WITH A LITTLE HELP BY NICHOLAS DE CUSA: ERASING INFINITY FROM PYHSICAL THEORIES

Arturo Tozzi
Center for Nonlinear Science, University of North Texas, Denton, Texas 76203, USA
1155 Union Circle, #311427
Denton, TX 76203-5017 USA
tozziarturo@libero.it

James F Peters
Department of Electrical and Computer Engineering, University of Manitoba
75A Chancellor's Circle Winnipeg, MB R3T 5V6 CANADA and
Department of Mathematics, Adıyaman University, 02040 Adıyaman, Turkey
James.Peters3@umanitoba.ca

The occurrence of infinite values in physical equations, such as singularity in the description of black holes, is a painstaking problem that causes many theories to break down and/or being incapable of describing extreme events. Different methods, such as re-normalization, have been used in the assessment of physical observables in order to erase the undesirable infinity. Here we propose a novel technique, based on geometrical considerations, that allows removal of infinity and achievement of physical theories void of such a problem. We compare finite quantities to curved lines endowed in positive-curvature manifolds, and intractable infinity to a straight line. In order to restore the equations and erase the straight line of infinity, we project the quantities onto curved lines endowed in negative-curvature manifolds.

In their theories and models dealing with formulas that describe finite, measurable quantities, physicists do not appreciatethe occurrence of unwished infinite values. Indeed, with the exception of various forms of conformal infinity (Frauendiener 2000; Sommers, 1978), mathematical infinity (indeterminate infinite results in which, for example, solutions of the gravitational field equations cannot be continued (Bergman, 1989)) prevents scientific issues to provide practical formulas that correspond to, or at least approximate, the real observables. For example, in case of bodies with infinite gravitational mass and/orenergy, equations become untractable and useless, since their results would be always the same, regardless of objects' position, mass and movement. In some cases, infinite results mean that the used theory is approaching the point where it fails. A classic example is the black holes' mathematical singularity: the commonest solutions of the general relativity's equations allow for zero-size finite mass distributions, leading to infinite density, e.g., a point where general relativity breaks down and is incapable of appropriately describing events. Other examples are the Newtonian gravity and Coulomb's law of electrostatics at r=0. Therefore, although infinity can be used in physics, scientists require for practical purposes the final result being physically meaningful: e.g., in quantum field theory, infinities are treated through procedures such as renormalization ('t Hooft, 1971).
Here we propose a different way to cope with the meaningless results of the infinity in the description of physical equations, holding out the possibility of a physical geometry (Peters, 2016) that is region-based instead of point -based.

INFINITY AS A STRAIGHT LINE IN GEOMETRY

As stated above, unqualified infinity cannot be any of the physical observables which we either can assess or measure: when we set out to investigate the infinity, we must leap beyond simple physical concepts and use mathematics. inspired by Nicholas de Cusa' treatment of infinity (de Cusa, 1440; Tozzi and Peters, 2017), here we show the way in which mathematical signs ought to be used in undertaking physical infinity. If we want to assess finite physical measurements leading to infinity, we need at first to consider finite mathematical figures and topological manifolds, together with their features and relations. Next, we must apply these relations in a projective way. Thirdly, we must thereafter, in a still more highly transformed way, apply the relations of these infinite figures to the general concept of mathematical infinity, which is altogether independent even of all figures and manifolds.
Let we start with the figure of mathematical infinite, which we will picture as a straight line. We maintain that, if there were an infinite line, it would be a straight one, or, for example, an infinite triangle, circle or sphere. Since the latter three figures display infinite sides, as will be shown, they can also be described in terms of infinite lines. First of all, an infinite line would be a straight one. The circle's diameter is a straight line, and its circumference is a curved line greater than the diameter. Therefore, if the curved line becomes less curved in proportion to the increased circle's circumference, then the maximum circle's circumference, which cannot be greater, is minimally curved and therefore maximally straight (**upper part of theFigure**). Hence, we can visually recognize that it is necessary for the maximum line to be maximally straight and minimally curved. Indeed, in the Figure, the arcs of the larger circle are less curved

