Structure, Properties and Implications of Complex Minkowski Spaces

Richard L. Amoroso[1] & Elizabeth A. Rauscher
[1]amoroso@noeticadvancedstudies.us

We consider the properties and implications of three $n > 4$ multidimensional geometries. These are Descartes geometry [1], the properties and implications of which are enumerated in [2-6]. Both macroscopic and microscopic implications of these geometries are presented. We also develop several forms of complex Minkowski space in terms of a generalized metric containing terms derived from real and imaginary coordinates. The metric of the space is real and therefore physical [7-17]. This geometry is found to be one-to-one with Kaluza-Klein geometry [18-20] in which there has been much recent interest in developing M-Theory, in particular in the apparent relationship between the gravitational and electromagnetic fields often called Quantum Gravity. We have discussed the properties and implications of complex geometries in a number of works. The basic structure of the geometries is based on the construction of complexified dimensions, consisting of orthogonal real and imaginary parts. We examine the implication of a complex 8-space geometry in which we introduce imaginary components for each real spatial dimension, $X = (x,y,z)$ and temporal dimension, t.

1. Some Predictions of Complex Geometries

The complexification of Minkowski space, M_4 yields an 8D geometry, $M_4 \subset \mathbb{C}_4$. The 8D space is the least number of dimensions to accommodate nonlocality and anticipatory incursion in complex symmetry. This 8-space is also Lorentz invariant. Additional dimensional spaces (XD) are also considered, such as 12D spaces that also yield an approach to a unification of macro and micro processes [7,11,18].

We have solved the Schrödinger equation and Dirac equation in this complex 8D Minkowski space [16,17] and also formulated a field theoretical model that has implications for both MHD and BCS type phenomena [19]. For additional symmetry considerations, we have also introduced a 12D space in which we consider a 3-component time which is complexified [11,18]. Let us briefly list some of the implications of $n > 4$ geometries.

- Remote connectedness properties exist between physical events and processes in spacetime.
- Anticipatory or precognitive-like processes are allowed in temporal processes.
- Superluminal 'signals' appear to exist in 4-space.
- Tachyonic 'particles' are predicted.
- Coherent nondispersive phenomena exist, such as plasma oscillations of individual particle states like 'ball lightning' or solar activity, possibly 'sun spots' [20,21].
- A mechanism for physical effects such as conductivity and dielectric properties of plasmas in 4-space based on vacuum polarization properties in complex space [19,20].
- A model for unification of electromagnetic and gravitational phenomena [12,18] through the one-to-one mapping of the spinor calculus and twister algebra of the complex space [22].
- A mechanism of formulating the so-called 'collapse of the wave function' in terms of the geometric structure of space and interpretation of the 'observer effect' [23].
- Possible interpretation of nonlinear effects in multidimensional geometries as an interpretation of

the mechanism of the collapse of the wave function to a particular state. One possible interpretation of such a model is that 'consciousness' generates 'geometric reality' (or constraints on reality) which relate to a particular 'potentia' (Heisenberg's term or de Broglie pilot wave) being actualized as a specific event, which is a possible mechanism of the physical effects or a manifestation of thought [14,22]. Such an interpretation is key to a model which may include *psi* phenomenon, since it now demonstrates a mechanism in which intention (goal) can be physically actualized [14].

- Application may be made to antenna theory [21].
- Formulation of certain processes in biological tissue is examined [24-27].

We list some examples of remote connectedness:

- Bell's theorem [28],
- Young's double slit experiment,
- Aharanov-Bohm experiment [29]
- Supercoherence phenomena, such as plasma coherent states and superfluidity [19,20],
- Remote perception [30].

Some examples of coherent 'non-dispersive' phenomena in which dispersion is overcome by recoherence are:

- Soliton-like phenomena of plasma-phonon-electron interactions [20].
- Solving the Dirac equation in complex 8-space and the Fermi-Dirac vacuum state model
- Complexifying $F_{\mu\nu}$ in 8-space, Non-Hertzian and Hertzian waves [12].
- Ball lightning is a coherent electrostatic soliton-like phenomenon. These phenomena are modeled after Prigogine [31] dissipative structures and catastrophe phenomena [32].
- Vortices in helium II represent soliton-like structures [22].

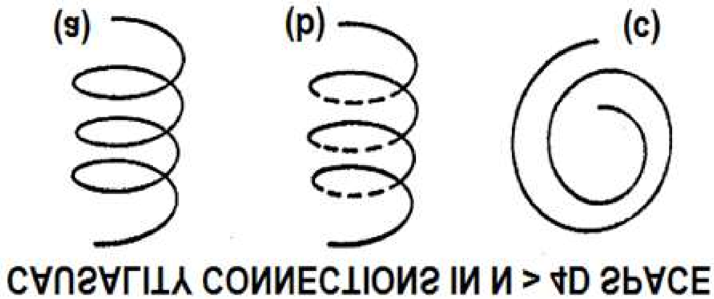

Figure 1. Causality in HD space. Representing a lightcone schematic. 1a represents closed time-like loops in 4-space where the vertical dimension is time, t and the horizontal dimension is $X = x,y,z$. 1b represents a non multiply connected world line in n > 4D where no CTL or multi-connected 'nows' exist. Figure 1c represents the view of Fig. 1b where the future time dimension comes out of the paper, X is in the plane of the paper and no multi-connected world lines exist and only single valued 'nows' exist.

Certain spacetime relationships that involve Closed Time-like Loops (CTL) paradoxes can be resolved utilizing formulations in terms of multidimensional geometries (Fig. 1). The issues involved are presented and extensively discussed in [3]. In the so-called twin paradox, only future time travel is possible in non-inertial frames because time dilation only occurs in the rapidly accelerating frame (Fig. 2).

For time machines of the relativistic Twin Paradox, time moves into the future. At each point along a world line in spacetime there are a number of potential states in which one is actualized with preferential probability (or equal as in the Schrödinger cat paradox). Time machines that move into the past from the future represent CTLs (Fig.3). Figure 4 represents various spacetime connections in a subset of 3D spacetime of the *X,ict* Minkowski 4D spacetime. Figure 5 represents various world line connections on the Minkowski light cone including an unconnected past and future, a single valued world line and a multivalued CTL world line. See Fig. 5.

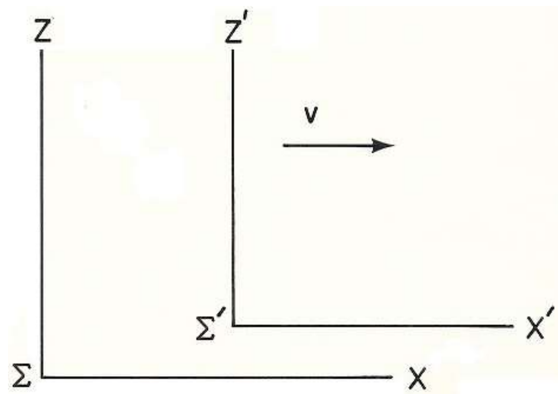

Figure. 2. The relative velocities of two frames of reference, Σ, taken as the rest frame and, Σ' as the moving frame, having a relative velocity, *v*. The *x,z* plane is represented as the abscissa and ordinate respectively in 2-space. Not represented is the *y* coordinate extending out of the plane in this representation of two relative 3-space Euclidian coordinates (*x,y,z*) for Σ and (*x',y'z'*) for Σ'.

In order to describe processes involving apparent future-to-past lightcone connections, one has paradoxes involving CTL, multi-valued 'nows' and 'accelerated times' which involve the paradox of moving more slowly than a rest frame! These paradoxes cannot be resolved in the usual Minkowski 4-space metric. In n > 4D spaces we have the possibility of the resolution of these paradoxes and the possibility of a more definitive formalism and description of some of the previously listed phenomena (Fig. 3).

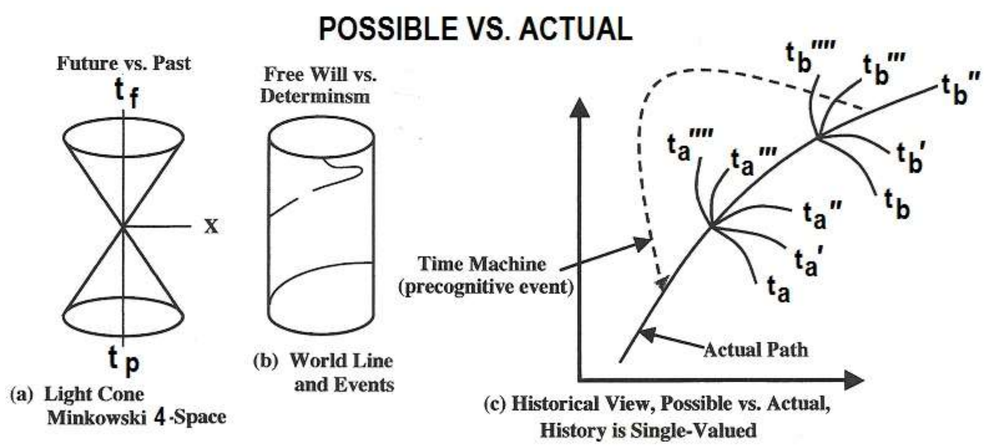

Figure 3. Possible versus actual. Several types of world lines are depicted. Figure 3a depicts a worldline with a single-valued "now", but Figs. 3b and 3c depict a multi-valued present. There is a dual world: constancy and change, absolute versus relativistic and Mach's Principle, and certainty versus uncertainty in terms of Einstein and Bohr $(\Delta x, \Delta p \geq \hbar)$.

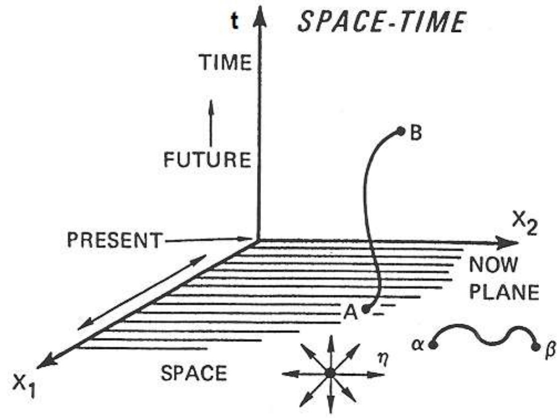

Figure 4. Our location is measured in space and time. In this figure we represent two dimensions of *x,y,z* as x_1 and x_2 and of time, *t* on the ordinate. Causal connections in real space are such that A can cause B by passage of time; at one time, α and β are correlated as effects at a distance. η represents instantaneous connections.

We have examined several forms of complex geometries. The complexification of Minkowski 4-space, M^4, gives rise to an 8D complex Minkowski space, $\hat{M}^4 \subset \mathbb{C}^4$ in which we take each of the 8D as an independent orthogonal dimension and that the real and imaginary components can be considered as two independent 4-space lightcones, $(X_{\mathrm{Re}}, t_{\mathrm{Re}})$ and $(X_{\mathrm{Im}}, t_{\mathrm{Im}})$; 4-space is then a slice through 8-space, rather than a subset or subspace formed by a projected geometry distorting the projection, causing variation in the defined variable length and vector orientation whereas orthogonal slices maintain uniformity (Fig. 4) [33,34].

