# Line-Surface Formulation of the Electromagnetic-Power-based Characteristic Mode Theory for Metal-Material Combined Objects — Part II

Renzun Lian

***Abstract*—In the Part I of Line-Surface formulation of the ElectroMagnetic-Power-based Characteristic Mode Theory for Metal-Material combined objects (LS-MM-EMP-CMT), the relevant fundamental principle had been established, and some very valuable complements and improvements are done in this Part II.**

**In this Part II, the traditional surface equivalent principle for a homogeneous material body whose boundary is only constructed by a closed surface is generalized to the line-surface equivalent principle of a homogeneous material body whose boundary can include some lines and open surfaces besides a closed surface; a new line-surface formulation of the input/output power operator for metal-material combined objects is given, and the new formulation is more advantageous than the formulation given in Part I; some more detailed formulations for establishing LS-MM-EMP-CMT are explicitly provided here, such as the formulations corresponding to the decompositions for currents and their domains and the formulations corresponding to variable unification.**

**In addition, a new concept *intrinsic resonance* is introduced in this paper, and then a new Characteristic Mode (CM) set, intrinsic resonant CM set, is introduced into the EMP-CMT family.**

***Index Terms*—Characteristic mode (CM), electromagnetic power, input power, interaction, intrinsic resonance, line-surface equivalent principle, metal-material combined object, output power, surface equivalent principle, the conservation law of energy, the decompositions for currents and their domains, variable unification.**

## I. Introduction

The fundamental principle of the ElectroMagnetic-Power-based Characteristic Mode Theory (EMP-CMT) for Metal-Material combined object (MM-EMP-CMT) was established in [1]-[2], and the object was constructed by metal line, metal surface, metal volume, and material body. The line and surface electric currents on metal line, metal surface, and the boundary of metal volume were utilized to express the various fields and powers related to metal part in [1]-[2], and the total field on material body and the equivalent surface currents on material boundary were utilized to express the various fields and powers related to material part in [1] and [2] respectively, so the formulations provided in [1] included line, surface, and volume variables, but only the line and surface currents appeared in [2]. Based on this, the [1] and [2] can be respectively called as the Line-Surface-Volume formulation of MM-EMP-CMT (LSV-MM-EMP-CMT) and the Line-Surface formulation of MM-EMP-CMT (LS-MM-EMP-CMT).

For LS-MM-EMP-CMT, the fundamental principles to decompose various currents and the domains on where currents exist, to select *basic variables* and *unify variables*, and to discretize input/output power operator and construct Input/Output-power-based Characteristic Mode (InpCM/OutCM) set had been carefully discussed in [2] (The definitions for the terminologies "basic variables" and "to unify variables / variable unification" can be found in [3]). However, some other valuable topics related to LS-MM-EMP-CMT were not carefully considered in [2], for example,

1) the formulations corresponding to variable unification were not explicitly provided in [2];

2) the case that the metal line is completely or partially submerged into the material body was not included in [2];

3) when the material boundary includes some lines and open surfaces besides a closed surface, the equivalent-current-based source-field relationships were not explicitly given in [2];

4) the arguments of the input/output power operator used in [2] are only line and surface currents, but there are some volume integrals in the operator. It is still a valuable topic how to obtain an input/output power operator which only includes line and surface currents and integrals but does not include any volume integral (i.e., how to establish a "real" LS-MM-EMP-CMT).

As a further supplement to the previous Part I of LS-MM-EMP-CMT, this Part II mainly focuses on completing and improving the Part I from the aspects mentioned above. At the same time, the traditional surface equivalent principle for a homogeneous material body whose boundary is a closed

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R. Z. Lian is with the School of Electronic Engineering, University of Electronic Science and Technology of China, Chengdu 611731, China. (e-mail: rzlian@vip.163.com).

surface is generalized to the line-surface equivalent principle for a homogeneous material body whose boundary can include some lines and open surfaces besides a closed surface. In addition, a new concept *intrinsic resonance* is introduced in this paper, and then a new CM set, *intrinsic resonant CM set*, is introduced into the EMP-CMT family.

This paper is organized as follows. For a general metal-material combined object illustrated in Fig. 1, the decompositions for currents and their domains and the line-surface equivalent principle are provided in Sec. II; the selection for basic variables and the source-field relationships are discussed in Sec. III; a new and "real" line-surface formulation of input/output power operator is provided in Sec. IV; the various power-based CM sets are constructed in Sec. V; in Sec. VI, the intrinsic resonance and the relevant concepts are introduced. In Sec. VII, the general formulations provided in Secs. II-V are specialized to the special forms corresponding to some typical examples, and some valuable engineering applications corresponding to these typical examples are simply discussed. Sec. VIII concludes this paper.

In what follows, the $e^{j\omega t}$ convention is used throughout, and the metal-material combined object is simply called as scatterer.

## II. To Decompose Currents and Their Domains and To Generalize Surface Equivalent Principle to Line-Surface Equivalent Principle

The scatterer focused on by this paper is constructed by the metal line part $L^{met}$, the metal surface part $S^{met}$, the metal volume part $V^{met}$, and the material volume part $V^{mat}$, and their boundaries are respectively denoted as $\partial L^{met}$, $\partial S^{met}$, $\partial V^{met}$, and $\partial V^{mat}$, and a typical example is illustrated in Fig. 1.

When an external excitation $\vec{F}^{inc}$ incidents on the scatterer, the line electric current $\vec{J}^{l}$, the surface electric current $\vec{J}^{s}_{met,surf}$, and the surface electric current $\vec{J}^{s}_{met,vol}$ will be excited on the $L^{met}$, $S^{met}$, and $\partial V^{met}$ respectively; the volume electric current $\vec{J}^{vop}$ and the volume magnetic current $\vec{M}^{vm}$ will be excited on the $V^{mat}$. The summation of $\vec{J}^{s}_{met,surf}$ and $\vec{J}^{s}_{met,vol}$ is simply denoted as $\vec{J}^{s}$, i.e., $\vec{J}^{s}=\vec{J}^{s}_{met,surf}+\vec{J}^{s}_{met,vol}$. These scattering currents $\{\vec{J}^{l},\vec{J}^{s}\}$ and $\{\vec{J}^{vop},\vec{M}^{vm}\}$ will generate scattering field $\vec{F}^{sca}$, and the summation of $\vec{F}^{inc}$ and $\vec{F}^{sca}$ is total field $\vec{F}^{tot}$, i.e., $\vec{F}^{tot}=\vec{F}^{inc}+\vec{F}^{sca}$. In fact, the $\vec{F}^{sca}$ can be divided into two parts, the $\vec{F}^{sca}_{met}$ generated by metal-based currents $\{\vec{J}^{l},\vec{J}^{s}\}$ and the $\vec{F}^{sca}_{mat}$ generated by material-based currents $\{\vec{J}^{vop},\vec{M}^{vm}\}$, and $\vec{F}^{sca}=\vec{F}^{sca}_{met}+\vec{F}^{sca}_{mat}$ based on superposition principle [4]. For the convenience of this paper, the field $\vec{F}^{tot}-\vec{F}^{sca}_{mat}$ on $\mathrm{int}V^{mat}$ is denoted as $\vec{f}^{inc}_{int}$, i.e., $\vec{f}^{inc}_{int}(\vec{r})=\vec{F}^{tot}(\vec{r})-\vec{F}^{sca}_{mat}(\vec{r})$ for any $\vec{r}\in\mathrm{int}V^{mat}$, here $F=E,H$ and correspondingly $f=e,h$, and the symbol "$\mathrm{int}V^{mat}$" represents the interior of domain $V^{mat}$ [5].

### *A. Some restrictions for $L^{met}$, $S^{met}$, and $\partial V^{met}$, from a practical point of view*

From a purely mathematical point of view, $L^{met}\subseteq \mathrm{cl}\,L^{met}$, and $S^{met}\subseteq \mathrm{cl}\,S^{met}$, and $V^{met}\subseteq \mathrm{cl}\,V^{met}$, here the symbol "cl" represents the closure of set [5]. However, from a practical point of view it is restricted in this paper that

$$\text{Restrction for } L^{met}:\quad L^{met} = \mathrm{cl}\,L^{met} \tag{1.1}$$

$$\text{Restrction for } S^{met}:\quad S^{met} = \mathrm{cl}\,S^{met} \tag{1.2}$$

$$\text{Restrction for } V^{met}:\quad V^{met} = \mathrm{cl}\,V^{met} \tag{1.3}$$

and these restrictions can be vividly understood as that there does not exist any "point-type hole" on $L^{met}$, "point-type hole and line-type hole" on $S^{met}$, and "point-type hole, line-type hole, and surface-type hole" on $V^{met}$. In addition, the restrictions (1.1) and (1.2) imply that $L^{met}=\partial L^{met}$, and $S^{met}=\partial S^{met}$ in three-dimensional Euclidean space $\mathbb{R}^3$. Based on the same consideration, it is also restricted in this paper that

$$\text{Restrction for } V^{mat}:\ \mathrm{cl}V^{mat}\setminus V^{mat}=\partial V^{mat}\cap\left(L^{met}\cup S^{met}\cup\partial V^{met}\right) \tag{1.4}$$

and the restriction (1.4) can be vividly understood as that there does not exist any air-filled "point-type hole, line-type hole, and surface-type hole" on $V^{mat}$; the "line-type hole" on $V^{mat}$ originates from the submergence of $L^{met}$ into $V^{mat}$, and the "surface-type hole" on $V^{mat}$ originates from the submergence of $S^{met}$ into $V^{mat}$. *In summary, the "holes" on $V^{mat}$, which is illustrated in Fig. 1, are metal-filled instead of being air-filled.*

In addition, it is also restricted in this paper that

$$\text{Restrction for } L^{met}:\quad L^{met} = \mathrm{cl}\left(L^{met}\setminus\left(S^{met}\cup V^{met}\right)\right) \tag{1.1'}$$

$$\text{Restrction for } S^{met}:\quad S^{met} = \mathrm{cl}\left(S^{met}\setminus V^{met}\right) \tag{1.2'}$$

The restriction (1.1') is equivalent to saying that the intersection between $L^{met}$ and $S^{met}\cup V^{met}$ can only be some discrete points, and cannot be any line; the restriction (1.2') is equivalent to saying that the intersection between $S^{met}$ and $V^{met}$ can only be some discrete points or lines, and cannot be any surface. The rationality of restrictions (1.1') and (1.2') had been carefully explained in [2], and it will not be repeated here.

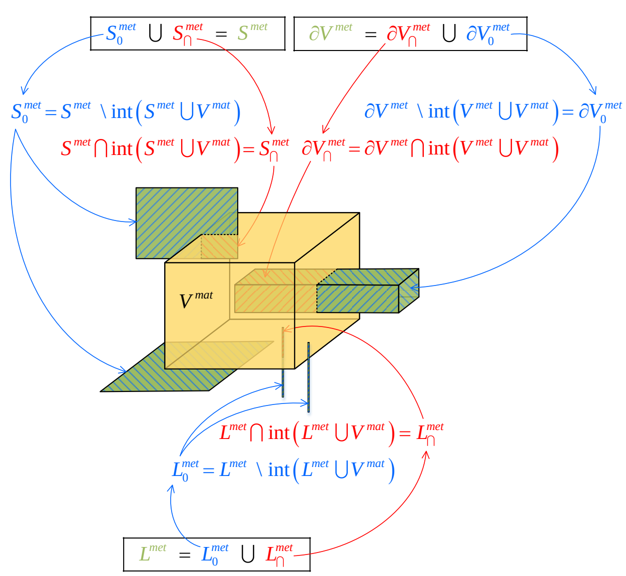



Fig. 1. The metal-material combined object considered in this paper, and the decomposition for its boundary.

In the following Secs. II-B and II-C, the decompositions for currents and their domains are done to prepare for the variable unification in Sec. III-A.