than the smaller ones. Therefore, the straight line will be the arc of the maximum circle, which cannot be greater. An infinite line is necessarily the straightest; and to it no curvature is opposed. In the same way, every manifold with positive curvature, such as, for example, a triangle, or a circumference, or a sphere, can be described in terms of an infinite line standing for a maximum triangle, or a maximum circle, or a maximum sphere. It will become clearer on the basis of the fact that an infinite line is whatever is present in the curvature of a finite line. We know that a line finite in length can be longer and straighter; and the maximum line is the longest and straightest. If these figures are describable by a finite line, and if an infinite line is all the things with respect to which a finite line is in infinity, then it follows that an infinite line stands also for a triangle, a circle, and a sphere.

How is it possible that an infinite line is a side of a triangle? It is evident that there can be only one minimum-curvature and infinite line. Moreover, since any two sides of any triangle cannot, if conjoined, be shorter than the third, this means that, in the case of a triangle whose one side is infinite, the other two sides are not shorter, i.e., they are both infinite. And because each part of what is infinite is infinite, for any triangle whose one side is infinite, the other sides must also be infinite. Since there cannot be more than one infinite thing, an infinite triangle cannot be composed of a plurality of lines, even though it is the greatest and simplest triangle. And because it is a triangle—something which it cannot be without three lines—it will be necessary that the one infinite line be three lines, and that the three lines be one most simple line. And similarly, regarding the angles: for there will be only one infinite angle; and this angle is three angles, and the three angles are one angle. Nor will this maximum triangle be composed of sides and angles; rather, the infinite line and angle are one and the same thing, so that the line is the angle, because the triangle is the line. The larger the one angle, the smaller are the other two. Now, any one angle can be increased almost but not completely up to the size of two right angles. Nevertheless, let us hypothesize that it is increased completely up to the size of two right angles, while the triangle remains nonethelessa triangle. In that case, it will be obvious that the triangle has one angle which is three angles and that the three angles are one. In the same manner, we can state that a triangle is a line. For any two sides of a quantitative triangle are, if conjoined, as much longer than the third side as the angle which they form is smaller than two right angles. Hence, the larger the angle, the less the lines and the smaller its surface. Therefore, if, by hypothesis an angle could be two right angles, the whole triangle would be resolved into a simple line. Hereby it is evident that an infinite line is a maximum triangle.

Next, is the maximum triangle also a circle and a sphere? We shall see that an infinite triangle is also an infinite circle. Let us postulate the triangle formed by rotating a line. If the infinite line is rotated until it comes all the way back to the starting point, an infinite circle would be formed, that is a straight line. Therefore, it is necessary that the infinite triangle be an infinite circle. And because the circumference is a straight line, it is not greater than the infinite line, for there is nothing greater than what is infinite. Nor are there two lines, because there cannot be two infinite things. Therefore, the infinite line, which is a triangle, is also a circle. Moreover, that an infinite line is a sphere becomes very obvious by applying the same reasoning: it follows that, from a coming around of a circle upon itself, a sphere is originated. And given that we previously proved that a manifold is a circle, a triangle and a line, we have now proved that it is also a sphere. And these are the results we set out to find.

Now that we have seen how an infinite line is actually and infinitely all that which is in the possibility of every finite line and manifold. By comparison, a triangle is educed from a line; but an infinite line, though a triangle, is not a triangle as is educed from a finite line; rather, the infinite line is actually an infinite triangle, which is identical with the infinite line. Hence, we notice here an important speculative consideration which, from the foregoing, can be inferred about infinity viz., that infinity is correlated with finite manifolds.

