An Elegant Solution to the Cosmological Constant Problem based on The Bohm-Poisson Equation

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Abstract

After applying the recently proposed Bohm-Poisson equation [1] to the observable Universe as a *whole*, and by introducing an ultraviolet (very close to the Planck scale) and an infrared (Hubble radius) scale, one can naturally obtain a value for the vacuum energy density which coincides *exactly* with the extremely small observed vacuum energy density, and explain the origins of its *repulsive* gravitational nature. Because Bohm's formulation of QM is by construction non-local, it is this non-locality which casts light into the crucial ultraviolet/infrared entanglement of the Planck/Hubble scales which was required in order to obtain the observed value of the vacuum energy density.

Exact solutions to the stationary spherically symmetric Newton-Schroedinger equation

$$i\hbar \frac{\partial \Psi(\vec{r},t)}{\partial t} \ = \ - \ \frac{\hbar^2}{2m} \nabla^2 \Psi(\vec{r},t) \ - \ \left(\ Gm^2 \ \int \ \frac{|\Psi(\vec{r}',t)|^2}{|\vec{r}-\vec{r}'|} \ d^3r' \right) \ \Psi(\vec{r},t) \tag{1}$$

were proposed recently in terms of integrals involving *generalized* Gaussians [1]. The energy eigenvalues were also obtained in terms of these integrals which agree with the numerical results in the literature. We proceeded to replace the nonlinear Newton-Schroedinger equation for a non-linear quantum-like Bohm-Poisson equation involving Bohm's quantum potential, and where the fundamental quantity is *no* longer the wave-function Ψ but the real-valued probability density ρ.

Bohm's quantum potential $V_Q = -\frac{\hbar^2}{2m}(\nabla^2 \sqrt{\rho}/\sqrt{\rho})$ has a *geometrical* derivation in terms of the Weyl scalar curvature produced by an ensemble density of paths associated with one, and only one particle [2]. This geometrization process of quantum mechanics allowed to derive the Schroedinger, Klein-Gordon [2] and Dirac equations [3]. Most recently, a related geometrization of quantum mechanics was proposed [4] that describes the time evolution of particles as geodesic lines in a curved space, whose curvature is induced by the quantum potential. This formulation allows therefore the incorporation of all quantum effects into the geometry of space-time, as it is the case for gravitation in the general relativity. Based on these results we proposed [1] the following nonlinear quantum-like Bohm-Poisson equation for *static* solutions $\rho = \rho(\vec{r})$, after reabsorbing a mass factor inside ρ so that ρ is now a mass-density,

$$\nabla^2 V_Q \ = \ 4\pi G m \rho \ \Rightarrow \ - \ \frac{\hbar^2}{2m} \ \nabla^2 \ (\frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}}) \ = \ 4\pi G m \rho \tag{2}$$

such that one could replace the nonlinear Newton-Schroedinger equation for the above non-linear quantum-like Bohm-Poisson equation (2) where the fundamental quantity is *no* longer the wave-function Ψ (complex-valued in general) but the real-valued probability density $\rho = \Psi^* \Psi$.

It has been proposed by [5], [6] to give up the description of physical states in terms of ensembles of state vectors with various probabilities, relying instead solely on the density matrix as the description of reality. The time evolution of ρ is governed by the Lindblad equation [1]. The authors [6] also investigated a number of unexplored features of quantum theory, including an interesting *geometrical* structure- which they called subsystem space- that they believed merits further study.

An infinite-derivative-gravity generalization of eq-(2) is [1]

$$- \ \frac{\hbar^2}{2m} \ (e^{-\frac{\sigma^2}{4}\nabla^2} \ \nabla^2) \ (\frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}}) \ = \ 4\pi G m \rho \tag{3}$$

the above equation is nonlinear *and* nonlocal.

If one wishes to introduce a temporal evolution to ρ via a Linblad-like equation, for instance, this would lead to an overdetermined system of differential equations for $\rho(\vec{r}, t)$. This problem might be another manifestation of the problem of time in Quantum Gravity. Naively replacing ∇^2 in eqs-(2,3) for the D'Alambertian operator $\partial_\mu \partial^\mu, \mu = 0, 1, 2, 3$ has the caveat that in QFT $\rho(x^\mu) = \rho(\vec{r}, t)$ *no* longer has the interpretation of a probability density (it is now related to the particle number current). For the time being we shall just focus on static solutions $\rho(\vec{r})$.

