The Surprising Proofs

By Leszek W. Guła

Lublin-POLAND

lwgula@wp.pl

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Abstract. The proof of the Fermat's Last Theorem. The proof of the theorem - *For all* $n \in \{3,5,7,...\}$ *and for all* $z \in \{3,7,11,...\}$ *and for all natural numbers* u, v: $z^n \neq u^2 + v^2$. The proof of the Goldbach's Conjecture. The proof of the Beal's Conjecture.

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I. INTRODUCTION

The cover of this issue of the Bulletin is the frontispiece to a volume of Samuel de Fermat's 1670 edition of Bachet's Latin translation of Diophantus's Arithmetica. This edition includes the marginalia of the editor's father, Pierre de Fermat. Among these notes one finds the elder Fermat's extraordinary comment in connection with the Pythagorean equation $x^2 + y^2 = z^2$ the marginal comment that hints at the existence of a proof (a demonstratio sane mirabilis) of what has come to be known as Fermat's Last Theorem. Diophantus's work had fired the imagination of the Italian Renaissance mathematician Rafael Bombelli, as it inspired Fermat a century later. [7]

The Goldbach's Conjecture is one of the oldest and best-known unsolved problems in number theory and all of mathematics. It states: Every even integer greater than 2 can be expressed as the sum of two primes. [2]

Let A, B, C, x, y and z be positive integers with $x; y; z > 2$. If $A^x + B^y = C^z$ then A, B and C have a common factor. [6] Beal's conjecture is a generalization of Fermat's Last Theorem. It states: If $A^x + B^y = C^z$, where A, B, C, x, y and z are positive integers and x, y and z are all greater than 2, then A, B and C must have a common prime factor. [1] Or – the below - slightly restated? The Beal Prize Fund is with US$1,000,000 to be awarded in a case presented a counterexample. [1]

II. THE PROOF OF THE FERMAT'S LAST THEOREM

Theorem 1. (FLT). *For all* $n \in \{3,4,5,...\}$ *and for all* $A, B, C \in \{1,2,3,...\}$: $A^n + B^n \neq C^n$.

Proof. Suppose that for some $n \in \{3,4,5,...\}$ and for some $A, B, C \in \{1,2,3,...\}$: $A^n + B^n = C^n$.

Then $(A + B > C \wedge A^2 + B^2 > C^2 \wedge ... \wedge A^{n-1} + B^{n-1} > C^{n-1})$, otherwise $A^n + B^n < C^n$.

Thus for some $A, B, C, C - A, C - B, v \in \{1,2,3,...\}$:

$$A + B - C = A - (C - A) = B - (C - B) = 2v > 0 \Longrightarrow$$

$$[(C - B) + 2v = A \wedge (C - A) + 2v = B \wedge A + B - 2v = C]. \qquad (1)$$

At present we can assume for generality of below that A, B and C are coprime.

Moreover $[A^2 + B^2 > C^2 \wedge (1)] \Longrightarrow 2v^2 > (C - A)(C - B)$. Every even number which is not the power of number 2 has odd prime divisor, hence sufficient that we prove FLT for $n = 4$ and for odd prime numbers $n \in \mathbb{P}$. [8]

A. Proof For $n = 4$. Without loss for the proof we can assume that B is even.

For some $C, A \in \{1,3,5, \dots\}$ and for some $B \in \{4,6,8, \dots\}$:

$$(C - A + A)^4 - A^4 = B^4 \Longrightarrow (C - A)^3 + 4(C - A)^2 A + 6(C - A)A^2 + 4A^3 = \frac{B^4}{C - A}.$$

Notice that

$$(C - A)^3 + 4(C - A)^2 A + 6(C - A)A^2 + 4A^3 = \frac{C^4 - A^4}{C - A} = \frac{(C^2 + A^2)(C + A)(C - A)}{C - A}.$$

For some $k \in \{1,2,3, \dots\}$ and for some coprime $e, c, d, h, m \in \{1,3,5, \dots\}$:

$$\left[\frac{B^4}{C - A} = \frac{(2^k ecd)^4}{2^{4k-2} d^4} = 4(ec)^4 \wedge h^4 = C - B \wedge 2^k d(2^{3k-2} d^3 + hm) = 2^k ecd = B\right].$$