*B. The decompositions for domains $L^{met}$, $S^{met}$, and $\partial V^{met}$ and the decompositions for currents $\vec{J}^{\,l}$ and $\vec{J}^{\,s}$*

The $L^{met}$, $S^{met}$, and $\partial V^{met}$ can be respectively decomposed as follows [1]-[2]

$$L^{met} = L_0^{met} \cup L_\cap^{met} \tag{2}$$

$$S^{met} = S_0^{met} \cup S_\cap^{met} \tag{3}$$

$$\partial V^{met} = \partial V_0^{met} \cup \partial V_\cap^{met} \tag{4}$$

here the $L_0^{met}$ and $L_\cap^{met}$ are defined as

$$L_0^{met} \triangleq L^{met} \setminus \operatorname{int}\left(L^{met} \cup V^{mat}\right) \tag{5.1}$$

$$L_\cap^{met} \triangleq L^{met} \cap \operatorname{int}\left(L^{met} \cup V^{mat}\right) \tag{5.2}$$

and the $S_0^{met}$ and $S_\cap^{met}$ are defined as

$$S_0^{met} \triangleq S^{met} \setminus \operatorname{int}\left(S^{met} \cup V^{mat}\right) \tag{6.1}$$

$$S_\cap^{met} \triangleq S^{met} \cap \operatorname{int}\left(S^{met} \cup V^{mat}\right) \tag{6.2}$$

and the $\partial V_0^{met}$ and $\partial V_\cap^{met}$ are defined as

$$\partial V_0^{met} \triangleq \partial V^{met} \setminus \operatorname{int}\left(V^{met} \cup V^{mat}\right) \tag{7.1}$$

$$\partial V_\cap^{met} \triangleq \partial V^{met} \cap \operatorname{int}\left(V^{met} \cup V^{mat}\right) \tag{7.2}$$

The $L_0^{met}$ and $L_\cap^{met}$ can be vividly understood as the part which is not submerged into $V^{mat}$ and the part which is submerged into $V^{mat}$, and the $S_0^{met}$ and $S_\cap^{met}$ can be similarly explained; the $\partial V_0^{met}$ and $\partial V_\cap^{met}$ can be vividly understood as the part which contacts with air and the part which contacts with material body. In addition, it is obvious that

$$L_0^{met} \cap L_\cap^{met} = \varnothing \tag{8}$$

$$S_0^{met} \cap S_\cap^{met} = \varnothing \tag{9}$$

$$\partial V_0^{met} \cap \partial V_\cap^{met} = \varnothing \tag{10}$$

Based on (2)-(4) and (8)-(10), the scattering electric currents $\vec{J}^{\,l}$ and $\vec{J}^{\,s}$ can be correspondingly decomposed as follows

$$\vec{J}^{\,l}(\vec{r}) = \vec{J}_0^{\,l}(\vec{r}) + \vec{J}_\cap^{\,l}(\vec{r}) \quad , \quad \left(\vec{r} \in L^{met}\right) \tag{11}$$

$$\vec{J}^{\,s}(\vec{r}) = \vec{J}_0^{\,s}(\vec{r}) + \vec{J}_\cap^{\,s}(\vec{r}) \quad , \quad \left(\vec{r} \in S^{met} \cup \partial V^{met}\right) \tag{12}$$

here the $\vec{J}_0^{\,l}$ and $\vec{J}_\cap^{\,l}$ are defined as

$$\vec{J}_0^{\,l}(\vec{r}) \triangleq \begin{cases} \vec{J}^{\,l}(\vec{r}) & , \quad \left(\vec{r} \in L_0^{met}\right) \\ 0 & , \quad \left(\vec{r} \in L_\cap^{met}\right) \end{cases} \tag{13.1}$$

$$\vec{J}_\cap^{\,l}(\vec{r}) \triangleq \begin{cases} 0 & , \quad \left(\vec{r} \in L_0^{met}\right) \\ \vec{J}^{\,l}(\vec{r}) & , \quad \left(\vec{r} \in L_\cap^{met}\right) \end{cases} \tag{13.2}$$

and the $\vec{J}_0^{\,s}$ and $\vec{J}_\cap^{\,s}$ are defined as

$$\vec{J}_0^{\,s}(\vec{r}) \triangleq \begin{cases} \vec{J}^{\,s}(\vec{r}) & , \quad \left(\vec{r} \in S_0^{met} \cup \partial V_0^{met}\right) \\ 0 & , \quad \left(\vec{r} \in S_\cap^{met} \cup \partial V_\cap^{met}\right) \end{cases} \tag{14.1}$$

$$\vec{J}_\cap^{\,s}(\vec{r}) \triangleq \begin{cases} 0 & , \quad \left(\vec{r} \in S_0^{met} \cup \partial V_0^{met}\right) \\ \vec{J}^{\,s}(\vec{r}) & , \quad \left(\vec{r} \in S_\cap^{met} \cup \partial V_\cap^{met}\right) \end{cases} \tag{14.2}$$

*C. The decomposition for $\partial V^{mat}$ and the line-surface equivalent principle for a homogeneous material body whose boundary includes some lines and open surfaces besides a closed surface*

As pointed out in (1), there does not exist any air-filled "point-type hole, line-type hole, and surface-type hole" on $V^{mat}$, so the $\partial V^{mat}$ can be decomposed into the following four parts

| | | | |
|---|---|---|---|
| Boundary Point Part | $: \partial V_{point}^{mat}$ | $= \varnothing$ | (15.1) |
| Boundary Line Part | $: \partial V_{line}^{mat}$ | $= L_\cap^{met}$ | (15.2) |
| Boundary Open Surface Part | $: \partial V_{open\ surf}^{mat}$ | $= S_\cap^{met}$ | (15.3) |
| Boundary Closed Surface Part | $: \partial V_{closed\ surf}^{mat}$ | $= \partial V^{mat} \setminus \left(L_\cap^{met} \cup S_\cap^{met}\right)$ | (15.4) |

It is obvious that the above four parts are pairwise disjoint, and that

(a) the boundary point part (i.e., the metal-filled "point-type hole" on $V^{mat}$) does not exist on $V^{mat}$, based on the restrictions in (1);

(b) the boundary line part (i.e., the metal-filled "line-type hole" on $V^{mat}$) originates from the submergence of metal line into material body, and it is constituted by some lines only, and it does not include any surface and discrete point;

(c) the boundary open surface part (i.e., the metal-filled "surface-type hole" on $V^{mat}$) originates from the submergence of metal surface into material body, and it is constituted by some open surfaces only, and it does not include any line, closed surface, and discrete point;

(d) the boundary closed surface part originates from the contact between material body and air, the contact between material body and metal line (the metal line is not submerged into material body), the contact between material body and metal surface (the metal surface is not submerged into material body), the contact between material body and metal body. The boundary closed surface part does not include any line, open surface, and discrete point. In fact, the boundary closed surface part $\partial V_{closed\ surf}^{mat}$ can be further decomposed as follows

$$\partial V_{closed\ surf}^{mat} = \partial V_0^{mat} \cup \partial V_\cap^{met} \tag{16}$$

here the $\partial V_\cap^{met}$ is defined as (7.2), and the $\partial V_0^{mat}$ is defined as follows

$$\begin{aligned} \partial V_0^{mat} &\triangleq \partial V_{closed\ surf}^{mat} \setminus \partial V_\cap^{met} \\ &= \left(\partial V^{mat} \setminus \left(L_\cap^{met} \cup S_\cap^{met}\right)\right) \setminus \partial V_\cap^{met} \\ &= \partial V^{mat} \setminus \left(L_\cap^{met} \cup S_\cap^{met} \cup \partial V_\cap^{met}\right) \end{aligned} \tag{17}$$

If the union of $\partial V^{mat}_{open\ surf}$ and $\partial V^{mat}_{closed\ surf}$ is denoted as $\partial V^{mat}_{surf}$ (i.e., the whole material boundary surface part is $\partial V^{mat}_{surf} = \partial V^{mat}_{open\ surf} \cup \partial V^{mat}_{closed\ surf}$ ), the whole material boundary $\partial V^{mat}$ can be detailedly decomposed as follows

$$\partial V^{mat} = \overbrace{\varnothing}^{\partial V^{mat}_{point}} \cup \overbrace{L^{met}_{\cap}}^{\partial V^{mat}_{line}} \cup \overbrace{\underbrace{S^{met}_{\cap}}_{\partial V^{mat}_{open\ surf}} \cup \underbrace{\partial V^{met}_{\cap} \cup \partial V^{mat}_{0}}_{\partial V^{mat}_{closed\ surf}}}^{\partial V^{mat}_{surf}} \tag{18}$$

**1) the equivalent surface currents on $\partial V^{mat}_{closed\ surf}$ (i.e., on $\partial V^{mat}_0 \cup \partial V^{met}_{\cap}$ )**

The equivalent surface currents $\{\vec{J}^{SE}_{closed\ surf}, \vec{M}^{SE}_{closed\ surf}\}$ on boundary closed surface part $\partial V^{mat}_{closed\ surf}$ are as follows

$$\vec{J}^{SE}_{closed\ surf}(\vec{r}) = \vec{J}^{SE}_0(\vec{r}) + \vec{J}^{SE}_{\partial V^{met}_{\cap}}(\vec{r}) \quad , \quad \left(\vec{r} \in \partial V^{mat}_{closed\ surf}\right) \tag{19.1}$$

$$\vec{M}^{SE}_{closed\ surf}(\vec{r}) = \vec{M}^{SE}_0(\vec{r}) + \vec{M}^{SE}_{\partial V^{met}_{\cap}}(\vec{r}) \quad , \quad \left(\vec{r} \in \partial V^{mat}_{closed\ surf}\right) \tag{19.2}$$

in which the $\{\vec{J}^{SE}_0, \vec{M}^{SE}_0\}$ are defined as [3], [6]-[8]

$$\vec{J}^{SE}_0(\vec{r}) \triangleq \left[\hat{n}_{\rightarrow mat}(\vec{r}) \times \vec{H}^{tot}(\vec{r}\,')\right]_{\vec{r}\,' \rightarrow \vec{r}} \quad , \quad \left(\vec{r} \in \partial V^{mat}_0\right) \tag{20.1}$$

$$\vec{M}^{SE}_0(\vec{r}) \triangleq \left[\vec{E}^{tot}(\vec{r}\,') \times \hat{n}_{\rightarrow mat}(\vec{r})\right]_{\vec{r}\,' \rightarrow \vec{r}} \quad , \quad \left(\vec{r} \in \partial V^{mat}_0\right) \tag{20.2}$$

and the $\{\vec{J}^{SE}_{\partial V^{met}_{\cap}}, \vec{M}^{SE}_{\partial V^{met}_{\cap}}\}$ are defined as [2]

$$\vec{J}^{SE}_{\partial V^{met}_{\cap}}(\vec{r}) \triangleq \left[\hat{n}_{\rightarrow mat}(\vec{r}) \times \vec{H}^{tot}(\vec{r}\,')\right]_{\vec{r}\,' \rightarrow \vec{r}} \quad , \quad \left(\vec{r} \in \partial V^{met}_{\cap}\right) \tag{21.1}$$

$$\vec{M}^{SE}_{\partial V^{met}_{\cap}}(\vec{r}) \triangleq \left[\vec{E}^{tot}(\vec{r}\,') \times \hat{n}_{\rightarrow mat}(\vec{r})\right]_{\vec{r}\,' \rightarrow \vec{r}} \quad , \quad \left(\vec{r} \in \partial V^{met}_{\cap}\right) \tag{21.2}$$

here $\vec{r}\,' \in \text{int}V^{mat}$ , and $\vec{r}\,'$ approaches to $\vec{r}$ as illustrated in the subscripts in (20)-(21); $\hat{n}_{\rightarrow mat}$ is the direction vector pointing to $\text{int}V^{mat}$ . It should be emphasized that the equivalent surface currents defined in [6] equal to the $\{-\vec{J}^{SE}_0, -\vec{M}^{SE}_0\}$ , because the direction vector used in [6] is $-\hat{n}_{\rightarrow mat}$ instead of $\hat{n}_{\rightarrow mat}$ .

**2) the equivalent surface currents on $\partial V^{mat}_{open\ surf}$ (i.e., on $S^{met}_{\cap}$ )**

The equivalent surface currents on boundary open surface part $S^{met}_{\cap}$ can be defined as follows [2]

$$\vec{J}^{SE}_{\pm}(\vec{r}) \triangleq \left[\hat{n}_{\pm}(\vec{r}) \times \vec{H}^{tot}(\vec{r}_{\pm})\right]_{\vec{r}_{\pm} \rightarrow \vec{r}} \quad , \quad \left(\vec{r} \in S^{met}_{\cap}\right) \tag{22.1}$$

$$\vec{M}^{SE}_{\pm}(\vec{r}) \triangleq \left[\vec{E}^{tot}(\vec{r}_{\pm}) \times \hat{n}_{\pm}(\vec{r})\right]_{\vec{r}_{\pm} \rightarrow \vec{r}} \quad , \quad \left(\vec{r} \in S^{met}_{\cap}\right) \tag{22.2}$$

here $\vec{r}_+, \vec{r}_- \in \text{int}V^{mat}$ , and $\vec{r}_+$ and $\vec{r}_-$ respectively approach to $\vec{r}$ from the plus and minus sides of $S^{met}_{\cap}$ [2]. The $\{\vec{J}^{SE}_+, \vec{M}^{SE}_+\}$ and $\{\vec{J}^{SE}_-, \vec{M}^{SE}_-\}$ can be more detailedly illustrated as the Fig. 2.