It is straightforward to verify that a spherically symmetric solution to eq-(2) in $D = 3$ is

$$\rho(r) \ = \ \frac{A}{r^4}, \ \ A \ = \ - \ \frac{\hbar^2}{2\pi G m^2} \ < \ 0 \tag{4}$$

At first glance, since $\rho(r) \leq 0$ one would be inclined to dismiss such solution as being unphysical. Nevertheless, we can bypass this problem by focusing instead on the *shifted*

[1]To be more precise it is the Gorini-Kossakowski-Sudarshan-Lindblad equation

density $\tilde{\rho}(r) \equiv \rho(r) - \rho_0$ obeying the Bohm-Poisson equation

$$- \frac{\hbar^2}{2m} \nabla^2 \left(\frac{\nabla^2 \sqrt{\tilde{\rho}}}{\sqrt{\tilde{\rho}}} \right) = 4\pi G m \tilde{\rho} \tag{5}$$

and whose solution for the *shifted* density is given by

$$\tilde{\rho} = A/r^4 = \rho(r) - \rho_o \leq 0, \quad \Rightarrow \rho(r) = \frac{A}{r^4} + \rho_0, \quad A = - \frac{\hbar^2}{2\pi G m^2} \tag{6}$$

It is not problematic that the terms inside the square roots are less than zero, since a common factor of $i = \sqrt{-1}$ appears both in the numerator and denominator, and hence it cancels out. The idea now is to focus on the *domain* of values where $\rho(r) \geq 0$. And, in doing so, it will allows to show that the value of ρ_0 can be made to coincide exactly with the (extremely small) observed vacuum energy density, by simply introducing an ultraviolet length scale l that is *very close* to the Planck scale, and infrared length scale L equal to Hubble scale R_H.

In particular, the ultraviolet scale l is chosen at the node of $\rho(r)$, so

$$\rho(r = l) = - \frac{\hbar^2}{2\pi G m^2} \frac{1}{l^4} + \rho_o = 0 \Rightarrow \rho_o = \frac{\hbar^2}{2\pi G m^2} \frac{1}{l^4} \tag{7}$$

The domain of physical values of r must be $r \geq l$ in order to ensure a positive-definite density $\rho(r) \geq 0$.

In natural units of $\hbar = c = 1$, introducing the infrared scale $L = R_H$ in the normalization condition (otherwise the mass would diverge) it yields

$$m = \int_l^{R_H} \rho(r)\, 4\pi r^2\, dr = \int_l^{R_H} \left(\frac{A}{r^4} + \rho_0 \right) 4\pi r^2\, dr = \int_l^{R_H} \left(- \frac{1}{2\pi G m^2} \frac{1}{r^4} + \rho_0 \right) 4\pi r^2\, dr \tag{8}$$

Upon performing the integral in eq-(8), after plugging in the value of ρ_0 derived from eq-(7), with the provision that $R_H >> l$, the dominant contribution to the integral stems solely from ρ_o, and one ends up with the following relationship

$$\frac{4\pi R_H^3}{3} \rho_o = \frac{4\pi R_H^3}{3} \frac{1}{2\pi G m^2 l^4} = m \Rightarrow m^3 = \frac{2}{3} \frac{R_H^3}{G l^4} \tag{9}$$

solving for m one gets

$$m = \left(\frac{2}{3 G l^4} \right)^{1/3} R_H \tag{10}$$

One can verify that when the ultraviolet scale l is chosen to be *very* close to the Planck scale, given by

$$l^4 = \frac{4}{3} L_P^4 \Rightarrow l = \left(\frac{4}{3} \right)^{1/4} L_P \tag{11}$$

and then upon inserting the values for m and l obtained in eqs-(10,11) into the expression for ρ_o *derived* in eq-(7), after setting $L_p^2 = 2G$, [2] it gives in natural units of $\hbar = c = 1$

$$\rho_o \ = \ \frac{1}{2\pi G m^2} \ \frac{1}{l^4} \ = \ \frac{1}{2\pi G} \ (\frac{3 \ G \ l^4}{2})^{2/3} \ \frac{1}{R_H^2 \ l^4} \ = \ \frac{3}{8\pi G} \ \frac{L_p^4}{R_H^2 \ L_p^4} \ = \ \frac{3}{8\pi G R_H^2} \qquad (12)$$

which is precisely equal to the observed vacuum energy density $\rho = (2\Lambda/16\pi G)$ associated with a cosmological constant $\Lambda = (3/R_H^2)$ and corresponding to a de Sitter expanding universe whose throat size is the Hubble radius R_H.

