Calculating Breit-Wigner Width of Hadrons

M. J. Germuska

Retired, 47 Clifden Road, Worminghall, HP18 9JR, UK. mgermuska@aol.com, phone: +(44) 1844 339754

Abstract

This paper shows how Breit-Wigner width of hadrons may be calculated using Vir Theory of Particles. The theory provides formulas for the relationship between mass and spin as well as for width and spin. The width of over 150 particles are calculated with such accuracy that the errors from the actual width are entirely attributable to the width measurement errors. The particles come from 16 families including the lightest family N and the heaviest family χY .

Keywords: particles, hadrons, phenomenology, width calculations, spin, Mendeleev-like tables

1. Introduction

Breit-Wigner width is one of the handful of the fundamental parameters that are provided for most particles in PDG data. The other such fundamental parameters are the mass, spin, parity and branching ratios. For the short-lived particles, usually referred to as resonances, Breit-Wigner width is related to the lifetime. A high value of the width corresponds to a small value for the lifetime.

The uncertainty principle provides a tool for characterizing the very short-lived products produced in high energy collisions in accelerators. The uncertainty principle in the form

$$\Delta E \Delta t \succ \frac{\hbar}{2}$$

suggests that for particles with extremely short lifetimes, there will be a significant uncertainty in the measured energy. The measurement of the mass energy of an unstable particle a large number of times gives a distribution of energies called a Breit-Wigner distribution. If the width of this distribution at half-maximum is labelled Γ , then the uncertainty in energy ΔE could be reasonably expressed as

$$\Delta E = \frac{\Gamma}{2} \succ \frac{\hbar}{2\tau}$$

where the particle lifetime τ is taken as the uncertainty in time $\tau = \Delta t$. Thus Breit-Wigner Γ is measured in the units of energy. In this paper we will use letter *w* instead of Γ .

Vir theory of particles [1] provides a formula for the relationship between mass and spin that has been used to construct Mendeleev-like tables of hadrons [2]. There is one or more tables for most hadron families. The tables comprise of 209 hadrons and their masses were calculated with such accuracy that the errors from the actual masses are entirely attributable to the mass measurement errors.

Using the width and spin formula the width was calculated separately for each table. Again the width was calculated with such accuracy that the errors from the actual width are entirely attributable to the width measurement errors. However, this time only 154 hadrons were used because PDG does not provide the width for hadrons with a long lifetime and even more significant is the number of hadrons for which PDG provides neither the lifetime nor the width.

2. Theoretical background

The Vir Theory of Particles is described in detail in paper [1]. Here we will outline only those features that are relevant to this paper. It has been proven mathematically using the Euler-Lagrange equations that the symmetric concave spinning top with the minimum moment of inertia, i.e. with the least resistance to spinning, has the shape of a solid of rotation generated by the profile function r(z) called Vir [3].

 $r(z) = a \left(\frac{a}{|z|}\right)^{\alpha}$ where $\alpha \succ 0$ $a \succ 0$ (2.1)

The parameter *a* gives the size, if z = a then r = a and vice versa, while the parameter α gives the shape. It has also been shown experimentally [4] that water and air vortices have the Vir shape with observed values of $\alpha = [0.6, 2.5]$.

Vir Theory of Particles assumes that particles are symmetric twin vortices in the relativistic ether, i.e. consisting of two vortices spinning about a common axis that are joined at the large ends. For the purpose of calculating the volume, mass and the moments of inertia we can treat a particle as a symmetric solid of revolution generated by the profile function r(z) in (2.1).

Since for symmetric twin vortices r(z) = r(-z) it is sufficient to consider r(z) only on the interval [0, Z] where Z is the height of a vortex half. For a profile curve r(z) the expressions for mass m and the moments of inertia around axes z and x, i.e. Iz and Ix are

$$m = 2\rho \pi \int_{0}^{Z} r(z)^{2} dz$$
 (2.2)

$$I_{z} = \rho \pi \int_{0}^{z} r(z)^{4} dz$$
 (2.3)

$$I_x = \rho \pi \int_0^Z \frac{1}{2} r(z)^4 + 2r(z)^2 z^2 dz$$
(2.4)

Since the shape of the vortices is the solid of rotation around axis z generated by the profile function r(z) the moment of inertia around axis y is the same as around axis x. For a Vir with an infinite radius r(z) at z = 0 the evaluation of the above integrals gives us

$$m = 2\rho \pi \frac{a^{2+2\alpha}}{1-2\alpha} Z^{1-2\alpha} \qquad \alpha < \frac{1}{2}$$
(2.5)

$$I_{z} = \rho \pi \frac{a^{4+4\alpha}}{1-4\alpha} Z^{1-4\alpha} \qquad \alpha < \frac{1}{4}$$
(2.6)

$$I_x = \rho \pi \frac{a^{2+2\alpha}}{3-2\alpha} Z^{3-2\alpha} \qquad \alpha < 3/2$$
(2.7)

Using the formulas above we find the following formula for mass m

$$m = b(2\sigma - 1)^{\beta} \tag{2.8}$$

where σ is the particle spin in the units of *h*, i.e. an integer or half integer, 1/2, 1, 3/2, 2, 5/2, etc.

$$\sigma = \frac{Ix}{Iz} \tag{2.9}$$

and

$$\beta = \frac{1 - 2\alpha}{2 + 2\alpha} \qquad 1/2 > \beta > 1/5 \qquad (2.10)$$

where

$$\alpha = \frac{1 - 2\beta}{2 + 2\beta} \qquad \qquad 0 < \alpha < 1/4 \qquad (2.11)$$

$$b = 2\rho\pi a^{3} \frac{1}{1 - 2\alpha} \left(\frac{1}{4} \frac{3 - 2\alpha}{1 - 4\alpha}\right)^{\beta}$$
(2.12)

When we know the parameters b and β we can find the corresponding parameters a and α

$$a = \left(\frac{3b}{2\rho\pi}\frac{\beta}{1+\beta}\right)^{\frac{1}{3}} \left(4\frac{5\beta-1}{5\beta+2}\right)^{\frac{\beta}{3}}$$
(2.13)

The relation (2.9) between Ix, Iz and σ comes about by considering a particle as a spinning top and applying to it the Euler equations of motion for rotating bodies. A spinning top that is not perfectly circular but slightly elliptical, like hurricanes, precesses in such a way that the axis of spin describes a circular but sinusoidal trajectory. Such a spinning top in air or water generates a circular wave that on completion of one revolution is in phase only if σ is an integer or half integer. Only under these conditions the wave does not destroy itself. Furthermore, the smallest possible value of σ is $\frac{1}{2}$ which corresponds to a planar disk, which we can roughly imagine as a paper disk spinning on a needle though its centre, or as the Saturn ring.