In sum, because infinite curvature is infinite straightness, this means that an infinite manifold can be described in opposite terms: it is not a thing and is not any other thing; it is not here and is not there; it is unqualifiedly free from all things and is beyond all things; is above the negation of all things. By a physicist’s standpoint, this explains why physical theories leading to infinite values are awfully problematic and difficult to cope with.

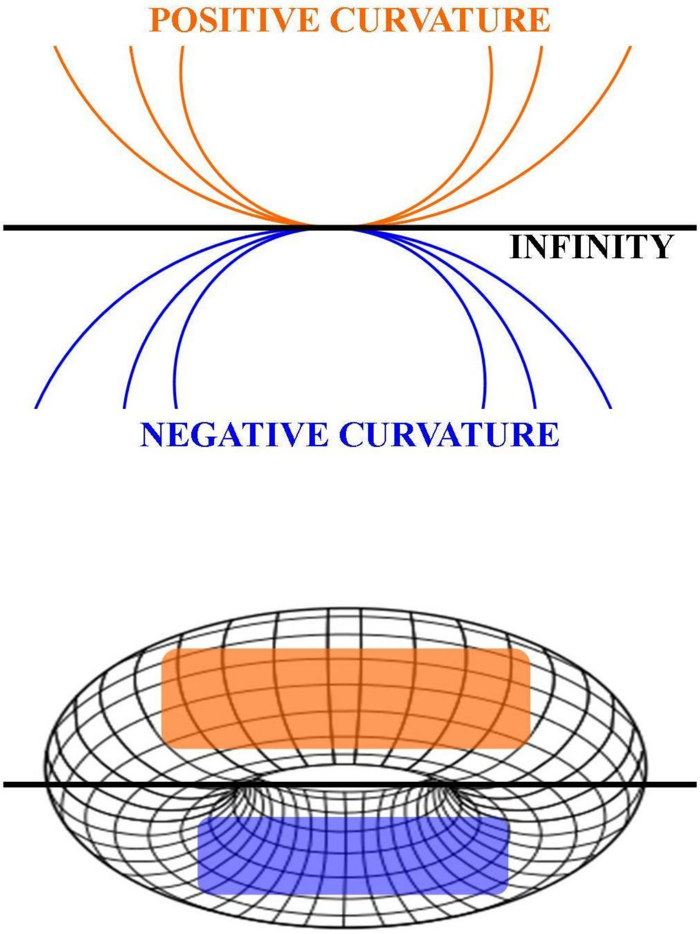
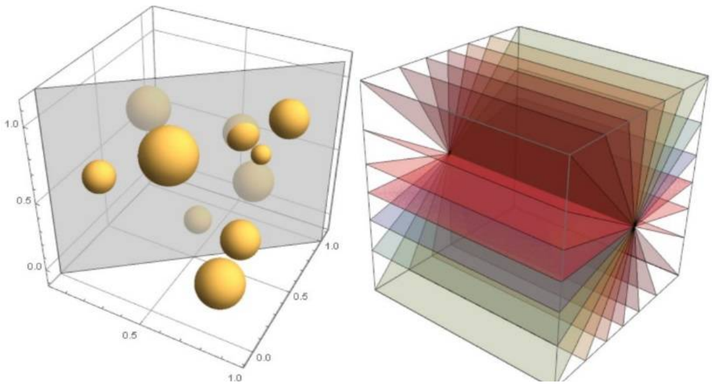


Figure 1. **Upper part**: given a physical system described by progressively increasing curves on a positive-curvature manifold, the occurrence of infinity (straight line) can be removed by taking into account progressively decreasing curves on a negative-curvature manifold. **Lower part**: by placing physical observables on a toroidal manifold, we achieve a correspondence between positive and negative curvatures, thus erasing the unwanted occurrence of infinity.

