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Multi criteria decision making using correlation coefficient under rough neutrosophic environment

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Abstract In this paper, we define correlation coefficient measure between any two rough neutrosophic sets. We also prove some of its basic properties.. We develop a new multiple attribute group decision making method based on the proposed correlation coefficient measure. An illustrative example of medical diagnosis is solved to demonstrate the applicability and effecriveness of the proposed method.

Keywords: Multi-attribute group decision making; Neutrosophic set; Rough set; Rough neutrosophic set; Correlation coefficient.

1 Introduction

Smarandache established the concept of neutrosophic set and neutrosophic logic [1] to deal uncertainty, inconsistency, incompleteness and indeterminacy in 1998. Smarandache [1] and Wang et. al. [2] studied single valued neutrosophic set (SVNS), a subclass of neutrosophic set to deal realistic problems in 2010. SVNSs have been widely studied and applied in different fields such as medical diagnosis [3], multi criteria decision making [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17], image processing [18, 19, 20], etc.

Pawlak [21] defined rough set to study intelligence systems characterized by inexact, uncertain or insufficient information. Broumi et al. [22, 23] defined rough neutrosophic set by combining the rough set and single valued neutrosophic set to deal with problems involving uncertain, imprecise, incomplete and inconsistent information existing in real world problems.

Decision making in rough neutrosophic environment is a new subfield of operational resesarch. In rough neutrosophic environment, Mondal and Pramanik [24] defined accumulated geometric operator to transform rough neutrosophic number (neutrosophic pair) to single valued neutrosophic number and developed a new multi-attribute decision-making (MADM) method based on grey relational analysis. Mondal and Pramanik [25] defined accuracy score function and proved its basic properties. In the same study, Mondal and Pramanik [25] presented a new MADM method in rough neutrosophic environment. Pramanik and Mondal [26] defined cotangent similarity measure of rough neutrosophic sets and proved its basic properties. In the same study, Pramanik and Mondal [26] presented its application to medical diagnosis. Pramanik and Mondal [27] proposed cosine similarity measure of rough neutrosophic sets and its application in medical diagnosis. Pramanik and Mondal [28] also proposed Dice and Jaccard similarity measures in rough neutrosophic environment and applied them for MADM. Mondal and Pramanik [29] studied cosine, Dice and Jaccard similarity measures for interval rough neutrosophic sets and presented MADM methods based on proposed rough cosine, Dice and Jaccard similarity measures in interval rough neutrosophic environment Mondal et al. [30] presented rough trigonometric Hamming similarity measures such as cosine, sine and cotangent rough similarity measures and proved their basic properties. In the same study, Mondal et al. [30] presented new MADM methods based on cosine, sine and cotangent rough similarity measures with illustrative example. Mondal et al. [31] proposed variational coefficient similarity measures under rough neutrosophic environment and proved some of their basic properties. In the same study, Mondal et al. [31] developed a new MADM method based on the proposed variational coefficient similarity measures and presented a comparison with four existing rough similarity measures namely, rough cosine similarity measure, rough dice similarity measure, rough cotangent similarity measure and rough Jaccard similarity measure for different values of the parameter λ . Mondal et al. [32] proposed rough neutrosophic aggregate operator and weighted rough neutrosophic aggregate operator to develop TOPSIS based MADM method in rough neutrosophic environment. Pramanik et al. [33] defined projection and bidirectional projection measures between rough neutrosophic sets. In the same study, Pramanik et al. [33] proposed two new multi criteria decision making (MCDM) methods based on neutrosophic projection and bidirectional projection measures respectively.

Mondal and Pramanik [34] proposed rough tri-complex similarity measure based MADM method in rough neutrosophic environment and proved some of its basic properties. In the same study, Mondal and Pramanik [34] presented comparison of obtained results for an illustrative MADM problem with other existing rough neutrosophic similarity measures.

Mondal et al. [35] defined rough neutrosophic hyper-complex set and rough neutrosophic hyper-complex cosine function and proved some of their basic properties. In the same study, Mondal et al. [35] also proposed rough neutrosophic hyper-complex similarity measure based MADM method.

