The Complexification of Maxwell's Equations

Richard L Amoroso[1] & Elizabeth A Rauscher
[1]amoroso@noeticadvancedstudies.us

In this chapter we demonstrate that complex electric and magnetic fields are consistent with a geometry consisting of complex spacetime. We thus demonstrate that complex spacetime coordinates are not inconsistent with electromagnetic phenomena and may point to a direction for its unification with gravitational phenomena, in the weak Weyl field limit. The particular case we examine in detail is for an electron in a field where we derive Coulomb's equation. We examine this unification using the Weyl geometry in the linear approximation of the gravitational field.

Should we not then use the equations of motion in high-energy as well as low energy physics? I say we should. A theory with mathematical beauty is more likely to be correct than an ugly one that fits some experimental data. –
Albert Einstein

1. Complex Electromagnetic Fields

The linear approximation of Weyl geometry [1-4] for the gravitational field is consistent with the conditions of the 5D Kaluza-Klein geometry [5,6]. We present the formalism for the complexification of the electric and magnetic fields in this approach. We obtain additional symmetry conditions on the classical form of Maxwell's equations; and we obtain a non-zero divergence condition for the magnetic field which may be identifiable with a magnetic monopole term.

The relationship of the geodesic world lines and the electromagnetic field lines involve the definition of the field line structure. The field lines represent equipotential surfaces or they are lines connecting equipotential surfaces on a field map. For the gravitational tensor potential, $g_{\mu\nu}$ this map is the geodesic path on the light cone, i.e., the path that a photon will take according to the least action principle. We can similarly define an electromagnetic vector potential in analogy to $g_{\mu\nu}$ which we denote, A_μ. We use the formalism of Weyl to describe the manner in which we can derive Maxwell's equations, and in particular, Coulomb's law from the properties of A_μ. We then expand this formalism to include electromagnetic field components with real and imaginary parts and discuss the implications of this formalism. We also relate this formalism into our complex spacetime multidimensional geometry and then demonstrate that a complex "space" can be represented as a multidimensional real space with complex rotation represented by a generalized Lorentz transformation, Θ. It is likely that the transformation Θ includes all the affine connections. See Fig. 1.

Inomata [7] and Rauscher [8-13] introduce a simple but elegant concept - complex components to the electric and magnetic field vectors. He starts from Maxwell's equations in their usual form for an electromagnetic media for electric charge, ρ_E and electric current, J_E. Then we write Maxwell's equations in their usual form [14] which build on the extensive work of Faraday and others [15]:

$$\nabla \cdot \underline{E} = 4\pi\rho_E \qquad \nabla \times \underline{E} = -\frac{1}{c}\frac{\partial \underline{B}}{\partial t}$$
$$\nabla \cdot \underline{B} = 0 \qquad \nabla \times \underline{B} = -\frac{1}{c}\frac{\partial \underline{E}}{\partial t} + \underline{J}_E \tag{1}$$

To introduce symmetry to Maxwell's equations one can introduce an imaginary "magnetic" charge, $i\rho$ or $i\rho_{\mathrm{Im}}$ and imaginary "magnetic" current, iJ_M or iJ_{Im}, where again $i \equiv \sqrt{-1}$ and ρ_M and J_M are real quantities. Upon substitution into Maxwell's equations, we have

$$\nabla \cdot \underline{E} = 4\pi\rho_E \qquad \nabla \times \underline{E} = -\frac{1}{c}\frac{\partial \underline{B}}{\partial t} + i\underline{J}_M$$
$$\nabla \cdot \underline{B} = 4\pi i\rho_M \qquad \nabla \times \underline{B} = -\frac{1}{c}\frac{\partial \underline{E}}{\partial t} + \underline{J}_E \tag{2}$$

In this form we see that there are no real terms for the magnetic charge or current in terms such as $4\pi i\rho_M$ and iJ_m. Now we can derive real forms of Maxwell's equations by introducing complex *E* and *B* fields and separating real and imaginary parts of the equations.

Consider both the electric and magnetic fields to be complex quantities, that is

$$\underline{E} = \underline{E}_{\mathrm{Re}} + i\underline{E}_{\mathrm{Im}}, \qquad \underline{B} = \underline{B}_{\mathrm{Re}} + i\underline{B}_{\mathrm{Im}} \tag{3}$$

where $E_{\mathrm{Re}}, E_{\mathrm{Im}}, B_{\mathrm{Re}}$ and B_{Im} are real quantities, then substitution of these two equations into the complex form of Maxwell's equations above yields, upon separation of real and imaginary parts, two sets of Maxwell-like equations where the real parts are the usual Maxwell's equations:

$$\nabla \cdot \underline{E}_{\mathrm{Re}} = 4\pi\rho_E \qquad \nabla \times \underline{E}_{\mathrm{Re}} = -\frac{1}{c}\frac{\partial \underline{B}_{\mathrm{Re}}}{\partial t}$$
$$\nabla \cdot \underline{B}_{\mathrm{Re}} = 0 \qquad \nabla \times \underline{B}_{\mathrm{Re}} = \frac{1}{c}\frac{\partial \underline{E}_{\mathrm{Re}}}{\partial t} + J_E \tag{4}$$

where $\rho_E = \rho_{\mathrm{Re}}$ and $\rho_M = \rho_{\mathrm{Im}}$ and $J_E = J_{\mathrm{Re}}$ -and $J_M = J_{\mathrm{Im}}$ for the imaginary parts:

$$\nabla \cdot \left(i\underline{E}_{\mathrm{Im}}\right) = 0 \qquad \nabla \times \left(i\underline{E}_{\mathrm{Im}}\right) = -\frac{1}{c}\frac{\partial\left(i\underline{B}_{\mathrm{Im}}\right)}{\partial t} + i\underline{J}_M$$
$$\nabla \cdot \left(i\underline{B}_{\mathrm{Im}}\right) = 4\pi i\rho_M \qquad \nabla \times \left(i\underline{B}_{\mathrm{Im}}\right) = \frac{1}{c}\frac{\partial\left(i\underline{E}_{\mathrm{Im}}\right)}{\partial t} \tag{5}$$

Note that the *i* drops from both sides of each equation, giving real equations in all cases.

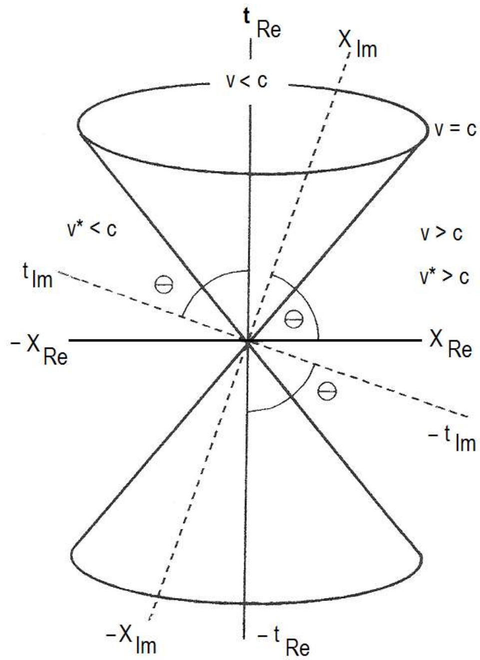

Figure 1 In the complex multidimensional space model we introduce, in addition to the usual orthogonal 4-space, four imaginary components, three spatial and one temporal. This is necessary in order to model remote connectedness and to retain the physical causality and symmetry conditions of conventional complex numbers. We can consider the eight orthogonal dimensions to be constituents of two intersecting light cones, one axis of real (*x,t*) and the other axis of imaginary (*y,t*) coordinates.

The real part of the electric and magnetic fields yield the usual Maxwell equations and the complex parts generates a "mirror" set of equations; for example, the divergence of the real component of the magnetic field is zero but the divergence of the imaginary part of the electric field is zero, and so forth. The imaginary part of the equations, the imaginary electric term replaces the real magnetic term, and vice versa. The structure of the real and imaginary parts of the fields form a symmetry in which electric real components are substituted by the imaginary part of the magnetic fields and the real part of the magnetic field being substituted by the imaginary part of the electric field in the second set of the equations [7,16].

The charge density and current density are expressed as complex quantities based on the separation of Maxwell's equations above. The complex generalized form for charge density and current is given as,

$$\rho = \rho_E + i\rho_M = \rho_{\text{Re}} + i\rho_{\text{Im}}$$

and (6)

$$J = J_E + iJ_M = J_{\text{Re}} + J_{\text{Im}}$$

where it may be possible to associate the imaginary complex charge with the magnetic monopole and, conversely, the electric current has an associated imaginary magnetic current.

The above definitions for the complex form of ρ and J appear to be interesting, where we let $\rho_E = \rho_{\text{Re}}$ and $\rho_M = \rho_{\text{Im}}$ and also $J_E = J_{\text{Re}}$ and $J_M = J_{\text{Im}}$ as before. For some interpretations we may not necessarily identify ρ_{Re} and J_{Re} as electric terms and ρ_{Im} and J_{Im} as magnetic terms. See

[7,16] as there are other ways to examine the complexification of the E and B fields.

By considering the "mirror" imaginary B_{Im} and E_{Im} fields of the real E_{Re} and B_{Re} field we may have an explanation of electrostatic cooling. Extensive research on this effect, and the theoretical approach to electromagnetic cooling has been conducted by Rauscher and Beal [17,18]. If J_E is neglected then we have the usual case where $\nabla \cdot E_{\mathrm{Re}} = 4\pi\rho_E$ and $\nabla \times E_{\mathrm{Im}} = 0$ so that no extra or anomalous terms appear. In [19], Dirac suggested a model similar to ours and to that of Inomata. Considering the imaginary part of Maxwell's equations in complex form we have $\nabla \cdot \underline{B}_{\mathrm{Re}} = 4\pi\rho_M$, where identification of $\rho_M = \rho_{\mathrm{Im}}$ is reasonable and where the i term is eliminated from both sides of the equation. Then B_{Re} and ρ_M are real and we consider only real derivatives in the del operation. Later we will examine the complex form of ∇ and perform complex derivatives where we use the transformations $x = x_{\mathrm{Re}} + x_{\mathrm{Im}}$ and $t = t_{\mathrm{Re}} + it_{\mathrm{Im}}$ and other complex metric forms.

If we take $\nabla \times \underline{B}_{\mathrm{Im}} = 0$ then we have $\partial \underline{E}_{\mathrm{Im}} / \partial t = 0$ and if also $\nabla \times \underline{E}_{\mathrm{Im}} = 0$ then we have $\partial \underline{B}_{\mathrm{Im}} / \partial t = c\underline{J}_{\mathrm{Im}}$. We identify the temporal change of the imaginary part of the magnetic field term. If we use the definition $\underline{B} = \underline{B}_{\mathrm{Im}} + i\underline{B}_{\mathrm{Im}}$ then we can take the total magnetic derivative as

$$\frac{\partial \underline{B}}{\partial t} = \frac{\partial \underline{B}_{\mathrm{Re}}}{\partial t} + \frac{\partial \underline{B}_{\mathrm{Im}}}{\partial t} = c\left(\underline{J}_{\mathrm{Re}} + i\underline{J}_{\mathrm{Im}}\right) \tag{7}$$

and again, we find the association of $\partial \underline{B}_{\mathrm{Im}} / \partial t = c\underline{J}_{\mathrm{Im}}$ We may be able to identify J_{Im} with a magnetic current, J_M, and associate a putative magnetic monopole current having one sign with the imaginary "mirror" part of the magnetic field. Before we proceed further with a physical interpretation of the imaginary component of the magnetic field, let us examine two issues in detail.

This formulation will assist us in understanding the physical interpretation of the complex model of Maxwell's equation. Currently we consider are the relationship between the complex form of E and B to the complex spacetime geometry and also the consideration of complex ($\underline{A}, \phi$) as a more useful and perhaps more primary interpretation of electromagnetic phenomena, rather than E and B.

2. Complex Electromagnetic Variables in Complex Multidimensional Spaces

We proceed from our 8D geometry. In [8,9], we defined the notation for the transformations, $x' \to x + ix$ and $t' \to t + i\tau$ which we have denoted as $x \to x_{\mathrm{Re}} + ix_{\mathrm{Im}}$ and $t \to t_{\mathrm{Re}} + it_{\mathrm{Im}}$. We can also denote $x \to x_1 + ix_2$ and $t \to t_1 + it_2$ in analogy to $E = E_1 + iE_2$ and $B = B_1 + iB_2$. We denote $E = E_{\mathrm{Re}} + iE_{\mathrm{Im}}$ and $B = B_{\mathrm{Re}} + iB_{\mathrm{Im}}$ as before.

In [8,9] we define a method for taking complex derivatives and apply this method to our examination of the Schrödinger equation in a complex Minkowski space. See Chap. 2. Because of the linear superposition principle approximation [10], we can solve the real and imaginary parts of the equation separately and sum them in the approximation of a small deviation from linearity. In the case of our calculation of the curl and divergence terms in Maxwell's equations we can no longer, to first order, make the linear approximation assumption.

