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Generalized Bipolar Neutrosophic Graphs of Type 1

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***Abstract*—In this research paper, based on the notion of generalized single valued neutrosophic graphs of type 1, we presented a new type of neutrosophic graphs called generalized bipolar neutrosophic graphs of type 1 (GBNG1) and presented a matrix representation for it and studied some properties of this new concept. The concept of GBNG1 is an extension of generalized fuzzy graphs of type 1 (GFG1) and generalized single valued neutrosophic of type 1 (GSVNG1).**

Keywords—Bipolar neutrosophic graph; Generalized bipolar neutrosophic graphs of type 1; Matrix representation.

I. INTRODUCTION

Smarandache [5] proposed the concept of neutrosophic set theory (in short NS) as a means of expressing the inconsistencies and indeterminacies that exist in most real–life problem. The proposed concept generalized the concept of fuzzy sets [13], intuitionistic fuzzy sets [11], interval-valued fuzzy sets [9] and interval-valued intuitionistic fuzzy sets [12]. In neutrosophic set, every element is characterized three membership degrees: truth membership degree T, an indeterminate membership degree I and a false membership degree F, where the degrees are totally independent, the three degree are inside the unit interval $]^{-}0, 1^{+}[$. To practice NS in real life problems, The single valued neutrosophic set was proposed by Smarandache in [5]. After, Wang et al.[8] discussed some interesting properties related to single valued neutrosophic sets. In [10], Deli et al. proposed the concept of bipolar neutrosophic sets and discussed some interesting properties. Some more literature about the extension of neutrosophic sets and their applications in various fields can be found in [6, 15, 26, 27, 28 29, 30, 31, 41, 42].

Graphs are models of relations between objects. The objects are represented by vertices and relations by edges. In a crisp graphs two vertices are either related or not related to each other, mathematically, the degree of relationship is either 0 or 1.While in fuzzy graphs, the degree of relationship takes values from [0, 1]. In the literature, many extensions of fuzzy graphs have been studied deeply such as bipolar fuzzy [1, 3, 16, 40]. All these types of graphs have a common property that each edge must have a membership value less than or equal to the minimum membership of the nodes it connects. Samanta et al. [35] proposed two concept of fuzzy graphs called generalized fuzzy graphs 1 (GFG1) and generalized fuzzy graphs 2 (GFG2) and studied some major properties such as completeness and regularity with proved results. When the description of the objects or relationships, or both happens to possess indeterminacy and inconsistency. The fuzzy graphs and theirs extensions cannot deal with it. So, for this purpose, Smarandache [4,7] proposed the two concepts of neutrosophic graphs, one based on literal indeterminacy (I) whereas the other is based neutrosophic truth-values (T, I, F) on to deal with such situations. Subsequently, Broumi et al. [23, 24, 25] introduced the concept of single valued neutrosophic graphs (in short SVNGs) and investigate some interesting properties with proofs and illustrations. Later on the same authors [32, 33] proposed the concept of bipolar single neutrosophic graphs (in short BSVNGs) and studying some interesting properties. Later on, others researchers proposed other structures of neutrosophic graphs [18, 19, 17, 20, 22, 23, 39]. Followed the concept of Broumi et al [23], several studies appeared in [2, 14, 21, 36, 37, 38]

Similar to the bipolar fuzzy graphs, which have a common property that each edge must have a membership value less than or equal to the minimum membership of the nodes it connects. Also, the bipolar neutrosophic graphs presented in the literature [32, 43] have a common property that edge positive truth-membership value is less than the minimum of its end vertex values, whereas the edge positive indeterminate-membership value is less than the maximum of its end vertex values or is greater than the maximum of its end vertex values. And the edge positive false-membership value is less than the minimum of its end vertex values or is greater than the maximum of its end vertex values. In [34], Broumi et al. have discussed the removal of the edge degree restriction of single valued neutrosophic graphs and presented a new class of single valued neutrosophic graph called generalized single valued neutrosophic graph of type1, which is a is an extension of generalized fuzzy graph of type 1 [35]. Motivated by the concept of generalized single valued neutrosophic graph of type 1(GSVNG1) introduced in [34]. The main contribution of this paper is to extend the concept of generalized single valued neutrosophic graph of type 1 to generalized bipolar neutrosophic graphs of type 1 (GBNG1) to model systems having an indeterminate information and introduced a matrix representation of GBNG1.