Because of superposition principle [4], the fields generated by both $\{\vec{J}^{SE}_+, \vec{M}^{SE}_+\}$ and $\{\vec{J}^{SE}_-, \vec{M}^{SE}_-\}$ are identical to the fields generated by $\{\vec{J}^{SE}_+ + \vec{J}^{SE}_-, \vec{M}^{SE}_+ + \vec{M}^{SE}_-\}$ , and then the $\{\vec{J}^{SE}_+ + \vec{J}^{SE}_-, \vec{M}^{SE}_+ + \vec{M}^{SE}_-\}$ is treated as a whole in this paper. In addition, considering of that both the domain of $\{\vec{J}^{SE}_+, \vec{M}^{SE}_+\}$ and the domain of $\{\vec{J}^{SE}_-, \vec{M}^{SE}_-\}$ are $S^{met}_{\cap}$ and that $\hat{n}_-(\vec{r}) = -\hat{n}_+(\vec{r})$ for any $\vec{r} \in S^{met}_{\cap}$ , the surface equivalent currents on the boundary open surface part $S^{met}_{\cap}$ can be defined as follows

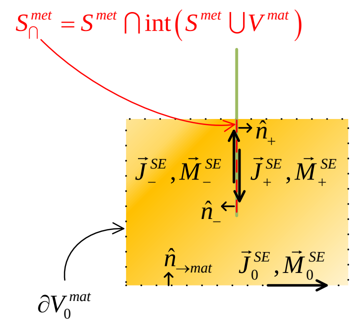



Fig. 2. The sectional view of the part $S^{met}_{\cap}$ , and the equivalent surface currents $\{\vec{J}^{SE}_+, \vec{M}^{SE}_+\}$ and $\{\vec{J}^{SE}_-, \vec{M}^{SE}_-\}$ .

$$\begin{aligned}\vec{J}^{SE}_{open\ surf}(\vec{r}) &\triangleq \vec{J}^{SE}_+(\vec{r}) + \vec{J}^{SE}_-(\vec{r}) \\ &= \hat{n}_+(\vec{r}) \times \left[\vec{H}^{tot}(\vec{r}_+) - \vec{H}^{tot}(\vec{r}_-)\right]_{\vec{r}_+, \vec{r}_- \rightarrow \vec{r}} \quad , \quad \left(\vec{r} \in S^{met}_{\cap}\right)\end{aligned} \tag{23.1}$$

$$\begin{aligned}\vec{M}^{SE}_{open\ surf}(\vec{r}) &\triangleq \vec{M}^{SE}_+(\vec{r}) + \vec{M}^{SE}_-(\vec{r}) \\ &= \left[\vec{E}^{tot}(\vec{r}_+) - \vec{E}^{tot}(\vec{r}_-)\right]_{\vec{r}_+, \vec{r}_- \rightarrow \vec{r}} \times \hat{n}_+(\vec{r}) \quad , \quad \left(\vec{r} \in S^{met}_{\cap}\right)\end{aligned} \tag{23.2}$$

**3) the equivalent line currents on $\partial V^{mat}_{line}$ (i.e., on $L^{met}_{\cap}$ )**

To efficiently introduce the equivalent line currents $\{\vec{J}^{LE}, \vec{M}^{LE}\}$ on the boundary line part $L^{met}_{\cap}$ , we firstly consider the example as illustrated in Fig. 3 (a) (i.e., a metal cylinder $V^{met}_{cylinder}$ is completely submerged into the material body), and then the $L^{met}_{\cap}$ illustrated in Fig. 3 (b) is viewed as the limitation of $V^{met}_{cylinder}$ when the radius of $V^{met}_{cylinder}$ approaches to zero.

The boundary of $V^{met}_{cylinder}$ is denoted as $\partial V^{met}_{cylinder}$ , and the surface scattering electric current on $\partial V^{met}_{cylinder}$ is denoted as $\vec{J}^{s}_{cylinder}$ . Obviously, the $\partial V^{met}_{cylinder}$ is a part of material boundary, and the material-based equivalent surface currents on $\partial V^{met}_{cylinder}$ are denoted as $\{\vec{J}^{SE}_{\partial V^{met}_{cylinder}}, \vec{M}^{SE}_{\partial V^{met}_{cylinder}}\}$ . If the radius of metal cylinder is denoted as $R^{met}_{cylinder}$ , the following limitations exist

$$\lim_{R^{met}_{cylinder} \rightarrow 0} \partial V^{met}_{cylinder} = L^{met}_{\cap} \tag{24}$$

$$\lim_{R^{met}_{cylinder} \rightarrow 0} \vec{J}^{s}_{cylinder} = \vec{J}^{l}_{\cap} \tag{25}$$

$$\lim_{R^{met}_{cylinder} \rightarrow 0} \vec{J}^{SE}_{\partial V^{met}_{cylinder}} = \vec{J}^{LE} \tag{26.1}$$

$$\lim_{R^{met}_{cylinder} \rightarrow 0} \vec{M}^{SE}_{\partial V^{met}_{cylinder}} = \vec{M}^{LE} \tag{26.2}$$

and then the equivalent line currents $\{\vec{J}^{LE}, \vec{M}^{LE}\}$ on the boundary line part $L^{met}_{\cap}$ can be defined as follows

$$\vec{J}^{LE}(\vec{r}) \triangleq \lim_{\vec{r}\,' \rightarrow \vec{r}} \oint_{C(\vec{r}\,')} \vec{H}^{tot}(\vec{r}\,') \cdot d\vec{l}\,' \quad , \quad \left(\vec{r} \in L^{met}_{\cap}\right) \tag{27.1}$$

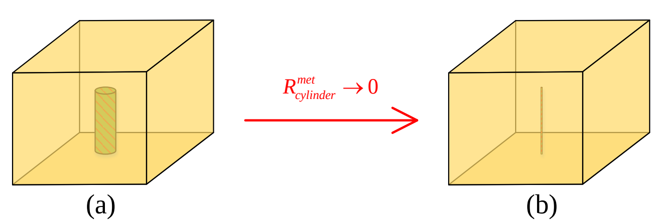



Fig. 3. (a) A metal cylinder is completely submerged into material body; (b) a metal line is completely submerged into material body.

$$\vec{M}^{LE}(\vec{r}) \triangleq -\lim_{\vec{r}'\to\vec{r}} \oint_{C(\vec{r}')} \vec{E}^{tot}(\vec{r}')\cdot d\vec{l}' \quad , \quad \left(\vec{r}\in L_{\cap}^{met}\right) \tag{27.2}$$

here the integral path $C(\vec{r}')$ is a circle constructed by the points $\vec{r}'$ which are in the set $\mathrm{int}V^{mat}$ and approach to the point $\vec{r}$ .

**4) Summary**

In summary, the whole material boundary $\partial V^{mat}$ can be decomposed into four parts as (15) or more detailedly decomposed into five parts as (18), and then the equivalent currents on $\partial V^{mat}$ can be correspondingly defined as (20), (21), (23), and (27). For simplifying the symbolic system of the following parts of this paper, the summation of $\vec{C}_{\partial V_{\cap}^{met}}^{SE}$ and $\vec{C}_{open\ surf}^{SE}$ is denoted as $\vec{C}_{\cap}^{SE}$ (because $\vec{C}_{\partial V_{\cap}^{met}}^{SE}$ and $\vec{C}_{open\ surf}^{SE}$ exist on the intersection between $\partial V^{mat}$ and $\partial V_{\cap}^{met}\cup S_{\cap}^{met}$ ), and the summation of $\vec{C}_{closed\ surf}^{SE}$ and $\vec{C}_{open\ surf}^{SE}$ is denoted as $\vec{C}^{SE}$ (because $\vec{C}_{closed\ surf}^{SE}$ and $\vec{C}_{open\ surf}^{SE}$ constitute the whole of equivalent surface currents), i.e.,

$$\text{Equivalent Currents on } \partial V^{mat} \ : \ \left\{ \vec{C}^{LE} \ , \overbrace{\overbrace{\vec{C}_0^{SE} \ , \vec{C}_{\partial V_{\cap}^{met}}^{SE}}^{\vec{C}_{closed\ surf}^{SE}} , \vec{C}_{open\ surf}^{SE}}^{\vec{C}^{SE}} \right\} \tag{28}$$

here $C = J, M$ .

In addition, it is obvious that the traditional surface equivalent principle [6] for the material body whose boundary is a closed surface can be viewed as the special case of the line-surface equivalent principle provided in this paper (when $L_{\cap}^{met}, S_{\cap}^{met} = \varnothing$ , i.e., $\partial V^{mat} = \partial V_{closed\ surf}^{mat}$ ). The source-field relationships corresponding to traditional surface equivalent principle can be found in [6] and the appendixes of [3] and [7]; the source-field relationships corresponding to line-surface equivalent principle are explicitly given in the following Sec. III-B.

## III. Basic Variables and Source-Field Relationships

As illustrated in [1]-[3] and [7]-[8], the selection for basic variables is an indispensable preprocessing step for constructing various power-based CM sets, and it is done in the following Sec. III-A, and then the basic-variables-based source-field relationships of a metal-material combined object are provided in the following Sec. III-B.

### A. Basic variables

Based on the above discussions, all the currents (except the volume scattering currents $\left\{\vec{J}^{vop}, \vec{M}^{vm}\right\}$ on $V^{mat}$ ) of a metal-material combined object are as follows

$$\text{Electric Currents} \ : \ \left\{ \overbrace{\overbrace{\vec{J}_0^{l} \ , \vec{J}_{\cap}^{l}}^{\vec{J}^{l}} , \overbrace{\vec{J}_0^{s} \ , \vec{J}_{\cap}^{s}}^{\vec{J}^{s}}}^{\text{scattering } \vec{J} \text{ on metal boundary}} \ , \overbrace{\vec{J}^{LE} , \overbrace{\overbrace{\vec{J}_0^{SE} , \vec{J}_{\partial V_{\cap}^{met}}^{SE}}^{\vec{J}_{closed\ surf}^{SE}} , \overbrace{\vec{J}_{+}^{SE} , \vec{J}_{-}^{SE}}^{\vec{J}_{open\ surf}^{SE}}}^{\vec{J}^{SE}}}^{\text{equivalent } \vec{J} \text{ on material boundary}} \right\} \tag{29.1}$$

$$\text{Magnetic Currents} \ : \ \left\{ \overbrace{\vec{M}^{LE} \ , \overbrace{\overbrace{\vec{M}_0^{SE} \ , \vec{M}_{\partial V_{\cap}^{met}}^{SE}}^{\vec{M}_{closed\ surf}^{SE}} , \overbrace{\vec{M}_{+}^{SE} \ , \vec{M}_{-}^{SE}}^{\vec{M}_{open\ surf}^{SE}}}^{\vec{M}^{SE}}}^{\text{equivalent } \vec{M} \text{ on material boundary}} \right\} \tag{29.2}$$

Due to the tangential boundary conditions of $\vec{H}^{tot}$ and $\vec{E}^{tot}$ on $S_{\cap}^{met}\cup\partial V_{\cap}^{met}$ , it has been pointed out in [2] that

$$\vec{J}_{\cap}^{s}(\vec{r}) = \vec{J}_{\cap}^{SE}(\vec{r}) = \begin{cases} \vec{J}_{open\ surf}^{SE}(\vec{r}) & , \ \left(\vec{r}\in S_{\cap}^{met}\right) \\ \vec{J}_{\partial V_{\cap}^{met}}^{SE}(\vec{r}) & , \ \left(\vec{r}\in \partial V_{\cap}^{met}\right) \end{cases} \tag{30.1}$$

$$0 = \vec{M}_{\cap}^{SE}(\vec{r}) = \begin{cases} \vec{M}_{open\ surf}^{SE}(\vec{r}) & , \ \left(\vec{r}\in S_{\cap}^{met}\right) \\ \vec{M}_{\partial V_{\cap}^{met}}^{SE}(\vec{r}) & , \ \left(\vec{r}\in \partial V_{\cap}^{met}\right) \end{cases} \tag{30.2}$$

In fact, it can be further proven that $\vec{M}_{+}^{SE}(\vec{r}) = 0 = \vec{M}_{-}^{SE}(\vec{r})$ for any $\vec{r}\in S_{\cap}^{met}$ , if the surface $S_{\cap}^{met}$ is viewed as the limitation of a thick metal slab.

Due to the same reasons to derive the second lines in (30.1) and (30.2), the following relations for the currents defined in Sec. II-C 3) can be derived

$$\vec{J}_{cylinder}^{SE}(\vec{r}) = \vec{J}_{cylinder}^{s}(\vec{r}) \ , \quad \left(\vec{r}\in\partial V_{cylinder}^{met}\right) \tag{31.1}$$

$$\vec{M}_{cylinder}^{SE}(\vec{r}) = 0 \ , \quad \left(\vec{r}\in\partial V_{cylinder}^{met}\right) \tag{31.2}$$

and then

$$\vec{J}^{LE}(\vec{r}) = \vec{J}_{\cap}^{l}(\vec{r}) \ , \quad \left(\vec{r}\in L_{\cap}^{met}\right) \tag{32.1}$$

$$\vec{M}^{LE}(\vec{r}) = 0 \ , \quad \left(\vec{r}\in L_{\cap}^{met}\right) \tag{32.2}$$

because of (25)-(26).

Based on the extinction theorem, the $\left\{\vec{J}^{LE}, \vec{J}^{SE}\right\}$ and $\left\{\vec{M}^{LE}, \vec{M}^{SE}\right\}$ can be related to each other [3], [7], so the basic variables of the metal-material combined object in Fig. 1 can be selected as follows

$$\text{Basic Variables} \ : \ \left\{ \vec{J}_0^{l} \ , \vec{J}_{\cap}^{l} \ , \vec{J}_0^{s} \ , \vec{J}_{\cap}^{s} \ , \vec{J}_0^{SE} \right\} \tag{33.1}$$

or equivalently selected as follows

$$\text{Basic Variables} \ : \ \left\{ \vec{J}_0^{l} \ , \vec{J}_{\cap}^{l} \ , \vec{J}_0^{s} \ , \vec{J}_{\cap}^{s} \ , \vec{M}_0^{SE} \right\} \tag{33.2}$$

because of the (30) and (32). It should be clearly pointed out that the above selection for the basic variables on $S_{\cap}^{met}$ is equivalent to the selection in [2], though they are different in form.

