Several Treasures of the Queen of Sciences

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Abstract**—**Theorem about the primitive Pythagorean triple. The proper proof of the Fermat's Last Theorem (FLT). The proof of the Goldbach's Conjecture.

Keywords**—**First Keyword: Diophantine Equations, Second Keyword: Fermat Equation, Third Keyword: Goldbach Conjecture, Fourth Keyword: Newton Binomial Formula.

MSC—Primary: 11D41, 11P32; Secondary: 11A51, 11D45, 11D61.

I. INTRODUCTION

The Guła's theorem is wider than the Pythagoras's theorem.

Theorem about the primitive Pythagorean triple gives all primitive solutions of the Pythagoras's equation, namely

$$\left(pq, \frac{p^2-q^2}{2}, \frac{p^2+q^2}{2}\right) = (u^2-v^2, 2uv, u^2+v^2).$$

In this work we have the proper proof of FLT. In [6] we have the proof of another hypothesis, not for $n = 4$.

The Goldbach's Conjecture is one of the oldest and best-known unsolved problems in number theory and all of mathematics. It states: Every even integer greater than 2 can be expressed as the sum of two primes. [1] Proof of the Goldbach's Conjecture is based on theorems **1** and **2**.

II. THE GUŁA'S THEOREM

Theorem 1 . *For each given* $g \in \{8,12,16,\dots\}$ *or for each given* $g \in \{3,5,7,\dots\}$ *there exist finitely many pairs* (u,v) *of positive integers such that*:

$g = \left(\frac{g+q^2}{2q}\right)^2 - \left(\frac{g-q^2}{2q}\right)^2 = (u+v)(u-v) = \frac{g}{q}(u-v) = \frac{g}{q}q = g \Longrightarrow g^2 = (u^2-v^2)^2 = (u^2+v^2)^2 - (2uv)^2$,

where $q \mid g$ *and* $q < \sqrt{g}$ *and* - $q, \frac{g}{q} \in \{2,4,6,\dots\}$ *with even* g *or* $q \in \{1,3,5,\dots\}$ *with odd* g. [2], [3] and [5]

This is the theorem.

Theorem 2 . *For each pair* (u,v) *of the relatively prime natural numbers* u *and* v *such that* $u-v$ *is positive and odd there exists exactly one a primitive Pythagorean triple* $(u^2-v^2, 2uv, u^2+v^2)$ *and each the primitive Pythagorean triple arises exactly from one pair* (u,v) *of the relatively prime natural numbers* u *and* v *such that* $u-v$ *is positive and odd. Hence–For each equation* $(p,q) = (u+v, u-v)$ *of the relatively prime odd natural numbers* p *and* q *such that* $p > q$, *and of the relatively prime natural numbers* u *and* v *such that* $u-v$ *is positive and odd there exists exactly one the primitive Pythagorean triple* $\left(pq, \frac{p^2-q^2}{2}, \frac{p^2+q^2}{2}\right) = (u^2-v^2, 2uv, u^2+v^2)$ *and each this primitive Pythagorean triple arises exactly from one equation* $(p,q) = (u+v, u-v)$ *of the relatively prime odd natural numbers* p *and* q *such that* $p > q$, *and of the relatively prime natural numbers* u *and* v *such that* $u-v$ *is positive and odd.*

This is the theorem.

III. THE PROPER PROOF OF THE FERMAT'S LAST THEORE

Theorem 3. *For all* $n \in \{3,4,5,\dots\}$ *and for all* $A, B, C \in \{1,2,3,\dots\}$: $A^n + B^n \neq C^n$.

Proof. Suppose that for some $n \in \{3,4,5,\dots\}$ and for some $A, B, C \in \{1,2,3,\dots\}$: $A^n + B^n = C^n$.

If $A + B \leq C$, then

$$A^{n-1} + B^{n-1} \leq C^{n-1} \Longrightarrow A^n + B^n < C^n,$$

which is inconsistent with $A^n + B^n = C^n$.

Thus it must be $(A + B > C \wedge A^{n-1} + B^{n-1} > C^{n-1})$.

Hence – For some $A, B, C, C-A, C-B, v \in \{1,2,3,\dots\}$:

$$A - (C-B) = B - (C-A) = 2v > 0 \Longrightarrow (C - B + 2v = A \wedge C - A + 2v = B \wedge A + B - 2v = C). \qquad (1)$$

At present we assume that A, B and C are coprime. Then only one number out of a hypothetical solutions $[A, B, C]$ is even. Thus we can assume that $A, C - B \in \{1,3,5,\dots\}$.