Pramanik and Mondal [36] defined bipolar rough neutrosophic sets and proved it basic properties.

The correlation coefficient is an important tool to judge the relation between two objects. The correlation coefficients [37, 38, 39, 40, 41, 42] have been widely employed to data analysis and classification, decision making, pattern recognition, and so on. Many researchers pay attention to correlation coefficients under fuzzy environments. Chiang and Lin [43] introduced the correlation of fuzzy sets. Hong [44] proposed fuzzy measures for a correlation coefficient of fuzzy numbers under Tw (the weakest t-norm)-based fuzzy arithmetic operations. As an extension of fuzzy correlations, Wang and Li [45] introduced the correlation and information energy of interval-valued fuzzy numbers. Gerstenkorn and Manko [46] developed the correlation coefficients of intuitionistic fuzzy sets IFSs). Hung and Wu [47] also proposed a method to calculate the correlation coefficients of IFSs by centroid method. Xu [48] developed another correlation measure of interval-valued intuitionistic fuzzy environment, and applied it to medical diagnosis. Ye [49] studied the fuzzy decision-making method based on the weighted correlation coefficient under intuitionistic fuzzy environment. Bustince and Burillo [50] and Hong [51] further developed the correlation coefficients for interval-valued intuitionistic fuzzy sets (IVIFSs). Hanafy et al. [52] introduced the correlation of neutrosophic data. Ye [53] presented the correlation coefficient of SVNSs based on the extension of the correlation coefficient of IFSs and proved that the cosine similarity measure of SVNSs is a special case of the correlation coefficient of SVNSs. Hanafy et al. [54] presented the centroid-based correlation coefficient of neutrosophic sets and investigated its properties. Broumi and Smarandache [55] defined correlation coefficient of interval neutrosophic set and investigated its properties.

In the literature no studies have been reported on MADM using correlation coefficient under rough neutrosophic environment. To fill the research gap, we propose correlation coefficient under rough neutrosophic environment and proved some of its basic properties. We also present a new MADM method based on proposed measure. We also present an illustrative numerical example to show the effectiveness and applicability of the proposed method.

Rest of the paper is organized as follows: Section 2 describes preliminaries of neutrosophic sets, SVNSs and rough neutrosophic set (RNS). Section 3 describes the correlation coefficient between SVNSs. Section 4 presents definition and properties of proposed correlation coefficient between RNSs. Section 5 presents a rough neutrosophic decision making method based on correlation coefficient. Section 6 presents an illustrative hypothetical medical diagnostic problem based on the proposed MADM method. Finally, section 7 presents concluding remarks and future scope of research.

2 Preliminaries

2.1 Neutrosophic sets In 1998, Smarandache offered the following definition of neutrosophic set(NS)[1].

Definition 2.1.1 [1]
Let X be a space of points(objects) with generic element in X denoted by x. A NS A in X is characterized by a truth-membership function T_A, an indeterminacy membership function I_A and a falsity membership function F_A. The functions T_A, I_A and F_A are real standard or non-standard subsets of $]0^-,1^+[$ that is $T_A : X \to]0^-,1^+[, I_A : X \to]0^-,1^+[$ and $F_A : X \to]0^-,1^+[$. It should be noted that there is no restriction on the sum of T_A, I_A and F_A i.e $0^- \le T_A + I_A + F_A \le 3^+$.

Definition 2.1.2 [1]
(Complement) The complement of a neutrosophic set A is denoted by C(A) and is defined by $T_{c(A)}(x)=\{1^+\}-T_A(x)$, $I_{c(A)}(x)=\{1^+\}-I_A(x)$, $F_{c(A)}(x)=\{1^+\}-F_A(x)$.

Definition 2.1.3 [1]
A neutrosophic set A is contained in another neutrosophic set B, denoted by $A \subseteq B$ iff inf $T_A(x) \le$ inf $T_B(x)$, sup $T_A(x) \le$ sup $T_B(x)$, inf $I_A(x) \ge$ inf $I_B(x)$, sup $I_A(x) \ge$ inf $I_B(x)$, inf $F_A(x) \ge$ inf $F_B(x)$ and sup $F_A(x) \ge$ sup $F_B(x)$ for all x in X.