We can define the divergence operation in the complex multidimensional geometry for a general vector $\underline{A} = \underline{A}_{\mathrm{Re}} + i\underline{A}_{\mathrm{Im}}$ (not to be confused with the vector potential, A_μ) where we have a vector form

$A(x,y,z) = \hat{x}A_x + \hat{y}A_y + \hat{z}A_z$ where each component A_x can be written as $A_x = A_{x\mathrm{Re}} + iA_{x\mathrm{Im}}$, etc. Then

$$\nabla \cdot \underline{A} = \left[\frac{\partial \underline{A}}{\partial x_{\mathrm{Re}}} + \frac{\partial \underline{A}}{\partial y_{\mathrm{Re}}} + \frac{\partial \underline{A}}{\partial z_{\mathrm{Re}}} + \frac{\partial \underline{A}}{\partial i x_{\mathrm{Im}}} + \frac{\partial \underline{A}}{\partial y_{\mathrm{Im}}} + \frac{\partial \underline{A}}{\partial z_{\mathrm{Im}}} \right] \tag{8}$$

We have $\underline{A} = A_{x\mathrm{Re}} + A_{y\mathrm{Re}} + A_{z\mathrm{Re}} + iA_{x\mathrm{Im}} + iA_{y\mathrm{Im}} + iA_{z\mathrm{Im}}$. Upon substitution we have twelve terms, six are real and six are imaginary. For $\nabla \cdot \underline{A}$, we have for $x \rightarrow x_{\mathrm{Re}} + ix_{\mathrm{Im}}$

$$\begin{aligned} \nabla \cdot \underline{A} = &+ \frac{\partial A_{x\mathrm{Re}}}{\partial_{x\mathrm{Re}}} + \frac{\partial A_{y\mathrm{Re}}}{\partial_{x\mathrm{Re}}} + \frac{\partial A_{z\mathrm{Re}}}{\partial_{x\mathrm{Re}}} - \frac{i\partial A_{x\mathrm{Im}}}{\partial_{x\mathrm{Re}}} - \frac{i\partial A_{y\mathrm{Im}}}{\partial_{x\mathrm{Re}}} - \frac{i\partial A_{z\mathrm{Im}}}{\partial_{x\mathrm{Re}}} \\ &- i\frac{\partial A_{x\mathrm{Re}}}{\partial_{x\mathrm{Im}}} - i\frac{\partial A_{y\mathrm{Re}}}{\partial_{x\mathrm{Im}}} - i\frac{\partial A_{z\mathrm{Re}}}{\partial_{x\mathrm{Im}}} + \frac{\partial A_{x\mathrm{Im}}}{\partial_{x\mathrm{Im}}} + \frac{\partial A_{y\mathrm{Im}}}{\partial_{x\mathrm{Im}}} + \frac{\partial A_{z\mathrm{Im}}}{\partial_{x\mathrm{Im}}} \end{aligned} \tag{9}$$

Also there are twelve terms for the partial derivatives in $y \rightarrow y_{\mathrm{Re}} + iy_{\mathrm{Im}}$ and another twelve in terms of the partial derivatives of $z \rightarrow z_{\mathrm{Re}} + iz_{\mathrm{Im}}$ We address the dependence of A and its components as $A(x_{\mathrm{Re}}, y_{\mathrm{Re}}, z_{\mathrm{Re}}, x_{\mathrm{Im}}, y_{\mathrm{Im}}, z_{\mathrm{Im}})$. Also we have dependence of A and its components on other components; for example, we can have $A_{x\mathrm{Re}}(x_{\mathrm{Re}}, \ldots, A_{y\mathrm{Re}}, A_{z\mathrm{Re}}, A_{x\mathrm{Im}}, \ldots)$. Let us assume that when we consider A as the general symbol for E and B, that they are dependent only on real and imaginary components of space and time. In such a case we also have another twelve terms for $t \rightarrow t_{\mathrm{Re}} + it_{\mathrm{Im}}$ totaling forty-eight terms.

We can use certain approximations to examine the forms of the complex electromagnetic fields in complex spacetime. We will see that more general forms are useful in examining energy transmission for transverse and longitudinal components. Consider the two divergent forms of Maxwell's equations. We have $\nabla \cdot \underline{E} = 4\pi\rho$ and $\nabla \cdot \underline{B} = 0$. If we then write $E = E_{\mathrm{Re}} + iE_{\mathrm{Im}}$ and also $B = B_{\mathrm{Re}} + iB_{\mathrm{Im}}$ we have $E(x_{\mathrm{Re}}, y_{\mathrm{Re}}, z_{\mathrm{Re}}, x_{\mathrm{Im}}, y_{\mathrm{Im}}, z_{\mathrm{Im}}, t_{\mathrm{Re}}, t_{\mathrm{Im}})$. However let us consider only that $E(x_{\mathrm{Re}}, x_{\mathrm{Im}})$, $B(x_{\mathrm{Re}}, x_{\mathrm{Im}})$ and $\rho(x_{\mathrm{Re}}, x_{\mathrm{Im}})$, or more specifically that $E_{x\mathrm{Re}}(x_{\mathrm{Re}}, x_{\mathrm{Im}})$ and $E_{\mathrm{Im}}(x_{\mathrm{Re}}, x_{\mathrm{Im}})$. Now

$$\begin{aligned} \nabla \cdot E &= \frac{\partial E_{x\mathrm{Re}}}{\partial x_{\mathrm{Re}}} + i\frac{\partial E_{x\mathrm{Im}}}{\partial x_{\mathrm{Re}}} - i\frac{\partial E_{x\mathrm{Re}}}{\partial x_{\mathrm{Im}}} + \frac{\partial E_{\mathrm{Im}}}{\partial x_{\mathrm{Im}}}\,. \\ &= 4\pi(\rho_{\mathrm{Re}} + i\rho_{\mathrm{Im}}) \end{aligned} \tag{10}$$

Collecting real and imaginary terms, we have two equations:

$$\frac{\partial E_{x\mathrm{Re}}}{\partial x_{\mathrm{Re}}} + \frac{\partial E_{x\mathrm{Im}}}{\partial x_{\mathrm{Im}}} = 4\pi\rho_{\mathrm{Re}}, \tag{11}$$

and

$$\frac{\partial E_{x\mathrm{Im}}}{\partial x_{\mathrm{Re}}} - \frac{\partial E_{x\mathrm{Re}}}{\partial x_{\mathrm{Im}}} = \rho_{\mathrm{Im}}. \tag{12}$$

Note now that the real and imaginary components are mixed.

In a similar manner we can write two similar expressions for $\nabla \cdot B = 0$ for real and imaginary components in complex space as

$$\frac{\partial B_{x\mathrm{Re}}}{\partial x_{\mathrm{Re}}} + \frac{\partial B_{x\mathrm{Im}}}{\partial x_{\mathrm{Im}}} = 0 \tag{13}$$

and

$$\frac{B_{x\mathrm{Im}}}{x_{\mathrm{Re}}} - \frac{B_{x\mathrm{Re}}}{x_{\mathrm{Im}}} = 0. \tag{14}$$

Again real and imaginary components are mixed, but since *i* exists on both sides of the second above equation, all four of the above equations are completely real. These equations are very restrictive in terms of purely spatial, and not temporal, dependence, and that $A_{x\mathrm{Re}}$ and $A_{x\mathrm{Im}}$ are taken as dependent on x_{Re} and x_{Im} where we take the term *A* as either *E* or *B*. In general, other terms such as $E_{x\mathrm{Re}}(y_{\mathrm{Re}}, t_{\mathrm{Im}}, E_{\mathrm{Re}})$, etc. can come into effect and we can approximate these by terms such as ε in *E* and β in *B* in the above equations so that terms in $\varepsilon_{\mathrm{Re}}(x_{\mathrm{Re}}, x_{\mathrm{Im}})$, etc., which appear as additional terms which we can consider to be small compared to the terms in the previous four equations. Perhaps terms such as $\beta(x_{\mathrm{Re}}, x_{\mathrm{Im}})$ and others might also act as effective terms. For example, we could write

$$\frac{\partial B_{x\mathrm{Re}}}{\partial x_{\mathrm{Re}}} + \frac{\partial B_{x\mathrm{Im}}}{\partial x_{\mathrm{Im}}} \equiv \frac{\partial B}{\partial x_{\mathrm{Re}}} \geq 0 \tag{15}$$

The above formalism does not represent strictly a projective geometry but is related to the concept that 4-space is a slice through a complex multidimensional space. We will make certain approximations which simplify the equations but they still remain nonlinear and give *E* and *B* fields of the form of $\mathrm{Sinh}^2(x)$, for parameter *x* [19-24].

We will examine in more detail how a projective geometrical form of the complex *E* and *B* fields form Hertzian as well as non-Hertzian waves. Then $\partial B_{x\mathrm{Re}} / \partial x_{\mathrm{Re}} = -\partial B_{x\mathrm{Im}} / \partial x_{\mathrm{Im}}$. The term on the right may be associated with a term in ρ such as ρ_{Im} or ρ_M. For example, we may have a form $\partial B_{x\mathrm{Re}} / \partial x_{\mathrm{Re}} = \rho_M$ or, in general, for the consideration of all components, $\nabla \cdot B = \rho_M$. The shadow imaginary terms to the real usual terms may supply insight as to new ways of interpreting conventional as well as novel electromagnetic phenomena. We will consider these issues in more detail in the Higgs field approximation

We turn our attention to the full detailed consideration of the set of derivatives involving complex $\underline{E}$ and $\underline{B}$ in complex spacetime. We use $\underline{E} = \underline{E}_{\mathrm{Re}} + i\underline{E}_{\mathrm{Im}}$ and $\underline{B} = \underline{B}_{\mathrm{Re}} + i\underline{B}_{\mathrm{Im}}$ and $x = x_{\mathrm{Re}} + ix_{\mathrm{Im}}$ and $t = t_{\mathrm{Re}} + t_{\mathrm{Im}}$; all terms such as $\underline{E}_{\mathrm{Re}}, \underline{E}_{\mathrm{Im}}, \underline{B}_{\mathrm{Re}}, \underline{B}_{\mathrm{Im}}$ and $x_{\mathrm{Re}}, x_{\mathrm{Im}}$ are real.

We use the Cauchy-Riemann relations [8-10]: $f(z) = u(x, y) + iv(x, y)$

and

$$f'(z) = \frac{\partial f}{\partial z} = \frac{\partial \mu}{\partial x} + i\frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} - i\frac{\partial \mu}{\partial y} \tag{16}$$

for $x = x + iy$

Now consider the definitions

$$\mu(x, y) \equiv E_{\mathrm{Re}}(x_{\mathrm{Re}}, x_{\mathrm{Im}}), \quad v(x, y) \equiv E_{\mathrm{Im}}(x_{\mathrm{Re}}, x_{\mathrm{Im}}). \tag{17}$$

Then

$$f(z) = E(X_{\mathrm{Re}} + iX_{\mathrm{Im}}) \tag{18}$$

for $z \equiv x_{\mathrm{Re}} + ix_{\mathrm{Im}}$.

We have the two equations for $f'(z)$:

$$f'(z) = \frac{df}{dz} = \frac{\partial E_{\mathrm{Re}}}{\partial x_{\mathrm{Re}}} + i\frac{\partial E_{\mathrm{Im}}}{\partial x_{\mathrm{Re}}} = \frac{\partial E_{\mathrm{Im}}}{\partial x_{\mathrm{Im}}} - \frac{i\partial E_{\mathrm{Re}}}{\partial x_{\mathrm{Im}}} \tag{19}$$

Returning to $\nabla \cdot \underline{E} = 4\pi\rho$, we have

$$\nabla \cdot \underline{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 4\pi\rho$$

or (20)

$$\nabla \cdot \underline{E} = \frac{\partial E_x}{\partial(x_{\mathrm{Re}} + ix_{\mathrm{Im}})} + \frac{\partial E_y}{\partial(y_{\mathrm{Re}} + iy_{\mathrm{Im}})} + \frac{\partial E_z}{\partial(z_{\mathrm{Re}} + iz_{\mathrm{Im}})} = 4\pi\rho$$

Using the Cauchy-Riemann relations there are two equations for $\nabla \cdot \underline{E}$,

$$\begin{aligned}&\frac{\partial E_{x\mathrm{Re}}}{\partial x_{\mathrm{Re}}} + i\frac{\partial E_{x\mathrm{Im}}}{\partial x_{\mathrm{Re}}} + \frac{\partial E_{y\mathrm{Re}}}{\partial y_{\mathrm{Re}}} + i\frac{\partial E_{y\mathrm{Im}}}{\partial y_{\mathrm{Re}}} + \frac{\partial E_{z\mathrm{Re}}}{\partial z_{\mathrm{Re}}} + i\frac{\partial E_{z\mathrm{Im}}}{\partial z_{\mathrm{Re}}} \\ &\quad = 4\pi(\rho_{\mathrm{Re}} + i\rho_{\mathrm{Im}})\end{aligned} \tag{21}$$

And

$$\begin{aligned}&\frac{\partial E_{\mathrm{Im}}}{\partial x_{\mathrm{Im}}} - i\frac{\partial E_{\mathrm{Re}}}{\partial x_{\mathrm{Im}}} + \frac{\partial E_{y\mathrm{Im}}}{\partial y_{\mathrm{Im}}} - \frac{i\partial E_{y\mathrm{Re}}}{\partial y_{\mathrm{Im}}} + \frac{i\partial E_{z\mathrm{Re}}}{\partial z_{\mathrm{Im}}} + \frac{i\partial E_{z\mathrm{Re}}}{\partial z_{\mathrm{Im}}} \\ &\quad = 4\pi(\rho_{\mathrm{Re}} + i\rho_{\mathrm{Im}})\end{aligned} \tag{22}$$

The above equations in terms of real spatial derivatives can be separated into real and imaginary terms as

$$\frac{\partial E_{\mathrm{Re}}}{\partial x_{\mathrm{Re}}} + \frac{\partial E_{y\mathrm{Re}}}{\partial y_{\mathrm{Re}}} + \frac{\partial E_{z\mathrm{Re}}}{\partial z_{\mathrm{Re}}} = 4\pi\rho_{\mathrm{Re}} \tag{23}$$

which is the usual Maxwell equation $(\nabla \cdot E_{\mathrm{Re}} = 4\pi\rho_{\mathrm{Re}})$. We also have the "mirror" equation

$$\frac{\partial E_{x\,\mathrm{Im}}}{\partial x_{\mathrm{Re}}}+\frac{\partial E_{y\,\mathrm{Im}}}{\partial y_{\mathrm{Re}}}+\frac{\partial E_{z\,\mathrm{Im}}}{\partial z_{\mathrm{Re}}}=4\pi\rho_{\mathrm{Im}} \qquad (24)$$

where the *i* is canceled. This equation appears to be $\nabla\cdot\underline{E}_{\mathrm{Im}}=4\pi\rho_{\mathrm{Im}}$ as before.