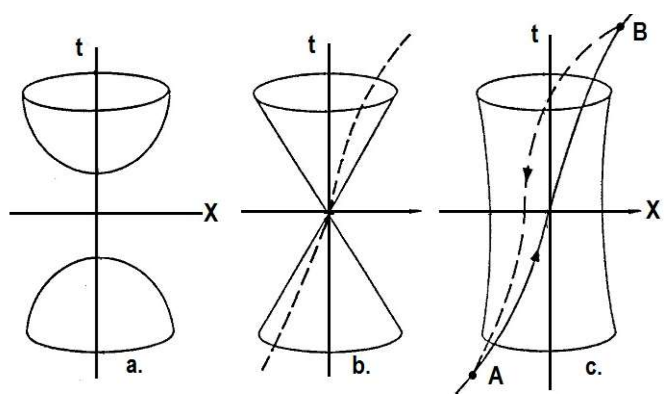

Figure 5. One can consider three classes of Minkowski diagrams with three types of causal connections of events along world lines. In Fig. 5a no connection exists between past and future. In Fig. 5b only one connection between past and future exists for a single valued now for the usual lightcone world line connection; and in Fig. 5c more than one connection of past and future exists as CTL. For example, one path to the future and another from the future hook into its point "B" past to point A.

We have also examined other forms of the complex geometric model [11,12]. For symmetry considerations, we consider an extension of the temporal variable as the pseudoscalar $\underset{\sim}{t} = t_x \hat{x} + t_y \hat{y} + t_z \hat{z}.$ Then we complexify each of these three temporal dimensions as $t_x = t_{x\mathrm{Re}} + i t_{x\mathrm{Im}}$, $t_y = t_{y\mathrm{Re}} + i t_{y\mathrm{Im}}$, $t_z = t_{z\mathrm{Re}} + i t_{z\mathrm{Im}}$. We have handled the complexification of spatial dimensions similarly in [7-18]. We now have a complex form of Minkowski space which is a 12D space with the

12D listed respectively as: $x_{\mathrm{Re}}, y_{\mathrm{Re}}, z_{\mathrm{Re}}, x_{\mathrm{Im}}, y_{\mathrm{Im}}, z_{\mathrm{Im}}$, $t_{x\mathrm{Re}}, t_{y\mathrm{Re}}, t_{z\mathrm{Re}}, t_{x\mathrm{Im}}, t_{y\mathrm{Im}}, t_{z\mathrm{Im}}$.

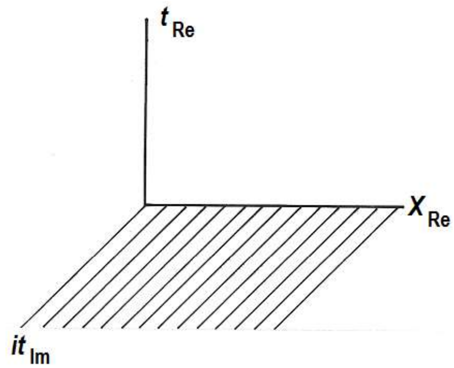

Figure. 6. The figure provides a relatively simple representation of real and imaginary components for a x_{Re}, t_{Re}, t_{Im} space. We extend this picture to a real and imaginary vertical time plane. t_{Re}, t_{Im} and a real and imaginary space plane of x_{Re}, x_{Im} for a 5D complex plane. See Fig. 7.

In considering certain classes of transformations we examine the manner in which moving and rest frames, ($\underline{A}, \phi$) form and transform. We consider the fact that both these geometries appear to demand remote connectedness and superluminal signal propagation in the subset real 4-space, M_4. In Fig. 6 we represent various of events in real space and time. In Fig. 3a is represented the usual Minkowski spacetime metric, which is like a 4D Pythagorean theorem for a right triangle where $h^2 = a^2 + b^2$, the sum of the square of the hypotenuse is equal to the sum of the other two sides.

2. Multidimensional Geometric Models and Macroscopic Remote Connectedness

It appears that a resolution of the problem of closed time-like loops (CTL) lies in developing a model in terms of a space of higher dimensionality, HD. What appears to be a closed loop in 4D spacetime may in fact *not* have an intersection in an HD space [8,11,18]. See Fig. 3. Normal macroscopic causality demands that no point in the forward lightcone is connected to another point outside the forward lightcone; that is, all signals are time-like [8,21]. Real events involve simultaneity which is defined by signals that do not exceed the velocity of light, $v \le c$ where v is the velocity of propagation and c is the velocity of light. Causality conditions for superluminal signals in constructing a Lorentz invariant quantum field theory are given in [7,8,12]. Tipler examines the problem of CTLs in general relativity for a rapidly rotating gravitational field [35]. The relationship of causality and locality conditions is discussed in [8].

- First, the case in which there is no connection of past and future is represented, i.e., there is no causal connection.

- Second, the usual Minkowski diagram for a single valued present. In quantum mechanical terms, the collapse of the wave function describing the system under consideration allows only one world line.

- Third, the present or ‘now’ condition is not single valued. The event wave function no longer collapses to a point, localized region of spacetime, and more than one world line can represent the present.

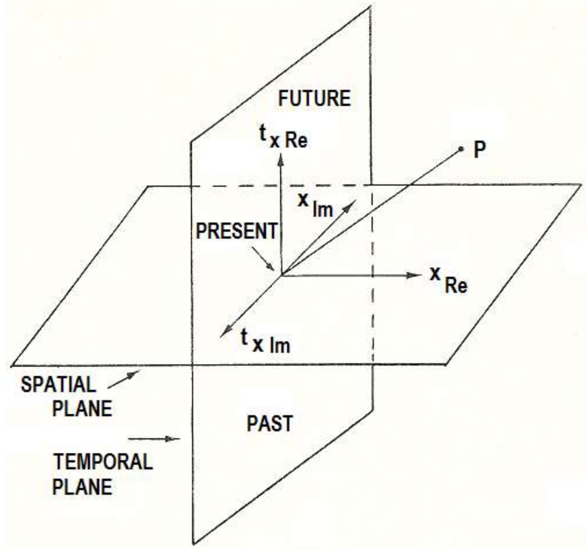

Figure 7. Spacetime of two intersecting complex planes in 8D described by the x components of space and time. Here $x = x_{\mathrm{Re}} + ix_{\mathrm{Im}}$ and time, t is $t = t_{x\mathrm{Re}} + it_{x\mathrm{Im}}$. Event P in this complex plane will be represented as: P($x + itx$). This figure can be extended to 3D of space and time where P will be represented as: P($x + itx, y + ity, z + itz$).

In fact, for point-like events, one could conceivably have an infinite number of world lines passing through the present. Everett, Graham and Wheeler have examined the quantum mechanical implications of a multi-valued universe theory [36]. More information about a future event may then be traced back to the present via another world line and that actual time sequencing experienced is associated with the first world line or possibly a third world line. See Fig. 3.

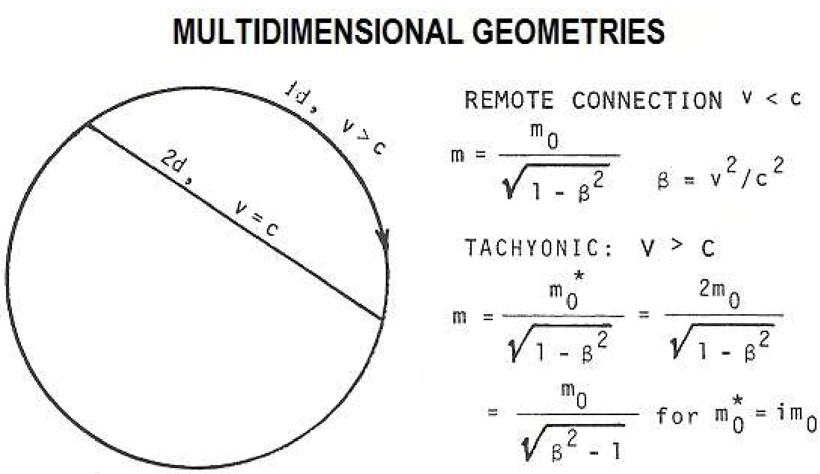

Figure 8. Represents a 1D circumference and a chord across the circle in a 2D space. In a lower dimensional space the velocity of propagation appears superluminal, $v > c$ and luminal in the hyperdimensional space, $v \leq c$. In analogy the velocity of propagation for Bell's Theorem in 4-space is $v > c$ or instantaneous, but in complex 8-space signaling can be luminal, $v = c$.

Of course, one of the major problems of a theory containing multivalue solutions is the difficulty in defining a reasonable and useful causal relationship. The 4-space description gives us CTL which yield difficulties in describing prior and post event occurrences [27], Fig. 1. Intuitively, considering HD geometric models appears to reconcile the problem of CTL. For example, a helical world line in a 3-space would be single valued but would appear to contain multiple intersections if viewed at a 45° angle to the vertical helical axis as represented in a 2D space. This representation would contain multiple intersections even with a large pitch of the angle to the perpendicular to axis radius and hence act like a CTL [29]. See Fig. 3.

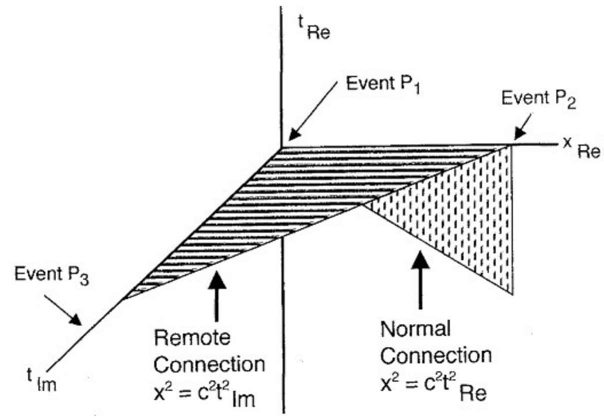

Figure 9. Complex time model of remote connectedness. We have the usual physical spatial separation of events on the x axis in the x_{Re}, t_{Re} plane which appears separated by a zero separation by 'moving" to the x_{Im}, t_{Im} plane. The separation between event P_1 and P_2 appears contiguous or simultaneously nonlocally correlated from the perspective of P_3. In an $n > 4$ space or an 8D space, nonlocal events can be correlated in such a manner as to not require standard signal propagation.

A number of HD geometries have been examined, in terms of reconciling complex anticipation and precognition-like signaling and causality as well as their possible relationship to superluminal signals [8,37-39]. In particular we have examined some 5 and 6D geometries where the additional dimensions, XD are space-like and time-like. In [8], instead of hypothesizing a model which involves energy transmission and associated problems of energy conservation, we chose to develop a model in which remote information is accessed in 4-space as though it was not remote in a HD geometry.

