Maxwell's theory of electrostatic is the basis of mathematics establishing a unified field equation

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Abstract

This article is based on the Maxwell's electromagnetic theory ，by the introduction of virtual particle cloud , and strict mathematics and physics verification ,firstly get the conclusion of gravitation field, nuclear power field and weak field all are result of the electrostatic field's different forms effect, and unify this four field force in an equation. This is the only unified field theory including gravitation at present.

Keywords: *Unified field theory　Gravitation field　Nuclear power field　Weak field　Electrostatic field　Virtual particle cloud*

1. Introduction

We know that the electromagnetic force and gravitation are long-range force, They are inversely proportional to the square distance，The role of strength differ 10^{37} times。 Nuclear force and weak force is short-range force, Its range is in 10^{15} meters and 10^{17} meters, For the establishment of the unified theory including these four forces , physicists tried to interaction as Perturbation to study, or as the transmission of interaction through the exchange of integer spin Bose particles, but they did not receive satisfactory results. Most physicists believe that new ideas must be made to gravity included in the unified theory of nature. In this paper, by considering the source was the role of charge charge charge around the role of virtual particle clouds, derived the electrostatic field equations from Maxwell's electromagnetic theory, including not only electricity, but also to the weak force, nuclear force and gravity unified in the field equation.

2. Complete theory of electrostatic

In the electrostatic field role theory, The electric field strength $\vec{E}$ on the P field points by the source charge Q stimulating is defined as negative gradient of electric potential φ ,is

$$\vec{E} = -\nabla\varphi \qquad (1)$$

To the formula into the divergence equation of static electric field

$$\nabla \cdot \vec{E} = -\rho / \varepsilon_0 \qquad (2)$$

Know that electric potential φ satisfy the Poisson equation in a vacuum space

$$\nabla^2\varphi = -\rho / \varepsilon_0 \qquad (3)$$

ε_0 is vacuum dielectric, ρ is charge density on the P field points.

If the assumption that there are a certain number of virtual particle with a positive charge and a certain number of virtual particle with negative charge around in each charge, we call virtual particle clouds. in the absence of the role of external electric field, these virtual particle cloud should be in a state of static equilibrium distribution, there is no the role of field strength to the charge by it surrounds. In the role of external electric field, the virtual particle cloud polarization will have a virtual electric dipole moment effect on the charge it surrounded, the value of the virtual electric dipole moment should be proportional to external electric field strength. According to this assumption ,for the charge Q being affected on the P field points, its peripheral virtual particle clouds will have a virtual electric dipole moment $\vec{P}$ on the P field points by the source charge Q affected. If the virtual polarization electric dipole moment $\vec{P}$ is defined as negative gradient of electric potential φ ,and assumes

$$\varphi_p = \chi_p \varphi \tag{4}$$

Where φ is the electric potential on the P field points by the source charge Q stimulating. Then the total electric potential of the being affected charge q on the P field points must be $\varphi + \varphi_p$ If assumes

$$\varphi + \varphi_p = \left(1 + \chi_p\right)\varphi = \varphi / \varepsilon(r) \tag{5}$$

Where $1 + \chi_p = 1/\varepsilon\left(r\right)$, the electric potential of the charge q should be

$$V = q\left(\varphi + \varphi_p\right) = q\varphi / \varepsilon(r) \tag{6}$$

Also have

$$\varphi = \varepsilon\left(r\right)V / q \tag{7}$$

But is not $\varphi = V / q$. So we know the potential energy between two charge should be satisfied equation

$$\nabla^2\left[\varepsilon\left(r\right)V / q\right] = -\rho / \varepsilon_0 \tag{8}$$

from style (3).