For the second equation for $\nabla\cdot E=4\pi\rho$ from the Cauchy-Riemann relation. We can write two equations in terms of the imaginary parts of space

$$\frac{\partial E_{\mathrm{Im}}}{\partial x_{\mathrm{Im}}}+\frac{\partial E_{y\,\mathrm{Im}}}{\partial y_{\mathrm{Im}}}+\frac{\partial E_{z\,\mathrm{Im}}}{\partial z_{\mathrm{Im}}}=4\pi\rho_{\mathrm{Re}}$$

and (25)

$$\frac{\partial E_{\mathrm{Re}}}{\partial x_{\mathrm{Im}}}+\frac{\partial E_{y\,\mathrm{Re}}}{\partial y_{\mathrm{Im}}}+\frac{\partial E_{z\,\mathrm{Re}}}{\partial z_{\mathrm{Im}}}=-4\pi\rho_{\mathrm{Im}}$$

in which we have multiplied through by *-i.*

Let us define a new *del* operator in terms of imaginary components of space. We define this as, ∇_{Im} and the usual *del* operator, ∇ interchangeably as ∇_{Re}. Then we have our latter two equations which become

$$\nabla_{\mathrm{Im}}\cdot\underline{E}_{\mathrm{Im}}=4\pi\rho_{\mathrm{Re}} \quad \text{and} \quad \nabla_{\mathrm{Im}}\cdot\underline{E}_{\mathrm{Re}}=-4\pi\rho_{\mathrm{Im}} \qquad (26)$$

giving us two more unique new equations. Note the minus sign in the density term in the above equation. Similarly, we can write a set of $\nabla\cdot\underline{B}=0$ and have $\nabla\cdot\underline{B}_{\mathrm{Re}}=0$, $\nabla\cdot\underline{B}_{\mathrm{Im}}=0$, $\nabla_{\mathrm{Im}}\cdot\underline{B}_{\mathrm{Re}}=0$ and $\nabla_{\mathrm{Im}}\cdot\underline{B}_{\mathrm{Im}}=0$. We can write forms such as $\nabla\cdot\underline{B}_{\mathrm{Re}}=\nabla\cdot\underline{B}_{\mathrm{Im}}$ where we identify the term $\nabla\cdot B_{\mathrm{Im}}$ as a monopole component. We discuss this further in terms of the Higgs solitons model.

The Higgs mechanism involves the carriers of the electroweak force, the $W^{\pm},Z^{0}$ Bosons which are hypothesized in analogy to the massless or near massless photon whereas standard hadrons, leptons and pions have mass which requires an explanation. Higgs et al [13,25,26] suggests that there was an undetected field, the Higgs field, filling the universe. The concept is that a massless Boson such as a photon could absorb a Higgs Boson and create a massive particle. Salam and Weinberg utilized the Higgs mechanism in a renormalized form to develop the electroweak theory [26]. It has been suggested that the CERN LHC Tevatron may produce enough energy to uncover the elusive Higgs particle. The question becomes, how does an all pervasive Higgs field filling the universe relate to the nature of the vacuum plenum?

Elsewhere we have given clear indications that a small photon mass, m_{γ} probably exists [13,25]. The physics community has thought this would interfere with Gauge Theory, but this is not the case because Gauge Theory is only an approximation. This is a key indicator of M-Theory where Planck's constant, $\hbar$ is no longer fundamental but must be modulated by string tension T_s [25]. M-Theory, is based essentially on one parameter, string tension, T_S

$$T_S = e / l = (2\pi\alpha')^{-1}; \tag{27}$$

where e is energy, l is length of the string and α the fine structure constant, $e^2 / \hbar c$ where this e is the electron charge. It is well known that the gauge condition is an approximation suggesting Planck's constant, $\hbar$ needs to be recalculated to satisfy the parameters of M-Theory [25]. Since our HAM cosmology is aligned with an extension of Einstein's energy-dependent spacetime metric $\hat{M}^{3(4)}$, (or the alternate (+++-) convention) the Stoney e^2 / c, an electromagnetic precursor to Planck's constant, [25] is therefore the choice for studying the recalculation. The factor added to $\hbar$ is string tension T_S, where T_0 can increase the size of $\hbar$ to the Larmour radius of the hydrogen atom in the small scale and lead to infinite size additional dimensionality cosmologically. Thus the fine-tuned Stoney, $\lambda\!\!\!^{-}$ and the cosmological constant, Λ adjust the microscopic and cosmological domain limits of H_R respectively. Equation (28) illustrates the initial historical basis for this distinction

$$l_P = \sqrt{\frac{\hbar}{mc} \cdot \frac{Gm}{c^2}} \quad or \quad l_S = \sqrt{\frac{e^2}{4\pi\varepsilon_0 mc^2} \cdot \frac{Gm}{c^2}}, \tag{28}$$

where $l_P \; and \; l_S$ are the length of the Planck and Stoney respectively.

One example for rescaling Planck's constant comes from Wolf [25]

$$\Delta x \Delta \rho = h \to h_0 \pm \Delta h. \tag{29}$$

He then suggests that

$$\Delta h = \frac{hv^2}{c} \tau_0 L_0 \tag{30}$$

where τ_0 and L_0 are time uncertainty and a discrete spacetime correction respectively. Wolf is able to speculate that this Planck rescaling has application to Neutron stars, CMBR and black hole formation. Our approach for a time, τ_0 and spacetime corrections, L_0 are different [25].

What does this mean for the Higgs mechanism? There are new topological conditions in Calabi-Yau mirror symmetry. With the addition of the parameters of string tension and string coupling to the fundamental structural-phenomenology of the nature of matter, mass arises in the 'topological charge' associated with the annihilation-creation vectors of the wave structure of matter in an extended view of the de Broglie-Bohm interpretation of quantum field theory. See Chap. 12.

We examine the equations involving the curl operation. When we calculate the curl of complex E and B fields in a complex geometry we have vector components and the curl operation becomes much more complicated. This is because, for a specific vector component, we have partial derivative terms as functions of other independent variables. We proceed from the standard form of the curl for a general vector $\underline{A} = \hat{x}A_x + \hat{y}A_y + \hat{z}A_z$ as

$$\nabla \times A = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$= \hat{x}\left[\frac{\partial}{\partial y}A_z - \frac{\partial}{\partial z}A_y\right] + \hat{y}\left[\frac{\partial}{\partial z}A_x - \frac{\partial}{\partial x}A_z\right] \qquad (31)$$

$$= \hat{z}\left[\frac{\partial}{\partial x}A_y - \frac{\partial}{\partial y}A_x\right]$$

where $\hat{x}, \hat{y}, \hat{z}$ are unit vectors and $\nabla \times A$ is a vector quantity. This is the usual three spatial dimensional quantity. The *del* operation, ∇ can be formed as the D'Alembertian operator, $\Box$ with $\Box = \nabla - (1/c^2)(\partial^2/\partial t^2)$ which includes *ct* terms. If we again write $\underline{A} = \underline{A}_{\text{Re}} + i\underline{A}_{\text{Im}}$ and also the complexified form of space and time, then we will have many more terms as part of the $\hat{x}_{\text{Re}}, \hat{y}_{\text{Re}}, \hat{z}_{\text{Re}}$ components as well as $\hat{x}_{\text{Im}}, \hat{y}_{\text{Im}}, \hat{z}_{\text{Im}}$ for $\hat{x}_{\text{Re}} \equiv \hat{x},$ etc. If we turn our attention to the curl expressions such as $\nabla \times E = -(1/c)(\partial B/\partial t)$ then we can consider *E* and *B* as cases of the general form of *A* (not to be confused with the vector potential of $(\underline{A}, \phi)$. The usual curl is derived for a 3 x 3 matrix. Consider the components $(\hat{x}_{\text{Re}}, \hat{y}_{\text{Re}}, \hat{z}_{\text{Re}}, \hat{x}_{\text{Im}}, \hat{y}_{\text{Im}}$ and $\hat{z}_{\text{Im}})$. Then we can write the generalized curl as

$$\nabla \times A = \begin{vmatrix} \hat{x}_{\text{Re}} & \hat{y}_{\text{Re}} & \hat{z}_{\text{Re}} & \hat{x}_{\text{Im}} & \hat{y}_{\text{Im}} & \hat{z}_{\text{Im}} \\ \dfrac{\partial}{\partial x_{\text{Re}}} & \dfrac{\partial}{\partial y_{\text{Re}}} & \dfrac{\partial}{\partial z_{\text{Re}}} & \dfrac{\partial}{\partial x_{\text{Im}}} & \dfrac{\partial}{\partial y_{\text{Im}}} & \dfrac{\partial}{\partial z_{\text{Im}}} \\ A_{x\text{Re}} & A_{y\text{Re}} & A_{z\text{Re}} & A_{x\text{Im}} & A_{y\text{Im}} & A_{z\text{Im}} \end{vmatrix} \qquad (32)$$

which forms a 3 x 6 matrix.

This generalized form is necessary for analyzing $\nabla \times E = -(1/c)(\partial B/\partial t)$ and $\nabla \times \underline{B} = -\underline{1}(\partial \underline{E}/\partial t) + \underline{J}$ for complex $\underline{E}$ and $\underline{B}$. (Note: We can handle coupling to other terms or additional terms can be handled as coupling to the usual terms which we can define as the coupling term g^2, as in [10] and Chap. 10).

Using the set of definitions, $E_x = E_{x\text{Re}} + iE_{xIM}$, $E_y = E_{y\text{Re}} + iE_{yIM}$, $E_z = E_{z\text{Re}} + iE_{zIM}$, $x = x_{\text{Re}} + ix_{\text{Im}}$, $y = y_{\text{Re}} + iy_{\text{Im}}$, $z = z_{\text{Re}} + iz_{\text{Im}}$, and also $\hat{e}_x = \hat{e}_{x\text{Re}} + i\hat{e}_{x\text{Im}}$, $\hat{e}_y = \hat{e}_{y\text{Re}} + i\hat{e}_{y\text{Im}}$, $\hat{e}_z = \hat{e}_{z\text{Re}} + i\hat{e}_{z\text{Im}}$. We formed a vector addition for the limit vector coordinates. We can also form the modulus length as $|\hat{e}_x|^2 = \hat{e}_{x\text{Re}}^2 + \hat{e}_{x\text{Im}}^2$. For unit dimensions, $|\hat{e}_x|^2 = 2$. Expressing the usual form of the curl of $\underline{E}$, we can use the above equations to calculate $\nabla \times \underline{E}$ as

$$\nabla \times E = \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \hat{e}_x\left[\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right] + \hat{e}_y\left[\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right] + \hat{e}_z\left[\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right]. \tag{33}$$

Using the above expression for complex forms of $\underline{E}$ and $\underline{x}$ we can write

$$\nabla \times E = (\hat{e}_{x\mathrm{Re}} + i\hat{e}_{x\mathrm{Im}})\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) + (\hat{e}_{y\mathrm{Re}} + i\hat{e}_{y\mathrm{Im}})\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right) + (\hat{e}_{z\mathrm{Re}} + i\hat{e}_{z\mathrm{Im}})\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right). \tag{34}$$

We can express the term in e_x as term I_{ex},

$$= (\hat{e}_{x\mathrm{Re}} + i\hat{e}_{x\mathrm{Im}})\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) = \hat{e}_{x\mathrm{Re}}\left(\frac{\partial E_z}{\partial y} - \hat{e}_{x\mathrm{Re}}\frac{\partial E_y}{\partial z}\right) + i\hat{e}_{x\mathrm{Im}}\left(\frac{\partial E_z}{\partial y} - i\hat{e}_{x\mathrm{Im}}\frac{\partial E_y}{\partial z}\right). \tag{35}$$

