realized (1905) that Maxwell's equations obey a special relativity principle - Physical law is the same for all observers in uniform relative motion, $x^{\mu}=\left(x^{0}, x^{1}, x^{2}, x^{3}\right)=(t, x, y, z)$. Then, his General Relativity required two indices: $g_{\mu \nu}(x)$ with line element $d s^{2}=g_{\mu \nu}(x) d x^{\mu} d x^{\nu}$. In 1919, Kaluza made his attempt to combine electromagnetism and general relativity by postulating a $5^{\text {th }}$ dimension with the additional HD coordinate, $x^{M}=\left(x^{0}, x^{1}, x^{2}, x^{3}, x^{4}\right)=(t, x, y, z, \theta)$, with line element: $d \hat{s}^{2}=\hat{g}_{M N}(x) d x^{M} d x^{N}$ where he then made a $4 \mathrm{D}+1 \mathrm{D}$ split. Klein assumed the 5 th dimension had circular topology so that the coordinate, $\theta$ is periodic: $0 \leq \theta \leq 2 \pi$. The KK $5^{\text {th }}$ dimension was assumed to be curled up at the Planck scale because it was unobserved - As mentioned above this is not the only interpretation for hidden XD!


Figure 2. a) Bottom, uncertainty principle arises by a 3-space knot shadow of XD topological M-brane degrees of freedom. b) SM line element, locus of semi-classical Riemann Bloch 2-spheres, $X_{1}, X_{2}$, as
basement of reality; Top, $1^{\text {st }}$ space-antispace mirror symmetric UFM step of quaternionic vertices cycling from 5D QM chaos to topological order as faces of 3-cube.

Figure 2 shows how a localized knot shadow hides XD behind the uncertainty principle. In Figure 2b we see the beginning of separation of the standard 3 -space line element into an extended KK cyclicality from order to chaos at the semi-quantum limit. The structure has an inherent beat frequency revealed by rf-modulation of the Dirac polarized vacuum.


Figure 3. a) Oppositely charged sub-elements rotating at $v \cong c$ around center 0 behaving as dipole bumps and holes on the surface of a covariant polarized Dirac vacuum, allowing rf-modulation. b) Topological Invariance must be included in any phase labeling therein.


Figure 4. a) Fundamental diagram changing lepton number transitions by two units, generalized for Majorana modes (MJM). b) Dual MJM cyclic modes for fusion of a-b Berry phase cycles (center) in graphene. c) Reduction schemes for L\&R-handed trefoil knots.

The Fano snowflake configuration (Figure 6) involutes to form a 2D hexagon (graphene) or vertices of a Euclidean ambiguous Necker 3-cube used to explore possible topological moves for fusion of ontological-phase transitions. In the context of graphene, Berry phase is the phase an eigenstate acquires after $p$ is forced to evolve a full circle at constant energy around a Dirac vortex point. When parallel transport creates a deficit angle in brane raising and lowering dynamics, in addition to Reidemeister moves, rotations, reflections or any other topological moves, other types of phase transition with lattice charge in anyon braid fusion channels apply. Half of the leptons are neutrinos, but unknown if they are Dirac or Majorana; finding neutrinoless double $\beta$-decay would demonstrate existence of the Majorana nature of neutrinos. Neutrinoless double $\beta$-decay occurs when two neutrons in a nucleus decay simultaneously, a fundamental diagram changing lepton number by two units (Figure 4a). We begin to explore a plethora of crossover links and moves cataloging various transformations applicable to anyon fusion channels studied to supervene the inaccessibility of topological braiding, $a \times b=\sum_{c} N_{a, b}^{c} C$, where $a \& b \rightarrow c$ [23-24]. We wish to illustrate fusion-duality as a Principle, by taking the more simplistic case of de Broglie fusion, coordinates $x_{1}, y_{1}, z_{1}$ and $x_{2}, y_{2}, z_{2}$ become $X=x_{1}+x_{2} / 2, Y=y_{1}+y_{2} / 2, Z=z_{1}+z_{2} / 2$. Then for identical particles of mass, $m$ without distinguishing coordinates, the Schrödinger equation (center of mass) is

$$
\begin{equation*}
-i \hbar \frac{\partial \psi}{\partial t}=\frac{1}{2 M} \Delta \psi, \quad M=2 m \tag{4}
\end{equation*}
$$

Eq. (4) corresponds to the present, Eq. (5a) the advanced wave and (5b) the retarded wave [3].