### B. Source-field relationships

The $\vec{f}_{int}^{inc}$ on $\mathrm{int}V^{mat}$ can be expressed in terms of the function of $\left\{\vec{J}^{LE}, \vec{J}_{\cap}^{SE}, \vec{J}_0^{SE}, \vec{M}_0^{SE}\right\}$ and the function of $\left\{\vec{J}_{\cap}^{l}, \vec{J}_{\cap}^{s}, \vec{J}_0^{SE}, \vec{M}_0^{SE}\right\}$ as follows [3], [7]-[8]

$$\begin{aligned}\vec{f}_{int}^{inc}(\vec{r}) &= f_{int}^{inc}(\vec{J}^{LE},0) + f_{int}^{inc}(\vec{J}_{\cap}^{SE},0) + f_{int}^{inc}(\vec{J}_0^{SE},\vec{M}_0^{SE}) \\ &= f_{int}^{inc}(\vec{J}_{\cap}^{l},0) + f_{int}^{inc}(\vec{J}_{\cap}^{s},0) + f_{int}^{inc}(\vec{J}_0^{SE},\vec{M}_0^{SE})\end{aligned} \tag{34}$$

for any $\vec{r} \in \text{int}V^{mat}$, here the second equality is due to (30.1) and (32.1), and

$$e_{int}^{inc}(\vec{X}_1,\vec{X}_2) = -j\omega\mu_0\mathcal{L}_0(\vec{X}_1) - \mathcal{K}_0(\vec{X}_2) \tag{35.1}$$

$$h_{int}^{inc}(\vec{X}_1,\vec{X}_2) = -j\omega\varepsilon_0\mathcal{L}_0(\vec{X}_2) + \mathcal{K}_0(\vec{X}_1) \tag{35.2}$$

The $\vec{F}^{tot}$ on $\text{int}V^{mat}$ can be expressed in terms of the function of $\{\vec{J}^{LE},\vec{J}_{\cap}^{SE},\vec{J}_0^{SE},\vec{M}_0^{SE}\}$ and the function of $\{\vec{J}_{\cap}^{l},\vec{J}_{\cap}^{s},\vec{J}_0^{SE},\vec{M}_0^{SE}\}$ as follows [3], [6]-[8]

$$\begin{aligned}\vec{F}^{tot}(\vec{r}) &= \mathcal{F}_{int}^{tot}(\vec{J}^{LE},0) + \mathcal{F}_{int}^{tot}(\vec{J}_{\cap}^{SE},0) + \mathcal{F}_{int}^{tot}(\vec{J}_0^{SE},\vec{M}_0^{SE}) \\ &= \mathcal{F}_{int}^{tot}(\vec{J}_{\cap}^{l},0) + \mathcal{F}_{int}^{tot}(\vec{J}_{\cap}^{s},0) + \mathcal{F}_{int}^{tot}(\vec{J}_0^{SE},\vec{M}_0^{SE})\end{aligned} \tag{36}$$

for any $\vec{r} \in \text{int}V^{mat}$, and

$$\mathcal{E}_{int}^{tot}(\vec{X}_1,\vec{X}_2) = -j\omega\mu\mathcal{L}_m(\vec{X}_1) - \mathcal{K}_m(\vec{X}_2) \tag{37.1}$$

$$\mathcal{H}_{int}^{tot}(\vec{X}_1,\vec{X}_2) = -j\omega\varepsilon_c\mathcal{L}_m(\vec{X}_2) + \mathcal{K}_m(\vec{X}_1) \tag{37.2}$$

The $\vec{F}_{mat}^{sca}$ on $\text{int}V^{mat}$ can be expressed in terms of the function of $\{\vec{J}^{LE},\vec{J}_{\cap}^{SE},\vec{J}_0^{SE},\vec{M}_0^{SE}\}$ and the function of $\{\vec{J}_{\cap}^{l},\vec{J}_{\cap}^{s},\vec{J}_0^{SE},\vec{M}_0^{SE}\}$ as follows [3], [7]-[8]

$$\begin{aligned}\vec{F}_{mat}^{sca}(\vec{r}) = {} & \vec{F}^{tot}(\vec{r}) - \vec{f}_{int}^{inc}(\vec{r}) \\ = {} & \mathcal{F}_{int}^{tot}(\vec{J}^{LE},0) + \mathcal{F}_{int}^{tot}(\vec{J}_{\cap}^{SE},0) + \mathcal{F}_{int}^{tot}(\vec{J}_0^{SE},\vec{M}_0^{SE}) \\ & - f_{int}^{inc}(\vec{J}^{LE},0) - f_{int}^{inc}(\vec{J}_{\cap}^{SE},0) - f_{int}^{inc}(\vec{J}_0^{SE},\vec{M}_0^{SE}) \\ = {} & \mathcal{F}_{int}^{tot}(\vec{J}_{\cap}^{l},0) + \mathcal{F}_{int}^{tot}(\vec{J}_{\cap}^{s},0) + \mathcal{F}_{int}^{tot}(\vec{J}_0^{SE},\vec{M}_0^{SE}) \\ & - f_{int}^{inc}(\vec{J}_{\cap}^{l},0) - f_{int}^{inc}(\vec{J}_{\cap}^{s},0) - f_{int}^{inc}(\vec{J}_0^{SE},\vec{M}_0^{SE})\end{aligned} \tag{38}$$

for any $\vec{r} \in \text{int}V^{mat}$. The $\vec{F}_{mat}^{sca}$ on $\text{ext}V^{mat}$ can be expressed as the function of $\{\vec{J}^{LE},\vec{J}_{\cap}^{SE},\vec{J}_0^{SE},\vec{M}_0^{SE}\}$ and the function of $\{\vec{J}_{\cap}^{l},\vec{J}_{\cap}^{s},\vec{J}_0^{SE},\vec{M}_0^{SE}\}$ as follows [3], [6]-[8]

$$\begin{aligned}\vec{F}_{mat}^{sca}(\vec{r}) &= \mathcal{F}_{ext}^{sca}(\vec{J}^{LE},0) + \mathcal{F}_{ext}^{sca}(\vec{J}_{\cap}^{SE},0) + \mathcal{F}_{ext}^{sca}(\vec{J}_0^{SE},\vec{M}_0^{SE}) \\ &= \mathcal{F}_{ext}^{sca}(\vec{J}_{\cap}^{l},0) + \mathcal{F}_{ext}^{sca}(\vec{J}_{\cap}^{s},0) + \mathcal{F}_{ext}^{sca}(\vec{J}_0^{SE},\vec{M}_0^{SE})\end{aligned} \tag{39}$$

for any $\vec{r} \in \text{ext}V^{mat}$, and

$$\mathcal{E}_{ext}^{sca}(\vec{X}_1,\vec{X}_2) = j\omega\mu_0\mathcal{L}_0(\vec{X}_1) + \mathcal{K}_0(\vec{X}_2) \tag{40.1}$$

$$\mathcal{H}_{ext}^{sca}(\vec{X}_1,\vec{X}_2) = j\omega\varepsilon_0\mathcal{L}_0(\vec{X}_2) - \mathcal{K}_0(\vec{X}_1) \tag{40.2}$$

here the symbol "$\text{ext}V^{mat}$" represents the exterior of domain $V^{mat}$ [3].

The operators $\mathcal{L}_0$, $\mathcal{K}_0$, $\mathcal{L}_m$, and $\mathcal{K}_m$ in (35), (37), and (40) are as follows [3], [6]-[9]

$$\mathcal{L}_{0/m}(\vec{X}) = \left(1+\frac{1}{k_{0/m}^2}\nabla\nabla\cdot\right)\int_\Omega G_{0/m}(\vec{r},\vec{r}')\vec{X}(\vec{r}')d\Omega' \tag{41.1}$$

$$\mathcal{K}_{0/m}(\vec{X}) = \int_\Omega \left[\nabla G_{0/m}(\vec{r},\vec{r}')\right]\times\vec{X}(\vec{r}')d\Omega' \tag{41.2}$$

here the integral domain $\Omega$ is the region where $\vec{X}$ exists, and $k_0 = \omega\sqrt{\mu_0\varepsilon_0}$, and $k_m = \omega\sqrt{\mu\varepsilon_c}$, and

$$G_{0/m}(\vec{r},\vec{r}') = \frac{1}{4\pi|\vec{r}-\vec{r}'|}e^{-jk_{0/m}|\vec{r}-\vec{r}'|} \tag{42}$$

In addition, it is well known that [9]

$$\begin{aligned}\vec{F}_{met}^{sca}(\vec{r}) &= \mathcal{F}_{met}^{sca}(\vec{J}^{l}) + \mathcal{F}_{met}^{sca}(\vec{J}^{s}) \\ &= \mathcal{F}_{met}^{sca}(\vec{J}_0^{l}+\vec{J}_{\cap}^{l}) + \mathcal{F}_{met}^{sca}(\vec{J}_0^{s}+\vec{J}_{\cap}^{s}) \\ &= \mathcal{F}_{met}^{sca}(\vec{J}_0^{l}) + \mathcal{F}_{met}^{sca}(\vec{J}_{\cap}^{l}) + \mathcal{F}_{met}^{sca}(\vec{J}_0^{s}) + \mathcal{F}_{met}^{sca}(\vec{J}_{\cap}^{s})\end{aligned} \tag{43}$$

for any $\vec{r} \in \mathbb{R}^3 \setminus (L^{met}\cup S^{met}\cup\partial V^{met})$, and

$$\mathcal{E}_{met}^{sca}(\vec{X}) = -j\omega\mu_0\mathcal{L}_0(\vec{X}) \tag{44.1}$$

$$\mathcal{H}_{met}^{sca}(\vec{X}) = \mathcal{K}_0(\vec{X}) \tag{44.2}$$

## IV. A New and "Real" Line-Surface Formulation of Input/Output Power Operator

In this section, the power operator of metal-material combined objects is expressed in terms of various currents mentioned in above sections.

For the metal-material combined object in Fig. 1, the input power $P^{inp}$ (the power done by incident field on scattering currents) and the output power $P^{out}$ are as follows [1]-[2]

$$\begin{aligned}P^{out} = P^{inp} = {} & (1/2)\left\langle \vec{J}^{l}\oplus\vec{J}^{s},\ \vec{E}^{inc}\right\rangle_{L^{met}\cup S^{met}\cup\partial V^{met}} \\ & +(1/2)\left\langle \vec{J}^{vop},\vec{E}^{inc}\right\rangle_{V^{mat}} + (1/2)\left\langle \vec{H}^{inc},\vec{M}^{vm}\right\rangle_{V^{mat}}\end{aligned} \tag{45}$$

here the inner product is defined as $<\vec{f},\vec{g}>_\Omega \triangleq \int_\Omega \vec{f}^*\cdot\vec{g}\,d\Omega$, and the superscript "$*$" represents the complex conjugate, and the operator "$\oplus$" is defined as

$$\left\langle \vec{A}\oplus\vec{B},\vec{C}\right\rangle_\Omega \triangleq \left\langle \vec{A},\vec{C}\right\rangle_\Omega + \left\langle \vec{B},\vec{C}\right\rangle_\Omega \tag{46.1}$$

$$\left\langle \vec{A},\vec{B}\oplus\vec{C}\right\rangle_\Omega \triangleq \left\langle \vec{A},\vec{B}\right\rangle_\Omega + \left\langle \vec{A},\vec{C}\right\rangle_\Omega \tag{46.2}$$

and the reason to utilize symbol "$\oplus$" instead of "$+$" is that the dimensions of line current $\vec{J}^{l}$ and surface current $\vec{J}^{s}$ are different. The first equality in (45) is due to the conservation law of energy [4], and the $P^{out}$ can also be written into a more convenient form as follows [1]-[2]

$$
\begin{aligned}
P^{out} = {} & \frac{1}{2}\oint\!\!\!\oint_{S_\infty}\left[\vec{E}^{sca}\times\left(\vec{H}^{sca}\right)^*\right]\cdot d\vec{S}+\frac{1}{2}\left\langle\sigma\vec{E}^{tot},\vec{E}^{tot}\right\rangle_{V^{mat}} \\
& +j\,2\omega\left\{\left[\frac{1}{4}\left\langle\vec{H}^{sca},\mu_0\vec{H}^{sca}\right\rangle_{\mathbb{R}^3}-\frac{1}{4}\left\langle\varepsilon_0\vec{E}^{sca},\vec{E}^{sca}\right\rangle_{\mathbb{R}^3}\right]\right. \\
& \left.+\left[\frac{1}{4}\left\langle\vec{H}^{tot},\Delta\mu\vec{H}^{tot}\right\rangle_{V^{mat}}-\frac{1}{4}\left\langle\Delta\varepsilon\vec{E}^{tot},\vec{E}^{tot}\right\rangle_{V^{mat}}\right]\right\}
\end{aligned} \tag{47}
$$

here the integral domain $S_\infty$ is a spherical surface at infinity.