Applying the Cauchy-Riemann relations to the terms in $\hat{e}_x$ we have

$$I\hat{e}_x = \hat{e}_{x\mathrm{Re}}\left(\frac{\partial E_{z\mathrm{Re}}}{\partial y_{\mathrm{Re}}} + i\frac{\partial E_{z\mathrm{Im}}}{\partial y_{\mathrm{Re}}}\right) - \hat{e}_{x\mathrm{Re}}\left(\frac{\partial E_{y\mathrm{Re}}}{\partial z_{\mathrm{Re}}} + i\frac{\partial E_{y\mathrm{Im}}}{\partial z_{\mathrm{Re}}}\right) + i\hat{e}_{x\mathrm{Im}}\left(\frac{\partial E_{z\mathrm{Re}}}{\partial y_{\mathrm{Re}}} + i\frac{\partial E_{z\mathrm{Im}}}{\partial y_{\mathrm{Re}}}\right) - i\hat{e}_{x\mathrm{Im}}\left(\frac{\partial E_{y\mathrm{Re}}}{\partial z_{\mathrm{Re}}} + i\frac{\partial E_{y\mathrm{Im}}}{\partial z_{\mathrm{Re}}}\right). \tag{36}$$

We also have another set of terms which we define as $I'_{\hat{e}x}$ from the other of the Cauchy-Riemann relations

$$I'_{\hat{e}x} = \hat{e}_{x\mathrm{Re}}\left(\frac{\partial E_{z\mathrm{Im}}}{\partial y_{\mathrm{Im}}} - i\frac{\partial E_{z\mathrm{Re}}}{\partial y_{\mathrm{Im}}}\right) - \hat{e}_{x\mathrm{Re}}\left(\frac{\partial E_{y\mathrm{Im}}}{\partial z_{\mathrm{Im}}} - i\frac{\partial E_{y\mathrm{Re}}}{\partial z_{\mathrm{Im}}}\right) + i\hat{e}_{x\mathrm{Im}}\left(\frac{\partial E_{z\mathrm{Im}}}{\partial y_{\mathrm{Im}}} - i\frac{\partial E_{z\mathrm{Re}}}{\partial y_{\mathrm{Im}}}\right) - i\hat{e}_{x\mathrm{Im}}\left(\frac{\partial E_{y\mathrm{Im}}}{\partial z_{\mathrm{Im}}} - i\frac{\partial E_{y\mathrm{Re}}}{\partial z_{\mathrm{Im}}}\right). \tag{37}$$

Separation into real and imaginary parts of $I_{\hat{e}x}$ and $I'_{\hat{e}x}$ can be per-formed. For $I_{\hat{e}x}$ we have

$$I_{\hat{e}x\,\mathrm{Re}} = \hat{e}_{x\,\mathrm{Re}}\left(\frac{\partial E_{z\,\mathrm{Re}}}{\partial y_{\mathrm{Re}}}\right) - \hat{e}_{x\,\mathrm{Re}}\left(\frac{\partial E_{y\,\mathrm{Re}}}{\partial z_{\mathrm{Re}}}\right) - \hat{e}_{x\,\mathrm{Im}}\left(\frac{\partial E_{z\,\mathrm{Im}}}{\partial y_{\mathrm{Re}}}\right) + \hat{e}_{x\,\mathrm{Im}}\left(\frac{\partial E_{y\,\mathrm{Im}}}{\partial z_{\mathrm{Re}}}\right)$$
$$= \hat{e}_{x\,\mathrm{Re}}\left(\frac{\partial E_{z\,\mathrm{Re}}}{\partial y_{\mathrm{Re}}} - \frac{\partial E_{y\,\mathrm{Re}}}{\partial z_{\mathrm{Re}}}\right) - \hat{e}_{x\,\mathrm{Im}}\left(\frac{\partial E_{z\,\mathrm{Im}}}{\partial y_{\mathrm{Re}}} + \frac{\partial E_{y\,\mathrm{Im}}}{\partial z_{\mathrm{Re}}}\right) \qquad (5.38)$$

and for $I'_{\hat{e}x}$ we have

$$I'_{\hat{e}x\,\mathrm{Im}} = i\hat{e}_{x\,\mathrm{Re}}\left(\frac{\partial E_{z\,\mathrm{Im}}}{\partial y_{\mathrm{Re}}}\right) - i\hat{e}_{x\,\mathrm{Re}}\left(\frac{\partial E_{y\,\mathrm{Im}}}{\partial z_{\mathrm{Re}}}\right) + i\hat{e}_{x\,\mathrm{Im}}\left(\frac{\partial E_{z\,\mathrm{Re}}}{\partial y_{\mathrm{Re}}}\right) - i\hat{e}_{x\,\mathrm{Im}}\left(\frac{\partial E_{y\,\mathrm{Re}}}{\partial z_{\mathrm{Re}}}\right)$$
$$= i\hat{e}_{x\,\mathrm{Re}}\left(\frac{\partial E_{z\,\mathrm{Im}}}{\partial y_{\mathrm{Re}}} - \frac{\partial E_{y\,\mathrm{Im}}}{\partial z_{\mathrm{Re}}}\right) - i\hat{e}_{x\,\mathrm{Im}}\left(\frac{\partial E_{z\,\mathrm{Re}}}{\partial y_{\mathrm{Re}}} - \frac{\partial E_{y\,\mathrm{Re}}}{\partial z_{\mathrm{Re}}}\right) \qquad (5.39)$$

We have eight terms for $I_{\hat{e}x}$ and also eight terms for $I'_{\hat{e}x}$. Therefore, there are sixteen terms for the $\hat{e}_x$ term of $\nabla \times \underline{E}$.

For all three components ($\hat{e}_x, \hat{e}_y$ and $\hat{e}_z$) of the curl, we have a total of forty-eight terms. Returning to $\hat{e}_x$ terms only then, let us consider these terms only in $\nabla \times \underline{E} = -1/c\,(\partial B/\partial t)$. From $I_{\hat{e}\,\mathrm{Re}}$, we have, using the separation of $\underline{B}$ into real and imaginary parts and using the *x* component only,

$$\hat{e}_{x\,\mathrm{Re}}\left(\frac{\partial E_{z\,\mathrm{Re}}}{\partial y_{\mathrm{Re}}} + \frac{\partial E_{y\,\mathrm{Re}}}{\partial z_{\mathrm{Re}}}\right) - \hat{e}_{x\,\mathrm{Im}}\left(\frac{\partial E_{z\,\mathrm{Im}}}{\partial y_{\mathrm{Re}}} + \frac{\partial E_{y\,\mathrm{Im}}}{\partial z_{\mathrm{Re}}}\right)$$
$$= -\frac{1}{c}\frac{\partial B_{x\,\mathrm{Re}}}{\partial x_{\mathrm{Re}}} \qquad (40)$$

where we use the expression as

$$\frac{\partial \underline{B}}{\partial t} = \frac{\partial B_x}{\partial t} + \frac{\partial B_y}{\partial t} + \frac{\partial B_z}{\partial t} \qquad (41)$$

and applying the Cauchy-Riemann relations to the *x* component of *B* we have for the temporal element $t = t_{\mathrm{Re}} + it_{\mathrm{Im}}$, for $\partial B_x / \partial t$, then $(\partial B_{x\,\mathrm{Re}}/\partial t_{\mathrm{Re}}) + i(\partial B_{x\,\mathrm{Im}}/\partial t_{\mathrm{Re}})$.

For real parts we consider the $\partial B_{x\,\mathrm{Re}}/\partial t_{\mathrm{Re}}$ term only, which we use in the above equation. We can define a term in terms of the imaginary directed component $\hat{e}_{x\,\mathrm{Im}}$; let

$$g^2 A_{\mathrm{Im}}\left(x_{\mathrm{Re}}, y_{\mathrm{Re}}, z_{\mathrm{Re}}\right) \equiv -\hat{e}_{x\,\mathrm{Im}}\left(\frac{\partial E_{z\,\mathrm{Im}}}{\partial y_{\mathrm{Re}}} + \frac{\partial E_{y\,\mathrm{Im}}}{\partial z_{\mathrm{Re}}}\right) \qquad (42)$$

so that the expression now reads

$$\hat{e}_{x\mathrm{Re}}\left(\frac{\partial E_{z\mathrm{Re}}}{\partial y_{\mathrm{Re}}}+\frac{\partial E_{y\mathrm{Re}}}{\partial z_{\mathrm{Re}}}\right)+g^2\mathrm{A}_{\mathrm{Im}}\left(x_{\mathrm{Re}},y_{\mathrm{Re}},z_{\mathrm{Re}}\right)=-\frac{1}{c}\frac{\partial B_{x\mathrm{Re}}}{\partial x_{\mathrm{Re}}}. \quad (43)$$

Terms not incorporated into the $g^2\mathrm{A}$ term comprise the usual Maxwell equation. We consider g^2 to be a coupling to a small order perturbation term given by $\mathrm{A}_{\mathrm{Im}}\left(x_{\mathrm{Re}},y_{\mathrm{Re}},z_{\mathrm{Re}}\right)$, where our components projected from the imaginary components of *E* and *B* lie on the imaginary axis $\hat{e}_{x\mathrm{Im}}$. Contributions from other imaginary dimensions of space and time can yield contributions that give rise to transverse components of the electromagnetic field and can contribute to energy transmission terms.

From $I_{\hat{e}x\mathrm{Im}}$ we can also form the equation

$$\hat{e}_{x\mathrm{Re}}\left(\frac{\partial E_{z\mathrm{Re}}}{\partial y_{\mathrm{Re}}}+\frac{\partial E_{y\mathrm{Im}}}{\partial z_{\mathrm{Re}}}\right)-\hat{e}_{x\mathrm{Im}}\left(\frac{\partial E_{z\mathrm{Re}}}{\partial y_{\mathrm{Re}}}-\frac{\partial E_{y\mathrm{Im}}}{\partial z_{\mathrm{Re}}}\right)=-\frac{1}{c}\left(\frac{\partial B_{x\mathrm{Im}}}{\partial t_{\mathrm{Re}}}\right) \quad (44)$$

where the "*i*'s" cancel from both sides. The terms in this equation are components of the $\hat{e}_{x\mathrm{Im}}$ direction.

Separation into real and imaginary parts are made for terms in $I'_{\hat{e}x}$ from the second coupling relation. For the real part we have

$$I'_{\hat{e}_{x\mathrm{Re}}}=\hat{e}_{x\mathrm{Re}}\frac{\partial E_{y\mathrm{Im}}}{\partial z_{\mathrm{Im}}}-\hat{e}_{x\mathrm{Re}}\frac{\partial E_{z\mathrm{Im}}}{\partial y_{\mathrm{Im}}}+\hat{e}_{x\mathrm{Im}}\frac{\partial E_{y\mathrm{Re}}}{\partial z_{\mathrm{Im}}}-\hat{e}_{x\mathrm{Im}}\frac{\partial E_{z\mathrm{Re}}}{\partial y_{\mathrm{Im}}} \quad (45)$$

and similarly for the imaginary parts $I'_{\hat{e}_{x\mathrm{Im}}}$. All these terms are in $x_{\mathrm{Im}},y_{\mathrm{Im}}$ and. z_{Im}. A similar process can be done for $I_{\hat{e}y},I'_{\hat{e}y}$ and $I_{\hat{e}z},I'_{\hat{e}z}$. In general, we can write

$$\nabla\times\left(\underline{E}_{\mathrm{Re}}+i\underline{E}_{\mathrm{Im}}\right)=\nabla\times\underline{E}_{\mathrm{Re}}+i\nabla\times\underline{E}_{\mathrm{Im}}$$

and (46)

$$\nabla_{\mathrm{Im}}\times\left(\underline{E}_{\mathrm{Re}}+i\underline{E}_{\mathrm{Im}}\right)=\nabla_{\mathrm{Im}}\times\underline{E}_{\mathrm{Re}}+i\nabla_{\mathrm{Im}}\times\underline{E}_{\mathrm{Im}}$$

For current purposes, we will not explore terms in ∇_{Im} which involve $\partial/\partial x_{\mathrm{Im}},\partial/\partial y_{\mathrm{Im}},\partial/\partial z_{\mathrm{Im}}$, etc. We will briefly discuss the relationship of the complex electric and magnetic fields, complex spacetime metrics [8,9] and the interpretation of models of the magnetic monopole.

3. Complex Electromagnetic Field Vectors, Virtual Energy States and Magnetic Monopole Interpretations

We will briefly discuss some issues related to magnetic monopole model interpretations. Let us start from the metric element measure for fields associated with electric and magnetic charge. Essentially, if monopoles exist they will fill in the zeroes in Maxwell's equations. Comparing $\nabla\cdot\underline{E}=4\pi\rho_R$ and $\nabla\cdot\underline{B}=0$ and $\nabla\times\underline{B}-(1/c)(\partial\underline{E}/\partial t)=(4\pi/c)\underline{J}_E$ to $\nabla\times\underline{E}-(1/c)(\partial\underline{B}/\partial t)=0$ indicates complete symmetry if the zeroes on the right side were replaced by ρ_B and J_B respectively. In relativistic notation we have for the electric current $J^{\nu}_{(E)}$, $\partial F^{\mu\nu}/\partial x_{\mu} = (4\pi/c)J^{\nu}_{(E)}$ and $\partial F^{\mu\nu}/\partial x_{\mu} = 0$. If monopole fields exist the right side of the second equation would be written in terms of a 4D magnetic current $J^{\nu}_{(B)}$.