May wish to make variable $r = 1/x$ instead of radial radius r in style (8) for getting function $\varepsilon\left(r\right)$,and use

$$\nabla^2 = \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}\right) = x^4\frac{d^2}{dx^2} \tag{9}$$

Expanding style (8)

$$\left(V/q\right)'' + 2\left[\varepsilon'\left(x\right)/\varepsilon\left(x\right)\right]\left(V/q\right)' + \left[\varepsilon''\left(x\right)/\varepsilon\left(x\right)\right]\left(V/q\right) = -\left(\rho/\varepsilon_0\right)x^{-4}\varepsilon^{-1}\left(x\right) \tag{10}$$

Where V is a function of x, x is countdown of the radius vector r.

It is clear wish to V can be included in the formula of Coulomb potential energy electrostatic in the case of $\varepsilon\left(x\right) = 1$, style (10) must be a constant coefficient differential equation. So coefficient $\varepsilon'\left(x\right)/\varepsilon\left(x\right)$ and $\varepsilon''\left(x\right)/\varepsilon\left(x\right)$ must be constant , and satisfies

$$\Delta = \left[2\varepsilon'\left(x\right)/\varepsilon\left(x\right)\right]^2 - 4\varepsilon''\left(x\right)/\varepsilon\left(x\right) = -4\left[\varepsilon'\left(x\right)/\varepsilon\left(x\right)\right]' = 0 \tag{11}$$

Also satisfies $\varepsilon'\left(x\right)/\varepsilon\left(x\right)$ equivalent to constant. If expressed the integral constant by R, there is

$$\varepsilon(x) = e^{Rx} = e^{R/r} \qquad (12)$$

To style (12) into the style (5) shows

$$\varphi + \varphi_p - \varphi_0 = \varphi / \varepsilon(r) = \varphi e^{-R/r} = \varphi - \frac{\varphi R}{r}\sum_{i=1}^{\infty}\frac{1}{n!}\left(-\frac{R}{r}\right)^{n-1}$$

It can clearly be seen， the second of the right side of the equation just is the negative gradient of electric potential give by polarization virtual electric dipole moment $\vec{P}$ on the P field points. This shows that style (5) is correct.

If substituting style (12) back style (10) , know that the equation of the electric field potential V satisfies is a Second-order constant coefficient non-homogeneous differential equation

$$(V/q)'' + 2R(V/q)' + R^2(V/q) = -(\rho/\varepsilon_0)x^{-4}e^{-Rx} \qquad (13)$$

In case of $\rho = 0$, if the solution of style (13) can be consistent with the Coulomb electrostatic potential energy

$$V = \frac{Qqx}{4\pi\varepsilon_0} = \frac{Qq}{4\pi\varepsilon_0 r} \qquad (14)$$

Easily get the electrostatic potential between the two charge from style (13) , It is

$$V = \frac{qQ}{4\pi\varepsilon_0}xe^{-Rx} = \frac{qQ}{4\pi\varepsilon_0 r}e^{-R/r} = q\varphi e^{-R/r} \qquad (15)$$

It is easily obtained the force between the two charges is

$$\vec{F} = -\nabla V = -\nabla\left(q\varphi e^{-R/r}\right) = -q\left(\nabla\varphi + \varphi\frac{R\vec{r}}{r^3}\right)e^{-R/r} = q\left(\vec{E} - \varphi\frac{\vec{r}}{r^2}\frac{R}{r}\right)e^{-R/r}$$

Also know from style (1)

$$\vec{E} = -\nabla\varphi = \varphi\frac{\vec{r}}{r^2} \qquad (16)$$

So have

$$\vec{F} = q\vec{E}\left(1 - \frac{R}{r}\right)e^{-R/r} \qquad (17)$$

Style (17) can transition to the electrostatic Coulomb formula in case of $R = 0$

R is a integral constant, It is determined from experimental or other conditions .But we know from style (15) and (17), if wish to electric potential and electric field strength in the absence of infinite singular point in case of $r \to 0$,R can not take the zero value, R non-zero means that there are minimum .There is a result can satisfy the requirement of there are minimum, which is assuming

There is relationship between R and the quality M、m of the two role objects.

$$R = \frac{\hbar}{(M+m)c} + \frac{G}{c^2}(M+m) \qquad (18)$$

Where M and m are the quality of the two role objects; $\hbar = h/2\pi$; h is Planck constant; c is the speed of light; G is Newton's gravity constant. At this point ,no matter M 、 m take what value, R is not less than $2\sqrt{\hbar G/c^3}$, Of course, whether correct also depend on experiment to test.