Moreover – For some pairwise (mutually) relatively prime $d, h, m \in \{1,3,5, \dots\}$ such that $d < h < m$:

$$2v^2 > (C - A)(C - B) = 2^{4k-2} d^4 h^4 \Longrightarrow v^2 > 2^{4k-3} d^4 h^4 \Longrightarrow v = 2^{k-1} mhd.$$

Therefore – For some relatively prime $e, c \in \{1,3,5, \dots\}$ such that $e > c$:

$$4(ec)^4 = (C^2 + A^2)(C + A) \Longrightarrow (C^2 + A^2 = 2e^4 \wedge C + A = 2c^4) \Longrightarrow$$

$$\begin{aligned}(C = x + y \wedge A = x - y \wedge C + A = 2x = 2c^4 \wedge x = c^4 \wedge x^2 + y^2 = e^4 \wedge x = c^4 \\ &= u^2 - v^2 \wedge y = 2uv \wedge e^2 = u^2 + v^2 \wedge e = p^2 + q^2 \wedge u = p^2 - q^2 \wedge v \\ &= 2pq) \\ &\Longrightarrow \{x = [(p^2 - q^2)^2 - (2pq)^2] = (c^2)^2 \in \mathbf{0} \wedge y \\ &= 4(p^2 - q^2)pq \wedge x^2 + y^2 \\ &= [(p^2 - q^2)^2 - (2pq)^2]^2 + 16(p^2 - q^2)^2(pq)^2 = (p^2 + q^2)^4 = e^4 \in \mathbf{1}\} \\ &\in \mathbf{0},\end{aligned}$$

inasmuch as on the strength of the Guła's Theorem [3] we have

$$(2pq)^2 = (p^2 - q^2)^2 - (c^2)^2 \Longrightarrow p^2 - q^2 = \frac{(2pq)^2 + (2q^2)^2}{2(2q^2)} = p^2 + q^2 \in \mathbf{0}.$$

This is the proof.

B. Proof For $n \in \mathbb{P}$. Without loss for the proof we assume that: A is odd and $4 \nmid B, C$. [4], [5]

The numbers C, B and A are coprime, therefore in view of (1) we will have – For some $n \in \mathbb{P}$ and for some $C, B, C - A \in \{1,2,3, \dots\}$ and for some $C - B, A, v \in \{1,3,5, \dots\}$:

$$
\begin{aligned}
&[(C-B+2\nu)^n=(C-B+B)^n-B^n \wedge\ (C-A+2\nu)^n \\
&\qquad =(C-A+A)^n-A^n \wedge\ (A+B-B)^n+B^n=(A+B-2\nu)^n] \Longrightarrow
\end{aligned}
$$

$$
\begin{aligned}
&\Bigg\{(C-B)^{n-2}\nu+(n-1)(C-B)^{n-3}\,\nu^2+\cdots+2^{n-2}\nu^{n-1}+\frac{2^{n-1}\nu^n}{n(C-B)} \\
&\quad =\frac{B}{2}\left[(C-B)^{n-2}+\frac{n-1}{2}(C-B)^{n-3}B+\cdots+B^{n-2}\right]\wedge(C-A)^{n-2}2\nu \\
&\quad +\frac{n-1}{2}(C-A)^{n-3}\,(2\nu)^2+\cdots+(2\nu)^{n-1}+\frac{(2\nu)^n}{n(C-A)} \\
&\quad =A\left[(C-A)^{n-2}+\frac{n-1}{2}(C-A)^{n-3}A+\cdots+A^{n-2}\right]\wedge(A+B)^{n-2}(-B) \\
&\quad +\frac{n-1}{2}(A+B)^{n-3}\,(-B)^2+\cdots+(-B)^{n-1} \\
&\quad =(A+B)^{n-2}(-2\nu)+\frac{n-1}{2}(A+B)^{n-3}\,(-2\nu)^2+\cdots+(-2\nu)^{n-1} \\
&\quad +\frac{(-2\nu)^n}{n(A+B)}\ \wedge\ n\mid\nu\ \wedge\ (n\mid A,C-B\ \veebar\ n\mid B,C-A\ \veebar\ n\mid A+B,C)\Bigg\}.
\end{aligned}
$$