Because $\vec{F}^{inc}=\vec{F}^{tot}-\vec{F}^{sca}_{mat}-\vec{F}^{sca}_{met}=\vec{f}^{inc}_{int}-\vec{F}^{sca}_{met}$ on $\mathrm{int}V^{mat}$, and $\left[\vec{E}^{inc}\right]_{\tan}=-\left[\vec{E}^{sca}\right]_{\tan}=-\left[\vec{E}^{sca}_{mat}+\vec{E}^{sca}_{met}\right]_{\tan}$ on $L^{met}\cup S^{met}\cup\partial V^{met}$ (here the subscripts "$\tan$" represent the tangential components of fields), the $P^{inp}$ in (45) can be rewritten as follows

$$
\begin{aligned}
P^{inp} = {} & -(1/2)\left\langle\vec{J}^l\oplus\vec{J}^s\,,\ \vec{E}^{sca}_{met}+\vec{E}^{sca}_{mat}\right\rangle_{L^{met}\cup S^{met}\cup\partial V^{met}} \\
& -\left[(1/2)\left\langle\vec{J}^{vop},\vec{E}^{sca}_{met}\right\rangle_{V^{mat}}+(1/2)\left\langle\vec{H}^{sca}_{met},\vec{M}^{vm}\right\rangle_{V^{mat}}\right] \\
& +\left[(1/2)\left\langle\vec{J}^{vop},\vec{e}^{inc}_{int}\right\rangle_{V^{mat}}+(1/2)\left\langle\vec{h}^{inc}_{int},\vec{M}^{vm}\right\rangle_{V^{mat}}\right]
\end{aligned} \tag{48}
$$

In addition, based on the conclusions given in [3] and [8] and the discussions given in the Sec. II-C of this paper, the following relations exist

$$
\begin{aligned}
& (1/2)\left\langle\vec{J}^{vop},\vec{E}^{sca}_{met}\right\rangle_{V^{mat}}+(1/2)\left\langle\vec{H}^{sca}_{met},\vec{M}^{vm}\right\rangle_{V^{mat}} \\
= {} & \frac{\mu\,\Delta\varepsilon_c^*}{\mu_0\varepsilon_0-\mu\varepsilon_c^*}\left[\frac{1}{2}\left\langle\vec{J}^{LE}\oplus\vec{J}^{SE},\,\vec{E}^{sca}_{met}\right\rangle_{\partial V^{mat}}+\frac{\mu_0}{\mu}\frac{1}{2}\left\langle\vec{M}_0^{SE},\,\vec{H}^{sca}_{met}\right\rangle_{\partial V_0^{mat}}\right] \\
& +\frac{\varepsilon_c\,\Delta\mu}{\varepsilon_0\mu_0-\varepsilon_c\mu}\left[\frac{\varepsilon_0}{\varepsilon_c}\frac{1}{2}\left\langle\vec{J}^{LE}\oplus\vec{J}^{SE},\,\vec{E}^{sca}_{met}\right\rangle^*_{\partial V^{mat}}+\frac{1}{2}\left\langle\vec{M}_0^{SE},\,\vec{H}^{sca}_{met}\right\rangle^*_{\partial V_0^{mat}}\right]
\end{aligned} \tag{49}
$$

and

$$
\begin{aligned}
& (1/2)\left\langle\vec{J}^{vop},\vec{e}^{inc}_{int}\right\rangle_{V^{mat}}+(1/2)\left\langle\vec{h}^{inc}_{int},\vec{M}^{vm}\right\rangle_{V^{mat}} \\
= {} & \frac{\mu\,\Delta\varepsilon_c^*}{\mu_0\varepsilon_0-\mu\varepsilon_c^*}\left[\frac{1}{2}\left\langle\vec{J}^{LE}\oplus\vec{J}^{SE},\,\vec{e}^{inc}_{int}\right\rangle_{\partial V^{mat}}+\frac{\mu_0}{\mu}\frac{1}{2}\left\langle\vec{M}_0^{SE},\,\vec{h}^{inc}_{int}\right\rangle_{\partial V_0^{mat}}\right] \\
& +\frac{\varepsilon_c\,\Delta\mu}{\varepsilon_0\mu_0-\varepsilon_c\mu}\left[\frac{\varepsilon_0}{\varepsilon_c}\frac{1}{2}\left\langle\vec{J}^{LE}\oplus\vec{J}^{SE},\,\vec{e}^{inc}_{int}\right\rangle^*_{\partial V^{mat}}+\frac{1}{2}\left\langle\vec{M}_0^{SE},\,\vec{h}^{inc}_{int}\right\rangle^*_{\partial V_0^{mat}}\right]
\end{aligned} \tag{50}
$$

Inserting the (49)-(50) into (48) and considering of that (34)-(44), the $P^{inp}$ in (48) can also be rewritten as (51).

Based on (30.1), (32.1), (35), (40), and (44), it is obvious that

$$
\left\langle\vec{J}^l\oplus\vec{J}^s,\mathcal{E}^{sca}_{ext}\left(\vec{J}^l_\cap,0\right)\right\rangle_{L^{met}\cup S^{met}\cup\partial V^{met}}=-\left\langle\vec{J}^l\oplus\vec{J}^s,\mathcal{E}^{sca}_{met}\left(\vec{J}^l_\cap\right)\right\rangle_{L^{met}\cup S^{met}\cup\partial V^{met}} \tag{52.1}
$$

$$
\left\langle\vec{J}^l\oplus\vec{J}^s,\mathcal{E}^{sca}_{ext}\left(\vec{J}^s_\cap,0\right)\right\rangle_{L^{met}\cup S^{met}\cup\partial V^{met}}=-\left\langle\vec{J}^l\oplus\vec{J}^s,\mathcal{E}^{sca}_{met}\left(\vec{J}^s_\cap\right)\right\rangle_{L^{met}\cup S^{met}\cup\partial V^{met}} \tag{52.2}
$$

and

$$
\left\langle\vec{J}^{LE}\oplus\vec{J}^{SE},\mathscr{e}^{inc}_{int}\left(\vec{J}^l_\cap,0\right)\right\rangle_{\partial V^{mat}}=\left\langle\vec{J}^{LE}\oplus\vec{J}^{SE},\mathcal{E}^{sca}_{met}\left(\vec{J}^l_\cap\right)\right\rangle_{\partial V^{mat}} \tag{53.1}
$$

$$
\left\langle\vec{J}^{LE}\oplus\vec{J}^{SE},\mathscr{e}^{inc}_{int}\left(\vec{J}^s_\cap,0\right)\right\rangle_{\partial V^{mat}}=\left\langle\vec{J}^{LE}\oplus\vec{J}^{SE},\mathcal{E}^{sca}_{met}\left(\vec{J}^s_\cap\right)\right\rangle_{\partial V^{mat}} \tag{53.2}
$$

$$
\left\langle\vec{M}_0^{SE},\mathscr{h}^{inc}_{int}\left(\vec{J}^l_\cap,0\right)\right\rangle_{\partial V_0^{mat}}=\left\langle\vec{M}_0^{SE},\mathcal{H}^{sca}_{met}\left(\vec{J}^l_\cap\right)\right\rangle_{\partial V_0^{mat}} \tag{53.3}
$$

$$
\left\langle\vec{M}_0^{SE},\mathscr{h}^{inc}_{int}\left(\vec{J}^s_\cap,0\right)\right\rangle_{\partial V_0^{mat}}=\left\langle\vec{M}_0^{SE},\mathcal{H}^{sca}_{met}\left(\vec{J}^s_\cap\right)\right\rangle_{\partial V_0^{mat}} \tag{53.4}
$$

Inserting the (52)-(53) into (51) and utilizing the (30.1) and (32.1), the $P^{inp}$ in (51) can be further simplified into the (54). In the following Sec. V, the operator (54) is transformed into its matrix form, and then the power-based CM sets are derived.

## V. POWER-BASED CHARACTERISTIC MODE SETS

In this section, the power-based CM sets of the metal-material combined object illustrated in Fig. 1 are constructed.

### A. From current space to expansion vector space

The currents $\{\vec{J}^l_0,\vec{J}^l_\cap\}$, $\{\vec{J}^s_0,\vec{J}^s_\cap\}$, and $\{\vec{J}^{SE}_0,\vec{M}^{SE}_0\}$ can be expanded as follows

$$
\vec{J}^l_{0/\cap}(\vec{r})=\sum_{\xi=1}^{\Xi^{\vec{J}^l_{0/\cap}}}a_\xi^{\vec{J}^l_{0/\cap}}\vec{b}_\xi^{\vec{J}^l_{0/\cap}}(\vec{r})=\bar{B}^{\vec{J}^l_{0/\cap}}\cdot\bar{a}^{\vec{J}^l_{0/\cap}}\quad,\quad\left(\vec{r}\in L^{met}_{0/\cap}\right) \tag{55}
$$

$$
\vec{J}^s_{0/\cap}(\vec{r})=\sum_{\xi=1}^{\Xi^{\vec{J}^s_{0/\cap}}}a_\xi^{\vec{J}^s_{0/\cap}}\vec{b}_\xi^{\vec{J}^s_{0/\cap}}(\vec{r})=\bar{B}^{\vec{J}^s_{0/\cap}}\cdot\bar{a}^{\vec{J}^s_{0/\cap}}\quad,\quad\left(\vec{r}\in S^{met}_{0/\cap}\cup\partial V^{met}_{0/\cap}\right) \tag{56}
$$

$$
\vec{C}^{SE}_0(\vec{r})=\sum_{\xi=1}^{\Xi^{\vec{C}^{SE}_0}}a_\xi^{\vec{C}^{SE}_0}\vec{b}_\xi^{\vec{C}^{SE}_0}(\vec{r})=\bar{B}^{\vec{C}^{SE}_0}\cdot\bar{a}^{\vec{C}^{SE}_0}\quad,\quad\left(\vec{r}\in\partial V^{mat}_0\right) \tag{57}
$$

in which $C=J,M$, and

$$
\bar{B}^Y=\left[\vec{b}_1^Y\quad,\quad\vec{b}_2^Y\quad,\quad\cdots\quad,\quad\vec{b}^Y_{\Xi^Y}\right] \tag{58.1}
$$

$$
\bar{a}^Y=\left[a_1^Y\quad,\quad a_2^Y\quad,\quad\cdots\quad,\quad a^Y_{\Xi^Y}\right]^T \tag{58.2}
$$

$$
\begin{aligned}
P^{inp} = {} & -(1/2)\left\langle\vec{J}^l\oplus\vec{J}^s\,,\ \mathcal{E}^{sca}_{met}\left(\vec{J}^l_0\right)+\mathcal{E}^{sca}_{met}\left(\vec{J}^l_\cap\right)+\mathcal{E}^{sca}_{met}\left(\vec{J}^s_0\right)+\mathcal{E}^{sca}_{met}\left(\vec{J}^s_\cap\right)\right\rangle_{L^{met}\cup S^{met}\cup\partial V^{met}} \\
& -(1/2)\left\langle\vec{J}^l\oplus\vec{J}^s\,,\ \mathcal{E}^{sca}_{ext}\left(\vec{J}^l_\cap,0\right)+\mathcal{E}^{sca}_{ext}\left(\vec{J}^{SE}_0,\vec{M}^{SE}_0\right)+\mathcal{E}^{sca}_{ext}\left(\vec{J}^s_\cap,0\right)\right\rangle_{L^{met}\cup S^{met}\cup\partial V^{met}} \\
& -\frac{\mu\,\Delta\varepsilon_c^*}{\mu_0\varepsilon_0-\mu\varepsilon_c^*}\left[\frac{1}{2}\left\langle\vec{J}^{LE}\oplus\vec{J}^{SE}\,,\ \mathcal{E}^{sca}_{met}\left(\vec{J}^l_0\right)+\mathcal{E}^{sca}_{met}\left(\vec{J}^l_\cap\right)+\mathcal{E}^{sca}_{met}\left(\vec{J}^s_0\right)+\mathcal{E}^{sca}_{met}\left(\vec{J}^s_\cap\right)\right\rangle_{\partial V^{mat}}+\frac{\mu_0}{\mu}\frac{1}{2}\left\langle\vec{M}^{SE}_0\,,\ \mathcal{H}^{sca}_{met}\left(\vec{J}^l_0\right)+\mathcal{H}^{sca}_{met}\left(\vec{J}^l_\cap\right)+\mathcal{H}^{sca}_{met}\left(\vec{J}^s_0\right)+\mathcal{H}^{sca}_{met}\left(\vec{J}^s_\cap\right)\right\rangle_{\partial V_0^{mat}}\right] \\
& -\frac{\varepsilon_c\,\Delta\mu}{\varepsilon_0\mu_0-\varepsilon_c\mu}\left[\frac{\varepsilon_0}{\varepsilon_c}\frac{1}{2}\left\langle\vec{J}^{LE}\oplus\vec{J}^{SE}\,,\ \mathcal{E}^{sca}_{met}\left(\vec{J}^l_0\right)+\mathcal{E}^{sca}_{met}\left(\vec{J}^l_\cap\right)+\mathcal{E}^{sca}_{met}\left(\vec{J}^s_0\right)+\mathcal{E}^{sca}_{met}\left(\vec{J}^s_\cap\right)\right\rangle^*_{\partial V^{mat}}+\frac{1}{2}\left\langle\vec{M}^{SE}_0\,,\ \mathcal{H}^{sca}_{met}\left(\vec{J}^l_0\right)+\mathcal{H}^{sca}_{met}\left(\vec{J}^l_\cap\right)+\mathcal{H}^{sca}_{met}\left(\vec{J}^s_0\right)+\mathcal{H}^{sca}_{met}\left(\vec{J}^s_\cap\right)\right\rangle^*_{\partial V_0^{mat}}\right] \\
& +\frac{\mu\,\Delta\varepsilon_c^*}{\mu_0\varepsilon_0-\mu\varepsilon_c^*}\left[\frac{1}{2}\left\langle\vec{J}^{LE}\oplus\vec{J}^{SE}\,,\ \mathscr{e}^{inc}_{int}\left(\vec{J}^l_\cap,0\right)+\mathscr{e}^{inc}_{int}\left(\vec{J}^{SE}_0,\vec{M}^{SE}_0\right)+\mathscr{e}^{inc}_{int}\left(\vec{J}^s_\cap,0\right)\right\rangle_{\partial V^{mat}}+\frac{\mu_0}{\mu}\frac{1}{2}\left\langle\vec{M}^{SE}_0\,,\ \mathscr{h}^{inc}_{int}\left(\vec{J}^l_\cap,0\right)+\mathscr{h}^{inc}_{int}\left(\vec{J}^{SE}_0,\vec{M}^{SE}_0\right)+\mathscr{h}^{inc}_{int}\left(\vec{J}^s_\cap,0\right)\right\rangle_{\partial V_0^{mat}}\right] \\
& +\frac{\varepsilon_c\,\Delta\mu}{\varepsilon_0\mu_0-\varepsilon_c\mu}\left[\frac{\varepsilon_0}{\varepsilon_c}\frac{1}{2}\left\langle\vec{J}^{LE}\oplus\vec{J}^{SE}\,,\ \mathscr{e}^{inc}_{int}\left(\vec{J}^l_\cap,0\right)+\mathscr{e}^{inc}_{int}\left(\vec{J}^{SE}_0,\vec{M}^{SE}_0\right)+\mathscr{e}^{inc}_{int}\left(\vec{J}^s_\cap,0\right)\right\rangle^*_{\partial V^{mat}}+\frac{1}{2}\left\langle\vec{M}^{SE}_0\,,\ \mathscr{h}^{inc}_{int}\left(\vec{J}^l_\cap,0\right)+\mathscr{h}^{inc}_{int}\left(\vec{J}^{SE}_0,\vec{M}^{SE}_0\right)+\mathscr{h}^{inc}_{int}\left(\vec{J}^s_\cap,0\right)\right\rangle^*_{\partial V_0^{mat}}\right]
\end{aligned} \tag{51}
$$