For a solid spinning top with a known moment of inertia Iz and angular velocity ω the angular momentum about axis z is simply Iz times ω . However, for a vortex there is no angular velocity, different parts of a vortex rotate at different angular velocity. Thus Vir theory of particles postulates that the angular momentum of a particle, i.e. its spin, is σ in (2.9) measured in the units of h.

To complete the outline of the theory we will need one more formula, namely the dependence of the height (depth) Z on σ

$$Z = a \left(\frac{3-2\alpha}{1-4\alpha} \frac{2\sigma-1}{4}\right)^{\frac{1}{2+2\alpha}}$$
(2.14)

Since α and σ are dimensionless quantities we have the height in the units of the parameter *a*.

9 June 2017

The mass formula (2.8) can be modified to take into account the structure of particles. Figures 1 and 2 show the structure of a baryon and a meson, the baryon has $\sigma = 2.5$ and the meson has $\sigma = 3$. The parameter σ is the spin of a particle when the entire particle rotates in the same direction. The horizontal lines in figure 1 show the heights for a baryon with $\sigma = 0.5$, 1.5 and 2.5. The line through the centre is the baryon disk with spin 0.5 but (virtually) no mass, which on its own cannot constitute a hadron. The horizontal lines in figure 2 show the heights for a meson with $\sigma = 1$, 2 and 3.



To obtain a baryon with spin 0.5 we assume that it has the shape, height and mass of $\sigma = 1.5$, but the disk rotates in the opposite direction. This is because in the absence of the disk the spin would be reduced by 0.5 and with the disk rotating in the opposite direction the spin is reduced by another 0.5. We find a similar phenomenon in hurricanes where the visible part (warm air) rises up from the ground (or sea) rotating anticlockwise, but when it ascends above the hurricane it suddenly turns and rotates clockwise. We can extend this idea, by rotating not only the disk but the adjacent parts in the opposite direction to the end parts we can obtain baryons spin 0.5 with the mass corresponding to $\sigma = 2.5$, 3.5, etc. This way for a given baryon σ we can obtain also any half integer spin *s* smaller than σ .

Similarly we can obtain a meson spin 0 starting with $\sigma = 2$ and have the ends rotating in the opposite direction to the middle section. We can do this starting from any even σ . However, when σ is odd then we can do it only if we are prepared to accept asymmetrical particles, where the top and bottom may have a different rotation pattern. For example the top rotating in the opposite direction to the bottom. More generally, if we wish to keep particles symmetrical we can obtain an even spin particle only from an even σ and odd spin particles only from an odd σ .

The horizontal lines in figures 1 and 2 separate the particles into notional "slices" with spin $\frac{1}{2}$. The slices that rotate in the opposite direction to the particle with spin *s* are called "contras" and the number of contras is denoted by the variable *c*. The relationship between σ , *s* and *c* is as follows

$$s = \sigma - c$$
 i.e. $\sigma = s + c$ (2.15)

Substituting this into the mass formula (2.8) we obtain

$$m = b [2(s+c)-1]^{\beta}$$
(2.16)

If we assume that all particles with a given (b, β) that have the same mass *m* also have the same Breit Wigner width *w* then the formula for *w* is the same as for *m* but with parameters (g, γ) instead of (b, β) .

$$w = g [2(s+c)-1]^{\gamma}$$
(2.17)

3. Particles data and error processing

Comprehensive data about hadrons are kept by the international organisation "Particles Data Group" (PDG) on a website hosted by the University of California [5]. Each particle for which credible data have been published has its own section in the "Particle Listings". Particles for which more accurate information has been compiled, usually including: "best mass estimate", "minimum mass", "maximum mass", "spin" and "parity" appear also in the "Particles Summary". The mass estimates are based on the published measurements that are included in the Particles Listings. The values of the best mass, min mass and max mass are not derived by purely statistical means, but also other considerations such as the overall consistency. For example, the values for Σ particles take into consideration the values of Λ particles that appear together in many reactions. The net result of such adjustments is that the best mass may not be the average of the min and max limits, hence the limits cannot be interpreted as the statistical standard deviation. In some extreme cases a mass limit coincides with the best mass, as for example in the case of $\Lambda(1830)$ where the best mass = max mass = 1830MeV.

In this paper, when the predicted mass falls outside the PDG limits we wish to estimate the seriousness of this departure. To do so we define the "relative prediction error" (RPE) that is the ratio of two distances from the best mass, namely the distance of the predicted value divided by the distance of the minimum or the maximum mass, as appropriate. In the example given above this can lead to an infinite RPE. In cases where the mass limits are less skewed, RPE is finite but may be equally unacceptable. Therefore when the predicted mass is outside the PDG limits we compute the standard deviation using the measurements in the PDG Listings. The computation method [6] gives the standard deviation that is usually smaller because by convention the error limits are those of the standard error, which is smaller than the standard deviation by the factor $1/\sqrt{N}$, where N is the number of measurements. We use the same method also for the particles for which there are no PDG limits, which applies to a vast majority of the particles not included in PDG Summary.

The PDG website is thoroughly updated every even year and partially updated every odd year. Most of the changes are small enough not to make a significant difference to our results. However, occasionally there are some very significant changes. For example for $\Lambda(1405)$ the best mass was 1407.0 MeV in 2004, 1406.0 in 2008, and 1405.1 in 2015, meanwhile the max mass decreased to 1406.4, which is smaller than the best mass in 2004! This paper uses the data updated in 2015, which being only a partial update may not always be most consistent. Therefore when we encounter RPE > 1 we first try the data from 2008 before resorting to computing the standard deviation.

As a rule unflavoured and strange hadrons with the mass greater or close to 3000 MeV appear only in PDG Listings without any limits, spin, parity and our computation usually results in an extremely large standard deviation, giving RPE < 1 for an unacceptably large range of values. There are seven such particles which due to the lack of reliable data will be excluded from any further consideration.

