First draft

Standard Model from Broken Scale Invariance in the Infrared

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Abstract

As we have recently shown, the *minimal fractal manifold* (MFM) describes the underlying structure of spacetime near or above the electroweak scale ($M_{EW} \approx 246$ GeV). Here we uncover the connection between quantum field operators and the MFM starting from the *operator product expansion* of high-energy Quantum Field Theory (QFT). The approach confirms that the Standard Model of particle physics (SM) stems from a symmetry breaking mechanism that turns the spacetime continuum into a MFM.

1. Multifractal description of high-energy QFT

QFT is plagued by several technical difficulties that challenge its consistency and predictive power in the ultraviolet (UV) region of energy scales. For example, [1]:

1) QFT operators $O(x)$ are singular at sharp points x, as they create or annihilate states with arbitrarily high energy from the vacuum,
2) Correlation functions $\langle 0|O(x_1)O(x_2)...|0\rangle$ are always singular, an outcome derived from the relativistic invariance and positivity of the Hilbert space,
3) Definite localization of quantum particles in space-time is impossible: Vectors having the form $\int d^4x\, f(x)O(x)|0\rangle$ are spread across the entire Hilbert space,

4) Interacting field theories contain arbitrarily many quantum particles associated with operators acting on the vacuum, $O(x)|0\rangle$.

In our opinion, these challenges call for a fresh perspective on the construction and interpretation of the high-energy Lagrangian, which describes a theory in a range well above the electroweak scale $(M_{EW} \approx 246\,GeV)$. The goal of this section is to elaborate on this viewpoint.

We start by recalling that all perturbative QFT's with well-behaved ultraviolet (UV) behavior are thought to be described by *operator product expansions* (OPE's). OPE's are a standard tool in the analysis of high-energy QFT, whose applications include quantum gauge and conformally invariant field theories [1-3]. The formulation of OPE's is based on the following prescription: Given a set of quantum fields labeled by the index A, namely $\{O_A\}$, a state Ψ represents an expectation value functional defined by the N point functions $\langle O_{A_1}(x_1)...O_{A_N}(x_N)\rangle_\Psi > 0$ and the OPE states that

$$\langle O_{A_1}(x_1)...O_{A_N}(x_N)\rangle_\Psi = \sum_B C^B_{A_1...A_N}(x_1,...,x_N)\langle O_B(x_N)\rangle_\Psi \tag{1}$$

The expansion (1) means that the product of N local fields located at nearby points $x_1,...,x_N$ is identical to the expectation value of another local field defined at the last point in the series (x_N). The numerical coefficient functions $C^B_{A_1...A_N}(x_1,...,x_N)$ are independent of Ψ and play the role of "structure constants" in the OPE algebra. In particular, the two-point function of perturbative Euclidean field theory is fully specified by the collection of

OPE coefficients which are state-independent, along with the 1-point function $\langle O_c(y)\rangle$, as in

$$\langle O_A(x)O_B(y)\rangle_\Psi = \sum_C C^{\,C}_{AB}(x-y)\langle O_c(y)\rangle \tag{2}$$

As shown in [6-7], the concept of OPE's sets up a direct link between multifractals and Lagrangian field theory, in that it maps the field operators to the moments of multifractal scaling according to

$$\overline{O^n_\delta(x_1)O^m_\delta(x_2)\ldots} = \langle O_n(x_1)O_m(x_2)\ldots\rangle_\delta \tag{3}$$

in which $\langle\cdot\rangle_\delta$ stands for the expectation value measured over a short-distance cutoff δ. Relation (3) is a typical an example of multifractal behavior since the expectation values of field operators scale as

$$\langle O_n\rangle_\delta \sim \left(\delta/\xi\right)^{x_n} \tag{4}$$

for vanishing cutoffs $(\delta \to 0)$ and diverging correlation lengths $(\xi \to \infty)$. In field-theoretic language, the spacetime cutoff represents the inverse of a large momentum cutoff $(\delta \sim \Delta^{-1}_{UV})$ and the correlation length the inverse of a mass parameter $(\xi \sim m^{-1})$. Thus,

$$\langle O_n\rangle_{\Delta_{UV}} \propto (m/\Lambda_{UV})^{x_n} \tag{5}$$

Elaborating further, one appeals to dimensional regularization to connect (5) with the infinitesimal deviation of spacetime dimension from $D=4$ viz. [9-13]

$$\boxed{\varepsilon = 4 - D \propto (m/\Delta_{UV})^2 \Rightarrow \langle O_n \rangle^2_{\Delta_{UV}} \propto \varepsilon^{x_n}} \tag{6}$$

Remarkably, it can be shown that, not only the expectation value of quantum fields, but masses and gauge charges obey similar power-law dependence on ε due to the "*minimal fractal manifold*" (MFM) structure of the four-dimensional continuum near or above the electroweak scale M_{EW}. It is this transition that explains both the hierarchical organization and self-contained nature of SM in a range of scales $O(M_{EW})$ [9-13].