Dirac hypothesizes that the pole strength of a magnetic monopole-like electric charge would be quantized and that a conservation principle for monopole strength would exist analogous to electric charge conservation principles. In [27] we examine the role of magnetic monopoles in a real multidimensional geometry. We demonstrate that the form of the quantized monopole introduced by Schwinger [27], in which the electric and magnetic charge is put on an equal footing, is consistent with the n-dimensional Descartes geometry [27,28]. If we have $e^2/\hbar c = \alpha \sim 1/137.037$, where α is the fine structure constant, we can form an analogous expression: $em/\hbar c = n$ where n is an integer 28]. This expression defines a quantized form of the magnetic monopole.

In the Dirac monopole model [19], (where m is the 'magnetic charge' which is termed g in Schwinger's notation) if the product of the pole strengths are given as $em = n\hbar c$ and $n = \frac{1}{2}$ (the smallest quantum value), then this gives $m \cong 68.5$ times the value of e. In the Schwinger model, n is taken as unity so that $m \cong 137e$. The latter value is the one usually considered in experimental explorations.

The set of assumptions for the Schwinger monopole is one of the simplest there is; it is the monopole structure for which most experimental detectors are designed to determine if monopoles exist. This picture brings into question the whole issue of the nature of charge as a quantized entity. We discuss the possibility of a more complete expression of charge as a quantum number in [24,28,29].

Teller [30] suggests that monopole detection will be made only in very high energy experiments. Alvarez [31], and his group conducted extensive monopole detection studies. Silvers presents some theoretical formulations that are relevant to the experimental detection of magnetic monopoles. Attempts have been made to find monopoles in moon rocks [32] by looking at heavy ionized radiation damage tracks [33]. Wheeler [34] has developed expressions for quantized charge which may have relevance to monopole formulation and detection.

4. Higgs Field Magnetic Monopole

Our model of plasma instabilities and superconductivity are based on the field theoretic approach. Both Abelian and non-Abelian fields are considered. The Abelian Higgs field can be represented as a 3D kink soliton which acts like a bare point soliton. We might identify such a system as a "vortex." In four dimensions we can identify a non-Abelian soliton as a static monopole [35].

The common definition in the quantum solutions of the sine-Gordon equation is that the institon is a finite action entity in space and time which is associated with the content of the vacuum. In elementary particle physics this institon state could be identified with the quark-gluon states. The soliton solution is an entity of finite energy in space [36] and time and is associated with the quark states in elementary particle physics.

The 3D Abelian Higgs confined field soliton, in the same absence of symmetry breaking, defines quark confinement [37]. The Lagrangian, L for the Higgs field is given as

$$L = \frac{1}{2} F_{\eta\nu} F^{\mu\nu} + \left| \frac{\partial\phi}{\partial x} + ieA\phi \right|^2 - \frac{g^2}{4} \left(|\psi|^2 - h^2 \right)^2 \qquad (47)$$

where g^2 is a coupling term (which acts like a potential) to a nonlinear field factor, and h^2 is an additional field term. The Higgs-Goldstone spontaneous splitting is due to the field shift term *h*.

The electromagnetic field $F_{\mu\nu}$ is given in terms of the four-vector potential by $F_{\eta\nu} = (\partial A_\mu / \partial x_\mu) - (\partial A_\nu / \partial x_\nu)$ where the vector potential, A_μ transforms as a gauge $A_\mu \to A_\mu - (1/e)h$ which defines the quantity *h*. The indexes μ and ν runs 1 to 4. The phase ψ represents the kink in the Higgs field in 3D. The form of ψ is given by its periodic form $\psi \propto e^{in\theta}$. We define the $A_\mu = (\underline{A}_j, \phi)$, where the index *j* runs 1 to 3, in their usual four space form. We use ϕ to represent the temporal component of the potential field A_μ where $\underline{A}_j$ is the vector potential.

Let us consider photon activation of pair production of a retarded (forward in time) and advanced (backward in time) potential waves in an analogy to the Cramer Transactional model [13]. The usual physical gauge condition gives $\psi = 0$ but for our coupling soliton theory, the kink $\psi \propto e^{in\theta}$ cannot be transformed away. The stability of the vortex solutions depends on the finite value of *n*. The gauge condition in the space with kink solitons becomes

$$A_\mu^{'} \to A_\mu - \frac{1}{e} n \frac{\partial \theta}{\partial x_\mu} \tag{48}$$

The Lagrangian, *L* gives the trajectories of the soliton where *A* is considered as the pair producing photon field. Solitons are coupled as a 1/*e* term and dominate as the coupling term g^2 becomes larger. See Chap. 8 for generalized extended Gauge conditions.

In [10,35] we discuss how soliton solutions to the nonlinear Schrödinger equation relate to the kink monopole soliton. It is actually through the relativistic formalism for the soliton solutions of the Dirac equation that we see that the kink soliton monopole is one such solution [35]. See Chap. 12. Both the Schrödinger [10] and Dirac equation are solved in the complex Minkowski space which contributes the nonlinear term leading to the soliton solutions. The soliton retains its identity in space and time and acts as a field particle that acts as a signal for remote connectedness events. The form of the soliton explains the source of the effect of the vacuum state virtual states. The exciton (pair production) couples to acoustic or acustiton modes giving rise to the soliton solution (Chaps. 10 and 12).

The Higgs field monopole relates to the symmetry term in the complex form of Maxwell equations. The current solution to the electromagnetic equations are of nonrelativistic form. The Higgs field method is a relativistic form. We will outline a relativistic complexification of Maxwell's equations.

5. Some Further Speculations on Monopole Structures

The relation $F_{\mu\nu} = (\partial A^\mu / \partial x_\nu) - (\partial A^\nu / \partial x_\mu)$ insures that the divergence of the *B* field is zero. In the condition where monopoles are allowed the condition on the relationship of $F_{\mu\nu}$ in terms of A^μ is relaxed. We can write an expression in terms of a monopole, field strength, *m*. Then we can write a for

$$F_{\mu\nu} = \frac{\partial A^\mu}{\partial x_\nu} - \frac{\partial A^\nu}{\partial x_\mu} - m\delta(f) f_{\mu\nu} \tag{49}$$

where *f* is an arbitrary given function of space, *x,y,z* and $f_{\mu\nu} = \{(\partial f^\mu / \partial x_\nu) - (\partial f^\nu / \partial x_\mu)\}$.

A number of tests for monopoles have been explored. Eberhard summarizes some of these, including the Price, Lexan controversial plate examination [38] Dirac has suggested a possible test using a soliton model. The form of the coupling constant, m, will then depend on the geometric form of the soliton. A quantum theory can be constructed for specific types of monopoles. We can define a form for m from the soliton model $A(\phi) = m/4\pi\rho$ where $\rho = \sqrt{x^2 + y^2}$ for an x directionally oriented solenoid axis and ϕ is the zenith angle of (ρ, θ, ϕ). Consider the flux Φ and we then use the monopole condition $\partial\Phi/\partial\phi = iem/2\pi$ [39,45]. If we consider the quantized flux condition in superconductivity vacua, such as $(\Psi) \neq 0$, then Ψ^+ acts as a creation operator and Ψ as a destruction operator for magnetic charge. Asymptotically we have $(\partial\Phi/\partial x^\mu) - ieA_\mu\Phi = 0$ with solutions of the form

$$\Phi \propto \Phi_0 e^{ie\int dx^\mu A_\mu(x)} \tag{50}$$

with the quantized condition for a closed path,

$$e\oint dx^\mu A_\mu(x) = 2\pi n \tag{51}$$

where n is an integer. So the quantized flux can be considered to be obeying the condition $2\pi n / e$. This condition holds for an infinite solenoid on the z axis (Aharanov-Bohm experiment).

More detailed consideration along this line may be fruitful to design a test for a possible monopole utilizing a solenoid configuration [41]. See Chap. 4. A more detailed examination of this picture and the suggested experiment by the Eyring Research Group should be made in which they suggest a test of the issues connecting $\underline{E}$ and $\underline{B}$ and $\underline{A}$ and ϕ [42]. In a suggested experiment by Mandelstam [43], gauge invariance and Poincairé invariance conditions need evaluation. The complexification of Maxwell's equations give us a detailed manner in which to formulate the nonlinear coupled terms, g^2.

6. The Structure of Non-Hertzian Waves in Complex Geometries and Electromagnetic Energy Transmission

Heinreich Hertz made two contributions that had a major influence on the interpretation of the nature and structure of electromagnetic waves. Maxwell had already shown the intimate relationship between electric and magnetic phenomena which had drawn together many of the discoveries by Faraday [15]. One of the two issues that Hertz put forward was that radio and light waves were part of the same phenomena; i.e. part of the electromagnetic spectra. The other was that electromagnetic waves were composed of the continuous orthogonal oscillations of electric and magnetic vector components transverse to the direction of motion. These oscillations traveled at the velocity of light (Maxwell) and the velocity of light is a constant in all frames of Einstein.

The former proposition of Hertz led to a coherent picture of many phenomena (such as radio, light, x-rays, and γ-rays) as part of the electromagnetic spectra. The condition on the vector oscillations of $\underline{E}$ and $\underline{B}$ may have been too restrictive and also that longitudinal components may exist and may have most significant implications [44-47]. Because of the great success of the former issue the second consideration was readily accepted. There was also a lack of understanding of Tesla's energy transmission ideas in his cryptic patents and also he was unable to complete vital tests of his ideas due to loss of funding from J.P. Morgan and his family [44]. Therefore the issue of longitudinal components of $\underline{E}$ and $\underline{B}$ and their possible interpretation as effects on A_μ or (A_μ, ϕ) was summarily dismissed from classical electromagnetic theory. The Aharonov-Bohm experiment appears to show that the (A_μ, ϕ) fields are detectable outside of the action of the $\underline{E}$ and $\underline{B}$ fields.

The ground wave and the ionospheric wave are set up in such a manner as to produce the predicted 1.57 ratio to the velocity of light which was stated by Tesla in one of his patents [44,47]. In his model Tesla treated the Earth as a finite capacitative reactance component surrounded by an ion shell of variable altitude, beginning at about 50 km in height, which represents a system whereby a resonant ringing signal can be set up and transmitted. Although the system represents a leaky capacitor with a Q of about 4 to 5 it is possible to set up a resonant state that appears as though a signal is transmitted and received from any two points on the Earth's surface. In actuality, according to the Rauscher-Van Bise model, the signal is not 'transmitted and received' but represents a nonlocal global coherent state. Any event which can 'wiggle' the static Earth-ionosphere magnetic flux is transmitted as both a local and nonlocal influence.

We will discuss in this section one model of non-Hertzian waves and suggest that there may be more modest tests of longitudinal wave effects and energy transmission than the major energy transmission program Tesla envisioned [44-48]. Some possible considerations for experiments may involve a solenoid Aharonov-Bohm type experiment and certain antenna designs for transmission and reception of significant signal, energy information and perhaps polarization experiments. See Chap. 4

If we consider the complex form of $\underline{E}$ and $\underline{B}$ then we can consider an orthogonal space in which the real components $\underline{E}_{\mathrm{Re}}$ and $\underline{B}_{\mathrm{Re}}$ are transverse projections to the direction of propagation of the wave and are the usual transverse components. The orthogonal components $\underline{E}_{\mathrm{Im}}$ and $\underline{B}_{\mathrm{Im}}$ (where $\underline{E}_{\mathrm{Im}}$ and $\underline{B}_{\mathrm{Im}}$ themselves are real) are projections on the direction of propagation of the wave and comprise the longitudinal components. These longitudinal components may act in an acoustic-like or acusticon motion $\underline{E}_{\mathrm{Re}}$, $\underline{B}_{\mathrm{Re}}$, $\underline{E}_{\mathrm{Im}}$ and $\underline{B}_{\mathrm{Im}}$ are all mutually orthogonal although models can be considered in which, although maxima of $\underline{E}_{\mathrm{Re}}$ and $\underline{B}_{\mathrm{Re}}$ are 90° out of phase, those of $\underline{E}_{\mathrm{Im}}$ and $\underline{B}_{\mathrm{Im}}$ can be in phase or 90° out of phase [49].

Longitudinal oscillations of $\underline{E}_{\mathrm{Im}}$ and $\underline{B}_{\mathrm{Im}}$ (See Fig. 2) appear as presence and absence of these fields varying from maximum projection of $\underline{E}$ and $\underline{B}$ to zero projection on the direction of propagation. The constraint conditions $\underline{E} = \underline{E}_{\mathrm{Re}} + i\underline{E}_{\mathrm{Im}}$ and $\underline{B} = \underline{B}_{\mathrm{Re}} + i\underline{B}_{\mathrm{Im}}$ but we can also express the relationship between transverse and longitudinal components as $\underline{E} = \underline{E}_{\mathrm{Re}} + ie\underline{E}_{\mathrm{Im}}$ and $\underline{B} = \underline{B}_{\mathrm{Re}} + ib\underline{B}_{\mathrm{Im}}$ where e and b can be chosen to be greater than or less than unity. This way we can determine the relationship between the magnitude of the transverse and longitudinal components. The existence of the imaginary components of $\underline{E}$ and $\underline{B}$ derive their existence from the imaginary components of space and time. Dependent relationships such as $\underline{E}_{\mathrm{Im}}(x_{\mathrm{Im}}, t_{\mathrm{Im}})$ can be found as well as $\underline{E}_{\mathrm{Im}}(x_{\mathrm{Re}}, x_{\mathrm{Im}}, t_{\mathrm{Re}}, t_{\mathrm{Im}})$ can be formed. Essentially though, $\underline{E}_{\mathrm{Im}}$ and $\underline{B}_{\mathrm{Im}}$ derive their meaning from the components $(x_{\mathrm{Im}}, t_{\mathrm{Im}})$ as previously discussed.