Obviously, the force direction is opposite in the two cases $r > R$ and $r < R$, R is the equilibrium point of the role force. According to style (18) know the role systems of different quality, different R .There is $R = 4.2\times10^{-16}m$ for the role system of proton-proton ; there is $R = 3.86\times10^{-13}m$ for the role system of proton and the electronic; there is $R = 7.72\times10^{-13}m$ for the role system of electronic and electronic.

3.Weak field is short-range effect which electrostatic theory attached

There are two linearly independent solutions of non-homogeneous equations with constant coefficients style (13) in case $\rho = 0$,in addition to long-range electrostatic potential energy power solutions of style (15), there is short-range electrostatic potential energy power solutions in case $r < r_m$,it is

$$V_{短}(r) = q\varphi_0 \frac{r_m}{r}\left(1 - \frac{r}{r_{\mathrm{m}}}\right)e^{-R/r + 2R/r_m} \qquad (19)$$

Where r_{m} is the most role distance of short-range electric field,

φ_0 is constant with dimensionless electric potential. According to the experimental determination the role of intensity of short-range electric field is the role of intensity of long-range electric field 10^{-11} times. The reasons leading to this result may be the relative dielectric constant in the role space of short-range electric field is e^{2R/r_m} , and satisfies $\varepsilon_0 e^{2R/r_m} = 1$. If this is the case can be made the constant with dimensionless electric potential

$$\varphi_0 = \frac{Q}{4\pi\varepsilon_0 r_m}e^{-2R/r_m} = \frac{Q}{4\pi r_m} \qquad (20)$$

Then style (19) can be written

$$V_{短}(r) = \frac{Qq}{4\pi r}\left(1 - \frac{r}{r_{\mathrm{m}}}\right)e^{-R/r} \qquad (21)$$

Style (21) is the role potential energy of weak field, the weak-field force derived from it is

$$\vec{F}_{短} = -\nabla V_{短}(r) = \frac{Qq\vec{r}}{4\pi r^3}\left(1 - \frac{R}{r} + \frac{R}{r_{\mathrm{m}}}\right)e^{-R/r} \qquad (22)$$

There is a most role distance r_{m} in short-range electric field (weak-field), there is role of short-range electric field only in case $r \le r_{\mathrm{m}}$, no role of short-range electric field but in case $r > r_{\mathrm{m}}$.

4. Nuclear power from the virtual particle cloud with contour contrary sign charge exist surrounding of charge particles

According to electric field strength $\vec{E}$ have infinite singular point at the origin, know that it is surely distributed inside and outside the two-tier virtual particle cloud with contour contrary sign charge within the charge particle, the inner layer particles cloud concentrated at the origin of the charge particle, the outer particle cloud are located in the vicinity of space. For inside and outside the two-tier virtual particle clouds brought the same charge, the relationship of potential energy V and ρ in style (13) should satisfy

$$\rho = -k^2 \varepsilon(r) \varepsilon_0 V / q = -k^2 \varepsilon_0 e^{Rx} V / q \qquad (23)$$

Also satisfies

$$(V/q)'' + 2R(V/q)' + R^2 (V/q) = k^2 x^{-4} (V/q) \qquad (24)$$

It is easy to get

$$V = -\xi \frac{Qq}{4\pi\varepsilon_0} x e^{-k/x - Rx} = -\xi \frac{Qq}{4\pi\varepsilon_0 r} e^{-kr - R/r} \qquad (25)$$

Where ξ is a constant to be determined.