Definition 2.1.4 [2]
Let X be a universal space of points (objects) with a generic element of X denoted by x. A single valued neutrosophic set A is characterized by a truth membership function $T_A(x)$, a falsity membership function $F_A(x)$ and

indeterminacy function $I_A(x)$ with $T_A(x)$, $I_A(x)$ and $F_A(x) \in [0,1]$ for all x in X.
When X is continuous, a SNVS A can be written as follows: A= $\int_X < T_A(x), I_A(x), F_A(x) > / x$ for all $x \in X$ and when X is discrete, a SVNS A can be written as follows : A= $\Sigma < T_A(x), I_A(x), F_A(x) > / x$ for all $x \in X$.
For a SVNS S, $0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3$.

Definition 2.1.5 [2]
The complement of a single valued neutrosophic set A is denoted by c(A) and is defined by $T_{c(A)}(x) = F_A(x)$, $I_{c(A)}(x) = 1 - I_A(x)$, $F_{c(A)}(x) = T_A(x)$.

Definition 2.1.6 [2]
A SVNS A is contained in the other SVNS B, denoted as $A \subseteq B$ iff, $T_A(x) \le T_B(x)$, $I_A(x) \ge I_B(x)$, $F_A(x) \ge F_B(x)$ for all x in X.

2.2 Rough Neutrosophic sets

Rough neutrosophic sets [22, 23] are the generalization of rough fuzzy sets [56, 57, 58] and rough intuitionistic fuzzy sets [59].

Definition 2.2.1 [22]
Let Y be a non-null set and R be an equivalence relation on Y. Let P be a neutrosophic set in Y with the membership function T_P, indeterminacy function I_P and non-membership function F_P. The lower and the upper approximations of P in the approximation space (Y, R) are respectively defined as:

$$\underline{N}(P) = \langle\langle x, T_{\underline{N}(P)}(x), I_{\underline{N}(P)}(x), F_{\underline{N}(P)}(x) > / y \in [x]_R, x \in Y >$$

and

$$\overline{N}(P) = \langle\langle x, T_{\overline{N}(P)}(x), I_{\overline{N}(P)}(x), F_{\overline{N}(P)}(x) > / y \in [x]_R, x \in Y >$$

where,

$$T_{\underline{N}(P)}(x) = \wedge z \in [x]_R T_P(Y), I_{\underline{N}(P)}(x) = \wedge z \in [x]_R I_P(Y), F_{\underline{N}(P)}(x) = \wedge z \in [x]_R F_P(Y)$$

and

$$T_{\overline{N}(P)}(x) = \vee z \in [x]_R T_P(Y), I_{\overline{N}(P)}(x) = \vee z \in [x]_R I_P(Y), F_{\overline{N}(P)}(x) = \vee z \in [x]_R F_P(Y).$$

So,
$0 \le T_{\underline{N}(P)}(x) + I_{\underline{N}(P)}(x) + F_{\underline{N}(P)}(x) \le 3$ and $0 \le T_{\overline{N}(P)}(x) + I_{\overline{N}(P)}(x) + F_{\overline{N}(P)}(x) \le 3$.

Here $\vee$ and $\wedge$ denote "max" and "min" operators respectively, $T_P(y)$, $I_P(y)$, and $F_P(y)$ are the degrees of membership, indeterminacy and non-membership of Y with respect to P.
Thus NS mapping, $\underline{N}, \overline{N} : N(Y) \to N(Y)$ are, respectively, referred to as the lower and upper rough NS approximation operators, and the pair $(\underline{N}(P), \overline{N}(P))$ is called the rough neutrosophic set in (Y, R).

Definition 2.2.2 [22]
If $N(P) = (\underline{N}(P), \overline{N}(P))$ is a rough neutrosophic set in (Y, R), the rough complement of N(P) is the rough neutrosophic set denoted by $\sim(N(P)) = ((\underline{N}(P))^c, (\overline{N}(P))^c)$, where $(\underline{N}(P))^c$ and $(\overline{N}(P))^c$ are the complements of neutrosophic sets $\underline{N}(P)$ and $\overline{N}(P)$ respectively.