New issues to address with the new formalism are primarily related to the possibility of non-Hertzian wave activity and transmission either in space or in a dielectric media. Possible means of "lossless" energy transmission or communication would necessarily involve non-Hertzian wave phenomena which does not attenuate in the usual $1/r^2$ diffusion mode. Of course laser light does not attenuate significantly in free space and is Hertzian and coherent, but a great amount of energy is not transmitted, nor can lasers be utilized (in their current form) to communicate with higher efficiency with undersea systems [50].

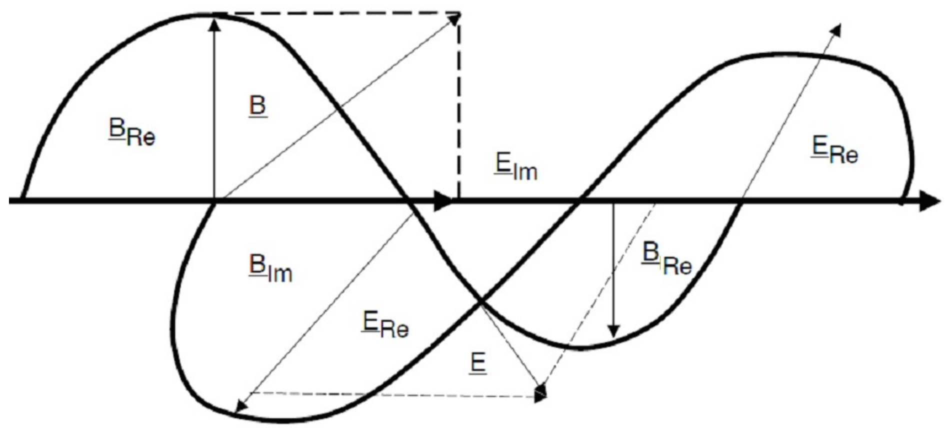

Figure 2 Hertzian and non-Hertzian waves. Where E and B are decoupled into two components $\underline{E}_{\mathrm{Re}}$ and $\underline{E}_{\mathrm{Im}}$ and $\underline{B}_{\mathrm{Re}}$ and $\underline{B}_{\mathrm{Im}}$.

Certain properties of the lasing phenomena do hold some clues for us because of its coherence properties. The possibility exists for utilization of the laser system (Hertzian wave) for remote communication, which can be formulated in terms of the remote connectedness properties of multidimensional geometries [8]. As stated before, phenomena such as Bell's interconnectedness theorem, Young's double slit phenomena, supercoherence phenomena and plasma instabilities (or coherent electron-electron states) etc. derive their properties and structures from the properties of remote connectedness and apparent superluminal connection in the 4-space as a subset of HD geometries [9,13].

Planck in his 1922 book on electromagnetic theory expresses the concept that energy is key to an understanding of Maxwell's equations and therefore proceeds from the Poynting vector, $\underline{S}$ (perpendicular to the vectors $\underline{E}$ and $\underline{B}$) which is in the direction of energy flow [50,51]. Note that S is also called the radiant vector. The electromagnetic energy field is calculated by the work to create the field on ergs

$$w = \frac{\varepsilon E^2 + \mu H^2}{8\pi} \qquad (52)$$

where ε is the dielectric constant or permittivity of the media and μ is the permeability (for free space or matter that is isotropic and non-ferromagnetic) and $c = 1/\sqrt{\varepsilon\mu}$ in matter and $c = 1/\sqrt{\varepsilon_0\mu_0}$ in vacuum. Then $\underline{S} = (c/4\pi)\underline{E}\times\underline{H}$, where $\underline{B} = \mu\underline{H}$ and the velocity in the direction of the Poynting vector is $\underline{v} = \underline{s}/w$ where $v = c$ is usually the case and where c is the velocity of light in vacuo. If we assume that each erg of moving energy has a mass of $1/c^2$, using $E = mc^2$ or a mass of about $1/(9\times10^{20})$ gm.; the energy in a cubic centimeter will have momentum equal to

$$\left(w/c^2\right)v = s/c^2 \qquad (53)$$

for $\underline{B} = \mu \underline{H}$, or similar expressions, depending on the media. In free space this is the magnitude of the momentum in unit volume so that the electromagnetic momentum in free space may be thought of as ordinary momentum possessed by the moving electromagnetic field.

The vectors $\underline{E}$ and $\underline{B}$ are represented as waves of electric and magnetic fields moving in a direction of propagation perpendicular to their amplitude variation. This variation is sinusoidal and transverse to the direction of propagation of the electromagnetic disturbance. For propagation in the *x* direction then, $E_y = f(x - vt)$, $E_x = E_z = 0$, and $H_z = \sqrt{\varepsilon / \mu}\ f(x - vt)$; $H_x = H_y = 0$ and $v = c / \sqrt{\varepsilon / \mu}$ so that the wave can be in a media or free space. Then we have a wave equation

$$-\sqrt{\frac{\varepsilon}{\mu}} \frac{\partial f}{\partial x} = \frac{\varepsilon}{c} \frac{\partial f}{\partial t} \tag{54}$$

for $E_y = A \sin 2\pi (\frac{x}{\lambda} vt)$ and $H_z = \sqrt{t/\mu}\ A \sin 2\pi (\frac{x}{\lambda} - vt)$ which are plane wave forms. Now let us briefly discuss possible longitudinal components.

It probably would not make sense to consider longitudinal vector modes along $\underline{S}$ but scalar modes may be perfectly acceptable. As indicated by other calculations, acoustic type collective excitations arise from coherent, collective, nonlinear phenomena. Consider the propagation of an acoustic type mode, which are described as a soliton, if interaction with a source term (or exciton term) exists. Such a mode will not involve a Poynting vector energy term and with a source term would not obey the usual $1/r^2$ dispersion. Actually, the recoherence from the nonlinear term overcomes the dispersion loss and disturbances do not eventually "wipe out", such as by water waves from a rock tossed into a pond, but retain their amplitude as in the soliton case [51]. Water waves cause interatomic friction and loss converts to heat in the water media. Electromagnetic energy disperses by dielectric (displacement currents), excitation of a media, and $1/r^2$ dispersion. The Hertzian wave momentum "pushes" through space.

The energy relationship for non-Hertzian waves is not of the form $S = (\varepsilon E^2 + \mu H^2)/8\pi$ does not fall off as $1/r^2$ with distance. The question then becomes, what is the energy content in standing and transmitted coherent non-dispersive waves such as solitons? Certain properties of ELF waves may not only depend on their extremely long wave lengths (~10^9 cm), but also on a possible mechanism for creating and transmitting extremely low frequency nonlinear waves. These may have some non-Hertzian properties particularly below 10KHz. These properties may explain low loss (non-attenuation) of wave energy and lack of frequency shifts when observed from different spatial locations in recently observed (since 1976) ELF phenomena [45]. The energy content is assumed to be distributed throughout the field in the direction of the Poynting vector, which is perpendicular to $\underline{E}$ and $\underline{B}$ and has a magnitude

$$S = \frac{c}{4\pi} EB \sin \theta. \tag{55}$$

As before the velocity of propagation in the direction of $\underline{S}$ is given as $v = S / \mathrm{E}$ and θ is the angle between $\underline{E}$ and $\underline{B}$.

The transverse mode may be associated with an acoustic-like wave of energy transmission. We have explored the manner in which acoustic modes reflect coherent, collective, nonlinear processes and relate to the coherent state, as modeled in the soliton physics of [10,24,45]. The soliton mode is pictured as a coupling of a collective acoustic mode to exciton (electron-positron) modes in a media. See Fig. 2.

Let us briefly examine a possible interpretation of a more general form of the electromagnetic field, E. We can consider complexification of E as $\mathrm{E} = \mathrm{E}_{\mathrm{Re}} + i\mathrm{E}_{\mathrm{Im}}$ Consider the terms $\mathrm{E} = \varepsilon_{\mathrm{Re}} + i\varepsilon_{\mathrm{Im}}$; $\mu = \mu_{\mathrm{Re}} + i\mu_{\mathrm{Im}}$; $E^2 = E_{\mathrm{Re}}^2 + E_{\mathrm{Im}}^2$ and $B^2 = B_{\mathrm{Re}}^2 + B_{\mathrm{Im}}^2$. We use the modulus of a vector form as $|E|^2 = EE^*$, for example. Then we can form E as

$$\mathrm{E}_{\mathrm{total}} = \frac{1}{8\pi}\Big\{\mathrm{E}_{\mathrm{Re}}E_{\mathrm{Re}}^2 + i\mathrm{E}_{\mathrm{Im}}E_{\mathrm{Re}}^2 + \mathrm{E}_{\mathrm{Re}}E_{\mathrm{Im}}^2 + i\mathrm{E}_{\mathrm{Im}}E_{\mathrm{Im}}^2 + \\ + \mu_{\mathrm{Re}}B_{\mathrm{Re}}^2 + i\mu_{\mathrm{Im}}B_{\mathrm{Re}}^2 + \mu_{\mathrm{Re}}B_{\mathrm{Im}}^2 + i\mu_{\mathrm{Im}}B_{\mathrm{Im}}^2\Big\}. \tag{56}$$

We collect the terms in E_{Re} and E_{Im} The usual terms in E are $\varepsilon_{\mathrm{Re}}E_{\mathrm{Re}}^2$ and $\mu_{\mathrm{Re}}B_{\mathrm{Re}}^2$. We also have real terms $\varepsilon_{\mathrm{Re}}E_{\mathrm{Im}}^2$ and $\mu_{\mathrm{Re}}B_{\mathrm{Im}}^2$ which comprise E_{Re}. The parts that comprise E_{Im} are given by $\varepsilon_{\mathrm{Im}}E_{\mathrm{Re}}^2$, $\varepsilon_{\mathrm{Im}}E_{\mathrm{Im}}^2$, etc., as

$$\mathrm{E}_{\mathrm{Im}} = \frac{1}{4\pi}\Big\{\varepsilon_{\mathrm{Im}}E_{\mathrm{Re}}^2 + \varepsilon_{\mathrm{Im}}E_{\mathrm{Im}}^2 + \mu_{\mathrm{Re}}B_{\mathrm{Re}}^2 + \mu_{\mathrm{Im}}B_{\mathrm{Im}}^2 + i\Big\}. \tag{57}$$

The traditional terms in E_{Re} as the usual terms as

$$\mathrm{E}_{\mathrm{Re}} = \frac{1}{4\pi}\Big\{\varepsilon_{\mathrm{Re}}E_{\mathrm{Re}}^2 + \mu_{\mathrm{Re}}B_{\mathrm{Re}}^2 + \varepsilon_{\mathrm{Re}}E_{\mathrm{Im}}^2 + \mu_{\mathrm{Re}}B_{\mathrm{Im}}^2\Big\}. \tag{58}$$

These latter two terms come from projected longitudinal components of the electromagnetic field. The usual components, $\varepsilon_{\mathrm{Re}}E_{\mathrm{Re}}^2$ and $\mu_{\mathrm{Re}}B_{\mathrm{Re}}^2$.

The corresponding longitudinal Poynting vector is given as

$$S' = \frac{c}{4\pi}EB\cos\theta \tag{59}$$

To be more precise, we have the usual transverse Poynting vector

$$S_{\mathrm{Re}} = \frac{c}{4\pi}E_{\mathrm{Re}}B_{\mathrm{Re}}\sin\theta \tag{60}$$

and the longitudinal Poynting vector

$$S_{\mathrm{Im}} = \frac{c}{4\pi}E_{\mathrm{Im}}B_{\mathrm{Im}}\cos\theta \tag{62}$$

In each case respectively the angle θ is defined between $\underline{E}_{\mathrm{Re}}$ and $\underline{B}_{\mathrm{Re}}$ or, in S_{Im} as between $\underline{E}_{\mathrm{Im}}$ and $\underline{B}_{\mathrm{Im}}$. These expressions depend on the assumption that both the transverse and longitudinal components are transmitted at the velocity of light, *c*, and that *c* retains its relationship with ε and μ .

The constraints on ε_{Im} and therefore on $\varepsilon_{\text{Im}} E_{\text{Im}}^2$, etc. terms, must be such as to retain the relationship between *c* and E_{total} and μ_{total}. If the acoustic (longitudinal acusticon) mode of transmission should occur at some other velocity, such as $v > c$, then we need to examine the whole issue of Lorentz invariance. See Chaps. 2 and 9. Feinberg [10] has demonstrated that $v > c$ signals for tachyonic particles with complex mass can occur [25] and arguments such as these have been demonstrated to be consistent with the complex Minkowski space [8,44]. See Chap. 2. In fact, the structure of the metric demands a superluminal signal. Note that Tesla described a non-Hertzian superluminal signal [44]. The form of the Poynting vector then reflects signaling, should it be detected, in which S_{Im} would depend on some general velocity $v > c$. The longitudinal acoustic mode then may require new considerations in experimental detection designs that involve some of the considerations in the concepts in tachyon detection. It may well be that the monopole is a tachyon and may therefore require similar approaches to those of attempted monopole detection [37] in which remote connection in the multidimensional Cartesian geometry is related to superluminal signals and magnetic monopoles. Also similar considerations are made for complex geometries [9].