for any $Y = \vec{J}_0^l, \vec{J}_\cap^l, \vec{J}_0^s, \vec{J}_\cap^s, \vec{J}_0^{SE}, \vec{M}_0^{SE}$. The superscript "$T$" in (58.2) represents the transpose of matrix.

## B. To unify variables in expansion vector space

Based on the methods given in [3] and [7]-[8], the expansion vector $\bar{a}^{\vec{M}_0^{SE}}$ can be expressed in terms of the expansion vectors $\{\bar{a}^{\vec{J}_0^{SE}}, \bar{a}^{\vec{J}_\cap^l}, \bar{a}^{\vec{J}_\cap^s}\}$ and the expansion vector $\bar{a}^{\vec{J}_0^{SE}}$ can be expressed in terms of the expansion vectors $\{\bar{a}^{\vec{J}_\cap^l}, \bar{a}^{\vec{J}_\cap^s}, \bar{a}^{\vec{M}_0^{SE}}\}$ as follows

$$\bar{a}^{\vec{M}_0^{SE}} = \bar{\bar{T}}^{\{\vec{J}_0^{SE}, \vec{J}_\cap^l, \vec{J}_\cap^s\} \to \vec{M}_0^{SE}} \cdot \bar{a}^{\{\vec{J}_0^{SE}, \vec{J}_\cap^l, \vec{J}_\cap^s\}} \tag{59}$$

$$\bar{a}^{\vec{J}_0^{SE}} = \bar{\bar{T}}^{\{\vec{J}_\cap^l, \vec{J}_\cap^s, \vec{M}_0^{SE}\} \to \vec{J}_0^{SE}} \cdot \bar{a}^{\{\vec{J}_\cap^l, \vec{J}_\cap^s, \vec{M}_0^{SE}\}} \tag{60}$$

here

$$\bar{a}^{\{\vec{J}_0^{SE}, \vec{J}_\cap^l, \vec{J}_\cap^s\}} = \begin{bmatrix} \bar{a}^{\vec{J}_0^{SE}} \\ \bar{a}^{\vec{J}_\cap^l} \\ \bar{a}^{\vec{J}_\cap^s} \end{bmatrix} \tag{61}$$

$$\bar{a}^{\{\vec{J}_\cap^l, \vec{J}_\cap^s, \vec{M}_0^{SE}\}} = \begin{bmatrix} \bar{a}^{\vec{J}_\cap^l} \\ \bar{a}^{\vec{J}_\cap^s} \\ \bar{a}^{\vec{M}_0^{SE}} \end{bmatrix} \tag{62}$$

and some different methods for obtaining the transformation matrices $\bar{\bar{T}}^{\{\vec{J}_0^{SE}, \vec{J}_\cap^l, \vec{J}_\cap^s\} \to \vec{M}_0^{SE}}$ and $\bar{\bar{T}}^{\{\vec{J}_\cap^l, \vec{J}_\cap^s, \vec{M}_0^{SE}\} \to \vec{J}_0^{SE}}$ can be found in [3] and [7]-[8], and two of them are specifically exhibited as follows

$$\begin{aligned} & \bar{\bar{T}}_{Magnetic\ Extinction}^{\{\vec{J}_0^{SE}, \vec{J}_\cap^l, \vec{J}_\cap^s\} \to \vec{M}_0^{SE}} \\ = & \left(\bar{\bar{\Phi}}^{M_0^{SE} H_{int}^{tot} M_0^{SE}}\right)^{-1} \cdot \left[\bar{\bar{\Phi}}^{M_0^{SE} H_{int}^{tot} J_0^{SE}} \quad \bar{\bar{\Phi}}^{M_0^{SE} H_{int}^{tot} J_\cap^l} \quad \bar{\bar{\Phi}}^{M_0^{SE} H_{int}^{tot} J_\cap^s}\right] \end{aligned} \tag{63}$$

$$\begin{aligned} & \bar{\bar{T}}_{Electric\ Extinction}^{\{\vec{J}_\cap^l, \vec{J}_\cap^s, \vec{M}_0^{SE}\} \to \vec{J}_0^{SE}} \\ = & \left(\bar{\bar{\Phi}}^{J_0^{SE} E_{int}^{tot} J_0^{SE}}\right)^{-1} \cdot \left[\bar{\bar{\Phi}}^{J_0^{SE} E_{int}^{tot} J_\cap^l} \quad \bar{\bar{\Phi}}^{J_0^{SE} E_{int}^{tot} J_\cap^s} \quad \bar{\bar{\Phi}}^{J_0^{SE} E_{int}^{tot} M_0^{SE}}\right] \end{aligned} \tag{64}$$

here the subscripts "$Magnetic\ Extinction$" and "$Electric\ Extinction$" in transformation matrices $\bar{\bar{T}}_{Magnetic\ Extinction}^{\{\vec{J}_0^{SE}, \vec{J}_\cap^l, \vec{J}_\cap^s\} \to \vec{M}_0^{SE}}$ and $\bar{\bar{T}}_{Electric\ Extinction}^{\{\vec{J}_\cap^l, \vec{J}_\cap^s, \vec{M}_0^{SE}\} \to \vec{J}_0^{SE}}$ mean that these two matrices are respectively derived from magnetic and electric extinction theorems [3], and the meanings of their superscripts are obvious, and the matrices and submatrices in (63)-(64) are as follows

$$\bar{\bar{\Phi}}^{M_0^{SE} H_{int}^{tot} M_0^{SE}} = \left[\phi_{\xi\zeta}^{M_0^{SE} H_{int}^{tot} M_0^{SE}}\right]_{\Xi^{\vec{M}_0^{SE}} \times \Xi^{\vec{M}_0^{SE}}} \tag{65.1}$$

$$\bar{\bar{\Phi}}^{M_0^{SE} H_{int}^{tot} J_0^{SE}} = \left[\phi_{\xi\zeta}^{M_0^{SE} H_{int}^{tot} J_0^{SE}}\right]_{\Xi^{\vec{M}_0^{SE}} \times \Xi^{\vec{J}_0^{SE}}} \tag{65.2}$$

$$\bar{\bar{\Phi}}^{M_0^{SE} H_{int}^{tot} J_\cap^l} = \left[\phi_{\xi\zeta}^{M_0^{SE} H_{int}^{tot} J_\cap^l}\right]_{\Xi^{\vec{M}_0^{SE}} \times \Xi^{\vec{J}_\cap^l}} \tag{65.3}$$

$$\bar{\bar{\Phi}}^{M_0^{SE} H_{int}^{tot} J_\cap^s} = \left[\phi_{\xi\zeta}^{M_0^{SE} H_{int}^{tot} J_\cap^s}\right]_{\Xi^{\vec{M}_0^{SE}} \times \Xi^{\vec{J}_\cap^s}} \tag{65.4}$$

and

$$\bar{\bar{\Phi}}^{J_0^{SE} E_{int}^{tot} J_0^{SE}} = \left[\phi_{\xi\zeta}^{J_0^{SE} E_{int}^{tot} J_0^{SE}}\right]_{\Xi^{\vec{J}_0^{SE}} \times \Xi^{\vec{J}_0^{SE}}} \tag{66.1}$$

$$\bar{\bar{\Phi}}^{J_0^{SE} E_{int}^{tot} J_\cap^l} = \left[\phi_{\xi\zeta}^{J_0^{SE} E_{int}^{tot} J_\cap^l}\right]_{\Xi^{\vec{J}_0^{SE}} \times \Xi^{\vec{J}_\cap^l}} \tag{66.2}$$

$$\bar{\bar{\Phi}}^{J_0^{SE} E_{int}^{tot} J_\cap^s} = \left[\phi_{\xi\zeta}^{J_0^{SE} E_{int}^{tot} J_\cap^s}\right]_{\Xi^{\vec{J}_0^{SE}} \times \Xi^{\vec{J}_\cap^s}} \tag{66.3}$$

$$\bar{\bar{\Phi}}^{J_0^{SE} E_{int}^{tot} M_0^{SE}} = \left[\phi_{\xi\zeta}^{J_0^{SE} E_{int}^{tot} M_0^{SE}}\right]_{\Xi^{\vec{J}_0^{SE}} \times \Xi^{\vec{M}_0^{SE}}} \tag{66.4}$$

The superscript "$M_0^{SE} H_{int}^{tot} M_0^{SE}$" on matrix $\bar{\bar{\Phi}}^{M_0^{SE} H_{int}^{tot} M_0^{SE}}$ means that the elements of this matrix is derived from testing the interior total magnetic field $\mathcal{H}_{int}^{tot}(0, \vec{b}_\zeta^{\vec{M}_0^{SE}})$ by using magnetic current basis $\vec{b}_\xi^{\vec{M}_0^{SE}}$, and the superscripts on the other matrices can be similarly explained. The elements in (65)-(66) can be computed as follows

$$\phi_{\xi\zeta}^{M_0^{SE} H_{int}^{tot} M_0^{SE}} = \left\langle \vec{b}_\xi^{\vec{M}_0^{SE}} \ , \ -j\omega\varepsilon_c \mathcal{L}_m\left(\vec{b}_\zeta^{\vec{M}_0^{SE}}\right) \right\rangle_{\partial V_0^{mat}} \tag{67.1}$$

$$\begin{aligned} \phi_{\xi\zeta}^{M_0^{SE} H_{int}^{tot} J_0^{SE}} &= \left\langle \vec{b}_\xi^{\vec{M}_0^{SE}} \ , \ -\hat{n}_{\to mat} \times \vec{b}_\zeta^{\vec{J}_0^{SE}} - \mathcal{K}_m\left(\vec{b}_\zeta^{\vec{J}_0^{SE}}\right) \right\rangle_{\partial V_0^{mat}} \\ &= \left\langle \vec{b}_\xi^{\vec{M}_0^{SE}} \ , \ -\frac{1}{2}\hat{n}_{\to mat} \times \vec{b}_\zeta^{\vec{J}_0^{SE}} - \text{P.V.}\,\mathcal{K}_m\left(\vec{b}_\zeta^{\vec{J}_0^{SE}}\right) \right\rangle_{\partial V_0^{mat}} \end{aligned} \tag{67.2}$$

$$\phi_{\xi\zeta}^{M_0^{SE} H_{int}^{tot} J_\cap^l} = \left\langle \vec{b}_\xi^{\vec{M}_0^{SE}} \ , \ -\mathcal{K}_m\left(\vec{b}_\zeta^{\vec{J}_\cap^l}\right) \right\rangle_{\partial V_0^{mat}} \tag{67.3}$$

$$\phi_{\xi\zeta}^{M_0^{SE} H_{int}^{tot} J_\cap^s} = \left\langle \vec{b}_\xi^{\vec{M}_0^{SE}} \ , \ -\mathcal{K}_m\left(\vec{b}_\zeta^{\vec{J}_\cap^s}\right) \right\rangle_{\partial V_0^{mat}} \tag{67.4}$$

and

$$\phi_{\xi\zeta}^{J_0^{SE} E_{int}^{tot} J_0^{SE}} = \left\langle \vec{b}_\xi^{\vec{J}_0^{SE}} \ , \ -j\omega\mu \mathcal{L}_m\left(\vec{b}_\zeta^{\vec{J}_0^{SE}}\right) \right\rangle_{\partial V_0^{mat}} \tag{68.1}$$

$$\phi_{\xi\zeta}^{J_0^{SE} E_{int}^{tot} J_\cap^l} = \left\langle \vec{b}_\xi^{\vec{J}_0^{SE}} \ , \ j\omega\mu \mathcal{L}_m\left(\vec{b}_\zeta^{\vec{J}_\cap^l}\right) \right\rangle_{\partial V_0^{mat}} \tag{68.2}$$

$$\phi_{\xi\zeta}^{J_0^{SE} E_{int}^{tot} J_\cap^s} = \left\langle \vec{b}_\xi^{\vec{J}_0^{SE}} \ , \ j\omega\mu \mathcal{L}_m\left(\vec{b}_\zeta^{\vec{J}_\cap^s}\right) \right\rangle_{\partial V_0^{mat}} \tag{68.3}$$