Much of what has been said above applies not only to Brite Wigner mass m but also to the width w. However, in general the relative measurement errors for the width are much greater, by an order of magnitude or more. In addition there are far more particles with no PDG width data at all, as well as the particles with insufficient data to compute the standard deviation.

The predicted width for a family of particles with given parameters b and β depends entirely on two parameters g and γ in formula (2.17). We use a purpose written MATLAB programme to obtain these parameters. Essentially, the programme minimises the sum of the squared RPEs, i.e. it obtains the best fit to the actual values.

4. Table of baryons N1

On the graphs below the best PDG width is marked by a blue circle for the positive parity and a red diamond for the negative parity. The min and max masses are marked by + signs. The headings give particle names and prediction errors, i.e. the ratios of the distances from the best mass to the prediction line and to the PDG minimum or maximum. The bottom prediction line corresponds to the row labelled "1" in the table. The higher lines correspond to the lower rows. Thus N(2600) with spin 2.5 on the top line has error -0.31, it is negative since the prediction line is below the particle.



FIGURE 3. Positive parity N1

FIGURE 4. Negative parity N1

	g				Sp	oin				γ	
0.0	0621	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	1.7	3
		zero								+	
С	0	mass								-	
0				N(1680)	N(1990)	N(2220)				+	Р
n	1			N(1675)		N(2250)				-	а
t			N(1720)	N(2000)				N(2700)		+	r
r	2		N(1700)		N(2190)		N(2600)			-	i
а										+	t
S	3	N(1650)		N(2060)						-	У
										+	
	4								unlikely	-	

TABLE 1. Mendeleev-like spin-width table of baryons N1

Width limits re-computed:Width limits 1st-computed:N(1990), N(2220)Missing from the mass table:N{939} there is no PDG width

5. Table of baryons N2



The spin-width graphs on this page show the particles from the corresponding spin-mass graphs.

FIGURE 5. Positive parity N2



	g				Spi	n				γ	
0.0)653	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	0.86	54
		zero		N(1860)						+	
С	0	mass	N(1520)		N(2190)					-	
0			N(1900)					N(2700)		+	Р
n	1	N(1535)	N(1875)				N(2600)			-	а
t		N(1880)								+	r
r	2	N(1895)	N(2120)							-	i
а		N(2100)	N(2300)							+	t
S	3									-	У
										+	
	4								unlikely	-	

TABLE 2. Mendeleev-like spin-width table of baryons N2

Width limits re-computed: Width limits 1st-computed: Missing from the mass table:

N(1860), N(1880), N(1895), N(2100), N(2120), N(2700)

6. Table of baryons $\Delta 1$



The spin-width graphs on this page shows the particles from the corresponding spin-mass graphs.

FIGURE 7. Positive parity $\Delta 1$



	g				Sp	oin)	/
0.	0388	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	0.9	76
			Δ{1232}		Δ(1950)		Δ(2420)			+	
С	0									-	
0			Δ(1600)	Δ(1905)		Δ(2300)				+	Р
n	1			Δ(1930)		Δ(2400)		Δ(2750)		-	а
t			Δ(1920)		Δ(2390)					+	r
r	2	Δ(1620)								-	i
а		Δ(1910)								+	t
S	3	Δ(1900)		Δ(2350)						-	у
										+	
	4	Δ(2150)								-	

TABLE 3. Mendeleev-like spin-width	table of baryons $\Delta 1$
------------------------------------	-----------------------------

Width limits re-computed: Width limits 1st-computed: Missing from the mass table:

 Δ {1232} Δ (1900), Δ (2300), Δ (2150), Δ (2350), Δ (2400), Δ (2750), Δ (2390)

7. Table of strange baryons $\Lambda 1a$

The spin-width graphs on this page show some particles from the spin-mass graphs $\Lambda 1$, the remaining particles are shown on the following page. $\Lambda(2585)$ is shown in black because PDG has not assign it the parity.



FIGURE 9. Positive parity $\Lambda 1a$

FIGURE 10. Negative parity $\Lambda 1a$

	g				Sp	oin)	(
0.0	00551	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	1.:	53
С	0									+ -	
0				Λ(1820)	Λ(2020)	Λ(2350)				+	Р
n	1			Λ(1830)	Λ(2100)		Λ(2585)			-	а
t			Λ(1890)	Λ(2110)						+	r
r	2									-	i
a		Λ(1810)								+	t
S	3									-	У
								-		+	
	4		Λ(2325)							-	

TABLE 4. Mendeleev-like spin-width table of strange baryons A1a

Width limits re-computed:Width limits 1st-computed: $\Lambda(2020), \Lambda(2325)$ Missing from the mass table: $\Lambda\{1116\}$ there is no PDG width

8. Table of strange baryons $\Lambda 1b$

The spin-width graph on this page show some particles from the spin-mass graph $\Lambda 1$, the remaining particles are shown on the previous page.



FIGURE 11. Positive and negative parity $\Lambda 1b$

	g				Sp	oin)	/
0.0	00152	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	1.0	67
										+	
С	0									-	
0										+	Р
n	1									-	а
t		Λ(1600)								+	r
r	2									-	i
а										+	t
S	3	Λ(1800)	A(2050)							-	У
										+	
	4									-	

TABLE 5. Mendeleev-like spin-width table of strange baryons $\Lambda 1b$

Width limits re-computed: Width limits 1st-computed: Missing from the mass table: $\Lambda(1520)$ does not fit

9. Table of strange baryons $\Lambda 2$

The spin-width graph on this page shows the particles from the corresponding spin-mass graph. Particle $\Lambda(2000)$ is shown in black because PDG has not been able to assign it the parity.