We assume that – For some $e,m,c,h\in\{1,3,5,\ldots\}$ such that n,e,m,c and h are coprime

$$2\nu^2>(C-A)(C-B)\Longrightarrow(\nu=nemch\ \wedge\ n\nmid emch).$$

B.1. Proof For Odd $A,B,C-B,$ if $n\mid A,C-B.$

For some $n\in\mathbb{P}$ and for some $e,m,c,h\in\{1,3,5,\ldots\}$ such that n,e,m,c and h are coprime:

$$
\begin{aligned}
&(n^{n-1}c^n+2nemch=A\wedge h^n+2nemch=B\wedge n^{n-1}c^n+h^n+4nemch=2^nm^n \\
&\qquad =A+B\wedge n^{n-1}c^n=C-B) \\
&\qquad \Longrightarrow(2^nm^n-h^n=\ n^{n-1}c^n+4nemch\ \wedge n\mid 2m-h\wedge n^2\mid 2^nm^n-h^n) \\
&\qquad \Longrightarrow n\mid emch,
\end{aligned}
$$

$$
\begin{aligned}
&[n^{n-1}c^n+2nemch=A\wedge h^n+2nemch=B\wedge n^{n-1}c^n+h^n+4nemch=2^nm^n \\
&\qquad =A+B\wedge n^{n-1}c^n=C-B] \\
&\qquad \Longrightarrow[2^nm^n-h^n=\ n^{n-1}c^n+4nemch\ \wedge n\mid 2m-h\wedge n^2\mid 2^nm^n-h^n] \\
&\qquad \Longrightarrow n\mid emch,
\end{aligned}
$$

which is inconsistent with $n\nmid emch.$ ♠

B.2. Proof For Odd $A,B,C-B,$ if $n\mid B,C-A.$

For some $n\in\mathbb{P}$ and for some $e,m,c,h\in\{1,3,5,\ldots\}$ such that n,e,m,c and h are coprime:

$$
\begin{aligned}
&[c^n+2nemch=A\wedge n^{n-1}h^n+2nemch=B\wedge c^n+n^{n-1}h^n+4nemch=2^nm^n \\
&\qquad =A+B\wedge c^n=C-B\wedge n^{n-1}h^n=C-A] \\
&\qquad \Longrightarrow[2^nm^n-c^n=\ n^{n-1}h^n+4nemch\wedge n\mid 2m-c\wedge n^2\mid 2^nm^n-c^n] \\
&\qquad \Longrightarrow n\mid emch,
\end{aligned}
$$

which is inconsistent with $n\nmid emch.$ ♠

B.3. Proof For Odd $A, B, C - B$, if $n \mid A + B, C$.

For some $n \in \mathbb{P}$ and for some $e, m, c, h \in \{1,3,5, \ldots\}$ such that n, e, m, c and h are coprime:

$$[c^n + 2nemch = A \wedge h^n + 2nemch = B \wedge c^n + h^n + 4nemch = n^{n-1}2^n m^n = A + B \wedge c^n$$
$$= C - B] \Longrightarrow [n \mid c + h \wedge n^2 \mid c^n + h^n] \Longrightarrow n \mid emch,$$

which is inconsistent with $n \nmid emch$. ♠

B.4. Proof For Even $B, C - A$, if $n \mid A, C - B$.

For some $n \in \mathbb{P}$ and for some $e, m, c, h \in \{1,3,5, \ldots\}$ such that n, e, m, c and h are coprime:

$$[n^{n-1}c^n + 2nemch = A \wedge 2^n h^n + 2nemch = B \wedge n^{n-1}c^n + 2^n h^n + 4nemch = m^n$$
$$= A + B \wedge 2^n h^n = C - A \wedge n^{n-1}c^n = C - B]$$
$$\Longrightarrow [m^n - 2^n h^n = n^{n-1}c^n + 4nemch \wedge n \mid m - 2h \wedge n^2 \mid m^n - 2^n h^n]$$
$$\Longrightarrow n \mid emch,$$

which is inconsistent with $n \nmid emch$. ♠

B.5. Proof For Even $B, C - A$, if $n \mid B, C - A$.