$$
\begin{equation*}
-i \hbar \frac{\partial \varphi}{\partial t}=\frac{1}{2 M} \Delta \varphi, \quad-i \hbar \frac{\partial \phi}{\partial t}=\frac{1}{2 M} \Delta \phi \tag{5}
\end{equation*}
$$

Extending Rauscher's concept for a complex 8 -space differential line element $d S^{2}=\eta_{\mu \nu} d Z^{\mu} d Z^{* \nu}$,
where indices run 1 to $4, \eta_{\mu \nu}$ is the complex 8 -space metric, $Z^{\mu}$ the complex 8 -space variable and where $Z^{\mu}=X_{\mathrm{Re}}^{\mu}+i X_{\mathrm{Im}}^{\mu}$ and $Z^{* \nu}$ is the complex conjugate, to 12 D continuous-state UFM multiverse space-time; we write just the dimensions for simplicity for space constraints $x_{\mathrm{Re}}, y_{\mathrm{Re}}, z_{\mathrm{Re}}, t_{\mathrm{Re}}, \pm x_{\mathrm{Im}}, \pm y_{\mathrm{Im}}, \pm z_{\mathrm{Im}}, \pm t_{\mathrm{Im}}$, where $\pm$ signifies Wheeler-Feynman/Cramer type future-past/retarded-advanced dimensions. This framework allows the application of hierarchical harmonic oscillator parameters for surmounting uncertainty [3,9].


Figure 5. a) Anyon topologically protected fusion channels, b) Basic pants cobordism, c) Golem, composition of cobordisms designed for ontological-phase fusion transformations.




Figure 6. a) Antennas (snowflakes) on a Fano plane represent b) vertices on the circumference of a hexagon/cube where c) center rotates unconnected so position 1 or 2 can create the front/rear vertices of an ambiguous Necker cube. b) Antennas 1-6 combine to form the outer vertices of a cube/hexagon depending on what dimensional phase the state is in.

An important feature of TQFTs is they do not presume fixed topology for space/spacetime; in dealing with an $n$-D TQFT, one is free to choose any $(n-1)$-D manifold to represent space at a given time. Given two such manifolds, $S$ and $S^{\prime}$, one is free to choose any $n \mathrm{D}$-manifold $M$ to represent the spacetime between $S$ and $S^{\prime}$. Mathematicians call $M$ a cobordism from $S$ to $S^{\prime}$ (Figure 5b,c). We write $M: S \rightarrow S^{\prime}$, because $M$ can be the process of time from moment $S$ to moment $S^{\prime}$. For example, Figure 5b depicts a 2D manifold $M$ going from a 1D manifold $S$ (pair of circles) to a 1D manifold $S^{\prime}$ (single circle). Crudely, $M$ represents two separate spaces colliding to form a single one. Seemingly outré, but physicists are willing to speculate about processes in which the topology of space changes with time [25]. Various operations can be performed on cobordisms; we describe two:

1) Compose two cobordisms $M: S \rightarrow S^{\prime}$ and $M^{\prime}: S^{\prime} \rightarrow S^{\prime \prime}$, obtaining cobordism $M^{\prime} M: S \rightarrow S^{\prime \prime}$, Figure 5c. The idea is that the passage of time corresponding to $M$ followed by the time corresponding to $M^{\prime}$ equals the time corresponding to $M^{\prime} M$. This is analogous to the idea that waiting $t$ seconds followed by waiting $t^{\prime}$ seconds is the same as waiting $t+t^{\prime}$ seconds. The difference in TQFT is we cannot measure time in seconds, because no background metric exists to let us count the passage of time. We track topological changes. Just as ordinary addition is associative, so is the composition of cobordisms: $\left(M^{\prime \prime} M^{\prime}\right) M=M^{\prime \prime}\left(M^{\prime} M\right)$. But, cobordism composition is not commutative - the famous noncommutativity of observables in QT [25].
2) Any ( $n-1$ ) D manifold $S$ representing space, there is a cobordism $1_{S}: S \rightarrow S$ called the identity cobordism, representing passage of time without topological change. For example, if $S$ is a circle, the identity cobordism $1_{S}$ is a cylinder. In general, the identity cobordism $1_{S}$ has the property that for any cobordism $M: S^{\prime} \rightarrow S$ we have $1_{S} M=M$, while for any cobordism $M: S \rightarrow S^{\prime}$ we have $M 1_{S}=M$ [25]. These properties say that an identity cobordism is analogous to waiting 0 seconds: if you wait 0 seconds and then wait $t$ more seconds, or wait $t$ seconds and then wait 0 more seconds, this is the same as waiting $t$ seconds. These operations just formalize of the notion of the passage of time in a context where the topology of spacetime is arbitrary and there is no background metric.
Atiyah's axioms relate this notion to QT as follows:

- A TQFT must assign a Hilbert space $Z(S)$ to each $(n-1)$ D manifold $S$. Vectors in this Hilbert space represent possible states of the universe given that space is the manifold $S$.
- The TQFT must assign a linear operator $Z(M): Z(S) \rightarrow Z\left(S^{\prime}\right)$ to each $n \mathrm{D}$ cobordism $M: S \rightarrow S^{\prime}$. This operator describes state change given that the portion of space-time between $S$ and $S^{\prime}$ is the manifold $M$ : If space is initially manifold $S$ and the state of the universe is $\psi$, with time passage corresponding to $M$, the state of the universe is $Z(M) \psi$ [25].
TQFTs must satisfy a list of properties. Two are: 1) A TQFT preserves composition: given cobordisms $M: S \rightarrow S^{\prime}$ and $M^{\prime}: S^{\prime} \rightarrow S^{\prime \prime}$, we must have $Z\left(M^{\prime} M\right)=Z\left(M^{\prime}\right) Z(M)$, where the right-hand side denotes the composite of the operators $Z(M)$ and $Z\left(M^{\prime}\right) .2$ ) It must preserve identities; given any manifold $S$ representing space, we must have $Z\left(1_{S}\right)=1_{Z(S)}$, where the right-hand side denotes the identity operator on the Hilbert space $Z(S)$ [25]. These axioms are not unreasonable if one ponders them a bit. The first says that the passage of time corresponding to the cobordism $M$ followed by the passage of time corresponding to $M^{\prime}$ has the same effect on a state as a combined passage of time corresponding to $M^{\prime} M$. The second says that a passage of time in which no topology change occurs has no effect at all on the state of the universe. This seems paradoxical at first, since it seems we regularly observe things happening even in the absence of topology change. However, this paradox is easily resolved: a TQFT describes a world quite unlike ours, one without local degrees of freedom. In such a world, nothing local happens, so the state of the universe can only change when the topology of space itself changes.
Loosely speaking, they all say that a TQFT maps structures in differential topology (study of manifolds) to corresponding structures in quantum theory. Atiyah took advantage of power between differential topology and quantum theory [25]. This analogy between differential topology and QT is the clue we should pursue for a deeper understanding of quantum gravity. At first glance, GR and QT look very different mathematically: one deals with space and spacetime, the other with Hilbert spaces and operators, not easy to combine; but TQFT suggests they are not so different. Quantum topology is technical, but it is obvious that differential topology and QT must merge to understand a backgroundfree QFT. Physics ignoring GR, treats space as a background for displaying world states. Similarly, spacetime is treated as a background for the process of change; these idealizations must be overcome in a background-free theory. In fact, concepts of space and state are two aspects of a unified whole; likewise, the concepts of spacetime and process [25]. In an alternative derivation of string tension, $T_{S}$ we met this challenge finding a unique background independent M-Theory [3], that after another decade led to OPTFT as the putative $3^{\text {rd }}$ regime integrating unquantized GR and UFM [4].
A photon, 2-component, 2D traveling plane wave projecting at right angles to the direction of propagation has a particulate radius not able to pass a slit $>\lambda$. We propose that behind the inherent backcloth of cyclic bumps and holes in the polarized Dirac vacuum (Figure 2a) [4], the uncertainty principle is hiding the XD topology of the MOU, which is not singular as in the SM because cyclic boost-compactification occurs continuously from asymptotic virtual $\hbar$ (shadow of uncertainty, Figure 2), to the Larmor radius of the hydrogen atom, making correspondence to dynamical Type-II M-
theoretic Calabi-Yau florets (multiply-connected Kahler manifold) undergoing translation, rotation, reflection as part of the process. Spectral lines characterize atoms by, $E=\hbar v=\hbar c / \lambda$ or wave number, $\sigma \equiv 1 / \lambda=E / \hbar c$ by discrete wavelengths confirmed by monochromatic $x$-ray bombardment. Excited states, $E_{2}$ decay to lower states, $E_{1}$ by emission of photon energy, $E_{1}-E_{2}$ frequency, $v$, wavelength, $\lambda$ and wave number, $E_{2}-E_{1}=\hbar v=\hbar c / \lambda=\hbar c \sigma$. By conditions hinted at in Figure 2 we propose new hyperspherical spectral lines below the lowest (ground state) Bohr orbit. Kowalski's interpretation from laser experiments [26] shows that emission and absorption between Bohr states takes place within a time interval equal to one period of the emitted-absorbed photon wave, the corresponding transition time is the time needed for the orbiting electron to travel one full orbit around the nucleus. Note that the same Lorentz conditions denoted in our tachyon measurement experiment apply directly to the TBS experiment with slight phase control alterations in the Cramer-like standingwave oscillation of the HD Calabi-Yau mirror symmetries [6]. Standard Hypervolume values for increasing $n$-dimensionality and radius, $r$ of a unit sphere or $n$-ball equal to 1 can be used to initially predict two TBS spectral lines hidden within the 6D Calabi-Yau dual 3-torus, the putative wavelengths can be calculated from the general hyperspherical $n$-volume equation, $\frac{1}{2 \pi^{2}}, 4,9346$ units for 4D, and $\frac{8}{15 \pi^{2}}, 5,2638$ units for 5D. If Randall-Sundrum are correct, the 6 D cavity will be degenerate, and the signal escape to infinity. We postulate a Manifold of Uncertainty (MOU) with finite XDradius corresponding to string theories T-Duality [3-5]. For preliminarily predictions we could calculate hyperspherical volume or surface area of 2D-5D MOU. The general $n$-volume equation is $V(n, r)=\pi \frac{n}{2} r^{n} / \Gamma\left(\frac{n}{2}+1\right)$, where $V_{n, r}$ is volume per number of dimensions, $n$ of radius $r$ and $\Gamma$ a factorial constant. We relate these $n$-volume equations to volumetric properties of the MOU for calculating an HD C-QED volume hierarchy for predicting new Tight-Bound State (TBS) spectral lines in hydrogen [4,21]. If LSXD exist, degeneracy would occur at the limit of $r$ discovered in the same manner the outermost energy level of an atom is detected when an outer electron acquires sufficient energy to escape to infinity.