FIGURE 12. Positive and negative parity $\Lambda 2$

	g				Sp	oin				2	(
0.	.0480	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	1.	70
										+	
С	0									-	
0					Λ(2020)					+	Р
n	1									-	а
t										+	r
r	2	Λ(1405)		Λ(2000)						-	i
а										+	t
S	3									-	У
										+	
	4									-	

TABLE 6. Mendeleev-like spin-width table of strange baryons $\Lambda 2$

Width limits re-computed: Width limits 1st-computed: Missing from the mass table:

ble: $\Lambda(1710)$ does not fit, it has only one PDG measurement

10. Table of strange baryons $\Sigma 1$

The spin-width graphs on this page show the particles from the corresponding spin-mass graphs. Particles $\Sigma(2250)$ and $\Sigma(2620)$ are shown in black because PDG has not been able to assign them the parity.



	g				Sp	oin)	(
0.	0165	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	1.	22
			Σ(1385)		Σ(2030)					+	
С	0			Σ(1775)	Σ(2100)	Σ(2250)		Σ(2620)		-	
0			Σ(1770)	Σ(2070)						+	Р
n	1									-	а
t										+	r
r	2	Σ(1750)								-	i
а		Σ(2000)								+	t
S	3									-	У
										+	
	4									-	

TABLE 7. Mendeleev-like spin-width table of strange baryons $\Sigma 1$

Width limits re-computed: Width limits 1st-computed: Missing from the mass table:

 $\Sigma(1770), \Sigma(2070), \Sigma(2250), \Sigma(2100), \Sigma(2620)$ $\Sigma(2455)$ does not fit, it has only one PDG measurement

11. Table of strange baryons $\Sigma 2$

The spin-width graphs on this page show the particles from the corresponding spin-mass graphs. Particles $\Sigma(1670)$, $\Sigma(1690)$, $\Sigma(2250)$ and $\Sigma(2620)$ are shown in black because PDG has not been able to assign them the parity.



FIGURE 15. Positive parity $\Sigma 2$



	g				Sp	oin)	(
0.	.0105	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	1.	15
										+	
С	0									-	
0						Σ(2250)				+	Р
n	1			Σ(1690)				Σ(2620)		-	а
t			Σ(1670)	Σ(1915)						+	r
r	2		Σ(1670)							-	i
а		Σ(1660)								+	t
S	3									-	У
		Σ(1880)						-		+	
	4	Σ(1900)	Σ(1940)							-	

TABLE 8. Mendeleev-like spin-width table of strange baryons $\Sigma 2$

Width limits re-computed:Width limits 1st-computed:Missing from the mass table: $\Sigma(1670), \Sigma(1690), \Sigma(1880), \Sigma(1900), \Sigma(1940), \Sigma(2620)$ $\Sigma(1940)$ does not fit, it has only one PDG measurement

12. Table of strange baryons $\Sigma 3$

The spin-width graphs on this page shows the particles from the corresponding spin-mass graphs. Particles $\Sigma(1560)$, $\Sigma(2250)$ and $\Sigma(2620)$ are shown in black because PDG has not been able to assign them the parity.



FIGURE 17. Positive parity $\Sigma 3$

FIGURE 18. Negative parity $\Sigma 3$

	g				Sp	oin)	/
0.0	00102	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	2.0	00
С	0									+ -	
0			Σ(1560)			Σ(2250)		Σ(2620)		+	Р
n	1		Σ(1580)							-	а
t			Σ(1840)	Σ(2080)						+	r
r	2									-	i
а									-	+	t
S	3									-	У
										+	
	4									-	



Width limits re-computed: Width limits 1st-computed: Missing from the mass table:

Σ(1560), Σ(1580), Σ(1840), Σ(2080), Σ(2620) $Σ{1193}$ does not fit, it has a exceptionally small width 10^{-15} .

13. Table of strange baryons $\Xi 1$

The spin-width graph on this page shows the particles from the corresponding spin-mass graph. Particles $\Xi(1690)$, $\Xi(2030)$, $\Xi(2250)$ and $\Sigma(2500)$ are shown in black because PDG has not been able to assign them the parity.



FIGURE 19. Positive and negative parity $\Xi 1$

	g				Sp	oin				2	(
0.0	00387	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	1.4	42
С	0									+ -	
0				E(2030)		Ξ(2500)				+	Р
n	1		Ξ(1690)		E(2250)					-	а
t										+	r
r	2									-	i
а										+	t
S	3									-	У
										+	
	4									-	

TABLE 10. Mendeleev-like spin-width table of strange baryons $\Xi 1$

Width limits re-computed: Width limits 1st-computed:

 $\Xi(1690), \Xi(2030), \Xi(2250), \Xi(2500)$ $\Xi\{1318\}$ does not fit, it has a exceptionally small width 10⁻¹⁵.

Missing from the mass table:

14. Table of strange baryons $\Xi 2$

The spin-width graph on this page shows the particles from the corresponding spin-mass graph. Particles $\Xi(1950)$, $\Xi(2120)$ and $\Sigma(2370)$ are shown in black because PDG has not been able to assign them the parity.



FIGURE 20. Positive and negative parity ± 2

	g				Sp	oin					(
0.0	00405	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	1.	27
			E(1530)		Ξ(2120)					+	
С	0			Ξ(1950)		E(2370)				-	
0										+	Р
n	1									-	а
t										+	r
r	2									-	i
а										+	t
S	3									-	У
										+	
	4									-	

TABLE 11. Mendeleev-like spin-width table of strange baryons $\Xi 2$

Width limits re-computed:Width limits 1st-computed: $\Xi(2120), \Xi(2370)$ Missing from the mass table:

15. Table of mesons a1 and p1

The spin-width graphs on this page show the particles from the corresponding spin-mass graphs.



C	0		ρ ₃ (1690)	ρ₅(2350)		-	
0						+	Р
n	1					-	а
t						+	r
r	2	ρ1(1700)				-	i
a						+	t
s	3					-	У
ĺ						+	
	4					-	



Width limits re-computed:	
Width limits 1st-computed:	a ₂ (1320)
Missing from the mass table:	ρ_1 {775} does not fit.

16. Table of mesons a2 and p2

The spin-width graph on this page shows the particles from the corresponding spin-mass graph.



FIGURE 23. Positive and negative parity a2 and $\rho 2$

	g				Sp	oin				γ	
0.	.0750	0	1	2	3	4	5	6	7	1.4	46
С	0									+	
0		a ₀ (980)								+	Р
n	1		ρ1(1450)							-	а
t		ao(1450)								+	r
r	2				-					-	i
а										+	t
s	3									-	У
										+	
	4									-	



Width limits re-computed:Width limits 1st-computed:Missing from the mass table:

17. Table of mesons f1a and ω 1a

The spin-width graphs on this page show some particles from the spin-mass graphs $f1-\omega 1$, the remaining particles are shown on the following page.