This paper is organized as follows: Section 2, focuses on some fundamental concepts related to neutrosophic sets, single valued neutrosophic sets, bipolar neutrosophic sets and generalized single valued neutrosophic graphs type 1. Section 3, provides the concept of generalized bipolar neutrosophic graphs of type 1 with an illustrative example. Section 4 deals with the representation matrix of generalized bipolar neutrosophic graphs of type 1 followed by conclusion, in section 5.

II.PRELIMINARIES

This section presented some definitions from [5,8,10, 32 34] related to neutrosophic sets, single valued neutrosophic sets, bipolar neutrosophic sets, and generalized single valued neutrosophic graphs of type 1, which will helpful for rest of the sections.

Definition 2.1 [5]. Let X be a space of points (objects) with generic elements in X denoted by x; then the neutrosophic set A (NS A) is an object having the form A = {< x: $T_A(x)$, $I_A(x)$, $F_A(x)$ >, x ∈ X}, where the functions T, I, F: X→]$^-$0,1$^+$[define respectively the truth-membership function, indeterminate-membership function, and false-membership function of the element x ∈ X to the set A with the condition:

$$^-0 \le T_A(x) + I_A(x) + F_A(x) \le 3^+ \quad (1)$$

The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of] $^-$0, 1$^+$[.

Since it is difficult to apply NSs to practical problems, Smarandache [5] introduced the concept of a SVNS, which is an instance of a NS and can be used in real scientific and engineering applications.

Definition 2.2 [8]. Let X is a space of points (objects) with generic elements in X denoted by x. A single valued neutrosophic set A (SVNS A) is characterized by truth-membership function $T_A(x)$, an indeterminate-membership function $I_A(x)$, and a false-membership function $F_A(x)$. For each point x in X, $T_A(x)$, $I_A(x)$, $F_A(x) \in$ [0, 1]. A SVNS A can be written as

$$A=\{< x: T_A(x),\ I_A(x),\ F_A(x)>, x \in X\} \quad (2)$$

Definition 2.3 [10]. A bipolar neutrosophic set A in X is defined as an object of the form A={<x, ($T_A^+(x)$,$I_A^+(x)$,$F_A^+(x)$,$T_A^-(x)$,$I_A^-(x)$,$F_A^-(x)$) >: x ∈ X}, where T_A^+, I_A^+, F_A^+:X ⟶ [1, 0] and T_A^-, I_A^-, F_A^-: X ⟶ [-1, 0]. The positive membership degree $T_A^+(x)$,$I_A^+(x)$,$F_A^+(x)$ denotes the truth membership, indeterminate membership and false membership of an element ∈ X corresponding to a bipolar neutrosophic set A and the negative membership degree $T_A^-(x)$, $I_A^-(x)$, $F_A^-(x)$ denotes the truth membership, indeterminate membership and false membership of an element ∈ X to some implicit counter-property corresponding to a bipolar neutrosophic set A. For convenience a bipolar neutrosophic number is represented by

$$A= < T_A^+, I_A^+, F_A^+, T_A^-, I_A^-, F_A^- > \quad (3)$$

Definition 2.4 [34]. Let V be a non-void set. Two functions are considered as follows:
$\rho=(\rho_T, \rho_I, \rho_F)$:V → $[0, 1]^3$ and
$\omega=(\omega_T, \omega_I, \omega_F)$:VxV → $[0, 1]^3$. We suppose
A= $\{(\rho_T(x),\rho_T(y)) \mid \omega_T(x, y) \ge 0\}$,
B= $\{(\rho_I(x),\rho_I(y)) \mid \omega_I(x, y) \ge 0\}$,
C= $\{(\rho_F(x),\rho_F(y)) \mid \omega_F(x, y) \ge 0\}$,

We have considered ω_T, ω_I and $\omega_F \ge 0$ for all set A,B, C , since its is possible to have edge degree = 0 (for T, or I, or F). The triad (V, ρ, ω) is defined to be generalized single valued neutrosophic graph of type 1 (GSVNG1) if there are functions α:A→ [0, 1] , β:B→ [0, 1] and δ:C→ [0, 1] such that
$\omega_T(x, y) = \alpha((\rho_T(x),\rho_T(y)))$
$\omega_I(x, y) = \beta((\rho_I(x),\rho_I(y)))$
$\omega_F(x, y) = \delta((\rho_F(x),\rho_F(y)))$ where x, y∈ V.

Here $\rho(x)= (\rho_T(x), \rho_I(x), \rho_F(x))$, x∈ V are the truth-membership, indeterminate-membership and false-membership of the vertex x and $\omega(x, y)$=($\omega_T(x, y)$, $\omega_I(x, y)$, $\omega_F(x, y)$), x, y∈ V are the truth-membership, indeterminate-membership and false-membership values of the edge (x, y).