For some $n \in \mathbb{P}$ and for some $e, m, c, h \in \{1,3,5, \ldots\}$ such that n, e, m, c and h are coprime:

$$[c^n + 2nemch = A \wedge n^{n-1}2^n h^n + 2nemch = B \wedge c^n + n^{n-1}h^n + 4nemch = m^n$$
$$= A + B \wedge c^n = C - B \wedge n^{n-1}2^n h^n = C - A]$$
$$\Longrightarrow [2^n m^n - c^n = n^{n-1}h^n + 4nemch \wedge n \mid 2m - c \wedge n^2 \mid 2^n m^n - c^n]$$
$$\Longrightarrow n \mid emch,$$

which is inconsistent with $n \nmid emch$. ♠

B.6. Proof For Even $B, C - A$, if $n \mid A + B, C$.

For some $n \in \mathbb{P}$ and for some $e, m, c, h \in \{1,3,5, \ldots\}$ such that n, e, m, c and h are coprime:

$$[c^n + 2nemch = A \wedge 2^n h^n + 2nemch = B \wedge c^n + 2^n h^n + 4nemch = n^{n-1}m^n$$
$$= A + B \wedge 2^n h^n = C - A] \Longrightarrow [n \mid c + h \wedge n^2 \mid c^n + 2^n h^n] \Longrightarrow n \mid emch,$$

which is inconsistent with $n \nmid emch$. ♠

Proof For $n \in \mathbb{P}$ **In Special Case**. For some $n \in \mathbb{P}$ and for some $p, q, w, r, x \in \{1,3,5, \ldots\}$ and for some $C, A \in \{3^2, 5^2, 7^2, \ldots\}$ such that $p > q$ and $w > r$ and p, q, w, r, x are coprime and $n \mid pq$: [4]

$$\Big[(2pq)^n = B^n = \left(C^{\frac{n}{2}}\right)^2 - \left(A^{\frac{n}{2}}\right)^2 \wedge\ C = (wr)^2 \wedge A = x^2 \wedge \frac{(2pq)^n + (x^2)^n}{2pq + x^2}$$
$$= \frac{(2pq)^n + (x^2)^n}{(r^2)^n} = \frac{(w^2\, r^2)^n}{(r^2)^n} = (w^2)^n \wedge (r^n)^2 - x^2$$
$$= 2pq\ \wedge\ (2 \mid pq \equiv 0)\Big] \in \mathbf{0}.\ \spadesuit$$

This is the proof.

III. PROOF OF - If $n \in \{3,5,7,\dots\}$ and $z \in \{3,7,11,\dots\}$, then for all $u,v \in \mathbb{N}1$: $z^n \neq u^2 + v^2$.

Theorem 2. *For all* $n \in \{3,5,7,\dots\}$ *and for all* $z \in \{3,7,11,\dots\}$ *the equation*

$$z^n = u^2 + v^2$$

has no primitive solutions $[z,u,v]$ *in* $\{1,2,3,\dots\}$.

Proof. Suppose that for some $n \in \{3,5,7,\dots\}$ and for some $z \in \{3,7,11,\dots\}$ the equation

$$z^n = u^2 + v^2$$

has primitive solutions $[z,u,v]$ in $\{1,2,3,\dots\}$. Then z,u and v are coprime and odd $u - v > 0$.

Without loss for the proof we can assume that $u > v$.

On the strength of the Guła's Theorem [3] we get

$$\text{Lside} = \left(\frac{z^n + d^2}{2d}\right)^2 = u^2 + \left(\frac{z^n - d^2}{2d}\right)^2 + v^2 = \text{Rside} \in \mathbf{0}$$

inasmuch as $4 \mid \text{Lside}$ and $4 \nmid \text{Rside}$ because the numbers $u, \frac{z^n - 1}{2}$ are odd or $v, \frac{z^n - 1}{2}$ are odd.

$$even\ \ \frac{z^n + d^2}{2d} = \frac{2m + 1 + 4s + 1}{2d} = \frac{2(m + 2s) + 2}{2d} = \frac{(m + 2s) + 1}{d},$$

where the numbers d, m are positive and odd and $s \in \{0,1,2,\dots\}$. This is the proof.