Let $\{3,5,7,\dots\} - \{(2a+b)b : a \in \mathrm{N} \wedge b \in [3,5,7,\dots]\} = \{3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,\dots\} = \mathbb{P}$.

Every even number which is not the power of number 2 has odd prime divisor, hence sufficient that we prove FLT for $n = 4$ and for odd prime numbers $n \in \mathbb{P}$. [6]

A. Proof For $n = 4$. Let the equation $A^4 + B^4 = C^4$ has primitive solutions $[A, B, C] = \left[A, B, \sqrt{c}\right]$. It is easy to verify that $c \in \{9,25,49,\dots\}$, inasmuch as $C \in \{3,5,7,\dots\}$, with $C = \sqrt{c}$, which is obviously. Therefore – For some $a, b \in \{1,2,3,\dots\}$ such that the numbers a and b are coprime and the number $a - b$ is positive and odd:

$$[(a^2+b^2)^2 - (2ab)^2 = (a^2-b^2)^2$$
$$= A^2 \wedge 2(a^2+b^2)2ab$$
$$= B^2 \wedge (a^2+b^2)^2 + (2ab)^2 = C^2$$
$$= c \wedge (A^2)^2 + (B^2)^2 = (C^2)^2\ ].$$

On the strength of the Theorem **1** we obtain

$$\left[C = \frac{(2ab)^2 + (2b^2)^2}{2 \cdot 2b^2} = a^2 + b^2 \wedge a^2 + b^2 = \frac{(2ab)^2 - (2b^2)^2}{2 \cdot 2b^2} = a^2 - b^2\right] \in \mathbf{0}. \square$$

B. Proof of Another Hypothesis. Suppose that the equation $A^4 + B^4 = c^2$. (2) has primitive solutions.

We assume that the number c is minimal. [6]

The hypothesis (2) and $A^4 + B^4 = C^4$ are different [2] **because** the number $c \in \{3,5,7,\dots\} \setminus \{9,25,49,\dots\}$, with $C \in \left\{\sqrt{3},\sqrt{5},\sqrt{7},\sqrt{11},\sqrt{13},\sqrt{15},\sqrt{17},\sqrt{19},\sqrt{21},\dots\right\}$.
On the strength of the Theorem **2** we obtain – For some coprime $p, q \in \{1,3,5,\dots\}$ and for some relatively prime $U, V \in \{1,2,3,\dots\}$ and for some $a, b \in \{1,2,3,\dots\}$ and for some mutually relatively prime $x, y, z \in \{1,2,3,\dots\}$ such that the numbers $U - V, x - y, a - b$ are positive and odd and $\mathbf{gcd}(a, b) = 1$:

$$\left[(pq)^2 = \frac{(p^2+q^2)^2}{4} - \frac{(p^2-q^2)^2}{4} = U^2 - V^2\right.$$
$$= (a^2-b^2)^2 = (a^2+b^2)^2 - (2ab)^2$$
$$= A^2 \wedge p = a + b \wedge q$$
$$= a - b \wedge \frac{p^4 - q^4}{2}$$
$$= \frac{p^2+q^2}{2}(p^2 - q^2) = 2UV$$
$$= 2(a^2+b^2)2ab = B^2 \wedge \frac{p^4+q^4}{2}$$
$$= \frac{(p^2+q^2)^2}{4} + \frac{(p^2-q^2)^2}{4} = U^2 + V^2$$
$$= (a^2+b^2)^2 + (2ab)^2 = c$$
$$= C^2 \wedge \frac{p^2+q^2}{2} = U = a^2 + b^2$$
$$= z^2 \wedge \frac{p^2-q^2}{2} = V = 2ab \wedge 4ab$$
$$= (2xy)^2 \wedge a = x^2 \wedge b$$
$$\left. = y^2 \wedge x^4 + y^4 = z^2 < c^2\right] \Longrightarrow z < c,$$

which is inconsistent with minimal c. $\square$

C. Proof For $n \in \mathbb{P}$. Without loss for this proof we can assume that $4 \nmid B, C$. In view of (1) we will have –