Definition 2.5 [32]. A bipolar single valued neutrosophic graph of a graph G^*= (V, E) is a pair G = (A, B), where A = (T_A^+ , I_A^+ , F_A^+ , T_A^- , I_A^-, F_A^-) is a bipolar single valued neutrosophic set in V and B = (T_B^+,I_B^+, F_B^+,T_B^- ,I_B^-, F_B^-) is a bipolar single valued neutrosophic set in $\tilde{V}^2$ such that
$T_B^+(v_i, v_j) \le \min(T_A^+(v_i),T_A^+(v_j))$
$I_B^+(v_i, v_j) \ge \max(I_A^+(v_i),I_A^+(v_j))$
$F_B^+(v_i, v_j) \ge \max(F_A^+(v_i),F_A^+(v_j))$ and
$T_B^-(v_i, v_j) \ge \max(T_A^-(v_i),T_A^-(v_j))$
$I_B^-(v_i, v_j) \le \min(I_A^N(v_i),I_A^-(v_j))$
$F_B^-(v_i, v_j) \le \min(F_A^N(v_i),F_A^-(v_j))$ for all $v_i v_j \in \tilde{V}^2$.

III. GENERALIZED BIPOLAR NEUTROSOPHIC GRAPH OF TYPE 1

In this section, based on the generalized single valued neutrosophic graphs of type 1 proposed by Broumi et al. [34], the definition of generalized bipolar neutrosophic graphs type 1 is defined as follow:

Definition 3.1. Let V be a non-void set. Two functions are considered, as follows:

ρ=(ρ_T^+,ρ_I^+,ρ_F^+,ρ_T^-, ρ_I^-,ρ_F^-):V→ $[-1, 1]^6$and

ω=(ω_T^+,ω_I^+, ω_F^+,ω_T^-, ω_I^-,ω_F^-):VxV→ $[-1, 1]^6$. We suppose

A= {($\rho_T^+(x)$,$\rho_T^+(y)$) |ω_T^+(x, y) ≥ 0},

B= {($\rho_I^+(x)$,$\rho_I^+(y)$) |ω_I^+(x, y) ≥ 0},

C= {($\rho_F^+(x)$,$\rho_F^+(y)$) |ω_F^+(x, y) ≥0},

D= {($\rho_T^-(x)$,$\rho_T^-(y)$) |ω_T^-(x, y) ≤ 0},

E= {($\rho_I^-(x)$,$\rho_I^-(y)$) |ω_I^-(x, y) ≤ 0},

F= {($\rho_F^-(x)$,$\rho_F^-(y)$) |ω_F^-(x, y) ≤ 0},

We have considered ω_T^+, ω_I^+,$\omega_F^+ \geq 0$ *and* ω_T^-,ω_I^-, $\omega_F^- \leq 0$ for all sets A,B, C , D, E, F since it is possible to have edge degree = 0 (for T^+ or I^+ or F^+,T^- or I^- or F^-).

The triad (V, ρ , ω) is defined to be generalized bipolar neutrosophic graph of type 1 (GBNG1) if there are functions α:A→ $[0, 1]$, β:B→ $[0, 1]$,δ:C→ $[0, 1]$and ξ:D→ $[-1, 0]$, σ:E→ $[-1, 0]$, ψ:F→ $[-1, 0]$such that

$\omega_T^+(x, y) = \alpha((\rho_T^+(x),\rho_T^+(y)))$,

$\omega_T^-(x, y) = \xi((\rho_T^-(x),\rho_T^-(y)))$,

$\omega_I^+(x, y) = \beta((\rho_I^+(x),\rho_I^+(y)))$,

$\omega_I^-(x, y) = \sigma((\rho_I^-(x),\rho_I^-(y)))$,

$\omega_F^+(x, y) = \delta((\rho_F^+(x),\rho_F^+(y)))$,

$\omega_F^-(x, y) = \psi((\rho_F^-(x),\rho_F^-(y)))$ where x, y ∈ V.

Here $\rho(x)$=($\rho_T^+(x)$, $\rho_I^+(x)$, $\rho_F^+(x)$,$\rho_T^-(x)$, $\rho_I^-(x)$, $\rho_F^-(x)$), x ∈ V are the positive and negative truth-membership, indeterminate-membership and false-membership of the vertex x and $\omega(x, y)$ =($\omega_T^+(x, y)$, $\omega_I^+(x, y)$, $\omega_F^+(x, y)$, $\omega_T^-(x, y)$, $\omega_I^-(x, y)$, $\omega_F^-(x, y)$),x,y∈ V are the positive and negative truth-membership, indeterminate-membership and false-membership values of the edge (x, y).