## 5. Experimental Design - Rf-Resonance Parameters on the Dirac Polarized Vacuum

Modulated harmonic Sagnac Effect rf-pulses oscillate electrons to couple with nucleons. Additional spin-spin pulse modulation couples to a putative de Broglie-Bohm-Cramer Kaluza-Klein cyclical beatfrequency, an inherent aspect of UFM semi-quantum limit space-time. With Dubois incursive oscillator parameters added, a QED cavity opens into XD/LSXD topology. These conditions in conjunction with a Bessel function discover additional TBS spectral lines.


Figure 7. a) Generalized schema for a TBS expriment. b) Simplistic illustrative functions for operting a TBS incursive harmonic oscillator. c) Rf-pulsed Sagnac Effect interference cycles.

From a common general example of a Bessel function, with $\alpha$ an arbitrary complex number:
$x^{2} \frac{d^{2} y}{d x^{2}}+x \frac{d y}{d x}+\left(x^{2}-\alpha^{2}\right) y=0$, we solve the Helmholtz equation in spherical coordinates by variable separation such that the radial equation takes the form

$$
x^{2} \frac{d^{2} y}{d x^{2}}+2 x \frac{d y}{d x}+\left[x^{2}-n(n+1)\right] y=0
$$

The spherical Bessel functions, $j_{n}, y_{n}$ are the two linearly independent solutions that relate to the ordinary Bessel functions, $J_{n}, Y_{n}$ as:

$$
\begin{equation*}
j_{n}(x)=\sqrt{\frac{\pi}{2 x}} J_{n+\frac{1}{2}}(x), \quad y_{n}(x)=\sqrt{\frac{\pi}{2 x}} Y_{n+\frac{1}{2}}(x)=(-1)^{n+1} \sqrt{\frac{\pi}{2 x}} J_{-n-\frac{1}{2}}(x) \tag{6}
\end{equation*}
$$