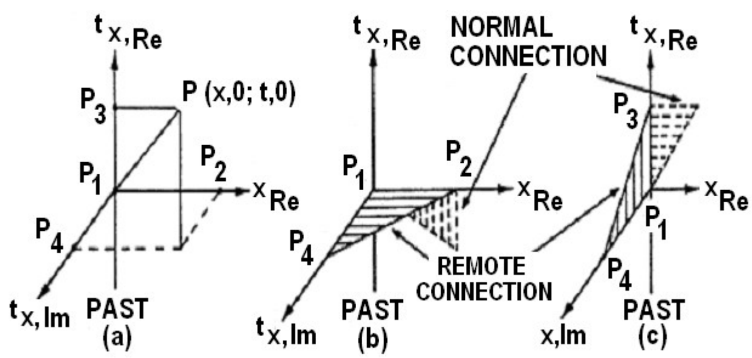

Figure 10. Four events in a complex plane. P_1 is at the origin. Event P is marked by non-zero spatial and temporal separation from the origin. P_1 and P_2 are separated in space but synchronous in time. P_1 and P_3 are separated in time, but there is no spatial separation. Event P_4 is located on the imaginary time axis; (b) Remote and normal connections of events P_1 and P_2 as viewed by an observer at P_4 such that space-like separation, $x(P_2) - x(P_1)$, between the events P_1 and P_2 is zero; (c) Remote and normal connections for zero time-like separation between the events P_3 and P_1 as viewed by an observer at P_4, such that, $t(P_3) - t(P_1) = 0$.

Relativity theory formally describes the relationship of macroscopic events in spacetime and, in particular, their causal connection is well specified. HD geometries appear to reconcile anticipation or precognition and causality and define a formalism in which the spatial and temporal separation of events in 4-space appear to be in juxtaposition in the HD geometry. This model can well accommodate information and perhaps energy transmission conditions as we will discuss in more detail in this volume. See Fig. 9 which represents a subset of the geometry we use in the present approach. There appears to be a reasonable relationship between these complex spaces and real 4, 5 and 6D spaces. The generalized causal relations in the complex space are consistent with the usual causality conditions, and exclude the CTL paradox. Multidimensional models appear to reconcile Maxwell's equations with the structure of general relativity in the weak gravitational field limit having some quantum mechanical features such as quantum nonlocality.

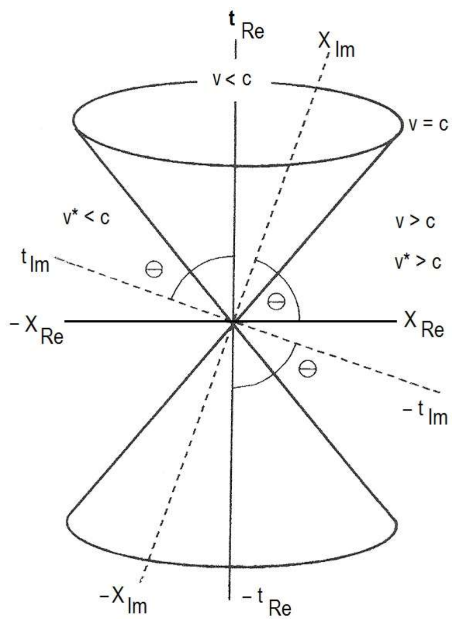

Figure 11. In the complex space multidimensional model, we introduce, in addition to the usual orthogonal 4-space, four imaginary components, three spatial and one temporal. This is necessary in order to model remote connectedness and to retain the causality and symmetry conditions in physics. We consider the eight orthogonal dimensions to be constituents of two intersecting lightcones, one real (x_{Re}, t_{Re}) and the other (y_{Im}, t_{Im}) coordinates.

We introduce a complex 8D matrix in which the real components comprise the usual 4-space of three real space components and a real-time component and four imaginary components composed of three imaginary space components and one imaginary time component. See Fig. 10. Hansen and Newman [33,34] and Rauscher [7-19] developed the properties of a complex Minkowski space and explored the properties of this geometry in detail. The formalism involves defining a complex space $Z^{\mu} = X^{\mu}_{\text{Re}} + iX^{\mu}_{\text{Im}}$ where the metric of the space is obtained for the line element $ds^2 = g_{\mu\nu} dZ^{\mu} dZ^{*\nu}$ where indices μ and ν run 1 to 4.

In defining conditions of causality for $ds^2 = 0$ for the metrical form we have the usual 4-space Minkowski metric with signature (+++-)

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} \tag{1a}$$

using units $c = 1$ and $dx_1 = dx,\ dx_2 = dy,\ dx_3 = dz$ and $dx_4 = cdt$ where the indices μ and ν run

1 to 4; where also

$$g_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \tag{1b}$$

which is a sixteen-element matrix where the trace, $tr = 2$.

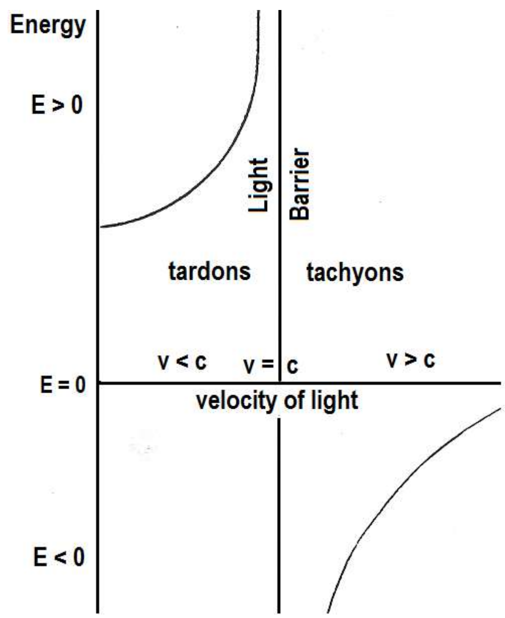

Figure 2.12. Tachyon and tardon signals are represented in the plot of energy versus velocity, as $v \to c,\ E \to \infty$. Perhaps tunneling through the velocity barrier from $v < c$ to $v > c$ can occur so that $E \neq \infty$.

In complex 8D space, we have for our differential line element with coordinates labeled $dZ^{\mu} = dX^{\mu}_{\text{Re}} + idX^{\mu}_{\text{Im}}$ (in which dZ is complex and dX_{Re} and dX_{Im} are themselves real), with a complex matrix where $\eta_{\iota\nu}$ is analogous to $g_{\mu\nu}$ such that

$$ds^2 = \eta_{\mu\nu} dZ^{\mu} dZ^{*\mu} \tag{2}$$

so that, for example, $dZ^{\mu} dZ^{*\mu} = (dX^{\mu}_{\text{Re}})^2 + (dX^{\mu}_{\text{Im}})^2$ where $\eta_{\mu\nu}$ is a 64-element matrix. We can write in general for real and imaginary space and time components:

$$\begin{aligned} ds^2 &= \left(dx^2_{\text{Re}} + dx^2_{\text{Im}}\right) + \left(dy^2_{\text{Re}} + dy^2_{\text{Im}}\right) \\ &\quad + \left(dz^2_{\text{Re}} + dz^2_{\text{Im}}\right) - c^2\left(dt^2_{\text{Re}} + dt^2_{\text{Im}}\right) \end{aligned} \tag{3}$$

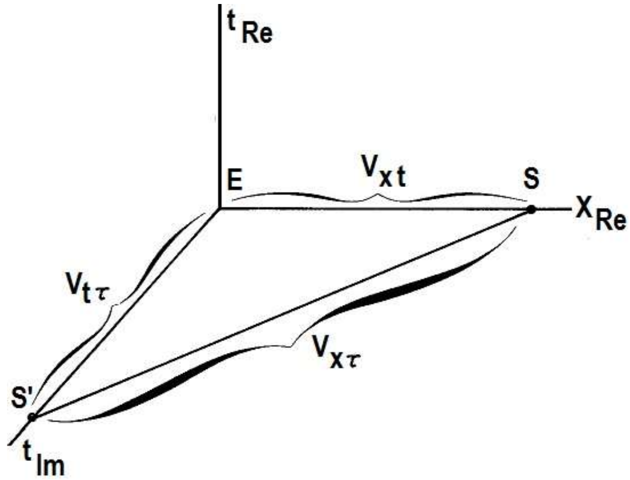

Figure 13. Real time separation between event E and event S on the real X axis can be made to appear contiguous by accessing the t_{Im} axis in 5D space as a subset of 8D space. The apparent velocity, v_{Re} is denoted as $v_x t = v_{x\mathrm{Re}} t_{\mathrm{Re}}$. Access to t_{Im} through a velocity $v_{t\mathrm{Re}} t_{\mathrm{Im}}$ along the t_{Im} axis run via this signal propagation can make $v_{x\mathrm{Re}} t_{\mathrm{Re}}$ appear instantaneous as $v_{x\mathrm{Re}} t_{\mathrm{Re}}$ goes to infinity. This figure corresponds to the remote connection points P_1 and P_2 via access to P_3 in Fig. 9 and also 9b.

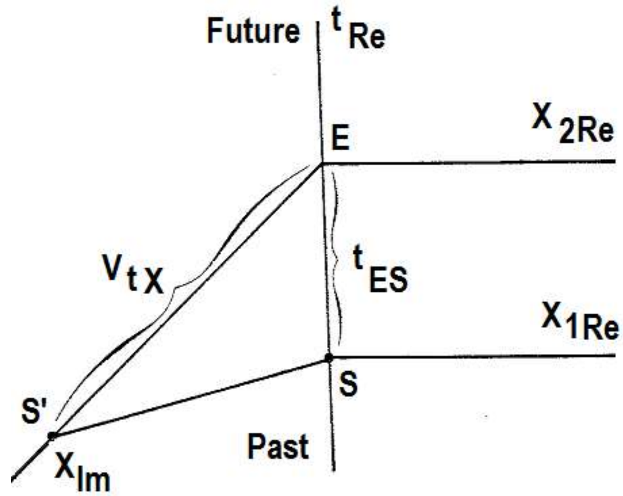

Figure 14. The separation of two events S and E along the real-time axis, t_{ES}. The anticipatory time separation does not violate CTL, if we have access to the imaginary space coordinates, $X_{\mathrm{Im}} = x_{\mathrm{Im}}, y_{\mathrm{Im}}, z_{\mathrm{Im}}$. The velocity of propagation on x_{Im}, t_{Re} space is $v_{t\mathrm{Re}}$, x_{Im}. Comparison with Fig. 9c the event P_1 corresponds to S, and P_3 to E, and S' to E at a velocity of vt_{Re}, x_{Im} from S' to P_4. Then E is an anticipatory event from the S frame of reference. At the vantage point of E at the future time can appear present and past events S can be anticipated when having access to S' is possible. The space comprises a 7D geometry.

In [7] we represent the three real spatial components, dx_{Re}, dy_{Re}, dz_{Re}, as dX and the three imaginary spatial components, dx_{Im}, dy_{Im}, dz_{Im} as dX_{Im} and similarly for the real-time component $dt_{\mathrm{Re}} \equiv dt$ and $dt_{\mathrm{Im}} = d\tau$. We then introduce complex spacetime-like coordinates as a space-like part $x_{\mathrm{Im}} = \chi$ and a time-like part $t_{\mathrm{Im}} = \tau$ as imaginary parts of X and t [8].