3 Correlation coefficient of SVNSs

Based on the correlation of intuitionistic fuzzy sets, Ye [53] defined the informational energy of a SVNS A, the correlation of two SVNSs A and B, and the correlation coefficient of two SVNSs A and B.

Definition 3.1 [53]
For a SVNS A in the universe of discourse X = {x_1, x_2,…, x_n}, the informational energy of the SVNS A is defined by

$$I(A) = \sum_{i=1}^{n} [T_A^2(x_i) + I_A^2(x_i) + F_A^2(x_i)]$$

Definition 3.2 [53]
For two SVNSs A and B in the universe of discourse X = {x_1, x_2,…, x_n}, correlation of the SVNSs A and B is defined as

$$C(A,B) = \sum_{i=1}^{n} [T_A(x_i)T_B(x_i) + I_A(x_i)I_B(x_i) + F_A(x_i)F_B(x_i)]$$

Definition 3. 3 [53]

The correlation coefficient of the SVNSs A and B is defined by the following formula: (1)

$$K(A, B) = \frac{C(A,B)}{[C(A,A).C(B,B)]^{1/2}} = \frac{\sum_{i=1}^{n} [T_A(x_i)T_B(x_i) + I_A(x_i)I_B(x_i) + F_A(x_i)F_B(x_i)]}{[\sum_{i=1}^{n} [(T_A(x_i))^2 + (I_A(x_i))^2 + (F_A(x_i))^2]]^{\frac{1}{2}} [\sum_{i=1}^{n} [(T_B(x_i))^2 + (I_B(x_i))^2 + (F_B(x_i))^2]]^{\frac{1}{2}}}$$

The correlation coefficient K(A, B) satisfies the following properties :
(1) K(A, B) = K(B, A);
(2) $0 \le K(A, B) \le 1$;
(3) K(A, B) = 1, if A = B.

4 Correlation coefficient of rough neutrosophic sets

Correlation coefficient between rough neutrosophic sets (RNSs) is yet to define in the literature. Therefore in this paper, we define correlation coefficient between RNSs.

Definition4.1. Assume that there are any two RNSs
$A = \langle(\underline{T_A(x_i)}, \underline{I_A(x_i)}, \underline{F_A(x_i)}), (\overline{T_A(x_i)}, \overline{I_A(x_i)}, \overline{F_A(x_i)})\rangle$ and
$B = \langle(\underline{T_B(x_i)}, \underline{I_B(x_i)}, \underline{F_B(x_i)}), (\overline{T_B(x_i)}, \overline{I_B(x_i)}, \overline{F_B(x_i)})\rangle$. Then the correlation between the RNSs A and B is defined as

$$C(A,B) = \sum_{i=1}^{n} [\delta T_A(x_i).\delta T_B(x_i) + \delta I_A(x_i).\delta I_B(x_i) + \delta F_A(x_i).\delta F_B(x_i)]$$

where

$$\delta T_A(x_i) = \frac{\underline{T}_A(x_i) + \overline{T}_A(x_i)}{2},$$

$$\delta I_A(x_i) = \frac{\underline{I}_A(x_i) + \overline{I}_A(x_i)}{2},$$

$$\delta F_A(x_i) = \frac{\underline{F}_A(x_i) + \overline{F}_A(x_i)}{2},$$

$$\delta T_B(x_i) = \frac{\underline{T}_B(x_i) + \overline{T}_B(x_i)}{2},$$

$$\delta I_B(x_i) = \frac{\underline{I_B}(x_i) + \overline{I_B}(x_i)}{2} \quad \text{and}$$