When written as Rayleigh's formulas

$$
\begin{equation*}
j_{n}(x)=(-x)^{n}\left(\frac{1}{x} \frac{d}{d x}\right)^{n} \frac{\sin (x)}{x}, y_{n}(x)=(-x)^{n}\left(\frac{1}{x} \frac{d}{d x}\right)^{n} \frac{\cos (x)}{x} \tag{7}
\end{equation*}
$$

the first spherical Bessel functions are

$$
\begin{equation*}
j_{0}(x)=\frac{\sin (x)}{x}, j_{1}(x)=\frac{\sin (x)}{x^{2}}-\frac{\cos (x)}{x}, j_{2}(x)=\left(\frac{3}{x^{2}}-1\right) \frac{\sin (x)}{x}-\frac{3 \cos (x)}{x^{2}} \tag{27}
\end{equation*}
$$

and

$$
\begin{align*}
& y_{0}(x)=-j_{-1}(x)=-\frac{\cos (x)}{x}, y_{1}(x)=-j_{-2}(x)=-\frac{\cos (x)}{x^{2}}-\frac{\sin (x)}{x}, \\
& y_{2}(x)=-j_{-3}(x)=\left(-\frac{3}{x^{2}}+1\right) \frac{\cos (x)}{x}-\frac{3 \sin (x)}{x^{2}} . \tag{27}
\end{align*}
$$

Figure 8. Bessel Function example necessary to synchronize coupling with the Dubois incursive oscillator in order access additional TBS beyond the first 4D QED cavity to 5 and 6D MOU cavities.

We think we know how, but surmounting uncertainty will not be trivial.
We show the $1^{\text {st }}$ three Bessel function solutions of the $1^{\text {st }}$ and $2^{\text {nd }}$ kind to illustrate our incursive oscillator process to locate the first TBS spectral line in hydrogen, expected to be relatively easy to find in comparison with finding the $2^{\text {nd }}$ and $3^{\text {rd }}$. The additional lines will be more challenging as there may be some unexpected complexity in the Bessel harmonic oscillator that must be overcome to completely surmount uncertainty. Restrictions related to this refinement have not revealed themselves to us as yet. They will require additional adjustment to the spin-spin coupling parameters of the algebra describing the HD hyperspherical volume. Parallel transport of the gravitational curvature deficit angle will kick in for the XD mirror symmetric brane topology, with noeon topological charge corrections (not quantized gravity) [3-5,7].
Completing the Noetic Transform (beyond Lorentz-Poincaré), clarifies the choice of linear combinations of Bessel solutions (6), which depends on their asymptotic behavior at $\infty$,

$$
\begin{equation*}
J_{n}(x) \cong \sqrt{\frac{\pi}{2 x}} \cos \left(x-\frac{\pi}{2} n-\frac{\pi}{4}\right) ; Y_{n}(x) \cong \sqrt{\frac{\pi}{2 x}} \sin \left(x-\frac{\pi}{2} n-\frac{\pi}{4}\right) \tag{10}
\end{equation*}
$$

thus

$$
\begin{equation*}
H_{n}^{ \pm}(x) \cong \sqrt{\frac{\pi}{2 x}} \exp \left[ \pm i\left(x-\frac{\pi}{2} n-\frac{\pi}{4}\right)\right] \tag{28}
\end{equation*}
$$

With the proper harmonic oscillator Bessel function solutions, the next step in the experimental design is to apply the Dubois incursive oscillator parameters as the final step in designing the synchrony of Sagnac effect rf-pulses. The incursive algorithms are numerically stable and the numerical simulation of the oscillator will show the conservation of the energy. Let us consider the harmonic pendulum as an illustrative example, with $m$ the oscillating mass and $k$ the spring constant, represented by the ordinary differential equations:

$$
\begin{equation*}
d x(t) / d t=v(t), \quad d v(t) / d t=-\omega^{2} x(t) \operatorname{dv}(\mathrm{t}) / \mathrm{dt} \tag{12}
\end{equation*}
$$

where $x(t)$ is the position and $v(t)$ the velocity as functions of the time $t$, and where the pulsation $\omega$ is related to $k$ and $m$ by $\omega^{2}=k / m$ [28]. The solution is given by

$$
\begin{equation*}
x(t)=x(0) \cos (\omega t)+[v(0) / \omega] \sin (\omega t), \quad v(t)=-\omega x(0) \sin (\omega t)+v(0) \cos (\omega t) \tag{13a,b}
\end{equation*}
$$

with the initial conditions $x(0)$ and $v(0)$. In the phase space, given by $(x(t), v(t))$, the solutions are given by closed curves (orbital stability). The period of oscillations is given by $T=2 \pi / \omega$. The energy $e(t)$ of the harmonic oscillator is constant and is given by

$$
\begin{equation*}
e(t)=k x^{2}(t) / 2+m v^{2}(t) / 2=k x^{2}(0) / 2+m v^{2}(0) / 2=e(0)=e_{0} \tag{14}
\end{equation*}
$$