Now we have the invariant line elements as

$$s^2 = |x'|^2 - c|t'|^2 = |x'|^2 - |t'|^2 \tag{4}$$

again where we choose units where $c^2 = c = 1$ and

$$x' = X_{\text{Re}} + iX_{\text{Im}} \tag{5}$$

and

$$t' = t_{\text{Re}} + it_{\text{Im}} \tag{6}$$

as our complex dimensional component [7,8]. We use

$$x'^2 = |x'|^2 = X_{\text{Re}}^2 + X_{\text{Im}}^2 \tag{7}$$

and

$$t'^2 = |t'|^2 = t_{\text{Re}}^2 + t_{\text{Im}}^2 . \tag{8}$$

Recalling that the square of a complex number is given as the modulus

$$|x'| = x'x'^* = \left(X_{\text{Re}} + iX_{\text{Im}}\right)\left(X_{\text{Re}} - iX_{\text{Im}}\right) \tag{9}$$

for X_{Re} and X_{Im} real. The fundamental key to this set of calculations is that the modulus of the product of complex numbers is real. Therefore, we have the 8-space line element

$$\begin{aligned} s^2 &= x_{\text{Re}}^2 - c^2 t_{\text{Re}}^2 + x_{\text{Im}}^2 - c^2 t_{\text{Im}}^2 \\ &= x_{\text{Re}}^2 - t_{\text{Re}}^2 + x_{\text{Im}}^2 - t_{\text{Im}}^2 \end{aligned} \tag{10}$$

Causality is defined by remaining on the right cone, in real spacetime, as

$$s^2 = x_{\text{Re}}^2 - c^2 t_{\text{Re}}^2 = x_{\text{Re}}^2 - t_{\text{Re}}^2 \tag{11}$$

using the condition c = 1. Then generalized causality in complex spacetime is defined by

$$s^2 = x_{\text{Re}}^2 - t_{\text{Re}}^2 + x_{\text{Im}}^2 - t_{\text{Im}}^2 \tag{12}$$

in the $x_{\text{Re}}, t_{\text{Re}}, x_{\text{Im}}, t_{\text{Im}}$ generalized light cone 8D space. See Fig. 11.

Let us calculate the interval separation between two events or occurrences Z_1 and Z_2 with real separation $\Delta x_{\text{Re}} = x_{2\text{Re}} - x_{1\text{Re}}$ and imaginary separation $\Delta x_{\text{Im}} = x_{2\text{Im}} - x_{1\text{Im}}$. Then the distance along the line element is $\Delta s^2 = \Delta(x_{\text{Re}}^2 + x_{\text{Im}}^2 - t_{\text{Re}}^2 - t_{\text{Im}}^2)$ and it must be true that the line interval is a real separation. Then

$$\begin{aligned} \Delta s^2 &= (x_{2,\text{Re}} - x_{1,\text{Re}})^2 + (x_{2,\text{Im}} - x_{1,\text{Im}})^2 \\ &\quad -(t_{2,\text{Re}} - t_{1,\text{Re}})^2 - (t_{2,\text{Im}} - t_{1,\text{Im}})^2 \end{aligned} \tag{13}$$

or

$$\Delta s^2 = (x_{2,\mathrm{Re}} - x_{1,\mathrm{Re}})^2 + (x_{2,\mathrm{Im}} - x_{1,\mathrm{Im}})^2 - (t_{2,\mathrm{Re}} - t_{1,\mathrm{Re}})^2 - (t_{2,\mathrm{Im}} - t_{1,\mathrm{Im}})^2 \tag{14}$$

Because of the relative signs of the real and imaginary space and time components and in order to achieve the causality connectedness condition between the two events, or Δs^2, we must "mix" space and time. That is, we use the imaginary time component to effect a zero space separation. We identify $(x_{1,\mathrm{Re}}, t_{1,\mathrm{Re}})$ with one spacetime event causally correlated with another spacetime event, $(x_{2,\mathrm{Re}}, t_{2,\mathrm{Re}})$ [8]. See Fig. 9. By introducing the imaginary time component, one can achieve a condition in which the apparent separation in the real physical plane defined by x_{Re}, t_{Re} is zero, given access to the imaginary time, t_{im}, or the x_{Re}, t_{im} plane yielding spatial nonlocality.

The lightcone metric representation may imply superluminal signal propagation between an event A transmitter and even in the four-real subset space by the event B (receiver) or two simultaneously remotely connected events. Separation will not appear superluminal in the 8-space representation. The causality conditions, which do not contain closed time-like loops, are for the complex 8-space geometry, where 4-space is a cut through the 8-space [8]. Newton examines causality conditions in 4-space with superluminal signals [40] and the problem of closed time-like loops posed by Feinberg's classic "Tachyon" paper [41,42]. These problems appear to be resolved by considering spaces of higher (> 4D) dimensions and are consistent with subliminal and superluminal signals. See Fig. 12.

3. The Lorentz Condition in Complex 8-Space Geometry and Tachyonic Signaling

In order to examine as the consequences of the relativity hypothesis that time is the fourth dimension of space, and that we have a particular form of transformation called the Lorentz transformation, we must define velocity in the complex space. That is, the Lorentz transformation and its consequences, the Lorentz contradiction and mass dilation, etc., are a consequence of time as the fourth dimension of space and are observed in three spaces [43]. These attributes of 4-space in 3-space are expressed in terms of velocity, as in the form $\gamma = \left(1-\beta^2\right)^{-1/2}$ for $\beta \equiv v_{\mathrm{Re}} / c$ where c is always taken as real.

If complex 8-space can be projected into 4-space, what are the con-sequences? We can also consider a 4D slice through the complex 8D space. Each approach has its advantages and disadvantages. In projective geometries information about the space is distorted or lost. What is the comparison of a subset geometry formed from a projected geometry or a subspace formed as a slice through an XD geometry? What does a generalized Lorentz transformation "look like"? We will define complex derivatives and therefore we can define velocity in a complex plane [8].

Consider the generalized Lorentz transformation in the system of x_{Re} and t_{Im} for the real time remote connectedness case in the $x_{\mathrm{Re}}, t_{\mathrm{Im}}$ plane. We define our substitutions from 4-to 8-space before us,

$$\begin{aligned} x &\to x' = x_{\mathrm{Re}} + i x_{im} \\ t &\to t' = t_{\mathrm{Re}} + i t_{im} \end{aligned} \tag{15}$$

and we represented the case for no imaginary component of x_{Re} or $x_{\mathrm{Im}} = 0$ where the $x_{\mathrm{Re}}, t_{\mathrm{Re}}$ plane comprises the ordinary 4-space plane.

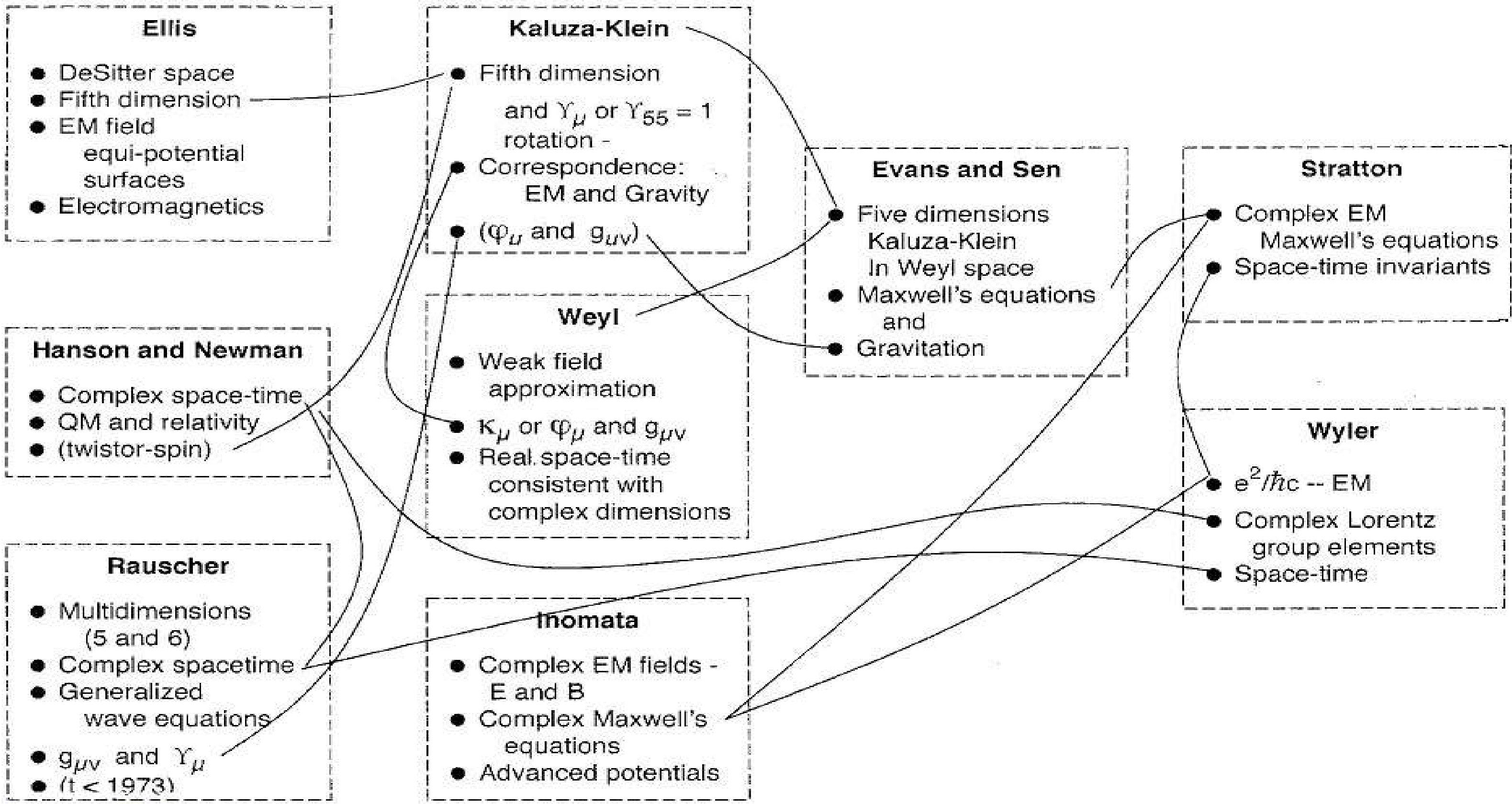

Figure 2.16. Relationship of multidimensional geometric theories. Comparing the differences between the concepts in these models is interesting because it may lead to unification electromagnetic and gravitational phenomena and model remote connectedness and nonlinear phenomena [7-12, 33,34,38,39, 46,50-54].

Let us recall that the usual Lorentz transformation conditions defined in four real space. Consider two frames of reference, Σ, at rest and Σ' moving at relative uniform velocity *v*. We call *v* the velocity of the origin of Σ' moving relative to Σ. A light signal along the *x* direction is transmitted by $x = ct$ or $x - ct = 0$ and also in Σ' as $x' = ct'$ or $x'-ct' = 0$, since the velocity of light in vacuo is constant in any frame of reference in 4-space. See Fig. 2. For the usual 4D Lorentz transformation, we have as shown in Eq. (6) and (8), $x = x_{\text{Re}}, t = t_{\text{Re}}$ and $v_{\text{Re}} = x_{\text{Re}} / t_{\text{Re}}$.