Golden Nyambuya proved (allegedly) the theorem – For all $n \in \{3,5,7,\dots\}$ the equation

$$z^n = u^2 + v^2$$

has no primitive solutions in $\{1,2,3,\dots\}$ with $z \in \{3,5,7,\dots\} - \{3^2, 5^2, 7^2, \dots\}$. [9]

Corollary 1. For some $n \in \{3,5,7,\dots\}$ and for some $z \in \{5,9,13,\dots\}$ and for some prime natural numbers u,v such that $u - v$ is positive and odd:

$$z^n = u^2 + v^2 \Longrightarrow (z^n)^2 = (u^2 + v^2)^2 = (u^2 - v^2)^2 + (2uv)^2.$$

This is the Corollary 1.

Example 1.

$$(5^3)^2 = (11^2 + 2^2)^2 = 117^2 + 44^2,$$

where $117 = 11^2 - 2^2 = u^2 - v^2$ and $44 = 2 \cdot 11 \cdot 2 = 2uv$.

This is the Example 1.

Example 2.

$$(17^3)^2 = (52^2 + 47^2)^2 = 495^2 + 4888^2,$$

where $495 = 52^2 - 47^2 = u^2 - v^2$ and $4888 = 2 \cdot 52 \cdot 47 = 2uv$.

This is the Example 2.

Example 3.

$$(29^3)^2 = (145^2 + 58^2)^2 = 17661^2 + 16820^2,$$

where $17661 = 145^2 - 58^2 = u^2 - v^2$ and $16820 = 2 \cdot 145 \cdot 58 = 2uv$.

This is the Example 3.

Example 4.

$$(41^3)^2 = (205^2 + 164^2)^2 = 15129^2 + 67240^2,$$

where $15129 = 205^2 - 164^2$ and $67240 = 2 \cdot 205 \cdot 164$.

This is the Example 4.

Example 5.

$$(13^5)^2 = (597^2 + 122^2)^2 = 341525^2 + 145668^2,$$

where $341525 = 597^2 - 122^2$ and $145668 = 2 \cdot 597 \cdot 122$. This is the Example 5.

Theorem 3. *For all* $n \in \{1,2,3, \dots\}$ *and for all* $m \in \{3,5,7, \dots\}$ *and for some* $p, q \in \{1,2,3, \dots\}$ *such that* $p > q$:

$$[m^n = p^2 - q^2 \wedge (p^2 - q^2)^2 + (2pq)^2 = (p^2 + q^2)^2].$$

This is the theorem.

Theorem 4. *For all* $n \in \{1,2,3, \dots\}$ *and for all* $m \in \{2,4,6, \dots\}$ *and for some* $p, q \in \{1,2,3, \dots\}$ *such that the number* $\frac{m^n}{2}$ *is even and bigger than two and* $p > q$:

$$[m^n = p^2 - q^2 \wedge (p^2 - q^2)^2 + (2pq)^2 = (p^2 + q^2)^2].$$

This is the theorem.

IV. THE PROOF OF THE GOLDBACH'S CONJECTURE

Conjecture 1 (Goldbach Conjecture). For all $Z \in \{6,8,10,\dots\}$ and for some $X, Y \in \mathbb{P}$:

$$Z = X + Y.$$

Proof.

$$\{6, 8,10, \dots\} = \{\mathbf{6}, 12, \mathbf{18}, 24, \mathbf{30}, 36, \mathbf{42}, 48, \mathbf{54}, 60, \mathbf{66}, 72, \mathbf{78}, 84, \mathbf{90}, \dots\} \cup$$

$$\{8, \mathbf{14}, 20, \mathbf{26}, 32, \mathbf{38}, 44, \mathbf{50}, 56, \mathbf{62}, 68, \mathbf{74}, 80, \mathbf{86}, 92, \mathbf{98}, 104, \mathbf{110}, \dots\} \cup$$

$$\{\mathbf{10}, 16, \mathbf{22}, 28, \mathbf{34}, 40, \mathbf{46}, 52, \mathbf{58}, 64, \mathbf{70}, 76, \mathbf{82}, 88, \mathbf{94}, 100, \mathbf{106}, 112, \dots\}.$$