The simulation of differential equations is impossible. This is only the discrete transformation which is computable with a recursive function. In differential equations, there is only the current time. In discrete systems, there is the current time $t$ and the interval of time $\Delta t=h$. The discrete time is defined as: $t_{k}=t_{0}+k h$ with $k=0,1,2, \ldots$ where $t_{0}$ is the initial value of the time and $k$ is the counter of the number for the interval of time $h$ [29]. The discrete variables are defined as $x_{k}=x\left(t_{k}\right)$ and $y_{k}=y\left(t_{k}\right)$. The discrete equations used in the harmonic oscillator case for computing the position and the velocity at consecutive moments have the general form

$$
\begin{equation*}
x_{k+1}=A x_{k}+B v_{k}, \quad v_{k+1}=C v_{k}-D \omega^{2} x_{k} \tag{15a,b}
\end{equation*}
$$

where $A, B, C$ and $D$ are coefficients with values specific to the numerical integration methods applied. In eliminating $v_{k}$ of Eq. (15a) in Eq. (15b), a second order discrete equation in $x_{k}$ is given by $x_{k+2}-(A+C) x_{k+1}+\left(A C+B D \omega^{2}\right) x_{k}=0$. The stability analysis for this discrete system can be performed by using the Z-transform, $z^{2}-(A+C) z+\left(A C+B D \omega^{2}\right)=0$ which presents two poles:

$$
\begin{equation*}
z_{1,2}=\left((A+C) \pm i \sqrt{-(A+C)^{2}+4\left(A C+B D \omega^{2}\right)}\right) / 2 \tag{16}
\end{equation*}
$$

that are complex when

$$
\begin{equation*}
(A+C)^{2}<4\left(A C+B D \omega^{2}\right) \tag{29}
\end{equation*}
$$

The position of the poles relative to the unit circle defines the system stability: A system is stable if the poles lie inside the unit circle, is unstable if the poles lie outside the unit circle and shows an orbital stability if the poles lie on the unit circle. The condition for stability is:

$$
\begin{equation*}
\left((A+C)^{2}-(A+C)^{2}+4\left(A C+B D \omega^{2}\right)\right) / 4 \leq 1 \tag{17b}
\end{equation*}
$$

or $A C+B D \omega^{2} \leq 1$ and the orbital stability must satisfy the strict equality

$$
\begin{equation*}
A C+B D \omega^{2}=1 \tag{17c}
\end{equation*}
$$

so, for the harmonic oscillator, the conditions for obtaining an orbital stability are given by relations (17a) and (17c), rewritten as

$$
\begin{equation*}
(A+C)^{2}<4 \text { and } A C+B D \omega^{2}=1 \tag{18a,b}
\end{equation*}
$$

in using the equality from the relation (17c), in the relation (18a) [29].
Considering $1^{\text {st }}$ the well-known Euler and Runge-Kutta integration methods, e.g. Scheid [30]; and then incursive methods will be analysed. In terms of incursive discrete algorithms, Dubois defines a generalized forward-backward discrete derivative, $D(w)=w D f+(1-w) D b$, where $w$ is a weight taking the values between 0 and 1 , and where the discrete forward and backward derivatives on a function $f$ are defined by $D_{f}(f)=\Delta^{+} f / \Delta t=\left[f_{k+1}-f_{k}\right] / h$ and $D_{b}(f)=\Delta^{-} f / \Delta t=\left[f_{k}-f_{k-1}\right] / h$. Then, a generalized incursive discrete harmonic oscillator is:

$$
\begin{equation*}
(1-w) x_{k+1}+(2 w-1) x_{k}-w x_{k-1}=h v_{k}, w v_{k+1}+(1-2 w) v_{k}+(w-1) v_{k-1}=-h \omega^{2} x_{k} . \tag{19a,b}
\end{equation*}
$$

When $w=0, D(0)=D_{b}$, this gives the first incursive equations:

$$
\begin{equation*}
x_{k+1}-x_{k}=h v_{k}, \quad v_{k}-v_{k-1}=-h \omega^{2} x_{k} \tag{20a,b}
\end{equation*}
$$