FIGURE 24. Positive and negative parity f1a and ω 1a

	g				Sp	in)	(
0.0	00848	0	1	2	3	4	5	6	7	1.4	49
						f4(2050)		f ₆ (2510)		+	
С	0		w 1(785)		w3(1670)		ω ₅ (2250)			-	
0										+	Р
n	1									-	а
t										+	r
r	2									-	i
а										+	t
S	3			r	r	r	1	r	1	-	У
										+	
	4									-	



Width limits re-computed:Width limits 1st-computed: $\omega_3(1670), f_4(2050)$ Missing from the mass table: $f_0(500), f_0(1370)$ no measurements approved by PDG

18. Table of mesons f1b and ω 1b

The spin-width graphs on this page show some particles from the spin-mass graphs $f1-\omega 1$, the remaining particles are shown on the previous page.



FIGURE 25. Positive and negative parity f1b and ω 1b

	g				Sp	oin)	(
0.	0563	0	1	2	3	4	5	6	7	1.0	09
				f ₂ (1270)						+	
С	0									-	
0										+	Р
n	1									-	а
t				f ₂ (1950)						+	r
r	2		w 1(1650)							-	i
а										+	t
S	3									-	У
										+	
	4									-	

TABLE 15. Mendeleev-like spin-width table of mesons f1b and ω 1b

Width limits re-computed: Width limits 1st-computed: Missing from the mass table:

19. Table of mesons f2 and $\omega 2$

The spin-width graph on this page shows the particles from the corresponding spin-mass graph.



FIGURE 26. Positive and negative parity f2 and $\omega 2$

	g				Sp	oin				γ	
0.	.0271	0	1	2	3	4	5	6	7	1.0	02
				f'2(1525)						+	
С	0									-	
0		f ₀ (980)								+	Р
n	1									-	а
t		f ₀ (1500)		f ₂ (2010)		f ₄ (2300)				+	r
r	2									-	i
a										+	t
S	3			•						-	У
				f ₂ (2340)						+	
	4									-	

TABLE 16. Mendeleev-like spin-width table of mesons f2 and $\omega 2$

Width limits re-computed:Width limits 1st-computed: $f_0(1500)$ Missing from the mass table:

20. Table of strange mesons K1a

The spin-width graphs on this page show some particles from the spin-mass graphs K1, the remaining particles are shown on the following page.



	g				Sp	oin				γ	
0	0.0491	0	1	2	3	4	5	6	7	0.676	5
	0		K*(892)	K*(1430)	K*(1780)	K*(2045)	K*(2380)			+	
C o	1									+	P a
n t	2									+	r i
r a	3			V(2250)					J	+	t y
S				K(2250)						+	
	4									-	

TABLE 17. Mendeleev-like spin-width table of strange mesons K1a

Width limits re-computed:	
Width limits 1st-computed:	K ₂ (2250)
Missing from the mass table:	$K_2(1770)$, $K_3(2320)$ no measurements approved by PDG

21. Table of strange mesons K1b

The spin-width graphs on this page show some particles from the spin-mass graphs K1, the remaining particles are shown on the preceding page.



FIGURE 29. Positive and negative parity K1b

	g				Sp	oin				γ	
0	.0536	0	1	2	3	4	5	6	7	0.701	
	0									+	
C o	1		K(1400) K*(1410)							+ -	P a
n t r	2	K*(1430)		K*(1980)						+	r i t
a s	3									+	y
	4							-		+	

TABLE 18. Mendeleev-like spin-width table of strange mesons K1b

Width limits re-computed: Width limits 1st-computed: Missing from the mass table:

K₁(1400), K*₁(1410) K₀(1460) no PDG measurements with min and max limits

22. Table of strange mesons K2

The spin-width graph on this page shows the particles from the corresponding spin-mass graph.



FIGURE 30. Positive and negative parity K2

	g				Sp	oin				γ	
0	.0536	0	1	2	3	4	5	6	7	0.701	l
	0									+	
C o	1					K(2500)				+ -	P a
n t r	2		K(1650) K*(1680)							+	r i t
a s	3									+	y
	4							-		+	

TABLE 19. Mendeleev-like spin-width table of strange mesons K2

Width limits re-computed: Width limits 1st-computed: Missing from the mass table:

 $K_{1}^{*}(1680)$ K_{0}^{495} there is no PDG width

23. Table of charmed baryons $\Lambda c1$

In the spin-mass table there are three particles but only one of them $\Lambda c(2940)$ has PDG width data. There is an infinite number of solutions in terms of the parameters g and gamma that would fit one particle. Hence the graph below is left empty.



FIGURE 31. Positive and negative parity Ac1

	g				Sp	in				γ	
		0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5		
										+	
С	0									-	
0					Ac(2940)					+	Р
n	1									-	а
t										+	r
r	2									-	i
a										+	t
S	3									-	У
										+	
	4									-	



Width limits re-computed: Width limits 1st-computed: Missing from the mass table:

 Λc {2286} has PDG Life Time, but no Width Λc (2595) has no PDG Width data

24. Table of charmed baryons $\Lambda c2$

In the spin-mass table there are three particles but only one of them $\Lambda c(2880)$ has PDG width data. There is an infinite number of solutions in terms of the parameters g and gamma that would fit one particle. Hence the graph below is left empty.



FIGURE 32. Positive and negative parity $\Lambda c2$

	g				Spi	n				2	Y
		0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5		
										+	
С	0									-	
0										+	Р
n	1									-	a
t										+	r
r	2									-	i
а										+	t
S	3									-	У
				Ac(2880)						+	
	4									-	

TABLE 21. Mendeleev-like spin-width table of charmed baryons $\Lambda c2$

Width limits re-computed: Width limits 1st-computed: Missing from the mass table: $\Lambda c(2625)$ and $\Lambda c(2765)$ have no PDG Width data 25. Table of charmed baryons $\Sigma c1$



FIGURE 33. Positive and negative parity $\Sigma c1$

	g				Spi	in				2	/
0.	00608	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	1.	81
										+	
С	0									-	
0		Σc (2455)								+	Р
n	1									-	а
t										+	r
r	2	Σc(2800)								-	i
а										+	t
S	3									-	У
										+	
	4									-	

TABLE 22. Mendeleev-like spin-width table of charmed baryons $\Sigma c1$

Width limits re-computed: $\Sigma c(2455), \Sigma c(2800)$ Width limits 1st-computed:Missing from the mass table: $\Sigma c(2765)$ has no PDG Width data

26. Table of charmed baryons $\Xi c1$

In the spin-mass table there are four particles but only one of them $\Xi c(2980)$ has the PDG width data. There is an infinite number of solutions in terms of the parameters g and gamma that would fit one particle. Hence the graph below is left empty.