Thus

$$[3] \cup [\mathbf{9}, \mathbf{15}, \mathbf{21}, \mathbf{27}, \mathbf{33}, \mathbf{39}, \mathbf{45}, \mathbf{51}, \mathbf{57}, \mathbf{63}, \dots] \cup$$

$$[7,13,19, \mathbf{25}, 31,37,43, \mathbf{49}, \mathbf{55}, 61,67,73,79, \mathbf{85}, \mathbf{91}, \dots] \cup$$

$$[5,11,17,23,29, \mathbf{35}, 41,47,53,59, \mathbf{65}, 71, \mathbf{77}, 83,89, \mathbf{95}, \dots] = [3,5,7, \dots].$$

Hence

$$\{6\} = \{Z\colon\ Z = X + Y \land X = Y = 3\}\ \lor\ \{8\} = \{Z\colon\ Z = X + Y \land X = 3 \land Y = 5\} \lor$$

$$\{14,20,26, \dots\} =$$

$$\{Z\colon\ Z = X + Y \land X \leq Y \land X, Y \in \mathbb{P} \land X, Y \in [7,13,19, \mathbf{25}, 31, \dots]\} \lor$$

$$\{10,16,22, \dots\} =$$

$$\{Z\colon\ Z = X + Y \land X \leq Y \land X, Y \in \mathbb{P} \land X, Y \in [5,11,17,23,29, \mathbf{35}, \dots]\} \lor$$

$$\{12,18,24, \dots\} =$$

$$\{Z\colon\ Z = X + Y \land X, Y \in \mathbb{P} \land X \in [5,11,17,23,29, \mathbf{35}, \dots] \land Y \in [7,13,19, \mathbf{25}, 31, \dots]\},$$

whence it implies that for all $Z \in \{6, 8,10, \dots\}$ and for some $X, Y \in \mathbb{P}$: $Z = X + Y$. [3], [4]

This is the proof.

V. THE PROOF OF THE BEAL'S CONJECTURE

Conjecture 2 (Beal Conjecture). *For all* $x, y, z \in \{3,4,5 \dots\}$ *the equation*

$$A^x + B^y = C^z$$

has no primitive solutions in $\{1,2,3, \dots\}$.

Proof. Suppose that for some $x, y, z \in \{3,4,5,...\}$ the equation $A^x + B^y = C^z$ has primitive solutions in $\{1,2,3,...\}$. Without loss for the proof we can assume that $A, C - B \in \{1,3,5,...\}$.

Then only one number out of (A, B, C) is even and $A = C^z - X$ and $B = C^z - Y$, where the positive natural numbers X, Y are coprime and $X - Y$ is odd, and $(A + B \leq C \wedge A^2 + B^2 < C^2)$ or $(A + B > C \wedge A^2 + B^2 > C^2)$ because the Jeśmanowicz Conjecture is true. [4]

Therefore we will have

$$(C^z - X)^x + (C^z - Y)^y = C^z \Longrightarrow$$

$$C^{zx} + xC^{zx-z}(-X) + \frac{x(x-1)}{2}(C^z)^{x-2}(-X)^2 + \cdots + xC^z(-X)^{x-1} + (-X)^x + (C^z)^y +$$

$$y(C^z)^{y-1}(-Y) + \frac{y(y-1)}{2}(C^z)^{y-2}(-Y)^2 + \cdots + yC^z(-Y)^{y-1} + (-Y)^y = C^z \Longrightarrow$$

$$(C^z)^{x-1} + x(C^z)^{x-2}(-X) + \frac{x(x-1)}{2}(C^z)^{x-3}(-X)^2 + \cdots + x(-X)^{x-1} + (C^z)^{y-1} +$$

$$y(C^z)^{y-2}(-Y) + \frac{y(y-1)}{2}(C^z)^{y-3}(-Y)^2 + \cdots + y(-Y)^{y-1} + \frac{(-X)^x + (-Y)^y}{C^z} = 1.$$