For some $n \in \mathbb{P}$ and for some $C, B, C - A \in \{1,2,3,\dots\}$ and for some $C - B, A, v \in \{1,3,5,\dots\}$:

$$\Bigg[(C - B + 2v)^n = (C - B + B)^n - B^n$$
$$\Longrightarrow (C-B)^{n-2}v + (n-1)(C-B)^{n-3}v^2 + \cdots + 2^{n-2}v^{n-1} + \frac{2^{n-1}v^n}{n(C-B)}$$
$$= \frac{B}{2}\left[(C-B)^{n-2} + \frac{n-1}{2}(C-B)^{n-3}B + \cdots + B^{n-2}\right] \wedge n \mid v$$
$$\wedge\ (n \mid B \veebar n \mid C)\Bigg] \wedge$$

$$\Bigg[(C - A + 2v)^n = (C - A + A)^n - A^n \Longrightarrow (C-A)^{n-2}2v$$
$$+ \frac{n-1}{2}(C-A)^{n-3}(2v)^2 + \cdots + (2v)^{n-1} + \frac{(2v)^n}{n(C-A)}$$
$$= A\left[(C-A)^{n-2} + \frac{n-1}{2}(C-A)^{n-3}A + \cdots + A^{n-2}\right] \wedge n \mid v$$
$$\wedge\ (n \mid A \veebar n \mid C)\Bigg] \wedge$$

$$\Bigg[A^n + B^n = C^n = (A + B - 2v)^n$$
$$= A^n + nA^{n-1}B + \frac{n(n-1)}{2}A^{n-2}B^2 + \cdots + nAB^{n-1} + B^n$$
$$+ n(A+B)^{n-1}(-2v) + \frac{n(n-1)}{2}(A+B)^{n-2}(-2v)^2 + \cdots + n(A+B)(-2v)^{n-1} + (-2v)^n$$
$$\Longrightarrow \Bigg[0 = AB\frac{\left(A^{n-2} + \frac{n-1}{2}A^{n-3}B + \cdots + B^{n-2}\right)}{A+B}$$
$$+ (A+B)^{n-2}(-2v) + \frac{n-1}{2}(A+B)^{n-3}(-2v)^2 + \cdots + (-2v)^{n-1} + \frac{(-2v)^n}{n(A+B)} \wedge n \mid v$$
$$\wedge \left(n \mid A \veebar n \mid B \veebar (n^{n-1} \mid A + B \wedge n \mid C)\right)\Bigg]\Bigg].$$

If $n \mid A \equiv 1$, then

$$[(n \mid A \veebar n \mid C) \equiv 1 \wedge (n \mid B \veebar n \mid C) \equiv 0 \wedge \left(n \mid A \veebar n \mid B \veebar (n^{n-1} \mid A+B \wedge n \mid C)\right) \equiv 1] \in \mathbf{0}.$$

If $n \mid B \equiv 1$, then

$$[(n \mid A \veebar n \mid C) \equiv 0 \wedge (n \mid B \veebar n \mid C) \equiv 1 \wedge \left(n \mid A \veebar n \mid B \veebar (n^{n-1} \mid A+B \wedge n \mid C)\right) \equiv 1] \in \mathbf{0}.$$

If $n \mid C \equiv 1$, then

$$[(n \mid A \veebar n \mid C) \equiv 1 \wedge (n \mid B \veebar n \mid C) \equiv 1 \wedge \left(n \mid A \veebar n \mid B \veebar (n^{n-1} \mid A+B \wedge n \mid C)\right) \equiv 1] \in \mathbf{1}.$$

For some $n \in \mathbb{P}$ and for some $e, m, c, h \in \{1,3,5, \dots\}$ such that n, e, m, c and h are coprime:

$$[nemch = v \wedge n \nmid emch \wedge (h^n = C - A \vee (2h)^n = C - A) \wedge c^n = C - B].$$

B.1. Proof For Odd $A, B, C - B$.

For some $n \in \mathbb{P}$ and for some $e, m, c, h \in \{1,3,5, \dots\}$ such that n, e, m, c and h are coprime:

$$[c^n + 2nemch = A \wedge h^n + 2nemch = B \wedge 2^n n^{n-1} m^n = A + B = c^n + h^n + 4nemch \wedge c^n + B = C] \Longrightarrow [2^n n^{n-1} m^n = c^n + h^n + 4nemch \wedge n \mid c^n + h^n] \Longrightarrow (n \mid c + h \wedge n^2 \mid c^n + h^n \wedge n \mid emch),$$

which is inconsistent with $n \nmid emch$. □

B.2. Proof For Even $B, C - A$.

For some $n \in \mathbb{P}$ and for some $e, m, c, h \in \{1,3,5, \dots\}$ such that n, e, m, c and h are coprime:

$$[c^n + 2nemch = A \wedge (2h)^n + 2nemch = B \wedge n^{n-1} m^n = A + B = c^n + (2h)^n + 4nemch \wedge c^n + B = C] \Longrightarrow [n^{n-1} m^n = c^n + (2h)^n + 4nemch \wedge n \mid c^n + (2h)^n] \Longrightarrow (n \mid c + 2h \wedge n^2 \mid c^n + (2h)^n \wedge n \mid emch),$$

which is inconsistent with $n \nmid emch$. This is the proof.