Two main issues come to mind. First, can information be transmitted by a superluminal acoustic wave?, and second, can energy be transmitted by a superluminal acoustic wave? We have previously demonstrated that collective coherent acoustic modes occur in matter in complex Minkowski spaces [24] and that acoustic modes coupled with vacuum state polarization may account for a variety of coherent phenomena such as plasma instabilities and superconductivity. These phenomena appear to depend on the remote connectivity of the manifold which is well described by the complex geometry.

Orthogonality of $\underline{E}_{\text{Re}}, \underline{B}_{\text{Re}}$ and $\underline{E}_{\text{Im}}, \underline{B}_{\text{Im}}$ is insured. A frequency dependent interaction between transverse and longitudinal components could lead to a standing wave, configuration. A self-reinforcing configuration could develop which would allow remote information transfer and interaction. Essentially such a model would be analogous to the coherence configuration of a laser but also have properties of nonlocality; possibly of energy 'transmission' or simultaneous information effects such as Bell's Theorem. See Chap. 4. Precise geometric transmitters (antennas) which form a nonlinear geometric array would be necessary to transmit the "acoustic" longitudinal components of the field.

Possible biological effects from ELF radiation may be due to nonlinear tissue "windowing" [21-23,52-72].Nonlinear properties of tissues in which lipoproteins may act as receiving antennas could explain biological activity to ELF or higher frequency electromagnetic fields [24] which are not explained by the usual thermal effects, where intensity is below the half degree threshold. Additional calculations and interpretations are in progress which relate to both the laser coherence remote information effects (communication) and possible models of nonlinear transmitter receivers for ELF radiation.

Maxwell and Hertz primarily respectively dealt theoretically and experimentally with radio frequencies (RF) and above. Light can be produced by the excitation from charged particles such as e^- and p^+ in the atmosphere such as from lightning in the visible and x-ray region such as the sprits and jets in the upper atmosphere and the aurora borealis which lies above them. These phenomena tend to perturb the Earths steady state fields, as well as from solar wind activity leading to ULF, VLF and ELF phenomena. Most research has been in the MHz and above frequency region and only recently studied in geophysics [22,44,45,73-81] and biological science [20,21,52-72]. Maxwell's equations are wave equations and well described phenomena down to the upper KHz region but not so well for the ELF and VLF region of the electromagnetic spectrum. Some of the principles of the applications to low frequency phenomena can be listed as follows: Note that the standard Maxwell's equations fail in this region below about 10KHz because not only are Hertzian waves involved but so are non-Hertzian waves as formulated in this chapter. Phenomena in geophysics and biology exhibit both Hertzian and non-Hertzian phenomena and apply to the low end of the electromagnetic spectrum. Particularly in biology collective neuronal processes in the brain and cardiac system exhibit non-Hertzian receiving and transmitting modalities. Antenna designs are based on Hertzian wave phenomena and hence are not designed to pick up and receive non-Hertzian systems. Rauscher and Van Bise have designed such a system [22,44,45,73-81] which detects ELF, VLF and ULF signals involving Hertzian and non-Hertzian waves up to 500 KHz. These researchers have also applied this research to understanding and developing interactive systems with biological tissue to enhance its function through resonance coupling [20,21, 52-72].

7. Summary and Concluding Remarks

Complexifying and expanding Maxwell's equations allows us to understand and expand upon our knowledge of low intensity and low frequency phenomena which is consistent with collective resonant recoherence of electric and magnetic transverse and longitudinal phenomena which can accommodate nonlocal interactions. With our new formulation, a number of systems can be reexamined and new ones developed. Some of the areas of research that can be examined and are being explored for technological development are:

- Energy systems and energy and information transmission and designing non-Hertzian antennas
- Better design and development and theoretical understanding of controlling and utilization of plasma energy
- More complete understanding of nonlocality in quantum processes and the development of quantum computation [13]
- A deeper and more complete and comprehensive understanding of the Earth and Earth's ionosphere and magnetosphere, understanding and data analysis using the T-1050 detection system (Rauscher-Van Bise design patent US 4-724-390) and analysis program for better solar wind, Earth and lunar processes [22,45] as well as design and science method and volcanic prediction as to location, approximate magnitude with warning cycles six weeks, two to three weeks and about two hours before the impending seismic event by deployment and operation of an array of earthquake detecting precursor sights [22,73-81].
- Emergence of new medical modalities which are non-invasive, gentler and medically effective and cost efficient. These involve methods of complete external cardiac normalization (US Patent 4-723-536 - non-invasive heart pacemaker) and pain reduction (US Patent 4-724-390) and elimination and brain wave effects that correct abnormal brain functions involving highly specific resonances tuned to biological tissue by pulsed magnetic fields. Biological maladaptive conditions not treatable by current medical modalities can be effectively treated with long term enhanced biological functional outcomes [20,21,23,58-71].

Some additional implications of complexification of Maxwell's equations for low, intermediate and high frequency phenomena:

- Relaxation and modification of gauge invariance conditions in which non-Hertzian or longitudinal waves occur. $A_\mu^{'} \to A_\mu - \frac{1}{e} n \frac{\partial \Lambda}{\partial x_\mu}$ is modified. See Chap. 8.
- Comparison to the Boltzman-Maxwell or Vlassov Magneto Hydrodynamic (MHD) equations which allows transverse and longitudinal components of $\underline{E}$ and $\underline{B}$ in a high temperature plasma around 10^6 °K.

- The usual condition, $F_{\mu\nu} = \dfrac{\partial \Lambda}{\partial y_\mu} - \dfrac{\partial \Lambda}{\partial x_\mu} \Rightarrow \nabla \cdot \underline{B} = 0$. In complexified modified form Maxwell's equations, $\nabla \cdot \underline{B} \neq 0$ have a monopole term mentioned earlier.
- Possible advanced potential 'pilot' wave mode of remote connection and Bell's Theorem. See Chap. 4.
- Possible explanation of the Aharonov-Bohm experiment where $\underline{A}, \varphi$ outside the effect of $\underline{E}$ and $\underline{B}$. We have $\underline{B} = \underline{B}_0 + \nabla \times \underline{A}$ and $\underline{E} = \underline{B}_0 - \nabla \phi$ for tensor potential, $\underset{\sim}{A}$, vector potential, $\underline{A}$ and scalar potential, ϕ. See Chap. 4.
- Finite but very small mass of the photon, m_γ has been postulated [25] and the mass of the neutrino has been detected experimentally [27,29].

Some examples of modifications of Maxwell's equations for ULF, ELF, VLF and LF frequency range are listed as follows:

- Maxwell's equations and the Hertzian electromagnetic wave assumptions are primarily applicable to $\omega \geq 10\text{KHz}$. For example in MHD oscillatory collections, electron motion produces electromagnetic waves that have both transverse and longitudinal components even in the RF region of the EM spectrum [45]. Also the 10KHz region and below apply to biological systems [20,21,52-72].
- For frequencies below 10KHz we can treat Maxwell's equations in the slowly varying soliton-like envelope approximation (SVEA) [20,21].
- We consider periodic variations of the magnetic field governed by nonlinear evolution equations with dispersion, diffusion and dissipative modes overcome by nonlinear recoherences, $\varphi_{xx} - \varphi_{tt} - \sin\varphi = \delta\varphi_t - \gamma\varphi_{xx}$ where the δ and γ terms represent wave dissipation losses and *x* and *t* are the usual independent spacetime variables and $\sin\varphi$ is the nonlinear term. Note φ_{xx} stands for $\partial^2\varphi / \partial x^2$ and φ_{tt} stands for $\partial^2\varphi / \partial t^2$.
- Analogy is made to the Korteweg-deVries equation in which nonlinear terms of the dispersive losses, φ_{xxx} yield soliton solutions [20,21].
- Both transverse and longitudinal modes of excitation are generated and a generalization of the usual gauge conditions are formulated such as to accommodate both Hertzian and non-Hertzian phenomena.
- The Lagrangian forms for the modified gauge conditions are of the form $L = 1/2(\varphi_{xx})^2 - 1/2(\varphi_{tt})^2 - \cos\varphi$ are made, which is written for a model of naturally occurring coherent time evolutionary soliton-like wave.
- Some forms of relaxation of the gauge invariance effect conditions on the divergence of the magnetic field, $\nabla \cdot \underline{B} = 0$ and hence relate to the possibility of a magnetic monopole.
- The separation of *E* and *B* for the ELF region of the spectrum represents what occurs in some types of biological tissue and atmospheric and ionospheric phenomena and other applications..

The *E* and *B* fields no longer primarily act in concert as an electromagnetic wave, but can act as electric and magnetic fields separately but in a coordinated manner which occurs in the detection of biological signaling as well as in the detection and analysis of ionospheric resonances, seismic and volcanic precursors and other low frequency, low intensity resonant phenomena.

References and Notes

[1] Weyl, H. (1918) *Math. Z.* 20, 384.
[2] Weyl, H. (1918*) Gravitation und Elektrizität, Sitzungsberichte d. Preuss. Akad. d. Wiss.*: 465.
[3] Weyl, H. (1920) *Phys. Z.* 21, 649.
[4] Weyl, H. (1957) Space-Time-Matter, Dover Publications, Inc.
[5] Kaluza, T. (1921) *Sitz. Berlin Preuss, Acad. Wiss*. 966, B. Hoffman, private communication, SRI, August 24, 1976 with EAR The Kaluza work was pointed out to Hoffman by A. Einstein in the 30's as a significant direction to investigage and Hoffman suggested these references to EAR at SRI International.
[6] Klein, O. (1926) *Z. Phys.* 37, 895; Additional constraints consistent with the nonlocal model in terms of a complex 8-space may be accomplished by introducing a Kaluza-Klein-like tempora 6D of the form $\gamma_{66} = -1$ suggested by EAR.
[7] Inomata, S. (1976) Consciousness and complex electromagnetic fields, Electrotechnical Laboratory, MITI, 5-4-1 Mukodai-cho, Tanaski-city, Tokyo.
[8] Ramon, C. & Rauscher, E.A. (1980) Found. Physics 10, 661.
[9] Rauscher, E.A. (1979) in The Iceland Papers, select papers on experimental and theoretical research on the physics of consciousness, Frontiers of Physics Conference, Amherst: Essentia Research Associates; and (1996) Ottawa: PACE.
[10] Rauscher, E.A. (1981) Conceptual changes in reality models from new discoveries in physics, Tecnic Research Laboratories publication PSRL-1076, M/B Conference, Palo Alto, Octobver 10-11; and (1981) Proceedings of the 1st International Symposium on Non-Conventional Energy Technology, October 23-24 1981 Ontario: University of Toronto; and Proceedings of the 10th International Conference on the Sciences, Vol II, pp. 1407-1431, New York: ICF Press.
[11] Rauscher, E.A. (1978) Bull. Amer. Phys. Soc. 23, 84.
[12] Rauscher, E.A. (1983) Electromagnetic Waves in Complex Geometries and Nonlinear Phenomena, Non-Hertzian Waves and Magnetic Monopoles, Millbrae: Tesla Books.
[13] Amoroso, R.L. & Rauscher, E.A. (2009) The Holographic Anthropic Multiverse: Formulating the Complex Geometry of Reality, Singapore: World Scientific.
[14] Maxwell, J.C. (1873) A Treatise on Electricity and Magnetism, Oxford: Clarendon Press, London, Vols. I &II); and (1865) A dynamic theory of the electromagnetic field, Royal Society Transactions, Vol. CLV, pg. 459. Appreciation to E. Byrd for supplying the author with a copy of this paper.
[15] Faraday, M. (1885) Experimental Researches in Electricity, Vols. I, II, III, in R. Taylor & W. Francis, (eds.) London: University of London.
[16] The alternative of defining and using $E' = E_{Re} + iB_{Im}$ and $B' = B_{Re} + i/c\ E_{Im}$ would not yield a description of the magnetic monopole in terms of complex quantities, but yields, for example, $\nabla \cdot (iB_{\mathrm{Im}}) = 0$ in the second set of equations. The proper gauge conditions are not well described by this approach.
[17] Rauscher, E.A. & Van Bise, W. (1984) Electrostatic cooling theoretical and experimental analysis and application ot heat transfer enhancement in 2219-T87 aluminum weldments, PSRL-1072P-1, TRL report for NASA Lockheed-Martin Aerospace Advanced Quality Technology Dept. New Orleans, Contract SC-W02846, December 1983, Part I, 84 pages, December 1984, Part II, 90 pages; This report received the US Air Force outstanding recognition.
[18] Beal, J. (1979) Space Shuttle Program, Lockheed-Martin, Martin Marietta, New Orleans; private communication with EAR
[19] Dirac, P.A.M. (1948) Phys. Rev. 74, 817.
[20] Rauscher, E.A. & Van Bise, W. (1988) External Magnetic Field Impulse Pacemaker Non-Invasive Method and Apparatus for Modulating Brain Signals through an External Magnetic or Electric Field to Pace the Heart and Reduce Pain, U.S. Patent Number 4.723.536 issued February 9.
[21] Rauscher, E.A. & Van Bise, W. 1989 Non-invasive Method and Apparatus for Modulating Brain Signals Through an External Magnetic or Electric Field to Reduce Pain. U.S. Patent Number 4.889.526 issued December 26.
[22] Rauscher, E.A. & Van Bise, W. (1988) Non-superconducting Apparatus for Detecting Magnetic and Electromagnetic Fields, U.S. Patent Number 4.724.390, Febraury 9.
[23] Rauscher, E.A., Van Bise, W. , & Kroening, R. (1989) Magnetic control of low back pain, Bull. Amer.