Hence – For some $x, y, z, X, Y, C \in \{3,4,5 ...\}$ such that $X - Y$ is odd and X, Y, C are coprime:

$$\frac{\left|(-X)^x + (-Y)^y\right|}{C^z} = \left|D\right| > 1.$$

Since $(A^x + B^y)/C^z = 1$, then we will have

$$\frac{A^x}{B^y} = \frac{\left(\frac{B - C^z}{B}\right)^y - D}{D - \left(\frac{A - C^z}{A}\right)^x} \wedge D = \frac{(-X)^x + (-Y)^y}{C^z} = \frac{(A - C^z)^x + (B - C^z)^y}{C^z} =$$

$$1 + xA^{x-1}(-1) + \frac{x(x-1)}{2}A^{x-2}(-1)^2C^z + \cdots + xA(-1)^{x-1}(C^z)^{x-2} + (-1)^x(C^z)^{x-1} +$$

$$yB^{y-1}(-1) + \frac{y(y-1)}{2}B^{y-2}(-1)^2C^z + \cdots + yB(-1)^{y-1}(C^z)^{y-2} + (-1)^y(C^z)^{y-1}.$$

Proof In Special Case. We assmume that the number C is minimal. For some $x, y, z \in \{3,4,5,...\}$ and for some $p, q \in \{1,3,5,...\}$ such that $p > q$ and p, q are coprime:

$$(pq)^{\frac{x}{2}} = A^{\frac{x}{2}} = \frac{1}{4}\left[\left(p^{\frac{x}{2}} + q^{\frac{x}{2}}\right)^2 - \left(p^{\frac{x}{2}} - q^{\frac{x}{2}}\right)^2\right] \Longrightarrow \left(\frac{p^x - q^x}{2} = B^{\frac{y}{2}} \wedge \frac{p^x + q^x}{2} = C^{\frac{z}{2}}\right) \Longrightarrow$$

$$(A^x + B^y = C^z \wedge C = c^2 \wedge B = b^2 \wedge c^z - b^y = q^x) \Longrightarrow c < C,$$

which is inconsistent with minimal C. ♠

Conclusion 1. If the bases in three powers we define as conclusions of the above implications that is to say from the above conditions, then we will not find such the arithmetic triple (A, B, C) for which our equation will be false. Yes that's is in the proof of FLT for odd n, so the Beal's Conjecture is true.

This is the proof.

SUPPLEMENT

Theorem 5. *For each fixed pair (u, v) of the relatively prime natural numbers u and v such that $u - v$ is positive and odd there exists exactly one a primitive Pythagorean triple (x, y, z) such that $(u^2 - v^2, 2uv, u^2 + v^2) = (x, y, z)$ and conversely – Any the primitive Pythagorean triple (x, y, z) such that $(x, y, z) = (u^2 - v^2, 2uv, u^2 + v^2)$ arises exactly from one pair (u, v) of the relatively prime natural numbers u and v such that $u - v$ is positive and odd.* This is the theorem.

Let $\boldsymbol{gcd}(U, V) = \boldsymbol{gcd}(u, v) = 1$ and $U - V,\ u - v \in \{1,3,5, \ldots\}$.

Suppose that for some $p, q, C \in \{1,3,5, \ldots\}$ and for some $B \in \{2,4,6, \ldots\}$ such that the numbers p, q, C and B are coprime and $q < p < C$: $(pq)^4 = C^2 - (B^2)^2$.

We assume that the number C is minimal.

On the strength of the Guła's Theorem [3] we obtain

$$B^2 = \frac{p^4 - q^4}{2} = \frac{p^2 + q^2}{2}(p^2 - q^2) \Longrightarrow \left(\frac{p^2 + q^2}{2} = w^2 \wedge p^2 - q^2 = r^2\right) \Longrightarrow w^2 = \frac{p^2 + q^2}{2}$$
$$= \frac{(u^2 + v^2)^2 + (u^2 - v^2)^2}{2} = u^4 + v^4 \Longrightarrow w < C,$$

which is inconsistent with minimal C. ♣

If

$$[U^2 - V^2 = A^2 \wedge 2UV = B^2 \wedge U^2 + V^2 = C^2 \wedge (A^2)^2 + (B^2)^2 = (C^2)^2],$$

then on the strength of the Guła's Theorem [3] we get

$$[V^2 = (2uv)^2 = U^2 - A^2 = C^2 - U^2 \wedge U = u^2 + v^2 \wedge u^2 - v^2 = A] \Longrightarrow$$

$$\left[C = \frac{(2uv)^2 + 2^2}{2 \cdot 2} = (uv)^2 + 1 \wedge u^2 + v^2 = U = \frac{(2uv)^2 - 2^2}{2 \cdot 2} = (uv)^2 - 1\right] \in \mathbf{0}. ♣$$

It's not true in [9] that FLT for $n = 4$ can be written equivalently as: $A^2 = C^4 - B^4$, where $A = 2UV$ or $A = U^2 - V^2$ because Fermat did not proved his own theorem for $n = 4$. [8]

The below we have the new proofs in the above two cases.