Take $R = 0$, style (25) compared to the nuclear power potential energy $V = -\frac{\hbar c}{r} e^{-kr}$

Given by Hideki Yukawa, know constant

$$\xi = 4\pi\varepsilon_0 \hbar c / Qq \qquad (26)$$

To style (26) into style (25), can get the expression of role potential of nuclear power and force are

$$V = -\frac{\hbar c}{r} e^{-kr - R/r} \qquad (27\text{-}1)$$

$$\vec{F} = -\nabla V = -\frac{\hbar c \vec{r}}{r^3} (1 + kr - R/r) e^{-kr - R/r} \qquad (27\text{-}2)$$

Easy to verify, nuclear power $\vec{F}(r)$ have two extreme points r_1、r_2,if assumes $k = 1/R$, can get $r_1 = R$ $\quad r_2 = \left(\sqrt[3]{2} - 1\right) R$

Why nuclear power will have the form of style (27)? This is related with the distribution of virtual particle clouds with charge existed in nuclear particles, according to

$$Q = \varepsilon_0 \oint \vec{E} \cdot d\vec{s} \qquad (28)$$

can be obtained in nuclear particles, the virtual particle cloud whose radius is r holds the electric charge

$$Q_S = -\varepsilon_0 \frac{d\varphi}{dr} \cdot 4\pi r^2 \qquad (29)$$

φ can be derived from style (25) into style (7)

$$\varphi = e^{R/r} V / q = -\frac{\hbar c}{rq} e^{-kr} \qquad (30)$$

To style (30) into style (29), can get

$$Q_S = -\frac{4\pi\varepsilon_0 \hbar c}{q} (1 + kr) e^{-kr} \qquad (31)$$

It easily can be seen from style (31) charge Q_S is not constant, it is increasing with r decreasing. When $r \to 0$, charge $Q_S \to -4\pi\varepsilon_0\hbar c/q$, this shows that distributing centrally inner virtual particle cloud with charge $Q_{内} = -4\pi\varepsilon_0\hbar c/q$ at the origin of charged particles.

In addition, it can be seen from style (31),in case $r >> 1/k$ $Q_S \to 0$, this shows that distributing outer virtual particle cloud with contour contrary sign charge with inner virtual particle cloud. With r increasing charges of inner and outer virtual particle cloud will gradually neutralize, make the total electricity gradually tend to zero.

Charge of outer virtual particle cloud can get from

$$Q_{外} = 4\pi\int_0^r \rho r^2 dr \tag{32}$$

Knowing from style (23)

$$\rho = -k^2\varepsilon_0 e^{R/r}V/q = \frac{k^2\varepsilon_0\hbar c}{qr}e^{-kr} \tag{33}$$

To style (33) into style (32)

$$Q_{外} = \frac{4\pi k^2\varepsilon_0\hbar c}{q}\int_0^r re^{-kr}dr = \left(4\pi\varepsilon_0\hbar c/q\right)\left[1-\left(1+kr\right)e^{-kr}\right] \tag{34}$$

Obviously, there is $Q_{外} = 0$ at the origin, and in case $r \to \infty$, just have $Q_{外} = -Q_{内}$.

It can be seen style (23) meet the request of the total charges brought by virtual particle cloud shows external neutral, which shows the theory of nuclear power role is self-consistent.

5、Universal gravitation is two neutral object the surplus effect of static electricity function

We know that if use vector and scalar to certainly express a field equation, ask for help of Luo Lun norm

$$\nabla\cdot\vec{A}+\frac{1}{c^2}\frac{\partial\varphi}{\partial t}=0 \tag{35}$$

May permit, $\vec{A}$、φ is based on fluctuations in the form of dissemination in space and to meet the wave equation

$$\begin{aligned}\nabla^2\vec{A}-\frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2}&=-\mu_0\vec{J}\\ \nabla^2\varphi-\frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2}&=-\rho/\varepsilon_0\end{aligned} \tag{36}$$