2. Emergence of SM from the minimal fractal manifold

Previous section has shown that the UV region of field theory may be modeled as a multifractal set with field operators unfolding from the non-vanishing deviation of spacetime dimensionality from $D = 4$ viz. $\varepsilon = 4 - D << 1$. Exploiting the analogy between multifractal sets and statistical physics [4- 5, 10-11], a reasonable interpretation of $\varepsilon << 1$ is that it encodes a *dimensional polarization* of spacetime. This polarization plays the role of an *effective order parameter* and it characterizes a phase transition occurring in the flow from UV to infrared (IR). In general, this scaling flow is *non-Markovian,* as it preserves the memory of consecutive scale transformations.

Elaborating on these insights, the coupling between field operators at successive polarizations $\varepsilon << 1$ along the flow trajectory may be described by analogy with the traditional one-dimensional Ising model. The simplest Hamiltonian reflecting this long-range and weak interaction takes the form [4]:

$$H(\{O\}_n) = -\frac{J}{2n} \sum_{i}^{n} \sum_{j \neq i}^{n} \langle O_i \rangle \langle O_j \rangle \tag{7}$$

where operator averaging is performed over the short-distance cutoff $\delta \sim \Delta_{UV}^{-1}$ and where $0 < J < 1$ denotes the "nearest neighbor" coupling. Replacing (6) in (7) yields

$$H(\{O\}_n) = -\frac{J}{2n}\sum_{i}^{n}\sum_{j\neq i}^{n}\varepsilon^{x_i}\varepsilon^{x_j} = -\frac{J}{2n}\sum_{i}^{n}\sum_{j\neq i}^{n}\varepsilon_i\varepsilon_j \tag{8}$$

It is seen from (7) and (8), that the expectation value of the Hamiltonian $\langle H(\{O\}_n)\rangle$ behaves as an extensive quantity, proportional to the number of scaling iterations n. By comparison with the concept of spin magnetization in the Ising model, the overall dimensional polarization is given by

$$\langle \varepsilon \rangle = \frac{\sum_k \varepsilon_k}{n} = \frac{\sum_k \langle O_k \rangle}{n} \tag{9}$$

and satisfies the self-consistent equation [4]

$$\frac{1}{2}\ln\left(\frac{1+\langle \varepsilon \rangle}{1-\langle \varepsilon \rangle}\right) = qJ\langle \varepsilon \rangle \tag{10}$$

There is a splitting of phases at the critical point $q = q_c$ such that (10) has a trivial solution $\langle \varepsilon \rangle = 0$ for $q < q_c$ while for $q > q_c$, $\langle \varepsilon \rangle$ grows with increasing q. In short,

$$\langle \varepsilon \rangle = \begin{cases} 0, & q < q_c \\ \langle \varepsilon \rangle_q, & q > q_c \end{cases} \tag{11}$$

Since q relates to an inverse temperature in the thermodynamic analog of multifractal sets ($q = T^{-1}$), a natural interpretation of (11) is as follows: large temperatures defining the UV sector of scales ($\delta \approx \Delta_{UV}^{-1} \to 0$) correspond to a vanishing overall dimensional polarization, while "cooling off" the flow above the critical point $(q > q_c)$ generates an "ordered" phase displaying non-zero polarization. The net effect of strong-gravity and strong quantum fluctuations in the UV is to smear off average deviations of spacetime dimensionality below the critical point $(q < q_c)$, yet this symmetry is broken once the flow evolves above this point. Considering [4-5, 10-11], one can further speculate that the emergence of q_c at some large $n >> 1$, fixes all parameters describing the multifractal set, in particular the free energy $\tau(q_c)$ and the generalized dimension

$$d_{q_c} = \frac{\tau(q_c)}{q_c - 1} \tag{12}$$

3. <u>Conclusions</u>

Needless to say, the derivation outlined here is far from being either rigorous or complete. However, it suggests a qualitative picture of how SM arises from dimensional polarization near the critical point, $\langle \varepsilon \rangle_{q_c} = \langle 4 - D \rangle_{q_c} << 1$, where the four-dimensional continuum turns into a MFM. It is also consistent with the *naturalness principle* advanced a while ago by 't Hooft [8], according to which a parameter should be small only if the underlying theory becomes more symmetric as that parameter tends to zero[1].

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[1] It is known that the SM Higgs sector is unnatural since its symmetry is not enhanced in the massless Higgs limit. Perturbative corrections drive the electroweak scale towards the Planck scale, leading to the so-called *fine-tuning problem*.

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