FIGURE 34. Positive and negative parity $\Xi c1$

	g				Spi	in)	/
		0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5		
										+	
С	0									-	
0				Ec(2980)						+	Р
n	1									-	а
t										+	r
r	2									-	i
а										+	t
S	3									-	У
										+	
	4									-	

TABLE 23. Mendeleev-like spin-width table of charmed baryons Ξ c1

Width limits re-computed:Width limits 1st-computed:Missing from the mass table: $\Xi c{2469}$ has PDG Life Time, but no Width $\Xi c(2790)$ has no PDG Width data

 $\Xi c(3123)$ has only one PDG width measurement





FIGURE 35. Positive and negative parity $\Xi c2$

	g				Sp	in)	/
0.	00048	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	0.9	36
										+	
С	0									-	
0										+	Р
n	1									-	а
t			Ec(2645)			Ec(3080)				+	r
r	2									-	i
а									-	+	t
S	3									-	У
										+	
	4									-	

TABLE 24. Mendeleev-like spin-width table of charmed baryons $\pm c2$

Width limits re-computed: Width limits 1st-computed: Missing from the mass table: $\Xi c(2815)$ has no PDG Width data

28. Table of charmed mesons D1

In the spin-mass table there are three particles but only one of them D(2400) has the PDG width data. There is an infinite number of solutions in terms of the parameters g and gamma that would fit one particle. Hence the graph below is left empty.



FIGURE 36. Positive and negative parity D1

	g				Sp	oin				γ	
		0	1	2	3	4	5	6	7		
	0									+	
	0									-	
C										+	Р
0	1									-	а
n		D (2400)								+	r ·
l r	2									-	1
1										+	v
s	3									-	y
								-		+	
	4									-	

TABLE 25. Mendeleev-like spin-width table of charmed mesons D1

Width limits re-computed: Width limits 1st-computed: Missing from the mass table:

D{1876} has PDG Life Time, but no Width D(2750) has only one author (with two measurements)

29. Table of charmed mesons D2



FIGURE 37. Positive and negative parity D2

	g				Sp	oin				γ	
0.	000992	0	1	2	3	4	5	6	7	1.95	
	0									+	
C o	1					D(2600)				+	P a
n t	2			D(2460)						+ -	r i
a s	3									+ -	y
	4							-		+	



Width limits re-computed:D(2600)Width limits 1st-computed:D(2007) has no PDG Width data

30. Table of charmed mesons Ds1

In the spin-mass table there are three particles but only one of them Ds(2700) has the PDG width data. There is an infinite number of solutions in terms of the parameters g and gamma that would fit one particle. Hence the graph below is left empty.



FIGURE 38. Positive and negative parity Ds1

	Size				Sp	oin				Shape	e
b	= 1.97	0	1	2	3	4	5	6	7	$\beta = 0.1$	99
	0									+	
_	0									-	
C										+	Р
0	1									-	а
n										+	r
t r	2		Ds(2700)							-	1 t
ı a										+	v
s	3									-	5
								-		+	
	4									-	

TABLE 27. Mendeleev-like spin-width table of charmed mesons Ds1

Width limits re-computed: Width limits 1st-computed: Missing from the mass table:

Ds{1968} has PDG Life Time, but no Width Ds(2460) has no PDG Width data

31. Table of charmed mesons Ds2

In the spin-mass table there are two particles but only one of them Ds(3040) has the PDG width data. There is an infinite number of solutions in terms of the parameters g and gamma that would fit one particle. Hence the graph below is left empty.



FIGURE 39. Positive and negative parity Ds2

	Size				Sp	oin				Shape	e
b	= 2.32	0	1	2	3	4	5	6	7	$\beta = 0.1$	69
	0									+	
С										-	Р
0	1									-	a
n t	2		Ds(3040)							+	r i
r	2									-	t
a										+	v
S	3									-	5
										+]
	4									-	

TABLE 28. Mendeleev-like spin-width table of charmed mesons Ds2

Width limits re-computed: Width limits 1st-computed: Missing from the mass table: Ds(2317) has no PDG Width data

32. Table of charmed mesons Ds3

In the spin-mass table there are two particles but only one of them Ds(2535) has the PDG width data. There is an infinite number of solutions in terms of the parameters g and gamma that would fit one particle. Hence the graph below is left empty.



FIGURE 40. Positive and negative parity Ds3

	g				Sp	oin				γ	
		0	1	2	3	4	5	6	7		
	0									+ -	
C o	1									+ -	P a
n t r	2		Ds(2535)							+ -	r i
a s	3									+	y
	4									+	

TABLE 29. Mendeleev-like spin-width table of charmed mesons Ds3

Width limits re-computed: Width limits 1st-computed: Missing from the mass table: Ds(2860) has only one PDG width measurement



33. Table of charmonium mesons X1

FIGURE 41. Positive and negative parity X1

	g				Sp	oin				γ	
().0097	0	1	2	3	4	5	6	7	1.27	
	0									+ -	
C	1		X(3900)	X(4500)	X(4250)					+ -	Р
0 n t	2									+ -	a r i
r a	3		X(4260)							+ -	t y
s	4	X(4240)								+ -	
	5		X(4430)							+ -	



Width limits re-computed: Width limits 1st-computed: Missing from the mass table: X(3872) has no PDG Width data



34. Table of charmonium mesons X2

FIGURE 42. Positive and negative parity X2

	g				Sp	oin				γ	
(0.0266	0	1	2	3	4	5	6	7	0.515	5
	0									+ -	
C	2	X(3940)								+ -	Р
n t	3									+ -	r i
r a	4		X(4360)							+	t y
s	5									+	
	6									+	
	7		X(4660)							+ -	

TABLE 31. Mendeleev-like spin-width table of charmonium mesons X2

Width limits re-computed: Width limits 1st-computed: Missing from the mass table: X(4660)

X(4160) has only one PDG Width measurement

35. Table of charmomium mesons $\chi c1$ and $\psi c1$

The particles in the family χ_c , ψ have Breit-Wigner width that is much smaller than all the previous families. The particle $\psi(1S)(3096)$ in second table has the smallest width = 0.00009 GeV. This is reflected in small values of the parameter g in all three tables.