Transform the equation in

$$\begin{aligned}\frac{\partial}{\partial x_j}&=ch\theta\frac{\partial}{\partial x'_j}-sh\theta\vec{k}_0\cdot\vec{e}_j\frac{\partial}{\partial x'_0}\\ \frac{\partial}{\partial x_0}&=ch\theta\frac{\partial}{\partial x'_0}-sh\theta\vec{k}_0\cdot\vec{e}_j\frac{\partial}{\partial x'_j}\end{aligned} \tag{37}$$

with a transformation of the covariance. Where $\vec{k}_0$ is the direction of electromagnetic wave propagation unit vector; $\vec{e}_j$ is the axis coordinate system with reference to the basis vectors; $x_0 = ct$； $x_0' = c't'$。

To make (35)-formula can also make use of (37) transform, may wish to make

$$A_0 = \varphi / c \qquad \vec{A} = A_J \vec{e}_j \tag{38}$$

(35) -formula can be written in the form of

$$\frac{\partial A_j}{\partial x_j} + \frac{\partial A_0}{\partial x_0} = 0 \tag{39}$$

The use of (37) type of (39)-type transformation, there are

$$ch\theta \left(\frac{\partial A_j}{\partial x_j'} + \frac{\partial A_0}{\partial x_0'} \right) - sh\theta \vec{k}_0 \cdot \vec{e}_j \left(\frac{\partial A_j}{\partial x_0'} + \frac{\partial A_0}{\partial x_j'} \right) = 0 \tag{40}$$

Thus, the Lorentz gauge can be written in the form of four-dimensional

$$\left(\frac{\partial}{\partial x_v'} - th\theta K_v \right) A_v = 0 \tag{41}$$

Which

$$K_v A_v = \vec{k}_0 \cdot \vec{e}_j \left(\frac{\partial A_j}{\partial x_0'} + \frac{\partial A_0}{\partial x_j'} \right) = -\frac{1}{c} \vec{k}_0 \cdot \vec{e}_j E_j = -\frac{1}{c} \vec{k}_0 \cdot \vec{E} \tag{42}$$

Which v = 0、j；j =1、2、3。

If the role of $\vec{n}$ particles that connect the two directions of the unit vector. Easy to verify, four-dimensional form of the Lorentz gauge (41) formula in

$$A_v \to A_v' = A_v \exp\left(i\vec{k}_0 \cdot \vec{n}\phi_0 \right) \tag{43}$$

the overall with invariance under the gauge transformation. Obviously, do not change with time of static electric field, cA_0' is the potential of its φ. Thus, in accordance with (43)-formula gauge transformation the overall requirements of potential φ should be the plural form

$$\varphi = cA_0' = \frac{Q_e}{4\pi\varepsilon_0 r} \exp\left(i\vec{k}_0 \cdot \vec{n}\phi_0 \right) \tag{44}$$

Also from formula (7), we can see that is located in φ field with electric charge q of the potential energy v satisfy

$$V / q = \varphi e^{-R/r} \tag{45}$$

Therefore, there must be

$$V = R_e \left[q^* \varphi \right] e^{-R/r} = \cos\left[\left(\vec{k}_0' - \vec{k}_0 \right) \cdot \vec{n}\phi_0 \right] \frac{Q_e q_0}{4\pi\varepsilon_0 r} e^{-R/r} \tag{46}$$

Which $q = q_0 \exp\left(i\, \vec{k}_0' \cdot \vec{n}\phi_0 \right)$; $\vec{k}_0'$ to $\vec{n}$ direction of the electric charge q_0 units in the direction of vector lines; $R_e[\ ]$ that take real part; “*”, the plural form of the conjugate q.

If $\vec{n}$ direction is set from Q_e to q_0, based on positive charge the direction of the electric field vector is the line back charge, negative charge of the direction of electric field lines point to the charge vector is provided (see Figure 1-1-1), we can see that when Q_e and q_0 charge with a different number, because $\vec{k}_0'$ and $\vec{k}_0$ in the same direction, as shown in Figure 1-1-2, there are $(\vec{k}_0' - \vec{k}_0) \cdot \vec{n} = 0$; when $\vec{k}_0'$ and $\vec{k}_0$ with the same charge number, because $\vec{k}_0'$ and $\vec{k}_0$ back, then

