An Elegant Solution to the Cosmological Constant Problem based on The Bohm-Poisson Equation

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Abstract

After applying the recently proposed Bohm-Poisson equation [1] to the observable Universe as a *whole*, and by introducing an ultraviolet (Planck) and infrared (Hubble) scale, one can naturally obtain a value for the vacuum energy density of the *same magnitude* as the extremely small observed vacuum energy density, and explain the origins of its *repulsive* gravitational nature. Because Bohm's formulation of QM is by construction non-local, it is this non-locality which casts light into the crucial ultraviolet/infrared entanglement of the Planck/Hubble scales which was required in order to obtain the observed value of the vacuum energy density.

Exact solutions to the stationary spherically symmetric Newton-Schroedinger equation

$$i\hbar \frac{\partial \Psi(\vec{r},t)}{\partial t} \;=\; -\;\frac{\hbar^2}{2m}\nabla^2\Psi(\vec{r},t) \;-\; \left(G m^2 \int \frac{|\Psi(\vec{r}',t)|^2}{|\vec{r}-\vec{r}'|}\; d^3r' \right)\; \Psi(\vec{r},t) \qquad (1)$$

were proposed recently in terms of integrals involving *generalized* Gaussians [1]. The energy eigenvalues were also obtained in terms of these integrals which agree with the numerical results in the literature. We proceeded to replace the nonlinear Newton-Schroedinger equation for a non-linear quantum-like Bohm-Poisson equation involving Bohm's quantum potential, and where the fundamental quantity is *no* longer the wave-function Ψ but the real-valued probability density ρ.

Bohm's quantum potential $V_Q = -\frac{\hbar^2}{2m}(\nabla^2\sqrt{\rho}/\sqrt{\rho})$ has a *geometrical* derivation in terms of the Weyl scalar curvature produced by an ensemble density of paths associated with one, and only one particle [2]. This geometrization process of quantum mechanics allowed to derive the Schroedinger, Klein-Gordon [2] and Dirac equations [3]. Most recently, a related geometrization of quantum mechanics was proposed [4] that describes the time evolution of particles as geodesic lines in a curved space, whose curvature is induced by the quantum potential. This formulation allows therefore the incorporation of all quantum effects into the geometry of space-time, as it is the case for gravitation in the general relativity. Based on these results we proposed [1] the following nonlinear quantum-like Bohm-Poisson equation for *static* solutions $\rho = \rho(\vec{r})$, after reabsorbing a mass factor inside ρ so that ρ is now a mass-density,

$$\nabla^2 V_Q \ = \ 4\pi G m \rho \ \Rightarrow \ - \ \frac{\hbar^2}{2m} \ \nabla^2 \ (\frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}}) \ = \ 4\pi G m \rho \tag{2}$$

such that one could replace the nonlinear Newton-Schroedinger equation for the above non-linear quantum-like Bohm-Poisson equation (2) where the fundamental quantity is *no* longer the wave-function Ψ (complex-valued in general) but the real-valued probability density $\rho = \Psi^* \Psi$.

It has been proposed by [5], [6] to give up the description of physical states in terms of ensembles of state vectors with various probabilities, relying instead solely on the density matrix as the description of reality. The time evolution of ρ is governed by the Lindblad equation [1]. The authors [6] also investigated a number of unexplored features of quantum theory, including an interesting *geometrical* structure- which they called subsystem space- that they believed merits further study.

An infinite-derivative-gravity generalization of eq-(2) is [1]

$$- \ \frac{\hbar^2}{2m} \ (e^{-\frac{\sigma^2}{4}\nabla^2} \ \nabla^2) \ (\frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}}) \ = \ 4\pi G m \rho \tag{3}$$

the above equation is nonlinear *and* nonlocal.