Example3.2: Let the vertex set be V={x, y, z, t} and edge set be E={(x, y),(x, z),(x, t),(y, t)}

Table 1. Positive and negative truth- membership, indeterminate-membership and false-membership of the vertex set.

	x	y	z	t
ρ_T^+	0.5	0.9	0.3	0.8
ρ_I^+	0.3	0.2	0.1	0.5
ρ_F^+	0.1	0.6	0.8	0.4
ρ_T^-	-0.6	-0.1	-0.4	-0.9
ρ_I^-	-0.4	-0.3	-0.2	-0.6
ρ_F^-	-0.2	-0.7	-0.9	-0.5

Let us consider functions $\alpha(m, n)$= max (m_T^+, n_T^+), $\beta(m, n)$=$\frac{(m_I^+ + n_I^+)}{2}$, $\delta(m, n)$= min (m_F^+, n_F^+), ξ(m ,n)= min (m_T^-, n_T^-), σ(m ,n)= $\frac{(m_I^- + n_I^-)}{2}$, and ψ(m ,n)= max (m_F^-, n_F^-), Here,

A={(0.5, 0.9), (0.5, 0.3), (0.5, 0.8),(0.9, 0.8)}

B={(0.3, 0.2), (0.3, 0.1), (0.3, 0.5), (0.2, 0.5)}

C={(0.1, 0.6), (0.1, 0.8), (0.1, 0.4), (0.6, 0.4)}

D={(-0.6, -1), (-0.6, -0.4), (-0.6, -0.9), (-1, -0.9)}

E = {(-0.4, -0.3), (-0.4, -0.2), (-0.4, -0.6), (-0.3, -0.6)}

F = {(-0.2, -0.7), (-0.2, -0.9), (-0.2, -0.5), (-0.7, -0.5)}

Then

Table 2. Positive and negative truth- membership, indeterminate-membership and false-membership of the edge set.

ω	(x, y)	(x, z)	(x, t)	(y, t)
ω_T^+(x,y)	0.9	0.5	0.8	0.9
ω_I^+(x,y)	0.25	0.2	0.4	0.35
ω_F^+(x,y)	0.1	0.1	0.1	0.4
ω_T^-(x,y)	-0.6	-0.6	-0.9	-0.9
ω_I^-(x,y)	-0.35	-0.3	-0.25	-0.45
ω_F^-(x,y)	-0.2	-0.2	-0.2	-0.5

The corresponding generalized bipolar neutrosophic graph of type 1 is shown in Fig.2

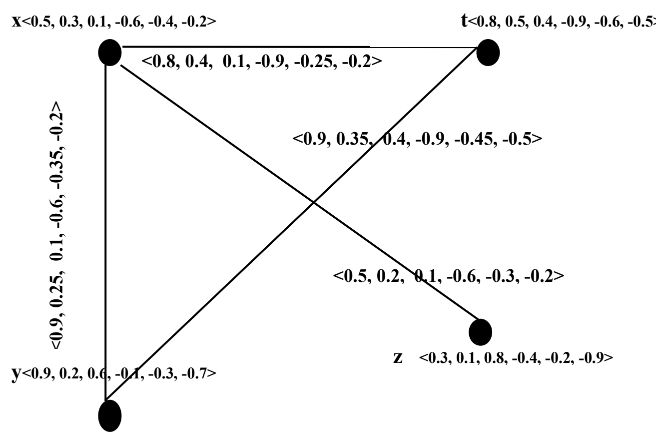

Fig 2. A BNG of type 1.

The easier way to represent any graph is to use the matrix representation. The adjacency matrices, incident matrices are the widely matrices used. In the following section, GBNG1 is represented by adjacency matrix.

IV. MATRIX REPRESENTATION OF GENERALIZED BIPOLAR NEUTROSOPHIC GRAPH OF TYPE 1

Because positive and negative truth- membership, indeterminate-membership and false-membership of the vertices are considered independents. In this section, we extended the representation matrix of generalized single valued neutrosophic graphs of type 1 proposed in [34] to the case of generalized bipolar neutrosophic graphs of type 1.