FIGURE 43. Positive and negative parity $\chi 1$ and $\psi 1$

	g		Spi	in			(
0.0	000070	0	1	2	3	2.	50
С	0			χc2(1P)(3556)		+	Р
o n	1		$\chi_{c1}(2P)(3511)$			+	a r
t						+	i
r a	2					-+	t y
s	3					-	-
	4					+	

TABLE 32. Mendeleev-like spin-width table of charmed mesons $\chi 1$ and $\psi 1$

Width limits re-computed: Width limits 1st-computed: Missing from the mass table: $\chi_{c1}(2P)(3511), \chi_{c2}(1P)(3556)$

 $\psi(3770)$ has $\chi_{c2}(2P)(3927)$ are in table 2 on the following page

36. Table of charmonium mesons $\chi c2$ and $\psi c2$

The for the family χ_c , ψ there are three spin-mass tables. The particle $\chi_{c2}(2P)(3927)$ appears in all three of them and the particle $\psi(3770)$ appears in two of them. However, when it comes to the spin-width these particle appear only in the second table, shown below. Thus the spin-width tables help us to refine the spin-mass tables.



FIGURE 44. Positive and negative parity $\chi 2$ and $\psi 2$

	g		Sp	in			(
0.0	00089	0	1	2	3	3.	15
0	0					+	D
C	0		ψ(1S)(3096)			-	Р
0						+	a
n	1					-	r
t				χc2(2P)(3927)		+	i
r	2		ψ(3770)			-	t
а						+	У
S	3					-	
						+	
	4					-	

TABLE 33. Mendeleev-like spin-width table of charmed mesons $\chi 2$ and $\psi 2$

Width limits re-computed: Width limits 1st-computed: Missing from the mass table:

 $\psi(1S)(3096), \psi(3770), \chi_{c2}(2P)(3927)$

 $\psi(4040)$ has only one PDG Width measurement



37. Table of charmonium mesons $\chi c3$ and $\psi c3$

FIGURE 45. Positive and negative parity $\chi 3$ and $\psi 3$

g		Spin					
0.0000064		0	3.15	2	3	3.69	
	0					+	
С	1		ψ(2S)(3686)			+	Р
o n	2					+	a r
t r	3	$\chi_{c0}(2P)(3915)$				+	i t
a s	4		ψ(4161)			+	У
	5					+	
	6		ψ(4415)			+	



Width limits re-computed: $\psi(2S)(3686), \chi_{c0}(2P)(3915), \psi(4161), \psi(4415)$ Width limits 1st-computed: $\chi_{c2}(2P)(3927)$ is in table 2 on the previous page

38. Table of beuatonium mesons $\chi b1$ and Y1

In the spin-mass table there are seven particles but none of them has adequate PDG width data. Hence the graph below is left empty.



FIGURE 45. Positive and negative parity $\chi b1$ and Y1

g		Spin					(
		0	1	2	3		
						+	
	0					-	
						+	
С	1					-	Р
0						+	а
n	2					-	r
t						+	i
r	3					-	t
а						+	У
S	4					-	
						+	
	5					-	
						+	
	6					-	



Width limits re-computed:
Width limits 1st-computed:
Missing from the mass table: $\chi_{b0}(1P)(9859), \chi_{b1}(1P)(9892), \chi_{b1}(2P)(10255), \chi_{b1}(3P)(10512), \chi_{b2}(2P)(10268)$ No PDG Width data

Y(1S)(9460), Y(3S)(10355) No PDG Width measurements

39. Table of beuatonium mesons $\chi b2$ and Y2

In the spin-mass table there are four particles but only one of them has adequate PDG width data. There is an infinite number of solutions in terms of the parameters g and gamma that would fit one particle. Hence the graph below is left empty.



FIGURE 46. Positive and negative parity $\chi b2$ and Y2

g		Spin					γ	
		0	1	2	3			
						+		
	0					-		
						+		
С	1					-	Р	
0						+	а	
n	2					-	r	
t						+	i	
r	3					-	t	
а						+	У	
S	4					-		
						+		
	5		Y(4S)(10580)			-		
						+		
	6					-		



Width limits re-computed: Width limits 1st-computed: Missing from the mass table: $\chi_{b0}(2P)(10232), \chi_{b2}(1P)(9912)$ No PDG Width data Y(2S)(10023) No PDG Width measurements

40. Table of beuatonium mesons χ b3 and Y3

In the spin-mass table there are two particles but only one of them has adequate PDG width data. There is an infinite number of solutions in terms of the parameters g and gamma that would fit one particle. Hence the graph below is left empty.



FIGURE 47. Positive and negative parity χ b3 and Y3

g		Spin					
		0	1	2	3		
						+	
	0					-	
						+	
С	1					-	Р
0						+	а
n	2					-	r
t						+	i
r	3					-	t
а						+	У
S	4					-	
						+	
	5		Y(11020)			-	
						+]
	6					-	

TABLE 37. Mendeleev-like table of beautonium mesons $\chi b3$ and Y3

Width limits re-computed:Width limits 1st-computed:Missing from the mass table:Y(2S)(10860) No PDG Width approved measurements

41. Analysis of the prediction errors

In section 2 we considered the quality of PDG data and pointed out that the mass limits for a particle are not obtained purely by statistical means. Therefore in situations where RPE is outside the PDG limits (26 particles) or the limits are absent (31 particles) we compute the standard deviation using the experimental results given in PDG Listings. This seems to correct the most skewed PDG limits and judging by the table below on the whole the PDG limits may be considered as the standard deviations.

Table 35 below shows the distribution of the absolute values of the Relative Prediction Errors for the plain and strange particles. The bottom line shows the percentages for the normal Gaussian random errors distribution. These percentages are very similar for the distribution of our relative prediction errors, hence these errors are almost entirely due to the random mass measurements errors.