Remark 1. If $n \in \{1,3,5, \dots\}$ and $c, h \in \{1,2,3, \dots\}$ and $n \mid c^n + h^n$ and $\mathbf{gcd}(c, h) = 1$, then

$$\left[\frac{(c+h-h)^n + h^n}{n} \wedge n \mid c + h \wedge n^2 \mid c^n + h^n\right],$$

which is obviously.

This is the remark.

IV. The Proof of the Goldbach's Conjecture

Conjecture 1. For all $u \in \{2,3,4, \dots\}$ and for some $p, q \in \mathbb{P} \cup \{2\}$: $2u = p + q$.

Proof. $4 = 2 + 2, 6 = 3 + 3$.

It is easy to verify that for each $u \in \{4,5,6, \dots\}$ there exists $v \in \{1,2,3, \dots\}$ and there exist $p, q \in \mathbb{P}$:

$$[\mathbf{gcd}(u, v) = 1 \wedge u + v = p \wedge u - v = q \wedge 2u = p + q \wedge 2v = p - q].$$

This is the proof.

On the strength of the Theorems **1**, **2**, and of the above proof of the Goldbach's Conjecture we obtain –

Therem 4. *For all* $p, q \in \mathbb{P}$ *and for some relatively prime* $u, v \in \{1,2,3, \dots\}$ *such that* $p > q$ *and* $u - v$ *is positive and odd*: [5]

$$pq = \left(\frac{p+q}{2}\right)^2 - \left(\frac{p-q}{2}\right)^2 = u^2 - v^2 \Longrightarrow \left[\left(pq, \frac{p^2 - q^2}{2}, \frac{p^2 + q^2}{2}\right) = (u^2 - v^2, 2uv, u^2 + v^2) \wedge p + q = 2u \wedge p - q = 2v \wedge p = u + v \wedge q = u - v \wedge (p + q = 2u = 8,10,12, \dots) \wedge (p - q = 2v = 2,4,6, \dots)\right].$$

Tis is the theorem.

References

[1] En.,Wikipedia, https://en.wikipedia.org/wiki/Goldbach%27s_conjecture

[2] L. W. Guła, Disproof the Birch and Swinnerton-Dyer Conjecture, American Journal of Educational Research, **Volume 4**, No 7, 2016, pp 504-506, doi: 10.12691/education-4-7-1 | Original Article electronically-published-on-May-3,2016 http://pubs.sciepub.com/EDUCATION/4/7/1/index.html

[3] L. W. Guła, Several Treasures of the Queen of Mathematics, International Journal of Emerging Technology and Advanced Engineering, **Volume 6**, Issue 1, January, 2016, pp 50-51 http://www.ijetae.com/files/Volume6Issue1/IJETAE_0116_09.pdf

[4] L. W. Guła, The Truly Marvellous Proof, International Journal of Emerging Technology and Advanced Engineering, **Volume 2**, Issue 12,December,2012,pp-96-97 http://www.ijetae.com/files/Volume2Issue12/IJETAE_1212_14.pdf

[5] L. W. Guła, International Journal of Innovation in Science and Mathematics Volume 6, Issue 1, ISSN (Online): 2347–9051, January, 2018. http://ijism.org/administrator/components/com_jresearch/files/publications/IJISM_708_FINAL.pdf

[6] W. Narkiewicz, Wiadomości Matematyczne XXX.1, Annuals PTM, Series II, Warszawa 1993, p. 3.

AUTHOR'S PROFILE.

Curriculum vitae

I was born in Lublin (Poland) on Ludolfina Day. 1975 - I finished Automotive Technical School in Lublin, Jan Długosz Avenue 10a. 1981 - I finished Technical Education at the Faculty of Pedagogy at Maria Curie Skłodowska University in Lublin. Master's Thesis in the field of psychology: ''Effects of professional orientation of eighth grade students'' (11, June). 1981 -1986 I was a teacher in a primary school in Lublin and since 1984 the appointed teacher. 1986 -1987 I was a miner 960 m underground. In the last century I have been making money in LZF Polfa in Lublin as a specialist for repairs in maintenance, where I prepare drawings for spare parts for SORTIMATS, MULTIVATES, INJECTORS or for other machines. I have not been making money for over 27 years. I live modestly. Soon I will be a pensioner. The adventure with the Queen of Sciences was very difficult. My works are available through the site http://lwgula.pl.tl/

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