The generalized bipolar neutrosophic graph (GBNG1) has one property that edge membership values (T^+, I^+, F^+, T^-, I^-, F^-) depends on the membership values(T^+, I^+, F^+, T^-, I^-,

F^-) of adjacent vertices. Suppose ζ=(V, ρ, ω) is a GBNG1 where vertex set V={v_1,v_2,…,v_n}. The functions

α :A→ [0, 1] is taken such that
$\omega_T^+(x,y)$ = $\alpha((\rho_T^+(x),\rho_T^+(y)))$, where x, y∈ V and
A= {$(\rho_T^+(x),\rho_T^+(y))$ |ω_T^+(x, y) ≥ 0},
β :B→ [0, 1] is taken such that
$\omega_I^+(x,y)$ = $\beta((\rho_I^+(x),\rho_I^+(y)))$, where x, y∈ V and
B= {$(\rho_I^+(x),\rho_I^+(y))$ |ω_I^+(x, y) ≥ 0},
δ :C→ [0, 1] is taken such that
$\omega_F^+(x,y)$ = $\delta((\rho_F^+(x),\rho_F^+(y)))$, where x, y∈ V and
C = {$(\rho_F^+(x),\rho_F^+(y))$ |ω_F^+(x, y) ≥ 0},
ξ :D→ [−1, 0] is taken such that
$\omega_T^-(x,y)$ = $\xi((\rho_T^-(x),\rho_T^-(y)))$, where x, y∈ V and
D= {$(\rho_T^-(x),\rho_T^-(y))$ |ω_T^-(x, y) ≤0},
σ :E→ [−1, 0] is taken such that
$\omega_I^-(x,y)$ = $\sigma((\rho_I^-(x),\rho_I^-(y)))$, where x, y∈ V and
E= {$(\rho_I^-(x),\rho_I^-(y))$ |ω_I^-(x, y) ≤ 0}, and
ψ:F→ [−1, 0] is taken such that
$\omega_F^-(x,y)$ =$\psi((\rho_F^-(x),\rho_F^-(y)))$, where x, y∈ V and
F = {$(\rho_F^-(x),\rho_F^-(y))$ |ω_F^-(x, y) ≤ 0},

The GBNG1 can be represented by (n+1) x (n+1) matrix $M_{G_1}^{T,I,F}$=[$a^{T,I,F}$(i, j)] as follows:
The positive and negative truth-membership(T^+ , T^-) , indeterminate-membership (I^+ , I^-) and false-membership (F^+, F^-), values of the vertices are provided in the first row and first column. The (i+1, j+1)-th-entry are the membership (T^+, T^-), indeterminate-membership (I^+, I^-)and the false-membership (F^+, F^-) values of the edge (x_i,x_j), i, j=1,…,n if i≠j.
The (i, i)-th entry is $\rho(x_i)$=($\rho_T^+(x_i)$, $\rho_I^+(x_i)$, $\rho_F^+(x_i)$, $\rho_T^-(x_i)$, $\rho_I^-(x_i)$, $\rho_F^-(x_i)$) where i=1,2,…,n. The positive and negative truth-membership (T^+, T^-), indeterminate-membership (I^+, I^-) and false-membership (F^+, F^-), values of the edge can be computed easily using the functions $\alpha, \beta, \delta, \xi$, σ and ψwhich are in (1, 1)-position of the matrix. The matrix representation of GBNG1, denoted by$M_{G_1}^{T,I,F}$, can be written as sixth matrix representation $M_{G_1}^{T^+}$, $M_{G_1}^{I^+}$,$M_{G_1}^{F^+}$,$M_{G_1}^{T^-}$, $M_{G_1}^{I^-}$, $M_{G_1}^{F^-}$.
The $M_{G_1}^{T^+}$ can be represented as follows:

Table3. Matrix representation of T^+-GBNG1

α	$v_1(\rho_T^+(v_1))$	$v_2(\rho_T^+(v_2))$	$v_n(\rho_T^+(v_n))$
$v_1(\rho_T^+(v_1))$	$\rho_T^+(v_1)$	$\alpha(\rho_T^+(v_1),\rho_T^+(v_2))$	$\alpha(\rho_T^+(v_1),\rho_T^+(v_n))$
$v_2(\rho_T^+(v_2))$	$\alpha(\rho_T^+(v_2),\rho_T^+(v_1))$	$\rho_T^+(v_2)$	$\alpha(\rho_T^+(v_2),\rho_T^+(v_2))$
…	….	…	…
$v_n(\rho_T^+(v_n))$	$\alpha(\rho_T^+(v_n),\rho_T^+(v_1))$	$\alpha(\rho_T^+(v_n),\rho_T^+(v_2))$	$\rho_T^+(v_n)$