Table	Number of	Absolut	e Values	of RPE
Name	Particles	(0, 1]	(1, 2]	(2, ∞)
N1	13	11	2	0
N2	13	12	1	0
$\Delta 1$	16	12	4	0
Λla	4	3	0	1
Λ1b	3	3	0	0
Λ2	3	3	0	0
Σ1	10	8	2	0
Σ2	10	8	2	0
Σ3	6	3	3	0
Ξ1	4	4	0	0
Ξ2	4	3	1	0
ap1	7	6	0	1
ap2	3	3	0	0
fw1a	5	4	1	0
fw1b	3	3	0	0
fw2	6	4	2	0
K1a	6	6	0	0
K1b	4	4	0	0
K2	3	3	0	0
All Particles	130	109	19	2
Percentage	100%	83.8%	14.6%	1.6%
Gauss	100%	84.1%	13.6%	2.3%

TABLE 38. Relative Prediction Errors for plain and strange particles

Table 39 on the next page shows the error distribution for the charmed and beauty particles. The percentages there are again very similar to the Gaussian distribution, hence all prediction errors are almost entirely due to the random mass measurements errors.

Table	Number of Absolute Values of F			of RPE
Name	Particles	(0, 1]	(1, 2]	$(2,\infty)$
AC1	0			
ΛC2	0			
ΣC1	2	1	1	0
ΞC1	0			
EC2	2	2	0	0
D1	0			
D2	2	2	0	0
DS1				
DS2				
DS3				
X1	6	6	0	0
X2	3	3	0	0
χψ1	2	2	0	0
χψ2	3	2	1	0
χψ3	4	2	1	1
χΥ1	0			
χΥ2	0			
χΥ3	0			
All Particles	24	20	3	1
Percentage	100%	83.3%	12.5%	4.2%
Gauss	100%	84.1%	13.6%	2.3%

 TABLE 39. Relative Prediction Errors for charmed particles

Taking into consideration all particles involved in the spin-width tables shown in this paper the error distribution of the prediction errors is such that they are almost entirely due to the random width measurements errors.

42. The relationship between mass and width

All particles in a given spin-width table have the same values of the parameters b, β and g, γ . From the formula (2.8) we have the following relationships involving these parameters

$$m = b(2\sigma - 1)^{\beta}$$
 $w = g(2\sigma - 1)^{\gamma}$ (39.1)

This can be written as

$$\left(\frac{m}{b}\right)^{\frac{1}{p}} = \left(2\sigma - 1\right) \qquad \left(\frac{w}{g}\right)^{\frac{1}{p}} = \left(2\sigma - 1\right) \tag{39.2}$$

Thus we have

$$\left(\frac{m}{b}\right)^{\frac{1}{\beta}} = \left(\frac{w}{g}\right)^{\frac{1}{\gamma}}$$
(39.3)

Hence we can express mass m in terms of width w and vice versa, i.e. calculate one from the other

$$m = b \left(\frac{w}{g}\right)^{\frac{\beta}{\gamma}} \qquad \qquad w = g \left(\frac{m}{b}\right)^{\frac{\gamma}{\beta}} \tag{39.4}$$

Looking at the formulas (39.1) we see that the particles with the same mass have the same σ and also the particles with the same width have the same σ . Thus the particles with the same mass have the same width, although not in general, but only those within a given spin-width table.

From the formula (2.15) we see that the value of the parameter σ is determined by spin *s* and contras *c*.

$$\sigma = s + c \tag{39.5}$$

Hence the particles with the same σ in a table all lie on an increasing diagonal, as can be seen on below

g					Sp	oin				γ	
0.00621		0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	1.7	3
		zero								+	
С	0	mass								-	
0				N(1680)	N(1990)	N(2220)				+	Р
n	1			N(1675)		N(2250)				-	а
t			N(1720)	N(2000)				N(2700)		+	r
r	2		N(1700)		N(2190)		N(2600)			-	i
а										+	t
S	3	N(1650)		N(2060)						-	У
										+	
	4								unlikely	-	

"TABLE 1. Mendeleev-like spin-width table of baryons N1"

Thus, for example, in Table N1 the particles N(1650), N(1720), N(1700), N(1680) and N(1675) all have the same mass and the same width.

43. Summary and conclusions

Using the Vir formula for Breit-Wigner width 154 particles were calculated with such accuracy that the errors from the actual width are entirely attributable to the width measurement errors.

The spin-mass tables contain 209 particles that is much more than the spin-width tables. This is because PDG does not provide the width for hadrons with a long lifetime and even more significant is the number of hadrons for which PDG provides neither the lifetime nor the width.

The following 16 particle families are included: N, Δ , Λ , Σ , Ξ , a ρ , f ω , K, Λ c, Σ c, Ξ c, D, Ds, X, $\chi\psi$, χY . They include the lightest family N and the heaviest family χY .

In each table the particles with the same mass have the same width. Such particles have the same value of the σ parameter and lie on an increasing diagonal in the table.

Each table has the same mass parameters b, β and the same width parameters g, γ . Using these four parameters we obtain a simple formula for calculating mass m from the width w and vice-versa.

$$m = b \left(\frac{w}{g}\right)^{\frac{\beta}{\gamma}} \qquad \qquad w = g \left(\frac{m}{b}\right)^{\frac{\gamma}{\beta}}$$

Acknowledgments

I would like to thank my wife Jo for her loving care without which this work would not have been possible. I thank God for everything.

References

[1] Germuska M J 2016 Vir Theory of Particles *viXra:1612.0236* [High Energy Particle *Physics*]

[2] Germuska M J 2016 Mendeleev-like Tables of Hadrons *viXra:1612.0244* [*High Energy Particle Physics*]

[3] Germuska M J, Germuska R B, Germuska M A 2016 Shapes with the Minimum Moment of Inertia *viXra:1612.0233 [General Mathematics]*

[4] Germuska M J 2016 The Shape of Water and Air Vortices *viXra:1612.0211* [*Classical Physics*]

[5] Particle Data Group (PDG) 2015 http://pdg.lbl.gov/2015/listings/contents_listings.html

[6] Germuska M J 2016 Standard Deviation for PDG Mass Data viXra:1612.0240 [Mathematics - Statistics]