The physical reason behind the choice of the ultraviolet scale l in eq-(11) is based on re-interpreting ρ_o as the uniform energy (mass) density inside a black hole region of Schwarzschild radius $R = 2Gm$

$$\rho_{bh} \ = \ \frac{m}{(4\pi/3)R^3} \ = \ \frac{3}{8\pi G R^2}, \quad L_P^2 = 2G, \quad \hbar = c = 1 \qquad (13)$$

when $R = 2Gm >> l$, equating the expression for ρ_o in eq-(7) to ρ_{bh} in eq-(13) gives

$$\frac{1}{2\pi G m^2 l^4} \ = \ \frac{1}{2\pi l^4} \ \frac{(2G)^2}{R^2} \ = \ \frac{1}{2\pi l^4} \ \frac{L_p^4}{R^2} \ = \ \frac{3}{8\pi G R^2} \ \Rightarrow \ l \ = \ (\frac{4}{3})^{1/4} \ L_p, \quad \hbar = c = 1 \qquad (14)$$

leading to eq-(11). Therefore, when $R = 2Gm >> l$, the value of l in eqs-(11,14) is always *very* close to the Planck scale, and *independent* of $R = 2Gm$, because the scale R has *decoupled* in eq-(14).

In this way, one can effectively view the observable universe as a "black-hole" whose Hubble radius R_H encloses a mass M_U given by $2GM_U = R_H$. From eq-(14) it follows that when $R = R_H$, the black hole density $\rho_{bh} = \rho_o = \rho_{obs}$ coincides with the observed vacuum energy density. It is well known that inside the black hole horizon region the roles of t and r are exchanged due to the switch in the signature of the g_{tt}, g_{rr} metric components. Cosmological solutions based on this $t \leftrightarrow r$ exchange were provided by the Kantowski-Sachs metric.

To sum up, after viewing the *whole* Universe as an effective lump of matter, whose mass density distribution $\rho(r) = Ar^{-4} + \rho_o$ (6) is explicitly derived from solving the Bohm-Poisson equation (5), one is able to *obtain* the observed vacuum energy density. Furthermore, the Bohm-Poisson equation allows us to find the correct physical interpretation of the vacuum energy density as a *repulsive* gravitational force. The reasoning goes as follows. A simple inspection of the left hand side of the Bohm-Poisson equation (5) for $\tilde{\rho} = \rho - \rho_o = Ar^{-4} \leq 0$, allows to multiply the numerator and denominator by $i = \sqrt{-1}$. Whereas in the right hand side one can simply rewrite $G\tilde{\rho} = (-G)(-\tilde{\rho})$, leading now to a Bohm-Poisson equation corresponding to a *positive* definite expression $-\tilde{\rho} = \rho_o - \rho = -Ar^{-4} \geq 0$, but with a *negative* gravitational constant $-G < 0$, associated to *repulsive* gravity.

[2] Some authors absorb the factor of 2 inside the definition of L_p

Concluding, after applying the Bohm-Poisson equation to the observable Universe as a *whole*, and by introducing an ultraviolet (very close to the Planck scale) and an infrared (Hubble) scale, one can naturally obtain a value for the vacuum energy density which coincides *exactly* with the extremely small observed vacuum energy density, and explain the origins of its *repulsive* gravitational nature. Is it a numerical coincidence or design ? Because Bohm's formulation of QM is by construction non-local, it is this non-locality which casts light into the crucial ultraviolet/infrared entanglement of the Planck/Hubble scales which was required in order to obtain the observed values of the vacuum energy density. Finally, we should add that of the many articles surveyed in the literature pertaining the role of Bohm's quantum potential and cosmology, [7], [8] we did not find any related to the Bohm-Poisson equation proposed in this work. A Google Scholar search provided the response "Bohm-Poisson equation and cosmological constant did not match any articles".

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