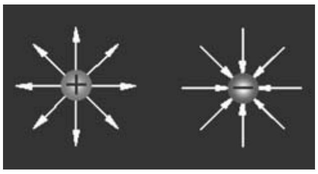

Figure1-1-1 **Positive and negative charge with the power line the relationship between direction of vector**

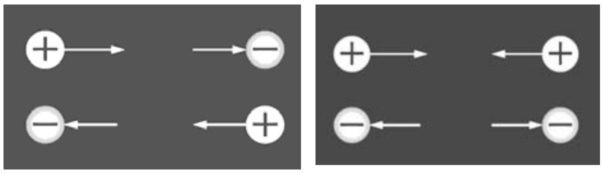

Figure1-1-2 **Charge between the two cases of the four role the**

there are $(\vec{k}_0' - \vec{k}_0) \cdot \vec{n} = 2$。Thus, from (46)-formula , we can see that the two different charge its electrostatic potential energy between the

$$V_{异} = -\frac{|Q_e q_0|}{4\pi\varepsilon_0 r} e^{-R/r} \tag{47}$$

Charge the 2 with the same electrostatic potential energy between $V_{同}$

$$V_{同} = \cos\left(2\phi_0\right) \frac{|Q_e q_0|}{4\pi\varepsilon_0 r} e^{-R/r} \tag{48}$$

It is clear that as long as it is $\phi_0 \neq 0$, the constant $|V_{异}| > |V_{同}|$。This means that between two neutral, there are electrostatic, the electrostatic effect is that of Newton (Newton) the role of gravity.

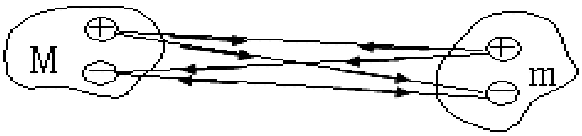

Figure 1-1-3 **Neutral between the two electrostatic**

As we all know, the material is from the protons, neutrons, composed of electronic elementary particles, protons a positive charge of a unit, the electronic unit with a negative charge, neutrons do not carry charge, and neutral substances containing a number of proton Constant equal to the number contained in the electronic. If the neutron is equivalent with a plus or minus two charge of elementary particles, can be seen from Figure 1-1-3, the two neutral between the total electrostatic potential energy should be V_m

$$V_m = 2\left(V_{异} + V_{同}\right) \tag{49}$$

If the mass of M neutral object with positive and negative charge are Q_e ,the quality of the neutral object with m positive and negative charge are between q_0 ,M and m, the total effect of the static $V_{异}$ gravitational potential energy and total $V_{同}$ the role of the electrostatic repulsion potential energy can be from (47) and (48) type are obtained, will be incorporated into the two-type (49) type, which can be between two neutral objects, the total electrostatic potential energy for the V_m

为

$$V_m = -2\left(1-\cos 2\phi_0\right)\frac{\left|Q_e q_0\right|}{4\pi\varepsilon_0 r}e^{-R/r} = -\sin^2\phi_0\frac{\left|Q_e q_0\right|}{\pi\varepsilon_0 r}e^{-R/r} \tag{50}$$

Where Q_e and q_0 can be obtained, that the neutral material with $\overline{M}$ nuclei molar (mol) quality, with N_0 that Avogadro (Avogadro • Amedeo) constant unit mass of $N_0/\overline{M}$, compared with neutral substances contained in the number of nuclei, so the quality of the M nuclei contain neutral number is $MN_0/\overline{M}$, as a result of a nuclear nuclei containing $\overline{M}$, and each also have a nuclear charge with a unit e. Therefore, the quality of the material M with a neutral overall positive and negative charge of electricity are

$$Q_e = \left(MN_0/\overline{M}\right)\overline{M}e = MN_0 e \tag{51}$$

Similarly, we can see that the quality of the material is neutral with the overall positive and negative charge of electricity are

$$q_0 = mN_0 e \tag{52}$$

Take (51), (52) into the two-type (50) where can be the role of Newton's gravity potential energy