---

$$\begin{aligned} P^{inp} = & -(1/2)\left\langle \left(\vec{J}_0^l + \vec{J}_\cap^l\right) \oplus \left(\vec{J}_0^s + \vec{J}_\cap^s\right) \ , \ \mathcal{E}_{met}^{sca}\left(\vec{J}_0^l\right) + \mathcal{E}_{met}^{sca}\left(\vec{J}_0^s\right) \right\rangle_{L^{met} \cup S^{met} \cup \partial V^{met}} \\ & -(1/2)\left\langle \left(\vec{J}_0^l + \vec{J}_\cap^l\right) \oplus \left(\vec{J}_0^s + \vec{J}_\cap^s\right) \ , \ \mathcal{E}_{ext}^{sca}\left(\vec{J}_0^{SE}, \vec{M}_0^{SE}\right) \right\rangle_{L^{met} \cup S^{met} \cup \partial V^{met}} \\ & -\frac{\mu\,\Delta\varepsilon_c^*}{\mu_0\varepsilon_0 - \mu\varepsilon_c^*}\left[ \frac{1}{2}\left\langle \vec{J}_\cap^l \oplus \left(\vec{J}_\cap^s + \vec{J}_0^{SE}\right) \ , \ \mathcal{E}_{met}^{sca}\left(\vec{J}_0^l\right) + \mathcal{E}_{met}^{sca}\left(\vec{J}_0^s\right) \right\rangle_{\partial V^{mat}} + \frac{\mu_0}{\mu}\frac{1}{2}\left\langle \vec{M}_0^{SE} \ , \ \mathcal{H}_{met}^{sca}\left(\vec{J}_0^l\right) + \mathcal{H}_{met}^{sca}\left(\vec{J}_0^s\right) \right\rangle_{\partial V_0^{mat}} \right] \\ & -\frac{\varepsilon_c\,\Delta\mu}{\varepsilon_0\mu_0 - \varepsilon_c\mu}\left[ \frac{\varepsilon_0}{\varepsilon_c}\frac{1}{2}\left\langle \vec{J}_\cap^l \oplus \left(\vec{J}_\cap^s + \vec{J}_0^{SE}\right) \ , \ \mathcal{E}_{met}^{sca}\left(\vec{J}_0^l\right) + \mathcal{E}_{met}^{sca}\left(\vec{J}_0^s\right) \right\rangle_{\partial V^{mat}}^* + \frac{1}{2}\left\langle \vec{M}_0^{SE} \ , \ \mathcal{H}_{met}^{sca}\left(\vec{J}_0^l\right) + \mathcal{H}_{met}^{sca}\left(\vec{J}_0^s\right) \right\rangle_{\partial V_0^{mat}}^* \right] \\ & +\frac{\mu\,\Delta\varepsilon_c^*}{\mu_0\varepsilon_0 - \mu\varepsilon_c^*}\left[ \frac{1}{2}\left\langle \vec{J}_\cap^l \oplus \left(\vec{J}_\cap^s + \vec{J}_0^{SE}\right) \ , \ \mathcal{e}_{int}^{inc}\left(\vec{J}_0^{SE}, \vec{M}_0^{SE}\right) \right\rangle_{\partial V^{mat}} + \frac{\mu_0}{\mu}\frac{1}{2}\left\langle \vec{M}_0^{SE} \ , \ \mathcal{h}_{int}^{inc}\left(\vec{J}_0^{SE}, \vec{M}_0^{SE}\right) \right\rangle_{\partial V_0^{mat}} \right] \\ & +\frac{\varepsilon_c\,\Delta\mu}{\varepsilon_0\mu_0 - \varepsilon_c\mu}\left[ \frac{\varepsilon_0}{\varepsilon_c}\frac{1}{2}\left\langle \vec{J}_\cap^l \oplus \left(\vec{J}_\cap^s + \vec{J}_0^{SE}\right) \ , \ \mathcal{e}_{int}^{inc}\left(\vec{J}_0^{SE}, \vec{M}_0^{SE}\right) \right\rangle_{\partial V^{mat}}^* + \frac{1}{2}\left\langle \vec{M}_0^{SE} \ , \ \mathcal{h}_{int}^{inc}\left(\vec{J}_0^{SE}, \vec{M}_0^{SE}\right) \right\rangle_{\partial V_0^{mat}}^* \right] \end{aligned} \tag{54}$$

$$\begin{aligned}\phi_{\xi\zeta}^{J_0^{SE} E_{int}^{tot} M_0^{SE}} &= \left\langle \vec{b}_{\xi}^{\vec{J}_0^{SE}} \ , \ \hat{n}_{\rightarrow mat} \times \vec{b}_{\zeta}^{\vec{M}_0^{SE}} + \mathcal{K}_m\left(\vec{b}_{\zeta}^{\vec{M}_0^{SE}}\right)\right\rangle_{\partial V_0^{mat}} \\ &= \left\langle \vec{b}_{\xi}^{\vec{J}_0^{SE}} \ , \ \frac{1}{2}\hat{n}_{\rightarrow mat} \times \vec{b}_{\zeta}^{\vec{M}_0^{SE}} + \mathrm{P.V.}\,\mathcal{K}_m\left(\vec{b}_{\zeta}^{\vec{M}_0^{SE}}\right)\right\rangle_{\partial V_0^{mat}}\end{aligned} \tag{68.4}$$

here the symbols " P.V. " in (67.2) and (68.4) represent the Cauchy principal value of integral.

### *C. The input power operator in expansion vector space (The matrix form of input power operator)*

Inserting (55)-(57) into (54), the input power $P^{inp}$ can be transformed into the following matrix form

$$P^{inp} = \left(\bar{a}^{\{\vec{J}_0^l,\vec{J}_0^s,\vec{J}_0^{SE},\vec{J}_\cap^l,\vec{J}_\cap^s,\vec{M}_0^{SE}\}}\right)^H \cdot \overline{\overline{P}}^{inp}_{\{\vec{J}_0^l,\vec{J}_0^s,\vec{J}_0^{SE},\vec{J}_\cap^l,\vec{J}_\cap^s,\vec{M}_0^{SE}\}} \cdot \bar{a}^{\{\vec{J}_0^l,\vec{J}_0^s,\vec{J}_0^{SE},\vec{J}_\cap^l,\vec{J}_\cap^s,\vec{M}_0^{SE}\}} \tag{69}$$

here the superscript " $H$ " represents the transpose conjugate of matrix, and the symbol " $\cdot$ " represents the matrix multiplication, and

$$\begin{aligned}&\overline{\overline{P}}^{inp}_{\{\vec{J}_0^l,\vec{J}_0^s,\vec{J}_0^{SE},\vec{J}_\cap^l,\vec{J}_\cap^s,\vec{M}_0^{SE}\}} \\ &= -\overline{\overline{P}}^{\{\vec{J}_0^l,\vec{J}_0^s,\vec{J}_\cap^l,\vec{J}_\cap^s\}\leftarrow\{\mathcal{E}_{met}^{sca}(\vec{J}_0^l),\mathcal{E}_{met}^{sca}(\vec{J}_0^s)\}} - \overline{\overline{P}}^{\{\vec{J}_0^l,\vec{J}_0^s,\vec{J}_\cap^l,\vec{J}_\cap^s\}\leftarrow\mathcal{E}_{ext}^{sca}(\vec{J}_0^{SE},\vec{M}_0^{SE})} \\ &\quad -\frac{\mu\,\Delta\varepsilon_c^*}{\mu_0\varepsilon_0-\mu\varepsilon_c^*}\left[\overline{\overline{P}}^{\{\vec{J}_0^{SE},\vec{J}_\cap^l,\vec{J}_\cap^s\}\leftarrow\{\mathcal{E}_{met}^{sca}(\vec{J}_0^l),\mathcal{E}_{met}^{sca}(\vec{J}_0^s)\}} + \frac{\mu_0}{\mu}\overline{\overline{P}}^{\vec{M}_0^{SE}\leftarrow\{\mathcal{H}_{met}^{sca}(\vec{J}_0^l),\mathcal{H}_{met}^{sca}(\vec{J}_0^s)\}}\right] \\ &\quad -\frac{\varepsilon_c\,\Delta\mu}{\varepsilon_0\mu_0-\varepsilon_c\mu}\left\{\frac{\varepsilon_0}{\varepsilon_c}\left[\overline{\overline{P}}^{\{\vec{J}_0^{SE},\vec{J}_\cap^l,\vec{J}_\cap^s\}\leftarrow\{\mathcal{E}_{met}^{sca}(\vec{J}_0^l),\mathcal{E}_{met}^{sca}(\vec{J}_0^s)\}}\right]^H + \left[\overline{\overline{P}}^{\vec{M}_0^{SE}\leftarrow\{\mathcal{H}_{met}^{sca}(\vec{J}_0^l),\mathcal{H}_{met}^{sca}(\vec{J}_0^s)\}}\right]^H\right\} \\ &\quad +\frac{\mu\,\Delta\varepsilon_c^*}{\mu_0\varepsilon_0-\mu\varepsilon_c^*}\left[\overline{\overline{P}}^{\{\vec{J}_0^{SE},\vec{J}_\cap^l,\vec{J}_\cap^s\}\leftarrow e_{int}^{inc}(\vec{J}_0^{SE},\vec{M}_0^{SE})} + \frac{\mu_0}{\mu}\overline{\overline{P}}^{\vec{M}_0^{SE}\leftarrow h_{int}^{inc}(\vec{J}_0^{SE},\vec{M}_0^{SE})}\right] \\ &\quad +\frac{\varepsilon_c\,\Delta\mu}{\varepsilon_0\mu_0-\varepsilon_c\mu}\left\{\frac{\varepsilon_0}{\varepsilon_c}\left[\overline{\overline{P}}^{\{\vec{J}_0^{SE},\vec{J}_\cap^l,\vec{J}_\cap^s\}\leftarrow e_{int}^{inc}(\vec{J}_0^{SE},\vec{M}_0^{SE})}\right]^H + \left[\overline{\overline{P}}^{\vec{M}_0^{SE}\leftarrow h_{int}^{inc}(\vec{J}_0^{SE},\vec{M}_0^{SE})}\right]^H\right\}\end{aligned} \tag{70}$$

and

$$\bar{a}^{\{\vec{J}_0^l,\vec{J}_0^s,\vec{J}_0^{SE},\vec{J}_\cap^l,\vec{J}_\cap^s,\vec{M}_0^{SE}\}} = \begin{bmatrix}\bar{a}^{\vec{J}_0^l}\\ \bar{a}^{\vec{J}_0^s}\\ \bar{a}^{\vec{J}_0^{SE}}\\ \bar{a}^{\vec{J}_\cap^l}\\ \bar{a}^{\vec{J}_\cap^s}\\ \bar{a}^{\vec{M}_0^{SE}}\end{bmatrix} = \begin{bmatrix}\bar{a}^{\vec{J}_0^l}\\ \bar{a}^{\vec{J}_0^s}\\ \bar{a}^{\{\vec{J}_0^{SE},\vec{J}_\cap^l,\vec{J}_\cap^s\}}\\ \bar{a}^{\vec{M}_0^{SE}}\end{bmatrix} = \begin{bmatrix}\bar{a}^{\vec{J}_0^l}\\ \bar{a}^{\vec{J}_0^s}\\ \bar{a}^{\vec{J}_0^{SE}}\\ \bar{a}^{\{\vec{J}_\cap^l,\vec{J}_\cap^s,\vec{M}_0^{SE}\}}\end{bmatrix} \tag{71}$$