There is a relationship between subliminal, time-like, and superluminal, space-like, interpretation of the remote connectedness phenomena, such as the nonlocality test of Bell's theorem.

$$
\begin{aligned}
x' &= \frac{x - vt}{\sqrt{1 - v^2/c^2}} = \gamma(x - vt) \\
y' &= y \\
z' &= z \\
t' &= \frac{t - (v/c^2)x}{\sqrt{1 - v^2/c^2}} = \gamma\left(t - \left(\frac{v}{c^2}x\right)\right)
\end{aligned}
\tag{16}
$$

for $\gamma = (1-\beta^2)^{-1/2}$ and $\beta = v/c$. Here *x* and *t* stand for x_{Re} and t_{Re} and *v* is the real velocity.

We consider the $x_{\text{Re}}, t_{\text{Im}}$ plane and write the expression for the Lorentz conditions for this plane. Since again t_{Im} like t_{Re} is orthogonal to x_{Im} and t'_{Im} is orthogonal to x'_{Im} we can write

$$
\begin{aligned}
x' &= \frac{x - ivt_{\text{Im}}}{\sqrt{1 - v^2/c^2}} = \gamma_v(x - vt_{\text{Im}}) \\
y' &= y \\
z' &= z \\
t' &= \frac{t - (v/c^2)x}{\sqrt{1 - v^2/c^2}} = \gamma_v\left(t - \left(\frac{v}{c^2}x\right)\right)
\end{aligned}
\tag{17}
$$

where γ_v represents the definition of γ in terms of the velocity *v*; also $\beta_{v\text{Im}} \equiv v_{\text{Im}} / c$ where *c* is always taken as real [7] where *v* can be real or imaginary.

In Eq. 17 for simplicity we let x', x, t' and t denote $x'_{\text{Re}}, x_{\text{Re}}, t'_{\text{Re}}$ and t_{Re} and we denote script v as v_{Im}. For velocity, v is $v_{\text{Re}} = x_{\text{Re}} / t_{\text{Re}}$ and $v = v_{\text{Im}} = ix_{\text{Im}} / it_{\text{Im}}$ where the *i* drops out so that $v = v_{\text{Im}} = x_{\text{Im}} / t_{\text{Im}}$ is a real value function. In all cases the velocity of light *c* is *c*. We use this alternative notation here for simplicity in the complex Lorentz transformation. The symmetry properties of the topology of the complex 8-space gives us the properties that allow Lorentz conditions in 4D, 8D and ultimately 12D space. The example we consider here is a subspace of the 8-space of $x_{\text{Re}}, t_{\text{Re}}, x_{\text{Im}}$ and t_{Im}. In some cases we let $x_{\text{Im}} = 0$ and just consider temporal remote connectedness and anticipation; but likewise, we can formulate remote, nonlocal connectedness solutions for $x_{\text{Im}} \neq 0$ and $t_{\text{Im}} = 0$ or $t_{\text{Im}} \neq 0$. The anticipatory case for $x_{\text{Im}} = 0$ is a 5D space as the space for $x_{\text{Im}} \neq 0$ *and* $t_{\text{Im}} = 0$ is a 7D space and for $t_{\text{Im}} \neq 0$ as well as the other real and imaginary spacetime dimensions, we have our complex 8D space. See Fig. 11.

It is important to define the complex derivative so that we can define velocity, v_{Im}. In the $x_{\text{Re}}t_{\text{Im}}$ plane then, we define a velocity of $v_{\text{Im}} = dx/dit_{\text{Im}}$. In the Section 4 we detail the velocity expression for v_{Im} and define the derivative of a complex function in detail [38]. For $v_{\text{Im}} = dx/idt_{\text{Im}} = -idx/dt_{\text{Im}} = -iv_{\text{Re}}$ for v_{Re} as a real quantity, we substitute into our $x_{\text{Re}}, t_{\text{Im}}$ plane Lorentz transformation conditions as

$$\begin{aligned} x' &= \frac{x_{\text{Re}} - v_{\text{Re}} t_{\text{Im}}}{\sqrt{1 + v_{\text{Re}}^2 / c^2}} \\ y' &= y \\ z' &= z \\ t'_{\text{Im}} &= \frac{t_{\text{Re}} - v_{\text{Re}} x_{\text{Re}}}{\sqrt{1 + v_{\text{Re}}^2 / c^2}} \end{aligned} \qquad (18)$$

These conditions will be valid for any velocity, $v_{\text{Re}} = - v$.

Let us examine the way this form of the Lorentz transformation relates to the properties of mass dilation. We will compare this case to the ordinary mass dilation formula and the tachyonic mass formula of Feinberg [41] which nicely results from the complex 8-space. See Fig. 7. In the ordinary x_{Re} t_{Re} plane then, we have the usual Einstein mass relationship of

$$m = \frac{m_0}{\sqrt{1 - v_{\text{Re}}^2 / c^2}} \quad \text{for} \quad v_{\text{Re}} \le c \qquad (19)$$

and we can compare this to the tachyonic mass relationship in the x_{Im}, t_{Im} plane

$$m = \frac{m_0^*}{\sqrt{1 - v_{\text{Re}}^2 / c^2}} = \frac{im_0}{\sqrt{1 - v_{\text{Re}}^2 / c^2}} = \frac{m_0}{\sqrt{v_{\text{Re}}^2 / c^2 - 1}} \qquad (20)$$

for v_{Re} now $v_{\text{Re}} \ge c$ and where m^* or m_{Im} stands for $m^* = im$ and we define m as m_{Re},

$$m = \frac{m_0}{\sqrt{1 + v^2 / c^2}} \qquad (21)$$

For m real (m_{Re}), we examine two cases on v as $v < c$ or $v > c$, so we let v be any value from $-\infty < v < \infty$, where the velocity, v, is taken as real, or v_{Re}.

Consider the case of v as imaginary (or v_{Im}) and examine the consequences of this assumption. Also we examine the consequences for both v and m imaginary and compare to the above cases. If we choose v imaginary or $v^* = iv$ (which we can term v_{Im}) the $v^{*2}/c^2 = -v^2/c^2$ and $\sqrt{1 + v^{*2}/c^2}$ becomes $\sqrt{1 - v^{*2}/c^2}$ or

$$m = \frac{m_0}{\sqrt{1 - v_{\text{Re}}^2 / c^2}} \qquad (22)$$

We get the form of this normal Lorentz transformation if v is imaginary $(v^* = v_{\text{Im}})$

If both v and m are imaginary, as $v^* = iv$ and $m^* = im$, then we have

$$m = \frac{m_0^*}{\sqrt{1+v^{*2}/c^2}} = \frac{im_0}{\sqrt{1-v^2/c^2}} = \frac{m_0}{\sqrt{v^2/c^2-1}} \tag{23}$$

or the tachyonic condition.

If' we go "off" into x_{Re} t_{Re} t_{im} planes, then we have to define a velocity "cutting across" these planes, and it is much more complicated to define the complex derivative for the velocities. For subliminal relative systems Σ and Σ' we can use vector addition such as $W = v_{Re} + iv_{Im}$ for $v_{Re} < c$, $v_{Re} < x$, $v_{Im} < c$ and $W < c$. In general, there will be four complex velocities. The relationship of these four velocities is given by the Cauchy-Riemann relations in the next section. These two are equivalent. The actual magnitude of v may be expressed as $v = [vv^*]^{\frac{1}{2}}\hat{v}$ (where $\hat{v}$ is the unit vector velocity) which can be formed using either of the Cauchy-Riemann equations. It is important that a detailed analysis not predict any extraneous consequences of the theory. Any possibly new phenomenon that is hypothesized should be formulated in such a manner as to be easily experimentally testable.

Feinberg suggests several experiments to test for the existence of tachyons [8,41,42]. He describes the following experiment. Consider in the laboratory, atom A, at time, t_0 is in an excited state at rest at x_1 and atom B is in its ground state at x_2. At time t_1 atom A descends to the ground state and emits a tachyon in the direction of B. Let E_1 be this event at t_1, x_1. Subsequently, at $t_2 > t_1$ atom B absorbs the tachyon and ascends to an excited state; this is event E_2, at t_2, x_2. Then at $t_3 > t_2$ atom B is excited and A is in its ground state. For an observer traveling at an appropriate velocity, $v < c$ relative to the laboratory frame, the events E_1 and E_2 appear to occur in the opposite order in time. Feinberg describes the experiment by stating that at t_2' atom B spontaneously ascends from the ground state to an excited state, emitting a tachyon which travels toward A. Subsequently, at t_1', atom A absorbs the tachyon and drops to the ground state.

It is clear from this that what is absorption for one observer is spontaneous emission for another. But if quantum mechanics is to remain intact so that we are able to detect such particles, then there must be an observable difference between them: The first depends on a controllable density of tachyons, the second does not. In order to elucidate this point, we should repeat the above experiment many times over. The possibility of reversing the temporal order of causality, sometimes termed 'sending a signal backwards in time' must be addresses [8,41,42]. Is this cause-effect statistical in nature? In the case of Bell's Theorem, these correlations are extremely strong whether explained by $v > c$ or $v = c$ signaling.

In [44], Bilaniuk, et al formulated the interpretation of the association of negative energy states with tachyonic signaling. From the different frames of reference, thus to one observer absorption is observed and to another emission is observed. These states do not violate special relativity. Acausal experiments in particle physics, such as for the S-Matrix, have been suggested by a number of researchers [45]. Another approach is through the detection of Cerenkov radiation, which is emitted by charged particles moving through a substance traveling at a velocity, $v > c$. For a tachyon traveling in free space with velocity, $v > c$. Cerenkov radiation may occur in a vacuum cause the tachyon to lose energy and become a tardon [4]. See Figs. 8 and 12.

4. Velocity of Propagation in Complex 8-Space

In this section we utilize the Cauchy-Riemann relations to formulate the hyperdimensional velocities of propagation in the complex plane in various slices through the hyperdimensional complex 8-space. In this model finite limit velocities, $v > c$ can be considered. In some Lorentz frames of reference, instantaneous signaling can be considered. In Fig. 13 is displayed the velocity connection between remote nonlocal events, and in Fig. 14 is displayed temporal separated events or anticipatory and real-time event relations.