$$\delta F_B(x_i) = \frac{\underline{F_B}(x_i) + \overline{F_B}(x_i)}{2}.$$

Definition 4.2.The correlation coefficient of the RNSs A and B is defined as

$$K(A,B)= \frac{C(A,B)}{[C(A,A).C(B,B)]^{1/2}}$$

$$= \frac{\sum_{i=1}^{n}[\delta T_A(x_i).\delta T_B(x_i) + \delta I_A(x_i).\delta I_B(x_i) + \delta F_A(x_i).\delta F_B(x_i)]}{(\sum_{i=1}^{n}[(\delta T_A(x_i))^2 + (\delta I_A(x_i))^2 + (\delta F_A(x_i))^2]^{\frac{1}{2}})(\sum_{i=1}^{n}[(\delta T_B(x_i))^2 + (\delta I_B(x_i))^2 + (\delta F_B(x_i))^2]^{\frac{1}{2}})} \quad ...(2)$$

The correlation coefficient K(A, B) satisfies the following properties :

(1) K(A, B) = K(B, A);

(2) $0 \le K(A,B) \le 1;$

(3) K(A, B) = 1, if A = B.

Proof

(i)

$$K(A,B) = \frac{C(A,B)}{[C(A,A).C(B,B)]^{1/2}}$$

$$= \frac{C(B,A)}{[C(B,B).C(A,A)]^{1/2}} = K(B,A)$$

(ii) As C(A, B) ≥ 0, C(A, A) ≥ 0, C(B, B) ≥ 0 so K(A, B) ≥ 0.

According to the Cauchy–Schwarz inequality:

$$(a_1 b_1 + ...+ a_n b_n)^2 \le (a_1^2 + ...+ a_n^2)(b_1^2 + ...+ b_n^2)$$

where a_i , $b_i \in R$ for i=1,......,n,

So $\frac{(a_1 b_1 + ...+ a_n b_n)}{(a_1^2 + ...+ a_n^2)^{\frac{1}{2}}(b_1^2 + ...+ b_n^2)^{\frac{1}{2}}} \le 1$

Replacing a_i by $\delta T_A(x_i)$ and b_i by $\delta T_B(x_i)$ we obtain

K(A, B) ≤1.

Therefore, 0≤ K(A, B) ≤ 1.

(iii) If A = B

then $K(A,B) = K(A,A) = \frac{C(A,A)}{[C(A,A).C(A,A)]^{1/2}}$

$= \frac{C(A,A)}{C(A,A)} = 1$

Hence proved.

Considering n = 1, we get the following: (3)

K(A,B)=

$$\frac{\delta T_A(x_i).\delta T_B(x_i) + \delta I_A(x_i).\delta I_B(x_i) + \delta F_A(x_i).\delta F_B(x_i)}{((\delta T_A(x_i))^2 + (\delta I_A(x_i))^2 + (\delta F_A(x_i))^2)^{\frac{1}{2}}((\delta T_B(x_i))^2 + (\delta I_B(x_i))^2 + (\delta F_B(x_i))^2)^{\frac{1}{2}}}$$

Which is the cosine similarity measure between two RNSs A and B [27].

Weighted correlation coefficient:

Let w = {w_1, w_2, …, w_n} be the weight vector of the elements x_i (i = 1, 2, …, n).

Then the weighted correlation coefficient between A and B is defined by the following formula:

$$K_W(A,B) = \frac{\sum_{i=1}^{n} w_i[\delta T_A(x_i).\delta T_B(x_i) + \delta I_A(x_i).\delta I_B(x_i) + \delta F_A(x_i).\delta F_B(x_i)]}{[(\sum_{i=1}^{n}\{w_i[(\delta T_A(x_i))^2 + (\delta I_A(x_i))^2 + (\delta F_A(x_i))^2]\}^{\frac{1}{2}})(\sum_{i=1}^{n}\{w_i[(\delta T_B(x_i))^2 + (\delta I_B(x_i))^2 + (\delta F_B(x_i))^2]\}^{\frac{1}{2}})]} \quad (4)$$

If w = {1/n, 1/n, …, 1/n}, then equation (4) reduces to equation (2).

Weighted correlation coefficient K_w(A, B) also satisfies the following properties:

(1) K_w(A, B) = K_w(B, A);

(2) $0 \le K_w(A,B) \le 1;$

(3) K_w(A, B) = 1, if A = B.