The $M_{G_1}^{I^+}$ can be represented as follows

Table 4. Matrix representation of I^+-GBNG1

β	$v_1(\rho_I^+(v_1))$	$v_2(\rho_I^+(v_2))$	$v_n(\rho_I^+(v_n))$
$v_1(\rho_I^+(v_1))$	$\rho_I^+(v_1)$	$\beta(\rho_I^+(v_1),\rho_I^+(v_2))$	$\beta(\rho_I^+(v_1),\rho_I^+(v_n))$
$v_2(\rho_I^+(v_2))$	$\beta(\rho_I^+(v_2),\rho_I^+(v_1))$	$\rho_I^+(v_2)$	$\beta(\rho_I^+(v_2),\rho_I^+(v_2))$
…	….	…	…
$v_n(\rho_I^+(v_n))$	$\beta(\rho_I^+(v_n),\rho_I^+(v_1))$	$\beta(\rho_I^+(v_n),\rho_I^+(v_2))$	$\rho_I^+(v_n)$

The $M_{G_1}^{F^+}$ can be represented as follows

Table5. Matrix representation of F^+-GBNG1

δ	$v_1(\rho_F^+(v_1))$	$v_2(\rho_F^+(v_2))$	$v_n(\rho_F^+(v_n))$
$v_1(\rho_F^+(v_1))$	$\rho_F^+(v_1)$	$\delta(\rho_F^+(v_1),\rho_F^+(v_2))$	$\delta(\rho_F^+(v_1),\rho_F^+(v_n))$
$v_2(\rho_F^+(v_2))$	$\delta(\rho_F^+(v_2),\rho_F^+(v_1))$	$\rho_F^+(v_2)$	$\delta(\rho_F^+(v_2),\rho_F^+(v_2))$
…	….	…	…
$v_n(\rho_F^+(v_n))$	$\delta(\rho_F^+(v_n),\rho_F^+(v_1))$	$\delta(\rho_F^+(v_n),\rho_F^+(v_2))$	$\rho_F^+(v_n)$

The $M_{G_1}^{T^-}$ can be represented as follows

Table6. Matrix representation of T^--GBNG1

ξ	$v_1(\rho_T^-(v_1))$	$v_2(\rho_T^-(v_2))$	$v_n(\rho_T^-(v_n))$
$v_1(\rho_T^-(v_1))$	$\rho_T^-(v_1)$	$\xi(\rho_T^-(v_1),\rho_T^-(v_2))$	$\xi(\rho_T^-(v_1),\rho_T^-(v_n))$
$v_2(\rho_T^-(v_2))$	$\xi(\rho_T^-(v_2),\rho_T^-(v_1))$	$\rho_T^-(v_2)$	$\xi(\rho_T^-(v_2),\rho_T^-(v_2))$
…	….	…	…
$v_n(\rho_T^-(v_n))$	$\xi(\rho_T^-(v_n),\rho_T^-(v_1))$	$\xi(\rho_T^-(v_n),\rho_T^-(v_2))$	$\rho_T^-(v_n)$

The $M_{G_1}^{I^-}$ can be represented as follows

Table7. Matrix representation of I^--GBNG1

σ	$v_1(\rho_I^-(v_1))$	$v_2(\rho_I^-(v_2))$	$v_n(\rho_I^-(v_n))$
$v_1(\rho_I^-(v_1))$	$\rho_I^-(v_1)$	$\sigma(\rho_I^-(v_1),\rho_I^-(v_2))$	$\sigma(\rho_I^-(v_1),\rho_I^-(v_n))$
$v_2(\rho_I^-(v_2))$	$\sigma(\rho_I^-(v_2),\rho_I^-(v_1))$	$\rho_I^+(v_2)$	$\sigma(\rho_I^-(v_2),\rho_I^-(v_2))$
…	….	…	…
$v_n(\rho_I^-(v_n))$	$\sigma(\rho_I^-(v_n),\rho_I^-(v_1))$	$\sigma(\rho_I^-(v_n),\rho_I^-(v_2))$	$\rho_I^-(v_n)$

The $M_{G_1}^{F^-}$ can be represented as follows

Table8. Matrix representation of F^--GBNG1

ψ	$v_1(\rho_F^-(v_1))$	$v_2(\rho_F^-(v_2))$	$v_n(\rho_F^-(v_n))$
$v_1(\rho_F^-(v_1))$	$\rho_F^-(v_1)$	$\psi(\rho_F^-(v_1),\rho_F^-(v_2))$	$\psi(\rho_F^-(v_1),\rho_F^-(v_n))$
$v_2(\rho_F^-(v_2))$	$\psi(\rho_F^-(v_2),\rho_F^-(v_1)$	$\rho_F^-(v_2)$	$\psi(\rho_F^-(v_2),\rho_F^-(v_2))$
…	….	…	…
$v_n(\rho_F^-(v_n))$	$\psi(\rho_F^-(v_n),\rho_F^-(v_1))$	$\psi(\rho_F^-(v_n),\rho_F^-(v_2))$	$\rho_F^-(v_n)$