It is important to define the complex derivative so that we can define the velocity, $v = v_{\mathrm{Im}}$. In the *xit* or $x_{\mathrm{Re}} i x_{\mathrm{Im}}$ plane then, we define a velocity of $v = dx / d(it_{\mathrm{Im}})$ We now examine in some detail the velocity of this expression, here $x = x_{\mathrm{Re}}$. In defining the derivative of a complex function we have two cases in terms of a choice in terms of the differential increment considered. Consider the orthogonal coordinates *x* and it_{Im}; then we have the generalized function, $f(x, t_{\mathrm{Im}}) = f(z)$ for $z = x + it_{\mathrm{Im}}$ and f(*z*) = $u(x, t_{\mathrm{Im}}) + iv(x, t_{\mathrm{Im}})$ where $u(x, t_{\mathrm{Im}})$ and $v(x_{\mathrm{Im}}, t_{\mathrm{Im}})$ are real functions of the rectangular coordinates *x* and t_{Im} of a point in space, $P(x, t_{\mathrm{Im}})$. Choose a case such as the origin $z_0 = x_0 + it_{0\mathrm{Im}}$ and consider two cases , one for real increments $h = \Delta x$ and imaginary increments $h = i\Delta t_{\mathrm{Im}}$. For the real increments $h = \Delta t_{\mathrm{Im}}$ we form the derivative $f'(z_0) \equiv df(z)/dz_{z_0}$ which is evaluated at z_0 as

$$f' = \lim_{\Delta x \to 0} \left\{ \frac{\mu\left(x_0 + \Delta x, t_{0\mathrm{Im}}\right) - \mu\left(x_0, t_{0\mathrm{Im}}\right)}{\Delta x} + i\frac{v\left(x_0 + \Delta x, t_{0\mathrm{Im}}\right) - v\left(x_0, t_{0\mathrm{Im}}\right)}{\Delta x} \right\} \tag{24a}$$

or

$$f'\left(z_0\right) = u_x\left(x_0, t_{0\mathrm{Im}}\right) + iv_x\left(x_0, t_{0\mathrm{Im}}\right) \text{ for } u_x \equiv \frac{\partial u}{\partial x} \text{ and } v_x \equiv \frac{\partial v}{\partial x}. \tag{24b}$$

Again $x = x_{\mathrm{Re}}$, $x_0 = x_{0\mathrm{Re}}$ and $v_x = v_{x\mathrm{Re}}$.

Now for the purely imaginary increment, $h = i\Delta t_{\mathrm{Im}}$ we have

$$f'\left(z_0\right) \lim_{\Delta t_{\mathrm{Im}} \to 0} \left\{ \frac{1}{i} \frac{\mu\left(x_0, t_{0\mathrm{Im}} + \Delta t_{\mathrm{Im}}\right) - \mu\left(x_0, t_{0\mathrm{Im}}\right)}{\Delta t_{\mathrm{Im}}} + \frac{v\left(x_0, t_{0\mathrm{Im}} + \Delta t_{\mathrm{Im}}\right) - v\left(x_0, t_{0\mathrm{Im}}\right)}{\Delta t_{\mathrm{Im}}} \right\} \tag{25a}$$

and

$$f'\left(z_0\right) = -iu_{t\mathrm{Im}}\left(x_0, t_{0\mathrm{Im}}\right) + v_{t\mathrm{Im}}\left(x_0, t_{0\mathrm{Im}}\right) \tag{25b}$$

for $u_{\mathrm{Im}} = u_{t\mathrm{Im}}$ and $v_{\mathrm{Im}} = v_{t\mathrm{Im}}$ then

$$u_{t\mathrm{Im}} \equiv \frac{\partial u}{\partial t_{\mathrm{Im}}} \text{ and } v_{t\mathrm{Im}} \equiv \frac{\partial v}{\partial t_{\mathrm{Im}}}. \tag{25c}$$

Using the Cauchy-Riemann equations

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial t_{\mathrm{Im}}} \text{ and } \frac{\partial u}{\partial t_{\mathrm{Im}}} = -\frac{\partial v}{\partial x} \tag{26}$$

and assuming all principle derivations are definable on the manifold and letting $h = \Delta x + i\Delta t_{\mathrm{Im}}$ we can use

$$f'(z_0)\lim_{h\to 0}\frac{f(z_0+h)-f(z_0)}{h} = \left.\frac{df(z)}{dz}\right|_{z_0} \tag{27a}$$

and

$$u_x(x_0,t_{0\mathrm{Im}}) + iv_x(x_0,t_{0\mathrm{Im}}) - \frac{\partial u(x_0,t_{0\mathrm{Im}})}{\partial x} + i\frac{\partial v}{\partial x}(x_0,t_{0\mathrm{Im}}) \tag{27b}$$

with v_x for x and t_{Re} that is $u_{\mathrm{Re}} = u_{x\mathrm{Re}}$, with the derivative form of the charge of the real space increment with complex time, we can define a complex velocity as,

$$f'(z_0) = \frac{dx}{d(it_{\mathrm{Im}})} = \frac{1}{i}\frac{dx}{dt_{\mathrm{Im}}} \tag{28a}$$

we can have $x(t_{\mathrm{Im}})$ where x_{Re} is a function of t_{Im} and $f(z)$ and using $h = i\Delta t_{\mathrm{Im}}$, then

$$f'(z_0) = x'(t_{\mathrm{Im}}) = \frac{dx}{dh} = \frac{dx}{idt_{\mathrm{Im}}}. \tag{28b}$$

Then we can define a velocity where the differential increment is in terms of $h = i\Delta t_{\mathrm{Im}}$. Using the first case as $u(x_0, t_{0\mathrm{Im}})$ and obtaining $dt_{0\mathrm{Im}}/\Delta x$ (with i's) we take the inverse. If u_x which is v_x in the $h \to i\Delta t_{\mathrm{Im}}$ case have both u_x and v_x, one can be zero.

In the next section, we present a brief discussion of $n > 4$D geometries. Like the complex 8D space, the 5D Kaluza-Klein geometries are subsets of the supersymmetry models. The complex 8-space deals in extended dimensions, but like the TOE models, Kaluza-Klein models also treat $n > 4$D as compactified on the scale of the Planck length, 10^{-33} cm [1-6].

In 4D space (Fig. 9) event point, P_1 and P_2 are spatially separated on the real space axis as $x_{0\mathrm{Re}}$ at point P_1 and $x_{1\mathrm{Re}}$ at point P_2 with separation $\Delta x_{\mathrm{Re}} = x_{1\mathrm{Re}} - x_{0\mathrm{Re}}$. From the event point P_3 on the t_{Im} axis we move in complex space from event P_1 to event P_3. From the origin, $t_{0\mathrm{Im}}$ we move to an imaginary temporal separation of $t_{1\mathrm{Im}}$ to $t_{2\mathrm{Im}}$ of $\Delta t_{\mathrm{Im}} = t_{2\mathrm{Im}} - t_{0\mathrm{Im}}$. The distance in real space and imaginary time can be set so that measurement along the t_{Im} axis yields an imaginary temporal separation Δt_{Im} subtracts out, from the spacetime metric, the temporal separation Δx_{Re}. In this case occurrence of events P1 and P2 can occur simultaneous, that is, the apparent velocity of propagation is instantaneous.

For the example of Bell's Theorem, the two photons leave a source nearly simultaneously at time, $t_{0\mathrm{Re}}$ and their spin states are correlated at two real spatially separated locations, $x_{1\mathrm{Re}}$ and $x_{2\mathrm{Re}}$ separated by $\Delta x_{\mathrm{Re}} = x_{2\mathrm{Re}} - x_{1\mathrm{Re}}$. This space-like separation, is forbidden by special relativity; however, in the complex space, the points $x_{1\mathrm{Re}}$ and $x_{2\mathrm{Re}}$ appear to be contiguous for the proper path 'traveled' to point at $t_{1\text{Im}}$ along the imaginary axis. Because of the possibility of proper spacetime adjustment or transformation which is possible in the complex plane, separate spacetime locations can appear contiguous in the hyperdimensional 8-space. Hence the upper limit of velocity propagation is instantaneous. See Table 1 and Figs. 13, and 14. By adjusting our imaginary spatial and temporal advantage, v_{Re} and v_{Im} can be variously adjusted and effect apparent causal conditions from the 8D space to the 4D space [14].

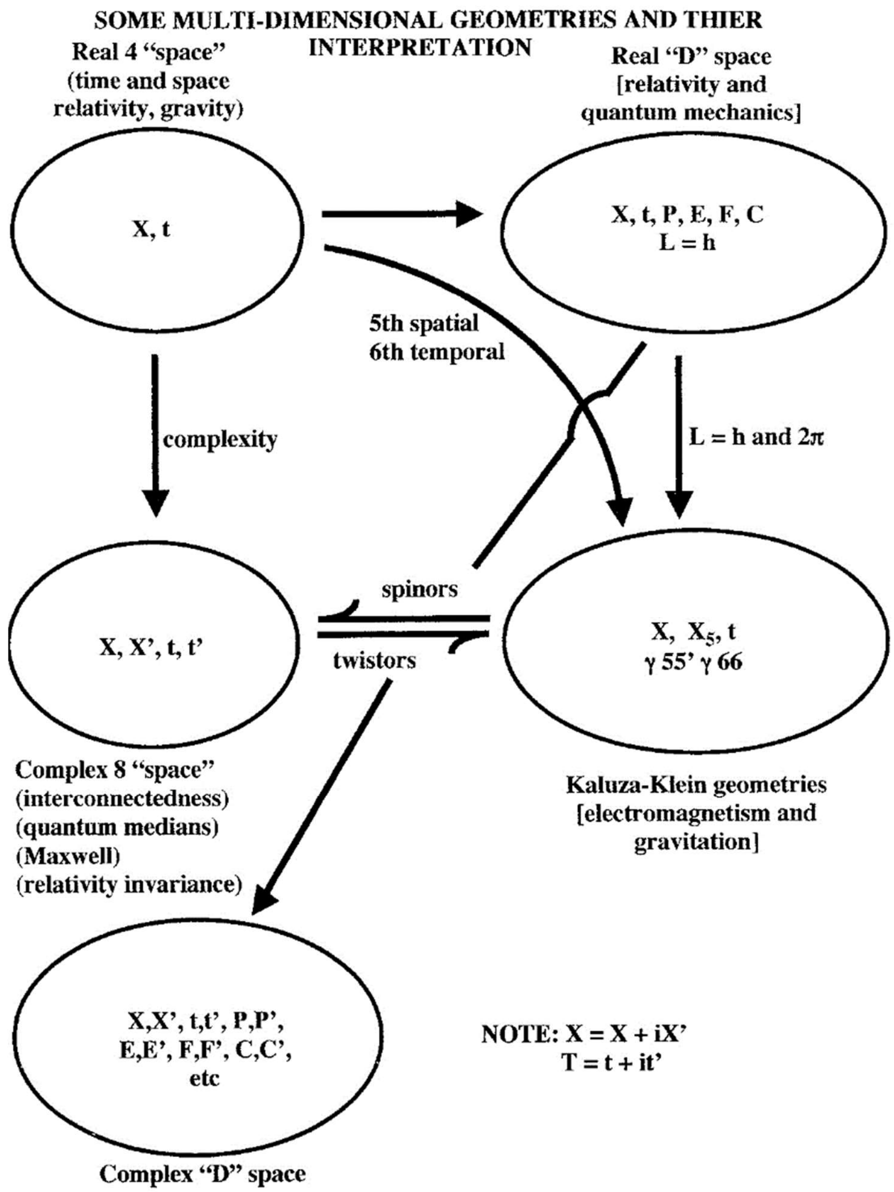

Figure 15. Representation of the usual 4-Space lightcone and four other multidimensional geometries. In the upper right is a representation of the 10D real Descartes geometry. Below and to the left of the usual 4D space is the complexified 8D space. To the lower right is the 5D and 6D Kaluza-Klein geometry and in the lower left is the complexified 10D Descartes space as a 20D complex Descartes space. Note that *X* represents *x,y,z* and P represents P_x, P_y, P_z in the upper right. The relationships of all these geometries are represented in this figure.