Proof

(i)

$$K_W(A,B) = \frac{\sum_{i=1}^{n} w_i[\delta T_A(x_i).\delta T_B(x_i) + \delta I_A(x_i).\delta I_B(x_i) + \delta F_A(x_i).\delta F_B(x_i)]}{[(\sum_{i=1}^{n}\{w_i[(\delta T_A(x_i))^2 + (\delta I_A(x_i))^2 + (\delta F_A(x_i))^2]\}^{\frac{1}{2}})(\sum_{i=1}^{n}\{w_i[(\delta T_B(x_i))^2 + (\delta I_B(x_i))^2 + (\delta F_B(x_i))^2]\}^{\frac{1}{2}})]}$$

$$= \frac{\sum_{i=1}^{n} w_i[\delta T_B(x_i).\delta T_A(x_i) + \delta I_B(x_i).\delta I_A(x_i) + \delta F_B(x_i).\delta F_A(x_i)]}{[(\sum_{i=1}^{n}\{w_i[(\delta T_B(x_i))^2 + (\delta I_B(x_i))^2 + (\delta F_B(x_i))^2]\}^{\frac{1}{2}})(\sum_{i=1}^{n}\{w_i[(\delta T_A(x_i))^2 + (\delta I_A(x_i))^2 + (\delta F_A(x_i))^2]\}^{\frac{1}{2}})]}$$

$$= K_w(B,A)$$

$$\sum_{i=1}^{n} w_i[\delta T_A(x_i).\delta T_B(x_i) + \delta I_A(x_i).\delta I_B(x_i) + \delta F_A(x_i).\delta F_B(x_i)] \ge 0,$$

(ii) As $\sum_{i=1}^{n}\{w_i[(\delta T_A(x_i))^2 + (\delta I_A(x_i))^2 + (\delta F_A(x_i))^2]\}^{\frac{1}{2}} \ge 0$

and $\sum_{i=1}^{n}\{w_i[(\delta T_B(x_i))^2 + (\delta I_B(x_i))^2 + (\delta F_B(x_i))^2]\}^{\frac{1}{2}} \ge 0$

so K_W(A,B) ≥ 0.

Using the weighted Cauchy–Schwarz inequality [60], we have

$$(w_1 a_1 b_1 + ...+ w_n a_n b_n)^2 \le (w_1 a_1^2 + ...+ w_1 a_n^2)(w_1 b_1^2 + ...+ w_n b_n^2)$$

where w_i , a_i , $b_i \in R$ for i =1, ... , n.

So $\frac{(w_1 a_1 b_1 + ...+ w_n a_n b_n)}{(w_1 a_1^2 + ...+ w_n a_n^2)^{\frac{1}{2}}(w_1 b_1^2 + ...+ w_n b_n^2)^{\frac{1}{2}}} \le 1$

Replacing a_i by $w_i \delta T_A(x_i)$ and b_i by $w_i \delta T_B(x_i)$ we obtain

K_W(A, B) ≤ 1.

Therefore, 0 ≤ K(A, B) ≤ 1.

(iii) If A = B, then

K(A,B) = K(A,A)

$$= \frac{\sum_{i=1}^{n} w_i[\delta T_A(x_i).\delta T_A(x_i) + \delta I_A(x_i).\delta I_A(x_i) + \delta F_A(x_i).\delta F_A(x_i)]}{[(\sum_{i=1}^{n}\{w_i[(\delta T_A(x_i))^2 + (\delta I_A(x_i))^2 + (\delta F_A(x_i))^2]\}^{\frac{1}{2}})(\sum_{i=1}^{n}\{w_i[(\delta T_A(x_i))^2 + (\delta I_A(x_i))^2 + (\delta F_A(x_i))^2]\}^{\frac{1}{2}})]}$$

$$= \frac{\sum_{i=1}^{n}\{w_i[(\delta T_A(x_i))^2 + (\delta I_A(x_i))^2 + (\delta F_A(x_i))^2]\}}{\sum_{i=1}^{n}\{w_i[(\delta T_A(x_i))^2 + (\delta I_A(x_i))^2 + (\delta F_A(x_i))^2]\}} = 1$$

Hence proved.