Remark 1: If$\rho_T^-(x)$=$\rho_I^-(x)$=$\rho_F^-(x)$0,the generalized bipolar neutrosophic graphs of type 1 is reduced to generalized single valued neutrosophic graph of type 1 (GSVNG1).

Remark 2: If $\rho_T^-(x)$=$\rho_I^-(x)$=$\rho_F^-(x)$0, and$\rho_I^+(x)$=$\rho_F^+(x)$ **=0** **,** the generalized bipolar neutrosophic graphs type 1 is reduced to generalized fuzzy graph of type 1 (GFG1).

Here the generalized bipolar neutrosophic graph of type 1 (GBNG1) can be represented by the matrix representation depicted in table 15.The matrix representation can be written as sixth matrices one containing the entries as T^+, I^+, F^+,T^-, I^-, F^- (see table 9, 10,11,12,13 and 14).

Table9. T^+- matrix representation of GBNG1

α = max(x, y)	x(0.5)	y(0.9)	z(0.3)	t(0.8)
x(0.5)	0.5	0.9	0.5	0.8
y(0.9)	0.9	0.9	0	0.9
z(0.3)	0.5	0	0.3	0
t(0.8)	0.8	0.9	0	0.8

Table10. I^+- matrix representation of GBNG1

β = (x+y)/2	x(0.3)	y(0.2)	z(0.1)	t(0.5)
x(0.3)	0.3	0.25	0.2	0.4
y(0.2)	0.25	0.2	0	0.35
z(0.1)	0.2	0	0.1	0
t(0.5)	0.4	0.35	0	0.5

Table11. F^+- matrix representation of GBNG1

δ= min(x, y)	x(0.1)	y(0.6)	z(0.8)	t(0.4)
x(0.1)	0.1	0.1	0.1	0.1
y(0.6)	0.1	0.6	0	0.4
z(0.8)	0.1	0	0.8	0
t(0.4)	0.1	0.4	0	0.4

Table12. T^-- matrix representation of GBNG1

ξ = min(x, y)	x(-0.6)	y(-0.1)	z(-0.4)	t(-0.9)
x(-0.6)	-0.6	-0.6	-0.6	-0.9
y(-0.1)	-0.6	-0.1	0	-0.9
z(-0.4)	-0.6	0	-0.4	0
t(-0.9)	-0.9	-0.9	0	-0.9

Table13. I^-- matrix representation of GBNG1

σ = (x+y)/2	x(-0.4)	y(-0.3)	z(-0.2)	t(-0.6)
x(-0.4)	-0.4	-0.35	-0.3	-0.25
y(-0.3)	-0.35	-0.3	0	-0.45
z(-0.2)	-0.3	0	-0.2	0
t(-0.6)	-0.5	-0.45	0	-0.6

Table14.F^-- matrix representation of GBNG1

ψ= max(x, y)	x(-0.2)	y(-0.7)	z(-0.9)	t(-0.5)
x(-0.2)	-0.2	-0.2	-0.2	-0.2
y(-0.7)	-0.2	-0.7	0	-0.5
z(-0.9)	-0.2	0	-0.9	0
t(-0.5)	-0.2	-0.5	0	-0.5

The matrix representation of GBNG1 can be represented as follows:

Table15. Matrix representation of GBNG1.