Table 1
Multidimensional Models: Macrocosm and Consciousness

- Einstein-Minkowski 4D space expressed as three spatial and one temporal dimension. This is the usual observed 3-space modeled on the Minkowski lightcone diagram [7,8].
- 4-space can be expanded to a multidimensional quantum gravity space of 11D; spacetime, momentum, mass-energy, force, velocity, acceleration, power, pressure, and rotation which comprise Descartes 10D space [4,5].
- Complex 8D space is generated by complexifying the usual 4-space using $\sqrt{-1}$, and has nonlocality and remote interconnectedness that can relate a twistor algebra to the spinor calculus of Kaluza-Klein 5D geometry and complexified Maxwell's equations and non-Hertzian phenomena.
- Kaluza-Klein geometry relates the Einstein-Minkowski 4-space of relativity to electromagnetic phenomena and complex 8-space.
- The Einstein-Minkowski 4-space, Kaluza-Klein 5-space, Rauscher 8-space, M-Theory 11-space and Amoroso 12-space relate topological geometries of modern particle physics to quantum theory and general and special relativity.
- These multidimensional models allow a domain to exist in the physical world for the action of local and nonlocal aspects of the reality of the observer.

5. Kaluza-Klein Geometries: A Possible Unification of Electromagnetic and Gravitational Phenomena

We will present a brief discussion of other multidimensional models and examine the manner in which they may relate to the complex 8D model which was presented in previous sections and in references [1-8]. In the last several decades there has been a great deal of interest in some specific types of 5D and 6D geometries. This revived interest is based on the work of two colleagues of Einstein, who received encouragement from him in the 1930's, Kaluza and Klein, who introduced a 5D covariant geometry which appears to have properties which suggest a method of unifying the electromagnetic, gauge-invariant field theories (Maxwell's equations) and the gravitational field [21,22] (gravitational potential). This particular multidimensional model appears to be useful to examine further because it not only demonstrates the relationship between electromagnetic phenomena and gravitational relativistic phenomena, but it appears to be consistent with the main body of physics [8,18]. The spinor calculus is an excellent framework for accounting for the coupling of the electromagnetic field to the gravitational field in a natural way rather than the usual phenomenological manner [8,37-39]. This approach is automatically accomplished by requiring periodicity of 5D spinor fields. The theory of spinors is used in unifying electromagnetic and gravitational phenomena based on the homomorphism between the group of Lorentz transformations in relativity and the group of unimodular linear transformations in Maxwell's theory [12]. It should be noted that this homomorphism is valid only in the weak Wey1 field approximation
for the gravitational field [46,47].

In addition to the general coordinate transformations of the four coordinates, x^{μ}, the preferred coordinate system permutation group is

$$x'^5 = x^5 + f\left(x^1, x^2, x^3, x^4\right) \qquad (29)$$

Using this condition and the 5D cylindrical metric or $ds^2 = (\gamma_{ik} dx^i dx^k)$ yields the form

$$ds^2 = (dx^5 + \gamma_{\mu 5} dx^\mu)^2 + g_{\mu\nu} dx^\mu dx^\nu \qquad (30)$$

where the second term is the usual 4-space metric. Greek indices μ, ν run 1 to 4 and Latin indices i, k run 1 to 5.

The quantity $\gamma_{\mu 5}$ in the above equation transforms like a gauge, *ds*

$$\gamma'_{\mu 5} = \gamma_{\mu 5} - \frac{\partial f}{\partial x^\mu}. \qquad (31)$$

where the function *f* is introduced as an arbitrary function. Returning to our 5D metric form in its five-compact form and 4D and 5D form gives

$$\gamma_{\mu\nu} = g_{\mu\nu} + \gamma_{\mu 5}\gamma_{\mu 5}. \qquad (32)$$

Starting from the metric form in a five "cylindrical" space $ds^2 = \gamma_{ik} dx^i dx^k$ where indices *i,k* run 1 to 5, we introduce the condition of cylindricity that can be described in a coordinate system in which the γ_{ik} are independent of x^5, that is

$$\frac{\partial \gamma_{ik}}{\partial x^5} = 0. \qquad (33)$$

Also, Kaluza-Klein assumed $\gamma_{55} = 1$ or the positive sign, $\gamma_{55} > 0$ for the condition of the fifth dimension for a 5D space, to ensure that the fifth dimension is metrically space-like [8]. We can also construct a 6D space for $\gamma_{66} = -1$ and $\gamma_{55} < 0.$ Geometrically one can interpret x^5 as an angle variable so that all values of x^5 differ by an integral multiple of 2π corresponding to the same point of the 5D space, if the values of the x^{μ} - are the same. For this specific case, each point of the 5D spaces passes exactly one geodesic curve which returns to the same point. In this case, there always exists a perpendicular coordinate system in which $\gamma_{55} = 1$ and,

$$\frac{\partial \gamma_{5\mu}}{\partial x^5} = 0. \qquad (34)$$

Other properties follow in which $g_{\mu\nu}$ and γ_{ik} are analogous and $\gamma_{\mu\nu} = g_{\mu\nu}$ and

$$\gamma_{55} = 1 + \gamma^{\mu\nu}\gamma_{\mu 5}\gamma_{\nu 5} \quad \text{also,} \quad \gamma^{\mu 5} = g^{\mu\nu}\gamma_{\nu 5} \qquad (35)$$

The gauge-like form alone is analogous to the gauge group, which suggests the identification of $\gamma_{\mu 5}$ with the electromagnetic potential ϕ_μ. We can write an expression for an antisymmetric tensor

$$\frac{\partial \gamma_{\mu 5}}{\partial x^\mu} - \frac{\partial \gamma_{\mu 5}}{\partial x^\nu} = f_{\mu\nu} \qquad (36)$$

which is an invariant with respect to the gauge transformation.

Using the independence of γ_{ik} of x_5 or $\partial\gamma_{ik} / \partial x^5 = 0$, the geodesics of the metric in five space can be interpreted by the expression

$$\frac{dx^5}{ds} + \gamma_{\mu 5}\frac{dx^\nu}{ds} = C \tag{37}$$

where C is a constant and s is a distance parameter. If we consider a generalized 5D curvature tensor, and using the form for $f_{\mu\nu}$ we can express it in terms of $f_{\mu\nu}$, the electromagnetic field strength

$$f_{\mu\nu} = \sqrt{\frac{16\pi G}{c4}}F_{\mu\nu} \tag{38a}$$

and then we can write

$$\gamma_{\mu 5} = \sqrt{\frac{16\pi G}{c4}}\phi_\mu \tag{38b}$$

The integration constant above can be identified as proportional to the ratio *e/m* of charge to mass of a particle traveling geodesics in the Kaluza-Klein space [37-39]; *c* is the velocity of light and *G* is the universal gravitational constant. The force term, $F = C^4 / G$ is found in Einstein's field equations in the stress energy tensor term and is identified as having cosmological significance by Rauscher [1].

Under specific conditions of the conformal mappings in the complex Minkowski space, one can represent twistors in terms of spinors. The spinor(s) will be said to "represent" the twistor. The twistor is described as a complex two-plane in the complex Minkowski space. References on twister theory and the spinor calculus are cited in [8,48]. Twistors and spinors can easily be related by the general Lorentz conditions in such a manner as to retain the condition that all signals are luminal in the complex space. The conformal invariance of tensor fields (which can be Hermitian) can be defined in terms of twistors and these fields can be identified with particles. See Chap. 11.

We can represent twistors in terms of a pair of spinors, μ^A and π_A which are said to represent the twistor, $\mathbb{Z}^\alpha$. Conditions for this representation are

- The null infinity condition for a zero-spin field, $\mathbb{Z}^\alpha \overline{\mathbb{Z}}_\alpha = 0$,
- Conformal invariance, and
- Independence of the origin.

Twistors and spinors are related by the general Lorentz conditions in such a manner as to retain the fact that all signals are luminal in the real 4-space, which does not preclude superluminal signals in an XD space [18].

The twistor is described as a 2-plane in complex Minkowski space, M_4. Twistors define the conformal invariance of the tensor field which can be identified with spin or spinless particles. For particles with spin *s* we have $\mathbb{Z}^\alpha \overline{\mathbb{Z}}_\alpha = 2s$. The twistor is derived from the imaginary part of the spinor field. The Kerr theorem comes out naturally. It is through the representation of spinors as twistors in complex Minkowski space that we can relate the complex 8-space model to the Kaluza-Klein geometries. In the 5D Kaluza-Klein geometries, the extra dimension, XD was considered to be a spatial rotational dimension in terms of $\gamma_{\mu 5}$.

The Hanson-Newman [33,34] and Rauscher [7-18] complex Minkowski space has introduced with it as an angular momentum, or helix or spiral dimension, called a twistor which is expressed in terms of spinors. We suggest that the problem of closed time-like loops may be resolved in terms of an additional dimension or dimensions which may, in one model, be represented by a helical world line in 5D and 6D space in such a manner that the world line does not collapse on itself and become multi-valued at a single spacetime point [39,44]. Note the twistor relates to the complex Schwarzschild metric yielding the Kerr (rotational) metric [32,45]. The Schwarzschild solution is seen as a "real slice" of a complex Minkowski space [32]. The complex Weyl tensor is viewed as a single complex field on the complex Minkowski space.

Some directions for further exploration of the relationship of our 8D model and the main body of physics may be made through the work of Hansen and Newman [33,34] and Kaluza and Klein [37-39]. Use of the Weyl weak field approximation may be used to examine the complex 8-space and electromagnetic phenomena [40] such as complex electric and magnetic fields which we explore further in the next section. Figure 12 presents a schematic of the relationship of some multidimensional geometries.

Basic to the Kaluza-Klein geometry is the series of papers published by Weyl [46,47] in which he forms a generalization of Riemannian geometry claiming to interpret all physical events in terms of gravitation and electromagnetism in terms of a "world metric" (note that this statement is much stronger than the Kaluza-Klein unification scheme, since it excludes strong and weak interactions). See Fig. 15. On Fig. 16 we present a comparison of some of the multi-dimensional n > 4D geometries and complexification theories and their interconnectedness to each other.

The gauge transformation of the Weyl space is formulated in terms of a quantity, ϕ_μ, rather than $g_{\mu\nu}$ where ϕ_μ is the four-vector potential. rather than strict gauge-invariance of $g_{\mu\nu}dx^\mu dx^\nu = 0$ (usually where $\phi_\mu = 0$). In Weyl's theory uses complex wave mechanics for electrically charged matter for the wave function, ψ . Then for a gauge transformation we have

$$\phi'_\mu = \phi_\mu - \frac{i\hbar c}{\varepsilon}\frac{\partial f}{\partial x^\mu} \tag{39}$$

where $\psi' = \psi e^{if(x)}$ and where he considers the invariance conditions on the imaginary exponent in ψ instead of the real exponent in $g_{\mu\nu}$. Weyl modified certain inconsistencies that occurred with relativity [46]. The Weyl theory most likely set up the considerations for the Kaluza-Klein model in their attempt to unify gravity and electromagnetism by relating $g_{\mu\nu}$ and ϕ^μ .