In the first case we will have – If

$$[2UV = A \wedge U^2 - V^2 = B^2 \wedge U^2 + V^2 = C^2 \wedge A^2 + (B^2)^2 = (C^2)^2],$$

then on the strength of the Guła's Theorem [3] we get

$$[V^2 = (2uv)^2 = U^2 - B^2 = C^2 - U^2 \wedge U = u^2 + v^2 \wedge u^2 - v^2 = B] \Longrightarrow$$

$$\left[C = \frac{(2uv)^2 + 2^2}{2 \cdot 2} = (uv)^2 + 1 \wedge u^2 + v^2 = U = \frac{(2uv)^2 - 2^2}{2 \cdot 2} = (uv)^2 - 1\right] \in \mathbf{0}. ♣$$

In the second case we have

$$\begin{aligned}[U^2 - V^2 = A \wedge 2UV = B^2 \wedge U^2 + V^2 = C^2 \wedge (U+V)^2(U-V)^2 &= (C^2)^2 - (B^2)^2 \\ &= (C^2 + B^2)(C^2 - B^2) \wedge (U+V)^2 = C^2 + B^2 \wedge (U-V)^2 \\ &= C^2 - B^2 \wedge U + V = u^2 + v^2 \wedge u^2 - v^2 = C \wedge 2uv = B] \Longrightarrow\end{aligned}$$

$$2UV = (2uv)^2 \Longrightarrow UV = 2u^2v^2 \Longrightarrow (U = u^2 \wedge V = 2v^2) \Longrightarrow U + V = u^2 + 2v^2,$$

which is inconsistent with $U + V = u^2 + v^2$. ♣

REFERENCES

[1]. AMS: http://www.ams.org/profession/prizes-awards/ams-supported/beal-prize-rules

[2] En. Wikipedia – https://en.wikipedia.org/wiki/Goldbach%27s_conjecture

[3]. Guła, L. W.: Disproof the Birch and Swinnerton-Dyer Conjecture, American Journal of Educational Research, **Volume 4**, No 7, 2016, pp 504-506, doi: 10.12691/education-4-7-1 | Original Article electronically published on May 3, 2016 – http://pubs.sciepub.com/EDUCATION/4/7/1/index.html

[4]. Guła, L. W.: Several Treasures of the Queen of Mathematics, International Journal of Emerging Technology and Advanced Engineering, **Volume 6**, Issue 1, January 2016, pp 50-51 – http://www.ijetae.com/files/Volume6Issue1/IJETAE_0116_09.pdf

[5]. Guła, L. W.: The Truly Marvellous Proof, International Journal of Emerging Technology and Advanced Engineering, **Volume 2**, Issue 12, December 2012, pp 96-97 – http://www.ijetae.com/files/Volume2Issue12/IJETAE_1212_14.pdf

[6]. Mauldin, R. Daniel: A Generalization of Fermat's Last Theorem: The Beal Conjecture and Prize Problem, Notices of the AMS, , **Volume 44**, No 11, December 1997, p. 1436 – http://www.ams.org/notices/199711/beal.pdf

[7]. Mazur, B.: Mathematical Perspectivies, Bulletin (New Series) of The American Mathematical Society, **Volume 43**, Number 3, July 2006, p. 399, Article electronically published on May 9, 2006 –

http://www.ams.org/journals/bull/2006-43-03/S0273-0979-06-01123-2/S0273-0979-06-01123-2.pdf

[8]. Narkiewicz, W.: Wiadomości Matematyczne XXX.1, Annuals PTM, **Series II**, Warszawa 1993, p. 3

[9]. Nyambuya, G. G.: On a Simpler, Much More General and Truly Marvellous Proof of Fermat's Last Theprem (II), Department of Applied Physics, National University of Science and Technology, Bulawayo, Republic of Zimbabwe, Preprint submitted to **viXra.org Version 3** - September 24, 2014, pp 6-12, – http://www.rxiv.org/pdf/1405.0023v3.pdf