When $w=1, D(1)=D_{f}$, this gives the second incursive equations:

$$
\begin{equation*}
x_{k}-x_{k-1}=h v_{k}, \quad v_{k+1}-v_{k}=-h \omega^{2} x_{k} \tag{21a,b}
\end{equation*}
$$

When $w=\frac{1}{2}, D\left(\frac{1}{2}\right)=\frac{\left[D_{f}+D_{b}\right]}{2}$, this gives the averaged (hyperincursive) equations:

$$
\begin{equation*}
x_{k+1}-x_{k-1}=2 h v_{k}, \quad v_{k+1}-v_{k-1}=-2 h \omega^{2} x_{k} . \tag{22a,b}
\end{equation*}
$$

These Eqs. (22a,b) integrate the two incursive equations. This deals with a deduction of this forwardbackwards discrete derivative, with the deduction of this time-symmetric discretization of the harmonic oscillator [30].
Next, simulation of the incursive and hyperincursive algorithms of the classical harmonic oscillator: First, for the simulation of the classical harmonic oscillator, the dimensionless variables $X, V$ and $H$, will be used [30], for the variables, $x, v$ and $h: X(k)=\sqrt{[k / 2]} x_{k}, V(k)=\sqrt{\frac{m}{2}} v_{k} \quad \tau=\omega t$ with $\omega=\sqrt{\frac{k}{m}}$ and $\Delta \tau=\omega \Delta t=\omega h=H$. So, the two incursive dimensionless harmonic oscillators are given by

$$
\begin{gather*}
X_{1}(k+1)=X_{1}(k)+H V_{1}(k), \quad V_{1}(k+1)=V_{1}(k)-H X_{1}(k+1)  \tag{23a,b}\\
V_{2}(k+1)=V_{2}(k)-H X_{2}(k), \quad X_{2}(k+1)=X_{2}(k)+H V_{2}(k+1) \tag{24a,b}
\end{gather*}
$$

and the hyperincursive dimensionless harmonic oscillator is given by

$$
\begin{equation*}
X(k+1)=X(k-1)+2 H V(k), \quad V(k+1)=V(k-1)-2 H X(k) . \tag{25a,b}
\end{equation*}
$$

So, this confirms that the incursive and hyperincursive algorithms are totally numerically stable with the conservation of energy [31]. When the parameters for the experiment are coordinated and the rfpulse sent into the MOU HD QED TBS hydrogen cavity, a positive result will retrieve a spectroscopic signal like the one represented in Figure 9. A negative result would send back 0 amplitude (no spectral line) $[4,21]$.

### 5.1. Experimental UFM OPTFT Protocols

Tight-Bound States (TBS): New spectral lines in hydrogen below lowest Bohr orbit. Until now spectral lines are measured in 3-Space. UFM predicts 2 to 5 new TBS Spectral Lines depending on the hyperspherical duality of the Manifold of Uncertainty (MOU) in 4D, 5D \& possibly 6D.


| $V(n, 1)$ |  |  |
| :--- | :--- | :---: |
| 0 | 0 | 0 |
| 1 | 2 | 2 |
| 2 | $\pi$ | $\approx 3.1416$ |
| 3 | $4 / 3 \pi$ | 4.1888 |
| 4 | $1 / 2 \pi^{2}$ | 4.9348 |
| 5 | $8 / 15 \pi^{2}$ | 5.2638 |
| 6 | Degenerate ? | $\infty$ |

Figure 9. a) Illustrating First 4D TBS spectral line in hydrogen emerging (Emitted) from the 4D spherical potential well in the Manifold of Uncertainty for $\alpha= \pm 1$ (adapted from [29]).
b) Standard Hypervolume values for increasing $n$-dimensionality and radius, $r$ of a unit sphere or $n$ ball equal to 1 . The 4D and 5D volumes can predict new TBS spectral lines.

The $1^{\text {st }}$ Bohr orbit in hydrogen is at $.5 \AA \&$ the $2^{\text {nd }}$ at $2 \AA$. We predict two new spectral lines between $.5 \AA \& 2 \AA$; with a $3^{\text {rd }}$ line likely to be degenerate, with the test signal escaping to infinity, or so-called LSXD Bulk. Experimental success is the gateway to $3^{\text {rd }}$ regime natural science would demonstrate MTheory and cause CERN type accelerators to become obsolete. Note: The protocol is low energy and tabletop by completely bypassing uncertainty $[4,21]$.

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