$$V_m = -N_0^{\,2}e^2\sin^2\phi_0\frac{Mm}{\pi\varepsilon_0 r}e^{-R/r} = -\frac{GMm}{r}e^{-R/r} \tag{53}$$

which $G = N_0^{\,2}e^2\sin^2\phi_0/\pi\varepsilon_0$, $R = G\left(M+m\right)/c^2$。 Force expression is

$$\vec{F} = -\nabla V_m = -\frac{GMm\vec{r}}{r^3}\left(1-R/r\right)e^{-R/r} \tag{54}$$

From above, we can see that the role of Newton's gravity is actually between two neutral objects electrostatic performance of the remaining effects.

6、Complete theory of static magnetic field

Electrostatic effect is similar to the discussion of the theory can also be set up in the vacuum of space in the static magnetic field theory. We know that in Maxwell's electromagnetic theory, by introducing the vector potential $\vec{A}$, the static magnetic field can be written $\vec{B}$

$$\vec{B} = \nabla\times\vec{A} \tag{55}$$

$\vec{A}$ vector potential which can be through the (55)-type equation on behalf of Admission

$$\nabla\times\vec{B} = \mu\vec{J} \tag{56}$$

and

$$\vec{A} = \vec{A}\left(\vec{X}\right) = \frac{\mu}{4\pi}\int_\infty \frac{\vec{J}\left(\vec{X}'\right)}{r}dv' \tag{57}$$

When the role of charge q magnetic field source relative to the speed $\vec{u}$ sport $\vec{A}$, the moving charge q would be an electric field and $\vec{E}_m$ the equivalent role in the body $\vec{u}\times\vec{B}+2\vec{\omega}\times\vec{A}$, which $2\vec{\omega} = \nabla\times\vec{u}$ 。With type (55) easy to find the equivalent of the divergence of electric field e is $\vec{E}_m$

$$\begin{aligned}\nabla\cdot\vec{E}_m &= \nabla\cdot\left(\vec{u}\times\vec{B}+2\vec{\omega}\times\vec{A}\right)\\ &= \nabla\cdot\left[\vec{u}\times\left(\nabla\times\vec{A}\right)\right]-\left(\nabla\times\vec{A}\right)\cdot\left(\nabla\times\vec{u}\right)\\ &= -\left(u\cdot\nabla\right)\left(\nabla\cdot\vec{A}\right)-\nabla^2\left(\vec{u}\cdot\vec{A}\right)\end{aligned} \tag{58}$$

Due to

$$\left(\vec{u}\cdot\nabla\right)\left(\nabla\cdot\vec{A}\right)=\left(\vec{u}\cdot\nabla\right)\left(\frac{\partial\varphi}{c\partial t}\right)=0 \tag{59}$$

It is

$$\nabla\cdot\vec{E}_m=\nabla\cdot\left(\vec{u}\times\vec{B}+2\vec{\omega}\times\vec{A}\right)=-\nabla^2\left(\vec{u}\cdot\vec{A}\right) \tag{60}$$

Obviously, the moving charge q feel the equivalent electric field

$$\vec{E}_m=\vec{u}\times\vec{B}+2\vec{\omega}\times\vec{A}=-\nabla\left(\vec{u}\cdot\vec{A}\right)=-\nabla\varphi_m \tag{61}$$

which

$$\varphi_m=\vec{u}\cdot\vec{A} \tag{62}$$

for the equivalent electric field potential。

Know that after the equivalent electric field potential, similar to the electrostatic theory in front of the discussion, the magnetic field $\vec{B}$ can charge q on the role of movement and force of expression of potential energy are

$$V_m=q\varphi_m e^{-R/r}=q\vec{u}\cdot\vec{A}e^{-R/r} \tag{63}$$

$$\vec{F}_m=-\nabla V_m==-q\left(1-\frac{R}{r}\right)e^{-R/r}\nabla\left(\vec{u}\cdot\vec{A}\right) \tag{64}$$

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