P.A.M. Dirac [49] generalized the complex scalar field to a complex two-component field $\binom{\mu}{\nu}$ in order to express the Schrödinger wave equation in a relativistic invariant form The complex two component field is called a spinor because it relates to the spin degrees of freedom that were needed because of the Zeeman spectral splitting in atoms. The question arises; does the spinor field result from the conditions in quantum physics or relativity theory? Using the irreducible representation of the underlying groups in relativity theory, Einstein and Mayer discovered that the real 4D representation in relativity reduces to the direct product of two 2D complex representations. The complex two component functions that are the basis of these representations are the spinor variables that Dirac discovered earlier to describe the electron and anti-electron or positron. Therefore, the spinor variable is the most fundamental expression of the theory of relativistic invariance. In this form then, relativity theory can be quantized as formulated by Dirac.

The hypergeometric Schrödinger equation is second order in space and first order in time. The Klein-Gordon equation is second order in both space and time; whereas the Dirac equation is first order in space and time, which is like a square root of the Klein-Gordon equation that has two solutions. That is, the Dirac equation has both a positive and a negative solution. For other multidimensional perspectives see Fig. 13.

Since the spinor invariant is complex, it corresponds to two invariance conditions; one real and the other the imaginary part of the two components of the spinor. We have seen that by introducing a complex Minkowski space we may be able to achieve a reasonable interpretation of some of the apparent paradoxes in quantum physics, relativity theory and electromagnetism. Non-locality and superluminal signal propagation are precisely formulated. We will examine the implications of complex geometries for electromagnetic phenomena, Bell's theorem, and other remote connectedness phenomena in Chap. 4. Also in Chap. 11 we address the relationship of 5R and 6D geometries with spinors, twistors and quaternions.

6. Additional Thoughts on Current Physical Theory

The formalism of the complex 8D space and the 5D Kaluza-Klein space are incorporated into the current grand unification theories (GUT), supersymmetry models, with gravity, and string theory (M-Theory where matter is considered to be made of vibrating strings and branes instead of paint like particles), that describes the unifications of the four force fields in particle physics and current models of the universe. The four fundamental forces are the strong nuclear force mediated by quarks, electromagnetic force, weak nuclear decay force and the gravitational force of General Relativity.

The Kaluza Klein model relates the electromagnetic and gravitational fields in which the photon (spin 1) mediates the electromagnetic field and the graviton (spin 2) mediates the gravitational field. This is why tensor analysis works. The electroweak force of the GUT model is mediated by $W^{\pm}, Z^0$ which are massive bosons for the electromagnetic and weak interactions. The mediators of the strong force are quarks and gluons. It becomes possible to relate the GUT theories (which only related the strong, electromagnetic and weak forces) to gravity via the use of the Kaluza-Klein geometry. These theories attempt to reduce “everything" to quarks and leptons mediated by the exchange of gauge bosons. This is currently termed the standard model.

References

[1] Rauscher, E.A. (1971) A Unifying Theory of Fundamental Processes, UCRL-20808 book, LBNL University of California Press.
[2] Rauscher, E.A. (1972) Closed Cosmological Solutions to Einstein’s Field Equations, Lett. Nuovo Cimento 3, 661.
[3] Rauscher, E.A. (1972) A Set of Generalized Heisenberg Relations and a New Form of Quantization, Lett. Nuovo Cimento 4, 757.
[4] Rauscher, E.A. (1972) A Possible Group Theoretical Representation of the Generalization Heisenberg Relations, Lett. Nuovo Cimento 5, 925.
[5] Rauscher, E.A. (1973) The Minkowski Metric for a Multidimesional Geometry, Lett. Nuovo Cimento 7, 361.
[6] Rauscher, E.A. (2005) Cosmogenises and quantum gravity, in R.L. Amoroso, B.Lehnert & J.P. Vigier (eds.) Beyond the standard model, pp. 43-72, Oakland: The Noetic Press.
[7] Rauscher, E.A. (1979) Superluminal Transformations in Complex Minkowski Spaces, LBL-9752; and (1980) Found. of Phys. 10, 661.
[8] Rauscher, E.A. (1979) The Iceland Papers: Select Papers On Experimental And Theoretical Research on Physics of Consciousness, Foreward by B. Josephson, and A. Puharich, Amherst: Essentia Research Associates, P.A.C.E.
[9] Rauscher, E.A. (1978) Complex Coordinate Geometries in General Relativity and Electromagnetism, Bull. Amer. Phys. Soc. 23, 84.
[10] Rauscher, E.A. (1981) Conceptual changes in reality from new discoveries in physics, pp. 1407-1431, proceeding of the 10th International Conferences on the Sciences, Volume II, the ICF Press, New York, E.P. Wigner, Comments on the support of the work of E.A. Rauscher, p. 1479.

[11] Ramon, C. & Rauscher, E.A. (1987) Remote connectedness in complex geometries, pp. 1423-1477, E.A. Rauscher, chair address pp. 1405-1407, Proceeding of the 11th Intl Conference on the Sciences, Volume II, New York: ICF Press.
[12] Rauscher, E.A. (1983) Electromagnetic Phenomena in Complex Geometries and Nonlinear Phenomena and Non-Hertzian Waves, Millbrae: Tesla Book Co.
[13] Rauscher, E.A. (1983) Electromagnetic and nonlinear phenomena in complex Minkowski spaces, paper presented at the University of California, Los Angeles, and Bull. Amer. Phys. Soc. 20, 351.
[14] Rauscher, E.A. and Targ, R. (2008) Integration of a complex spacetime and nonlocality, pp. 121-146, D.P. Sheehan (ed.) AIP proceedings 863.
[15] Rauscher, E.A. and Targ, R. (2002) Why only four dimensions will not explain non-locality, Journal of Scientific Explorations 16, 655.
[16] Rauscher, E.A. and Amoroso, R.L. (2009) Relativistic physics in complex Minkowski space, nonlocality, and quantum physics, in M.C. Duffy & J., Levey (eds.) Ether Spacetime and Cosmology, pp. 23-45, Montreal: Aperion.
[17] Rauscher, E.A. & Amoroso, R.L. (2008) The Schrödinger equation in complex Minkowski space, nonlocality and anticipatory systems, in D.M. Dubois (ed.) pp. 370-388, International Jour. Computing Anticipation Systems.
[18] Amoroso, R.L. & Rauscher, E.A. (2009) The Holographic Anthropic Multiverse, Singapore: World Scientific Pub.
[19] Haramein, N. & Rauscher, E.A. (2005) Collective coherent plasma modes in the media surrounding black holes, in R.L. Amoroso, B. Lehnert & J.P. Vigier (eds.) Searching for Unity in Physics, Oakland: The Noetic Press.
[20] Rauscher, E.A. (1968), Electron interactions and quantum plasma physics, J. Plasma Phys. 2, 517.
[21] Rauscher, E.A. and Van Bise, W. (1988) Non-superconducting Apparatus for Detecting Magnetic and Electromagnetic Fields, U.S. Patent, 4,724,390, February 9.
[22] Penrose, P. and Newman, E.J., (1978) Proc. Roy. Soc. 363, 445.
[23] Rauscher, E.A. (2010), NORA
[24] Rauscher, E.A. and Van Bise, W. (1988) External Magnetic Field Impulse Pacemaker Not Invasive Method and Apparatus for Modulating Brain Signals Through an External Magnetic or Electric Field to Pace the Heart and Reduce Pain, U.S. Patent Number 4,723,536, issued February 9.
[25] Rauscher, E.A. and Van Bise, W. Non-invasive Method and Apparatus for Modulating Brain Signals Through an External Magnetic or Electric Field to Reduce Pain, U.S. Patent Number 4,889,526, issued December 26, 1989.
[26] Rauscher, E.A. and Van Bise, W. (1988) Magnetic field interaction with macro biological systems with applications to effects on physiology, in J.M. Srinivasan (ed.) Medicine Around the World, pp. 205-218, Phoenix: Gabrial Press, Print World, Nov., Proceedings from conference at the Madras Institute of Magnetobiology 1987).
[27] Amoroso, R.L. (2009), Universal quantum computing: anticipatory parameters predicting bulk implementation, in D.M. Dubois (ed.) pp. 254-282, International Jour. of Computing Anticipatory Systems.
[28] Bell, J.S. (1964), Physics 1, 195.
[29] Aharonov, Y. and Bohm, D. (1959) Phys. Rev. 115,485.
[30] Silver Threads: 25 Years of Parapsychology Research (1993), eds. B. Kane, J. Millary and D. Brown and W. Harman Araeges, Conn.
[31] Prigogine, A. (1993) Chaotic Dynamics and Transport in Fluids and Plasmas: Research Trends in Physics Series, New York: American Institute of Physics.
[32] Rauscher, E.A. (1973) Superconductivity and Superfluidity in Multidimensional Geometries, LBNL, LBL-1572.
[33] Hansen, R.O. and Newman, E.T. (1975) Gen. Rel. and Grav. 6, 216.
[34] Newman, E.T. (1973) J. Math. Phys. 14, 774.
[35] Tipler, F.J. (1974) Phys. Rev. D9, 2203.
[36] B.S. DeWitt and N. Graham (eds.) (1973) The Many Worlds Interpretation of Quantum Mechanics, with papers by H,. Everett, N. Graham and J.A. Wheeler, Princeton: Pinceton Univ. Press.
[37] Kaluza, Th., (1921) *sitz. Berlin Press, A. Kad. Wis*. 1968.
[38] Klein, O. (1926) *Z. Phys.* 37, 805.
[39] Klein, O. (1927) *Z. Phys.* 41, 407.
[40] Newton, R.G. (1967) Phys. Rev. 162,1274.

[41] Feinberg, G. (1967) Phys. Rev. 159, 1089; Feinberg, G. (1970) Particles that go faster than light, Sci Am, 222, p. 68.
[42] Benford, G.A., Book, D.LO. and Newcomb, (1970), Tachyonic Anti-telephone, Phys. Rev. D2, 263.
[43] Rauscher, E.A. and Amoroso, R.L. (2006) The physical implications of multidimensional geometries and measurement, pp. 161-177, in D.M. Dubois (ed.) Internatiobnal Journal of Computing Anticipation Systems 19.
[44] Bilaniuk, O.M.P, Sundersham, E.C.G, & Deshpande, V.K. (1962) Am J Physics, 30, 718.
[45] Chew, G. (1964) Analytic S-Matix, Benjamin Pub.
[46] Weyl, H. (1918), Math. Z. 20, 384.
[47] Weyl, H. (1957), Space-Time-Matter, Dover Pub.
[48] Penrose, R. (1967) J. Math. Phys. 8, 345.
[49] Dirac, P.A.M. (1928) The Quantum Theory of the Electron, Proceedings of the Royal Society of London, Series A, 117, 778, pp. 610-624.
[50] de Sitter, W. (1916) Monthly Notices, RAS 77,155.
[51] Evan, G.T. and Sen, D.K. (1973) J. Math. Phys. 14, 1668.
[52] Stratton, T.A. (1941) Electromagnetic Theory, McGraw Hill.
[53] Wyler, A. (1969) Acad. Sc. Paris, Series A, 269, 743.
[54] Wyler, A. (1971) Acad. Sc. Paris, Series A, 271, 186.