5 Rough neutrosophic decision making based on correlation coefficient

Let $A_1, A_2, \ldots, A_m$ be a set of elements (/objects / persons), $C_1, C_2, \ldots, C_n$ be a set of criteria for each element and $E_1, E_2, \ldots, E_k$ are the alternatives for each element.

Step 1. The relation between elements A_i (i = 1, 2, ... , m) and the criteria C_j (j = 1, 2, ... ,n) is presented in Table 1 in terms of RNSs.

Table1 : Relation between elements and criteria

$$\begin{array}{c} \\ A_1 \\ A_2 \\ \ldots \\ A_m \end{array} \begin{array}{c} \begin{matrix} C_1 & C_2 & \ldots & C_n \end{matrix} \\ \begin{bmatrix} X_{11} & X_{12} & \ldots & X_{1n} \\ X_{21} & X_{22} & \ldots & X_{2n} \\ \ldots & \ldots & \ldots. & \ldots \\ X_{m1} & X_{m2} & \ldots & X_{mn} \end{bmatrix} \end{array}$$

where

$$X_{ij} = <(\underline{T}_{ij}, \underline{I}_{ij}, \underline{F}_{ij}), (\overline{T}_{ij}, \overline{I}_{ij}, \overline{F}_{ij})>$$

with $0 \le \underline{T}_{ij} + \underline{I}_{ij} + \underline{F}_{ij} \le 3$ and $0 \le \overline{T}_{ij} + \overline{I}_{ij} + \overline{F}_{ij} \le 3$.

The relation between criterion C_i (i = 1, 2, ... , n) and the alternative E_j (j = 1, 2, ... ,k) is presented in Table 2 in terms of RNSs.

Table 2 : Relation between criteria and alternatives

$$\begin{array}{c} \\ C_1 \\ C_2 \\ \ldots \\ C_m \end{array} \begin{array}{c} \begin{matrix} E_1 & E_2 & \ldots & E_k \end{matrix} \\ \begin{bmatrix} Y_{11} & Y_{12} & \ldots & Y_{1k} \\ Y_{21} & Y_{22} & \ldots & Y_{2k} \\ \ldots & \ldots & \ldots. & \ldots \\ Y_{n1} & Y_{n2} & \ldots & Y_{nk} \end{bmatrix} \end{array}$$

where

$$Y_{ij} = <(\underline{T}_{ij}, \underline{I}_{ij}, \underline{F}_{ij}), (\overline{T}_{ij}, \overline{I}_{ij}, \overline{F}_{ij})>$$

with

$0 \le \underline{T}_{ij} + \underline{I}_{ij} + \underline{F}_{ij} \le 3$ and $0 \le \overline{T}_{ij} + \overline{I}_{ij} + \overline{F}_{ij} \le 3$.

Step 2. Determine the correlation measure between Table 1 and Table 2 using equation 2. The obtained values are presented in Table 3.

Table 3 : Correlation coefficient between table1 and table2

$$\begin{array}{c} \\ A_1 \\ A_2 \\ \ldots \\ A_m \end{array} \begin{array}{c} \begin{matrix} E_1 & E_2 & \ldots & E_k \end{matrix} \\ \begin{bmatrix} p_{11} & p_{12} & \ldots & p_{1k} \\ p_{21} & p_{22} & \ldots & p_{2k} \\ \ldots & \ldots & \ldots. & \ldots \\ p_{m1} & p_{m2} & \ldots & p_{mk} \end{bmatrix} \end{array}$$

Step 3. From Table 3, for each element A_i (i = 1, 2, ..., m), find the maximum correlation value of the i-th row (i = 1, 2, ..., m). If the maximum value occurs at j-th column (j = 1, 2, ..., k) (see Table 3), then E_j will be the best alternative for the element A_i (i =1, 2, ..., m).

Step 4. End.