Phys. Soc. 34, 109.
[24] Haramein, N. & Rauscher, E.A. (2005) Collective coherent oscillation plasma modes in surrounding media of black holes, in R.A. Amoroso, B. Lehnert & J.P. Vigier (eds.) Beyond the Standard Model: Searching for Unity in Physics, pp. 279-331, Oakland: The Noetic Press.
[25] Randall, L. (2005) Warped Passages: Unraveling the Mysteries of the Unvierse Hidden Deimensions, New York: Harper Collins.
[26] Veltman, M. (2003) Elementary Particle Physics, Singapore: World Scientific.
[27] Rauscher, E.A. (1971) A Unifying Theory of Fundamental Processes, UCRL-20808, UCB / LBNL report.
[28] Ford, K.W. (1963) Magnetic monopoles, Sci. Am., December, p. 122.
[29] Vigier, J.P. (2002) Selected papers, Jean-Piere Vigier and the Causal Stochastic Interpretation of Quantum Mechanics, S. Jeffers, B. Lehnert, N. Abramson & Chebotarov, (eds.) Montreal, Aperion; and private communications with RLA.
[30] Teller, E. (1966) Quarks and magnetic monopoles, Talk at the LLNL, January 1966; and private communication with EAR.
[31] Alvarez, L.W. et al., (1970) A magnetic monopole detector utilizing superconducting elements, UCRL-19756; and private communication.
[32] Silvers, D. (1979) Possible bending of a magnetic monopole to a particle with electric charge and magnetic dipole moment, UCRL-19794.
[33] Price, B. (1978) private communication, University of California, Berkeley, February 1971, June 1978.
[34] Wheeler, J. (1962) Geometrodynamics, New York: Academic Press; and fruitfull private communication with EAR.
[35] Rauscher, E.A. & Amoroso, R.L. (2010) Solutions to the Dirac equation in complex Minkowski spaces, in progress.
[36] Maharana, J. (1982) Proc. Theoret. Phys. 68, 277.
[37] Nimanton, P. (1979) Nucl. Phys. B158, 141.
[38] Eberhard, P. (1975) Status of the search for magnetic monopoles, presented at the Division of Particles and Fields, August 27, 1975, and LBL-4289; private communication with EAR.
[39] Dirac, P.A.M. (1931) Proc. Roy. Soc. A133, 60.
[40] Alvarez, L.W. (1975) Talk at the Stanford lepton/photon symposium, August 1975; and LBL-4260; private communication with EAR.
[41] Mandelstam, S. (1976) Talk LBNL, March 1; and private communication EAR.
[42] Discussions and communications with R. Curtin & D.L. Faust, Eyring Research Institute, Inc., Provo, Utah.
[43] Hertz, H. (1955) The Principles of Mechanics, , p.1899, New York: Dover.
[44] Tesla, N. (1989) Nikola Tesla Complete Patents, Vols. I & II; and Nikola Tesla Selected Patent Wrappers, Vols. I-IV, compiled by J.T. Ratzlaff , Milbrae: Tesla Books; and Colorado Springs Notes as well as various papers and articles.
[45] Rauscher, E.A. & Van Bise, W. (2002) Fundamental excitatory modes of the earth and earth ionosphere resonant cavity, pp. 233-268, in Thomas Valone (ed.) Harnessing the Wheelwork of Nature, Tesla's Science of Energy, Kempton: Adventures Unlimited Press.
[46] Tesla, N. (1905) Art of Transmitting Electrical Energy through Natural Mediums, US Patent 787,412, April 18.
[47] In calculating the velocity ratio of air and ground waves, one approach is to consider an air (earth ionosphere) wave travelling at v_2 and a through-the-earth wave traveling at v_1. Consider two waves emitted from the same location on the earth's surface, one in the air and the other through the earth and both traversing paths in the same time so as to come back to the emission location as reinforced. The path length for the air wave is πD and the through-earth wave is 2D. For equal time of travel, the velocity becomes $v_2 / v_1 = \pi / 2 = 1.57$. In this analysis, the greater velocity wave, v_2, is the air wave. If v_1 is chosen to be the velocity, then the relative velocity (v_2) is $\pi / 2$=1.57 time the speed of light. We could also consider the velocity v_2 as the velocity of light and then v_1 is $2 / \pi$=0.64 smaller than the velocity of light.
In Tesla's patents he makes it clear that the ground wave is the more rapidly travelling wave and the air wave is an electromagnetic wave travelling at the velocity of light. The above analysis is therefore not consistent with Tesla's model. In fact, there would be a mixing and reinforcing of a phonon/earth wave and an electro-magnetic wave in the rarefield air and interaction. Therefore the above simple geometric problem does not apply. The problem, in fact, invokes phonon (longitudinal) and transverse electromagnetic wave interactions, as discussed

in the next section.
[48] Rauscher, E.A. (1984) Multi-Dimensional Formalism of Maxwell's Equations and Solutions to Maxwell's Equation in Complex Geometries, Lecture presented to Colorado College. Colorado Springs, CO. August 11, 1984. Sponsored by the International Tesla Society and the Pikes Peak Section of IEEE, Colorado Springs. Meeting conceived, sponsored, and chaired by E.A. Rauscher.
[49] Rauscher, E.A. (1968) J. Plasma Phys. 2, 517.
[50] The laser system utilizes an external energy flux to produce stimulated emission, whereas coherence in super-conductivity is achieved through the effective Cooper pair interaction at low temperatures. The laser system is more like the I. Prigogine self-ordering system. See [15] for further details.
[51] Scott, A. (1980) The birth of a paradigm, in P.L.E. Uslenghi (ed.) Non-Linear Electromagnetics, pp. 3563, New York: Academic Press.
[52] Becker, R.O. & Selden, G. (1985) The Body Electric: Electromagnetics and the Foundation of Life, New York: William Marrow Press; and private communication between R.O. Becker, EAR & Van Bise, W. .
[53] Adey, W.R. (1980) Frequency and power windowing in tissue interactions with weak electromagnetic fields, Proceedings of the IEEE 68, 119C.
[54] Pressman, A.S., (1968) Electromagnetic Fields and Living Nature, Moscow, English translation (1970) Electromagnetic Fields and Life, New York: Plenum.
[55] Van Bise, W. (1928) Low power radio frequency and microwave effects on human electroencephalogram and behavior, Physiological Chem. and Physics 10 387.
[56] Stocklin, P.L. & Stocklin, B.F. (1981 low power microwave effects on the human electroencephalogram: supporting the results of Bise, Physical Chem and Physics 13, 175.
[57] Van Bise, W. (1977) Radiofrequency induced interference response in the human nervous system, pp. 1221, Radiation Health; and Hearings Before the committee on commerce, science, and transporatation, United States Senate, 95th congress, Serial No. 95-19 June.
[58] Rauscher, E.A. (2010) Mathematical Medicine: A New Medical Model, Treatment Modality and Methods for Non-Invadive Medicine, book in progress.
[59] Rauscher, E.A. (1988) Theories and Experiments in Healing II ICAT. International Congress on Medical Thherapies. Sao Paulo, Brazil, September.
[60] Rauscher, E.A. & Van Bise, W.L. (1987) Magnetic field interaction with macro biological systems with applications to effects on physiology, medicine around the world, in T.M. Srinivasan (ed.) pp. 205-218, Phoenix: Gabriel Press; Print World, November 1988, Proceeding from conference Madras Inst. of Magnetobiology 1987.
[61] Rauscher, E.A. & Van Bise, W. (1989) Magnetic flux control of low back pain, Proceedings of the Association for the Advancement of Medical Instrumentation (AAMI), St. Louis, pp. 76, May.
[62] Van Bise, W.L. & Rauscher, E.A. (2001) Magnetic field impulse cardiovascular stimulation for normalizing a arrhythmias and/or heart block BEMS, European Bioelectromagnetics Association, 23rd Annual Meeting, pp. 155, St. Paul, June 10-14.
[63] Rauscher, E.A. & Van Bise, W.L. (2001) Pulsed magnetic field treatment and chronic back pain, BEMS, European Bioelectromagnetic Association, 23rd Annual Meeting, pp. 56, St. Paul, June 10-14.
[64] Rauscher, E.A. & Van Bise, W.L. (2002) Medical application of pulsed magnetic fields in medicine, Presentation talk to the Department of Bioelectromagnetism, Beijing Polytecnic Institute University, Beijing, October 4.
[65] Rauscher, E.A. & Van Bise, W. (1987) Magnetic field flux induction into the visual system of humans, Institute for Electrical and Electronic Engineers IEEE Annual Conference on Engineering in Medicine and Biology Society, pp. 1590, Boston, November.
[66] Van Bise, W. & Rauscher, E.A. (1988) multiple extremely low frequency magnetic and electromagnetic field effects on human electroenchphalogram and behavior, pp. 57-58, Annual Review of Research on Biological Effects of Electric and Magnetic Fields form the Generation, Delivery and use of Electricity, DOE. EPRI; and U.S. National Inst. of Environmental Health Sciences, Tucson, September.
[67] Rauscher, E.A. (1981) Probing into control mechanisms in cellular healing processes, A San Francisco Initiative, Doctors Research Group, PACE Canadian Journal p. 8 February.
[68] Rauscher, E.A. (1981) The Biophysics of Electrical Pathways and Magnetic Fluxes in the Human Heart Wall The Doctors Research Group. PACE Canadian Journal page 9, February.
[69] Rauscher, E.A. (1982) Theoretical and field studies of ELF radiation and coherent nonlinear phenomena, Intl. Symp. on the Biological Effects of Electromagnetic Radiations, Learned Societies Conf., University of

Ottawa, June.
[70] Rauscher, E.A. (1985) The mathematical theory of the magnetic piezoelectric resonator, MPR-7944-T device, for treatment of pain, Tecnic Research Laboratory report PSRL-12764, April.
[71] Van Bise, W. & Rauscher, E.A. (1999) The relatinship of extremely low frequency electomagnetic and magnetic fields associated with seismic and volcanic activity and natural and artificial ionospheric disturbances, 2nd Magnetic, Electric and Electromagnetic Methods in Seismology and Vulcanology, Ministry of Interior, Public Administration and General Secretariat for Civil Protection, General Secretariat for Research and Technology. National Institute for Geophysics (NIG) and Institute of Advanced Methodologies for Environmental Analysis, IMAAA-CNR. Chania, Greece, September.
[72] Han, M., Yang, X., Lee, J., Alla C.H. & Muneoka, K. (2008) Development and regeneration in the neonatal digit tip in mice, Developmental Biology, 315 125.
[73] Rauscher, E.A. & Van Bise, W. (1987) Observations of local and global earth-ionospheric excitation for earthquake and volcanic prediction, Bull. Am. Phys. Soc. 32, 67.
[74] Van Bise, W. & Rauscher, E.A. (1987) Instrumentation and techniques for analysis of extremely low frequency (ELF) magnetic field impulses preceding geologic events, Bull. Am. Phys. Soc. 32, 67B.
[75] Van Bise, W. & Rauscher, E.A. (1988) Fundamental excitatory modes of the earth and earth-ionosphere resonant cavity, Field Physics Division. Tecnic Research Laboratories Report. PSRL-702C-II. July, Proceedings of the IEEE. Colorado Springs Section, pp. 3-34.
[76] Van Bise, W.L. & Rauscher, E.A. (2003) Detection and analysis of precursor magnetic signatures, Preceding the Turkey and Greek Seismicity, Geophysical Research Abstracts 5, 14637.
[77] Van Bise, W.L. & Rauscher, E.A. (2002) Nonlinear coherent collective resonant plasma propagation modes in the ionosphere, Bull. Am. Phys. Soc. 47, 169.
[78] Rauscher, E.A. & Van Bise, W. (1999) The relationship of extremely low frequency electromagnetic and magnetic fields associated with seismic and volcanic natural activity and artificial ionospheric disturbances, pp. 459-487, in M. Hayakawa (ed.) Atmospheric and Ionospheric Electromagnetic Phenomena Associated with Earthquakes, Tokyo: Terra Scientific Pub. Co. (TERRAPUB).
[79] Van Bise, W. & Rauscher, E.A. (1989) Non-superconducting systems for detecting and analyzing low intensity pure magnetic fields, Tecnic Research Laboratories Report PSRL-5476B; and Bull. Am. Phys. Soc. 34 p. 109.
[80] Van Bise, W.L. & Rauscher, E.A. (1996) Ambient electromagnetic fields as possible seismic and volcanic precursors, pp. 221-242, I M. Hayakawa & Y. Fujinowa (eds.) Electromagntic Phenomena Related to Earthquake Prediction, Tokyo Terra Sci. Pub. Co. (TERRAPUB).
[81] Rauscher, E.A. (with W.L. Van Bise's ghost) The Living Earth: Forces that Shape Seismic and Volcanic Occurrences, in progress.