If one wishes to introduce a temporal evolution to ρ via a Linblad-like equation, for instance, this would lead to an overdetermined system of differential equations for $\rho(\vec{r}, t)$. This problem might be another manifestation of the problem of time in Quantum Gravity. Naively replacing ∇^2 in eqs-(2,3) for the D'Alambertian operator $\partial_\mu \partial^\mu, \mu = 0, 1, 2, 3$ has the caveat that in QFT $\rho(x^\mu) = \rho(\vec{r}, t)$ *no* longer has the interpretation of a probability density (it is now related to the particle number current). For the time being we shall just focus on static solutions $\rho(\vec{r})$.

It is straightforward to verify that a spherically symmetric solution to eq-(2) in $D = 3$ is

$$\rho(r) \ = \ \frac{A}{r^4}, \ \ A \ = \ - \ \frac{\hbar^2}{2\pi G m^2} \ < \ 0 \tag{4}$$

At first glance, since $\rho(r) \leq 0$ one would be inclined to dismiss such solution as being unphysical. Nevertheless, we can bypass this problem by focusing instead on the *shifted* density $\tilde{\rho}(r) \equiv \rho(r) - \rho_0$ obeying the Bohm-Poisson equation

$$- \ \frac{\hbar^2}{2m} \ \nabla^2 \ (\frac{\nabla^2 \sqrt{\tilde{\rho}}}{\sqrt{\tilde{\rho}}}) \ = \ 4\pi G m \tilde{\rho} \tag{5}$$

[1] To be more precise it is the Gorini-Kossakowski-Sudarshan-Lindblad equation

and whose solution for the *shifted* density is given by

$$\tilde{\rho} \ = \ A/r^4 \ = \ \rho(r) \ - \ \rho_o \ \leq 0, \quad \Rightarrow \ \rho(r) \ = \ \frac{A}{r^4} \ + \ \rho_0, \quad A \ = \ - \ \frac{\hbar^2}{2\pi G m^2} \tag{6}$$

It is not problematic that the terms inside the square roots are less than zero, since a common factor of $i = \sqrt{-1}$ appears both in the numerator and denominator, and hence it cancels out. The idea now is to focus on the *domain* of values where $\rho(r) \geq 0$. And, in doing so, it will allows to show that ρ_0 is of the *same* order of magnitude as the (extremely small) observed vacuum energy density, after introducing an ultraviolet and infrared length scale which are close in values to the Planck L_p and Hubble scale R_H, respectively. In particular, the ultraviolet Planck scale L_p is chosen such that

$$\rho(r = L_p) \ = \ - \ \frac{\hbar^2}{2\pi G m^2} \ \frac{1}{L_p^4} \ + \ \rho_o \ = \ 0 \ \Rightarrow \ \rho_o \ = \ \frac{\hbar^2}{2\pi G m^2} \ \frac{1}{L_p^4} \tag{7}$$

The domain of physical values of r must be $r \geq L_P$ in order to ensure a positive-definite density $\rho(r) \geq 0$. By inserting the observed vacuum energy density $\rho_o = \frac{3}{8\pi L_p^2 R_H^2}$ (in units $\hbar = c = 1$) into eq-(7) one arrives at

$$\rho_o \ = \ \frac{3}{8\pi L_p^2 R_H^2} \ = \ \frac{3}{8\pi} \ (\frac{L_p}{R_H})^2 \ \frac{1}{L_p^4} \ = \ \frac{1}{2\pi G m^2} \ \frac{1}{L_p^4} \ \Rightarrow$$

$$G m \ = \ \frac{2}{\sqrt{3}} \ R_H \ = \ 1.154 \ R_H, \quad G = L_p^2, \quad \hbar = c = 1 \tag{8}$$

Therefore, the value of Gm is quite close to R_H, which is compatible with the value of the mass of the observable universe M_U given by $GM_U = R_H$, and which in Planck mass units is $M_U = (R_H/L_p) M_p \sim 10^{61} M_p$.