$(\alpha, \beta, \delta, \xi, \sigma, \psi)$	x(0.5, 0.3, 0.1, -0.6,-0.4, -0.2)	y(0.9, 0.2, 0.6, -0.1,-0.3, -0.7)	z(0.3, 0.1, 0.8, -0.4,-0.2, -0.9)	t(0.8, 0.5, 0.4, -0.9, -0.6, -0.5)
x(0.5, 0.3, 0.1, -0.6,-0.4, -0.2)	(0.5, 0.3, 0.1, -0.6,-0.4,-0.2)	(0.9, 0.25, 0.1, -0.6,-0.35,-0.2)	(0.5, 0.2, 0.1, -0.6,-0.3,-0.2)	(0.8, 0.4, 0.1, -0.9,-0.5,-0.2)
y(0.9, 0.2, 0.6, -0.1,-0.3, -0.7)	(0.9, 0.25, 0.1, -0.6,-0.35,-0.2)	(0.9, 0.2, 0.6, -0.1,-0.3,-0.7)	(0, 0, 0, 0, 0, 0)	(0.9, 0.35, 0.4, -0.9,-0.45,-0.5)
z(0.3, 0.1, 0.8, -0.4,-0.2, -0.9)	(0.5, 0.2, 0.1, -0.6,-0.3,-0.2)	(0, 0, 0, 0, 0, 0)	(0.3, 0.1, 0.8, -0.4,-0.2,-0.9)	(0, 0, 0, 0, 0, 0)
t(0.8, 0.5, 0.4, -0.9, -0.6, -0.5)	(0.8, 0.4, 0.1, -0.9,-0.5,-0.2)	(0.9, 0.35, 0.4, -0.9,-0.45,-0.5)	(0, 0, 0, 0, 0, 0)	(0.8, 0. 5, 0.4, -0.9,-0.6,-0.5)

Theorem 1. Let $M_{G_1}^{T^+}$ be matrix representation of T^+-GBNG1, then the degree of vertex
$D_{T^+}(x_k)$=$\sum_{j=1,j\neq k}^{n} a_{T^+}(k+1, j+1)$,$x_k \in$ V or
$D_{T^+}(x_p)$=$\sum_{i=1,i\neq p}^{n} a_{T^+}(i+1, p+1)$, $x_p \in$ V.

Proof: It is similar as in theorem 1 of [34].

Theorem 2. Let $M_{G_1}^{I^+}$ be matrix representation of I^+-GBNG1, then the degree of vertex
$D_{I^+}(x_k)$ =$\sum_{j=1,j\neq k}^{n} a_{I^+}(k+1, j+1)$,$x_k \in$ V or
$D_{I^+}(x_p)$ =$\sum_{i=1,i\neq p}^{n} a_{I^+}(i+1, p+1)$, $x_p \in$ V.
Proof: It is similar as in theorem 1 of [34].

Theorem 3. Let $M_{G_1}^{F^+}$ be matrix representation of F^+-GBNG1, then the degree of vertex
$D_{F^+}(x_k)$ =$\sum_{j=1,j\neq k}^{n} a_{F^+}(k+1, j+1)$,$x_k \in$ V or
$D_{F^+}(x_p)$ =$\sum_{i=1,i\neq p}^{n} a_{F^+}(i+1, p+1)$, $x_p \in$ V.

Proof: It is similar as in theorem 1 of [34]

Theorem 4. Let $M_{G_1}^{T^-}$ be matrix representation of T^--GBNG1, then the degree of vertex
$D_{T^-}(x_k)$ =$\sum_{j=1,j\neq k}^{n} a_{T^-}(k+1, j+1)$,$x_k \in$ V or
$D_{T^-}(x_p)$ =$\sum_{i=1,i\neq p}^{n} a_{T^-}(i+1, p+1)$, $x_p \in$ V.

Proof: It is similar as in theorem 1 of [34].

Theorem 5. Let $M_{G_1}^{I^-}$ be matrix representation of I^--GBNG1, then the degree of vertex
$D_{I^-}(x_k)$ =$\sum_{j=1,j\neq k}^{n} a_{I^-}(k+1, j+1)$,$x_k \in$ V or
$D_{I^-}(x_p)$ =$\sum_{i=1,i\neq p}^{n} a_{I^-}(i+1, p+1)$, $x_p \in$ V.
Proof: It is similar as in theorem 1 of [34].

Theorem 6. Let $M_{G_1}^{F^-}$ be matrix representation of F^--GBNG1, then the degree of vertex
$D_{F^-}(x_k) = \sum_{j=1, j\neq k}^{n} a_{F^-}(k+1, j+1), x_k \in$ V or
$D_{F^-}(x_p) = \sum_{i=1, i\neq p}^{n} a_{F^-}(i+1, p+1)$, $x_p \in$ V.
Proof: It is similar as in theorem 1 of [34]

V. CONCLUSION

In this article, we have extended the concept of generalized single valued neutrosophic graph of type 1 (GSVNG1) to generalized bipolar neutrosophic graph of type 1(GBNG1) and showed a matrix representation of it. The concept of GBNG1 can be applied to the case of tri-polar neutrosophic graphs and multi-polar neutrosophic graphs. In the future works, we plan to study the concept of completeness, the concept of regularity and to define the concept of generalized bipolar neutrosophic graphs of type 2.

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