6 Medical Diagnosis Problem

We consider a medical diagnosis problem for illustration of the proposed method. Medical diagnosis comprises of inconsistent, indeterminate and incomplete information though increased volume of information available to doctors from new medical technologies. The proposed correlation coefficients among the patients versus symptoms and symptoms versus diseases will provide medical diagnosis. Let P = {P_1, P_2, P_3} be a set of patients, D = {Viral fever, Malaria, Stomach problem, Chest problem} be a set of diseases and S = {Temperature, Headache, Stomach pain, Cough, Chest pain} be a set of symptoms. Using proposed method the doctor is to examine the patient and to determine the disease of the patient in rough neutrosophic environment.

Based on the proposed approach the considered problem is solved using the following steps:

Step 1. Construction of the rough neutrosophic decision matrix

Table 4: (Relation-1) The relation between Patients and Symptoms

	Temperature	Headache	Stomach pain	cough	Chest pain
P_1	<(.6,.4,.3), (.8,.2,.1)>	<(.4,.4,.4), (.6,.2,.2)>	<(.5,.3,.2), (.7,.1,.2)>	<(.6,.2,.4), (.8,.0,.2)>	<(.4,.4,.4), (.6,.2,.2)>
P_2	<(.5,.3,.4), (.7,.3,.2)>	<(.5,.3,.3), (.7,.3,.3)>	<(.5,.3,.4), (.7,.1,.4)>	<(.5,.3,.3), (.9,.1,.3)>	<(.5,.3,.3), (.7,.1,.3)>
P_3	<(.6,.4,.4), (.8,.2,.2)>	<(.5,.2,.3), (.7,.0,.1)>	<(.4,.3,.4), (.8,.1,.2)>	<(.6,.1,.4), (.8,.1,.2)>	<(.5,.3,.3), (.7,.1,.1)>

Table 5: (Relation-2) The relation among Symptoms and Diseases

	Viral Fever	Malaria	Stomach problem	Chest problem
Temperature	<(.6,.5,.4), (.8,.3,.2)>	<(.1,.4,.4), (.5,.2,.2)>	<(.3,.4,.4), (.5,.2,.2)>	<(.2,.4,.6), (.4,.4,.4)>
Headache	<(.5,.3,.4), (.7,.3,.2)>	<(.2,.3,.4), (.6,.3,.2)>	<(.2,.3,.3), (.4,.1,.1)>	<(.1,.5,.5), (.5,.3,.3)>
Stomach pain	<(.2,.3,.4), (.4,.3,.2)>	<(.1,.4,.4), (.3,.2,.2)>	<(.4,.3,.4), (.6,.1,.2)>	<(.1,.4,.6), (.3,.2,.4)>
cough	<(.4,.3,.3), (.6,.1,.1)>	<(.3,.3,.3), (.5,.1,.3)>	<(.1,.6,.6), (.3,.4,.4)>	<(.5,.3,.4), (.7,.1,.2)>

Chest pain	<(.2,.4,.4), (.4,.2,.2)>	<(.1,.3,.3), (.3,.1,.1)>	<(.1,.4,.4), (.3,.2,.2)>	<(.4,.4,.4), (.6,.2,.3)>

Step 2. Determination of correlation coefficient between table 1 and table 2

Table 6: The correlation measure between Relation-1 and Relation-2

	Viral Fever	Malaria	Stomach problem	Chest problem
P_1	**0.95135**	0.91141	0.84518	0.87465
P_2	**0.95033**	0.94374	0.86228	0.91731
P_3	**0.93473**	0.89549	0.82559	0.85937

Step 3. Ranking the alternatives

According to the values of correlation coefficient of each alternative shown in Table 3, the highest correlation measure occurs in column1(i.e. for the diseases viral fever. Therefore, all three patients P_1, P_2, P_3 suffer from viral fever.

7 Conclusion

In this paper, we have proposed correlation coefficient and weighted correlation coefficient between rough neutrosophic sets and proved some of their basic properties. We have developed a new multi criteria decision making method based on the correlation coefficient measure. We presented an illustrative example in medical diagnosis. We hope that the proposed method can be applied in solving realistic multi criteria group decision making problems in rough neutrosophic environment.

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Received: July 13, 2017. Accepted: August 1, 2017.