The infrared scale L is fixed by the normalization condition

$$m \ = \ \int_{L_p}^{L} \ \rho(r) \ 4\pi r^2 \ dr \ = \ \int_{L_p}^{L} \ (\frac{A}{r^4} \ + \rho_0) \ 4\pi r^2 \ dr \ = \ \int_{L_p}^{L} \ \left(- \frac{1}{2\pi G m^2} \ \frac{1}{r^4} \ + \ \rho_0 \right) \ 4\pi r^2 \ dr \tag{9}$$

Upon performing the integral in eq-(9), using eq-(8), and after some straightforward algebra one arrives at the relationship

$$\frac{1}{L_p} \ \left(\frac{1}{3} \ (\frac{L}{L_p})^3 \ + \ \frac{L_p}{L} \ - \ \frac{4}{3} \right) \ = \ \frac{1}{2L_p} \ (\frac{2}{\sqrt{3}})^3 \ (\frac{R_H}{L_p})^3 \tag{10}$$

which furnishes the value of the infrared scale

$$L \ \simeq \ (\frac{4}{\sqrt{3}})^{1/3} \ R_H \ = \ 1.321 \ R_H \tag{11}$$

in terms of the Hubble scale. As expected, the value of $L \simeq R_H$ is of the *same* order of magnitude as the Hubble scale.

On the other hand, if one were to set a priori the value of $L = R_H$ to coincide *precisely* with the Hubble scale, from the two eqs-(7,9) one would have obtained (in units $\hbar = c = 1$) the following values

$$Gm \ = \ (\frac{2}{3})^{1/3} \ R_H, \quad \rho_o \ = \ \frac{1}{2\pi} \ (\frac{3}{2})^{2/3} \ (\frac{L_p}{R_H})^2 \ \frac{1}{L_p^4} \tag{12}$$

for the mass m *and* the vacuum energy density ρ_0. The ratio of the value of ρ_o obtained in eq-(12) with the observed value of ρ_o associated with a de Sitter expanding universe is $\frac{4}{3} \ (\frac{3}{2})^{2/3} = 1.746$; i.e. the values are of the *same* order of magnitude. This result is to be contrasted with the 10^{122} discrepancy associated with the cosmological constant problem. A third scenario is to fix a priori the values of $Gm = R_H$, and $\rho_o = \frac{3}{8\pi L_p^2 R_H^2}$, and then from eqs-(7,9) derive the values of the ultraviolet and infrared scales, which will also turn out to be quite close to the Planck and Hubble scale, respectively, as expected.

Not only one is able to *derive* very acceptable results for the vacuum energy density from the Bohm-Poisson equation, but one also finds the correct physical interpretation of the vacuum energy density as a *repulsive* gravitational force. The reasoning goes as follows. A simple inspection of the left hand side of the Bohm-Poisson equation (5) for $\tilde{\rho} = \rho - \rho_o = Ar^{-4} \leq 0$, allows to multiply the numerator and denominator by $i = \sqrt{-1}$. Whereas in the right hand side one can simply rewrite $G\tilde{\rho} = (-G)(-\tilde{\rho})$, leading now to a Bohm-Poisson equation corresponding to a *positive* definite expression $-\tilde{\rho} = \rho_o - \rho = -Ar^{-4} \geq 0$, but with a *negative* gravitational constant $-G < 0$, associated to *repulsive* gravity.

Concluding, after applying the Bohm-Poisson equation to the observable Universe as a *whole*, and by introducing an ultraviolet (Planck) and infrared (Hubble) scale, one can naturally obtain a value for the vacuum energy density of the *same* magnitude as the extremely-small observed vacuum energy density, and explain the origins of its *repulsive* gravitational nature. Is it numerical coincidence or design ? Because Bohm's formulation of QM is by construction non-local, it is this non-locality which casts light into the crucial ultraviolet/infrared entanglement of the Planck/Hubble scales which was required in order to obtain the observed values of the vacuum energy density.

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