HOW TO COPE WITH INFINITY IN PHYSICS: AN EXAMPLE

In the previous section, we reached the result that infinity can be expressed by a straight line with zero-curvature. How to use this observation, in the mathematical treatment of physical finite systems whose equations lead to the unwanted infinity? When we have a physical system of equations, we may project it into a positive-curvature manifold through well-described geometrical procedures (Frankel, 2011). If the equations tend towards infinity, and therefore give meaningless results, we are allowed to remove the infinity, by projecting the same physical equations on a negative-curvature manifold. The passage and the relationships between the positive- and negative-curvature manifolds are described in **the lower part of the Figure 1**, for sake of simplicity, as taking place on the opposite sides (one convex, one concave) of a single multidimensional torus.
Solutions to the problem of infinity were introduced in Geroch et al. (1972), leading to computable solutions (Frauendiener 2000; Zenginoglu, 2007). The basic approach is to rewrite metrics defined in terms of points at infinity with tractable, computable metrics that sidestep infinite planes and points at infinity such as those shown in **Figure 2**.



X(a) X(b)

Figure 2 Conformable infinity at computable points near, but not at ideal points. **Left side**: Tractable point on spheres in spacetime that are sliced by an infinite plane. Ideal points on the infinite plane are replaced with non-ideal points that are computable. **Right side**: Those parts of a sheaf of infinite places that intersect with computable lines are computable up to but not including the ideal points.

Here is an example. The Minkowski spacetime metric in polar coordinates is $g = dt^2 - dr^2 - r^2 d\sigma^2$, where $d\sigma^2$ is the metric of the unit sphere. To achieve confomal rescaling of g, the null coordinates $u = t - r$ and $v = t + r$ are introduced, to obtain

$$g = dudv - \frac{1}{4}(v-u)^2 \, d\sigma^2.$$

The coordinates u and v range over the complete real line, provided $v - u \geq 0$. This infinite range is compactified, using $u = \tan U, v = \tan V$, and new null coordinates ranging over $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, resulting in

$$g = \frac{1}{4\cos^2 U \cos^2 V}\left[4dUdV - \sin^2(V-U)d\sigma^2\right],$$

which is not defined at $U = \pm\frac{\pi}{2}, V = \pm\frac{\pi}{2}$. To resolve this problem, a new metric is extracted from g, namely,

$$g_{new} = 4dUdV - \sin^2(V-U)d\sigma^2.$$

g_{new} is the tractable metric of the Einstein cylinder, which means that Minkowski space is now conformable embedded into the Einstein cyliner (Frauendiener 2000).

CONCLUSION: SOLVING INFINITY

It is assumed by physicists, due to pragmatic issues, that no measurable quantity or event might have infinite values. Indeed, any physical theory needs to provide operational formulas that correspond, to or at least approximate, reality (*e.g.*, Iyer and Petters, 2007). As an example, if any object of infinite gravitational mass were to exist, any use of the formula in order to quantify the gravitational force would lead to an useless infinite result. The formula would be useful neither to compute the force between two objects of finite mass, nor to compute their motions. Sometimes, an infinite result of a physical quantity may mean that the theory being used is approaching the point where it fails.
Here we provide a novel conceptual framework able to theoretically solve the problem of the occurrence of infinity in physical equations. We achieve at first a physical system's description on, say, the convex surface of a manifold. In order to avoid the occurrence of infinite when the curvature approaches the zero, we project such description to the concave surface of a manifold, through a long list of available and well-studied procedures of vectorial and tensorial transport: e.g., Ehresmann connections (Tozzi et al., 2017), parallel transport on Riemannian manifolds (Sengupta et al., 2017), and so on. In other words, we need at first to project a physical phenomenon which tends to intractable infinity

onto a positive-curvature manifold, such as a triangle, or a sphere, or a donut-like manifold, arriving at a physical geometry useful in the physical sciences (see, e.g., Peters 2016). Indeed, when the phenomenon's calculations tend towards infinity, we achive an unwanted flat line. To go back to a tractable description, we need now to introduce a negative-curvature manifold, corresponding to a modified physical phenomenon, where finite operations might take place.

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