The various matrices in (70) are as follows

$$\overline{\overline{P}}^{\{\vec{J}_0^l,\vec{J}_0^s,\vec{J}_\cap^l,\vec{J}_\cap^s\}\leftarrow\{\mathcal{E}_{met}^{sca}(\vec{J}_0^l),\mathcal{E}_{met}^{sca}(\vec{J}_0^s)\}} = \begin{bmatrix}\overline{\overline{\Phi}}^{J_0^l E_{met}^{sca} J_0^l} & \overline{\overline{\Phi}}^{J_0^l E_{met}^{sca} J_0^s} & 0 & 0 & 0 & 0\\ \overline{\overline{\Phi}}^{J_0^s E_{met}^{sca} J_0^l} & \overline{\overline{\Phi}}^{J_0^s E_{met}^{sca} J_0^s} & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ \overline{\overline{\Phi}}^{J_\cap^l E_{met}^{sca} J_0^l} & \overline{\overline{\Phi}}^{J_\cap^l E_{met}^{sca} J_0^s} & 0 & 0 & 0 & 0\\ \overline{\overline{\Phi}}^{J_\cap^s E_{met}^{sca} J_0^l} & \overline{\overline{\Phi}}^{J_\cap^s E_{met}^{sca} J_0^s} & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix} \tag{72}$$

$$\overline{\overline{P}}^{\{\vec{J}_0^l,\vec{J}_0^s,\vec{J}_\cap^l,\vec{J}_\cap^s\}\leftarrow\mathcal{E}_{ext}^{sca}(\vec{J}_0^{SE},\vec{M}_0^{SE})} = \begin{bmatrix}0 & 0 & \overline{\overline{\Phi}}^{J_0^l E_{ext}^{sca} J_0^{SE}} & 0 & 0 & \overline{\overline{\Phi}}^{J_0^l E_{ext}^{sca} M_0^{SE}}\\ 0 & 0 & \overline{\overline{\Phi}}^{J_0^s E_{ext}^{sca} J_0^{SE}} & 0 & 0 & \overline{\overline{\Phi}}^{J_0^s E_{ext}^{sca} M_0^{SE}}\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & \overline{\overline{\Phi}}^{J_\cap^l E_{ext}^{sca} J_0^{SE}} & 0 & 0 & \overline{\overline{\Phi}}^{J_\cap^l E_{ext}^{sca} M_0^{SE}}\\ 0 & 0 & \overline{\overline{\Phi}}^{J_\cap^s E_{ext}^{sca} J_0^{SE}} & 0 & 0 & \overline{\overline{\Phi}}^{J_\cap^s E_{ext}^{sca} M_0^{SE}}\\ 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix} \tag{73}$$

$$\overline{\overline{P}}^{\{\vec{J}_0^{SE},\vec{J}_\cap^l,\vec{J}_\cap^s\}\leftarrow\{\mathcal{E}_{met}^{sca}(\vec{J}_0^l),\mathcal{E}_{met}^{sca}(\vec{J}_0^s)\}} = \begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ \overline{\overline{\Phi}}^{J_0^{SE} E_{met}^{sca} J_0^l} & \overline{\overline{\Phi}}^{J_0^{SE} E_{met}^{sca} J_0^s} & 0 & 0 & 0 & 0\\ \overline{\overline{\Phi}}^{J_\cap^l E_{met}^{sca} J_0^l} & \overline{\overline{\Phi}}^{J_\cap^l E_{met}^{sca} J_0^s} & 0 & 0 & 0 & 0\\ \overline{\overline{\Phi}}^{J_\cap^s E_{met}^{sca} J_0^l} & \overline{\overline{\Phi}}^{J_\cap^s E_{met}^{sca} J_0^s} & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix} \tag{74}$$

$$\overline{\overline{P}}^{\vec{M}_0^{SE}\leftarrow\{\mathcal{H}_{met}^{sca}(\vec{J}_0^l),\mathcal{H}_{met}^{sca}(\vec{J}_0^s)\}} = \begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ \overline{\overline{\Phi}}^{M_0^{SE} H_{met}^{sca} J_0^l} & \overline{\overline{\Phi}}^{M_0^{SE} H_{met}^{sca} J_0^s} & 0 & 0 & 0 & 0\end{bmatrix} \tag{75}$$

$$\overline{\overline{P}}^{\{\vec{J}_0^{SE},\vec{J}_\cap^l,\vec{J}_\cap^s\}\leftarrow e_{int}^{inc}(\vec{J}_0^{SE},\vec{M}_0^{SE})} = \begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & \overline{\overline{\Phi}}^{J_0^{SE} e_{int}^{inc} J_0^{SE}} & 0 & 0 & \overline{\overline{\Phi}}^{J_0^{SE} e_{int}^{inc} M_0^{SE}}\\ 0 & 0 & \overline{\overline{\Phi}}^{J_\cap^l e_{int}^{inc} J_0^{SE}} & 0 & 0 & \overline{\overline{\Phi}}^{J_\cap^l e_{int}^{inc} M_0^{SE}}\\ 0 & 0 & \overline{\overline{\Phi}}^{J_\cap^s e_{int}^{inc} J_0^{SE}} & 0 & 0 & \overline{\overline{\Phi}}^{J_\cap^s e_{int}^{inc} M_0^{SE}}\\ 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix} \tag{76}$$

$$\overline{\overline{P}}^{\vec{M}_0^{SE}\leftarrow h_{int}^{inc}(\vec{J}_0^{SE},\vec{M}_0^{SE})} = \begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & \overline{\overline{\Phi}}^{M_0^{SE} h_{int}^{inc} J_0^{SE}} & 0 & 0 & \overline{\overline{\Phi}}^{M_0^{SE} h_{int}^{inc} M_0^{SE}}\end{bmatrix} \tag{77}$$

The submatrices in (72) are as follows

$$\overline{\overline{\Phi}}^{J_0^l E_{met}^{sca} J_0^l} = \left[\phi_{\xi\zeta}^{J_0^l E_{met}^{sca} J_0^l}\right]_{\Xi^{\vec{J}_0^l}\times\Xi^{\vec{J}_0^l}} \tag{78.1}$$

$$\overline{\overline{\Phi}}^{J_0^l E_{met}^{sca} J_0^s} = \left[\phi_{\xi\zeta}^{J_0^l E_{met}^{sca} J_0^s}\right]_{\Xi^{\vec{J}_0^l}\times\Xi^{\vec{J}_0^s}} \tag{78.2}$$

$$\overline{\overline{\Phi}}^{J_0^s E_{met}^{sca} J_0^l} = \left[\phi_{\xi\zeta}^{J_0^s E_{met}^{sca} J_0^l}\right]_{\Xi^{\vec{J}_0^s}\times\Xi^{\vec{J}_0^l}} \tag{78.3}$$

$$\overline{\overline{\Phi}}^{J_0^s E_{met}^{sca} J_0^s} = \left[\phi_{\xi\zeta}^{J_0^s E_{met}^{sca} J_0^s}\right]_{\Xi^{\vec{J}_0^s}\times\Xi^{\vec{J}_0^s}} \tag{78.4}$$

$$\overline{\overline{\Phi}}^{J_\cap^l E_{met}^{sca} J_0^l} = \left[\phi_{\xi\zeta}^{J_\cap^l E_{met}^{sca} J_0^l}\right]_{\Xi^{\vec{J}_\cap^l}\times\Xi^{\vec{J}_0^l}} \tag{78.5}$$

$$\overline{\overline{\Phi}}^{J_\cap^l E_{met}^{sca} J_0^s} = \left[\phi_{\xi\zeta}^{J_\cap^l E_{met}^{sca} J_0^s}\right]_{\Xi^{\vec{J}_\cap^l}\times\Xi^{\vec{J}_0^s}} \tag{78.6}$$

$$\overline{\overline{\Phi}}^{J_\cap^s E_{met}^{sca} J_0^l} = \left[\phi_{\xi\zeta}^{J_\cap^s E_{met}^{sca} J_0^l}\right]_{\Xi^{\vec{J}_\cap^s}\times\Xi^{\vec{J}_0^l}} \tag{78.7}$$

$$\overline{\overline{\Phi}}^{J_\cap^s E_{met}^{sca} J_0^s} = \left[\phi_{\xi\zeta}^{J_\cap^s E_{met}^{sca} J_0^s}\right]_{\Xi^{\vec{J}_\cap^s}\times\Xi^{\vec{J}_0^s}} \tag{78.8}$$

and the submatrices in (73) are as follows

$$\overline{\overline{\Phi}}^{J_0^l E_{ext}^{sca} J_0^{SE}} = \left[\phi_{\xi\zeta}^{J_0^l E_{ext}^{sca} J_0^{SE}}\right]_{\Xi^{\vec{J}_0^l}\times\Xi^{\vec{J}_0^{SE}}} \tag{79.1}$$

$$\overline{\overline{\Phi}}^{J_0^l E_{ext}^{sca} M_0^{SE}} = \left[\phi_{\xi\zeta}^{J_0^l E_{ext}^{sca} M_0^{SE}}\right]_{\Xi^{\vec{J}_0^l}\times\Xi^{\vec{M}_0^{SE}}} \tag{79.2}$$

$$\overline{\overline{\Phi}}^{J_0^s E_{ext}^{sca} J_0^{SE}} = \left[\phi_{\xi\zeta}^{J_0^s E_{ext}^{sca} J_0^{SE}}\right]_{\Xi^{\vec{J}_0^s}\times\Xi^{\vec{J}_0^{SE}}} \tag{79.3}$$

$$\overline{\overline{\Phi}}^{J_0^s E_{ext}^{sca} M_0^{SE}} = \left[\phi_{\xi\zeta}^{J_0^s E_{ext}^{sca} M_0^{SE}}\right]_{\Xi^{\vec{J}_0^s}\times\Xi^{\vec{M}_0^{SE}}} \tag{79.4}$$

$$\overline{\overline{\Phi}}^{J_\cap^l E_{ext}^{sca} J_0^{SE}} = \left[\phi_{\xi\zeta}^{J_\cap^l E_{ext}^{sca} J_0^{SE}}\right]_{\Xi^{\vec{J}_\cap^l}\times\Xi^{\vec{J}_0^{SE}}} \tag{79.5}$$

$$\overline{\overline{\Phi}}^{J_{\cap}^{l}E_{ext}^{sca}M_{0}^{SE}} = \left[\phi_{\xi\zeta}^{J_{\cap}^{l}E_{ext}^{sca}M_{0}^{SE}}\right]_{\Xi^{\vec{J}_{\cap}^{l}}\times\Xi^{\vec{M}_{0}^{SE}}} \tag{79.6}$$

$$\overline{\overline{\Phi}}^{J_{\cap}^{s}E_{ext}^{sca}J_{0}^{SE}} = \left[\phi_{\xi\zeta}^{J_{\cap}^{s}E_{ext}^{sca}J_{0}^{SE}}\right]_{\Xi^{\vec{J}_{\cap}^{s}}\times\Xi^{\vec{J}_{0}^{SE}}} \tag{79.7}$$

$$\overline{\overline{\Phi}}^{J_{\cap}^{s}E_{ext}^{sca}M_{0}^{SE}} = \left[\phi_{\xi\zeta}^{J_{\cap}^{s}E_{ext}^{sca}M_{0}^{SE}}\right]_{\Xi^{\vec{J}_{\cap}^{s}}\times\Xi^{\vec{M}_{0}^{SE}}} \tag{79.8}$$

and the submatrices $\overline{\overline{\Phi}}^{J_{\cap}^{l}E_{met}^{sca}J_{0}^{l}}$, $\overline{\overline{\Phi}}^{J_{\cap}^{l}E_{met}^{sca}J_{0}^{s}}$, $\overline{\overline{\Phi}}^{J_{\cap}^{s}E_{met}^{sca}J_{0}^{l}}$, and $\overline{\overline{\Phi}}^{J_{\cap}^{s}E_{met}^{sca}J_{0}^{s}}$ in (74) are given in the (78.5)-(78.8), and the other submatrices in (74) are as follows

$$\overline{\overline{\Phi}}^{J_{0}^{SE}E_{met}^{sca}J_{0}^{l}} = \left[\phi_{\xi\zeta}^{J_{0}^{SE}E_{met}^{sca}J_{0}^{l}}\right]_{\Xi^{\vec{J}_{0}^{SE}}\times\Xi^{\vec{J}_{0}^{l}}} \tag{80.1}$$

$$\overline{\overline{\Phi}}^{J_{0}^{SE}E_{met}^{sca}J_{0}^{s}} = \left[\phi_{\xi\zeta}^{J_{0}^{SE}E_{met}^{sca}J_{0}^{s}}\right]_{\Xi^{\vec{J}_{0}^{SE}}\times\Xi^{\vec{J}_{0}^{s}}} \tag{80.2}$$

and the submatrices in (75) are as follows

$$\overline{\overline{\Phi}}^{M_{0}^{SE}H_{met}^{sca}J_{0}^{l}} = \left[\phi_{\xi\zeta}^{M_{0}^{SE}H_{met}^{sca}J_{0}^{l}}\right]_{\Xi^{\vec{M}_{0}^{SE}}\times\Xi^{\vec{J}_{0}^{l}}} \tag{81.1}$$

$$\overline{\overline{\Phi}}^{M_{0}^{SE}H_{met}^{sca}J_{0}^{s}} = \left[\phi_{\xi\zeta}^{M_{0}^{SE}H_{met}^{sca}J_{0}^{s}}\right]_{\Xi^{\vec{M}_{0}^{SE}}\times\Xi^{\vec{J}_{0}^{s}}} \tag{81.2}$$

and the submatrices in (76) are as follows

$$\overline{\overline{\Phi}}^{J_{0}^{SE}e_{int}^{inc}J_{0}^{SE}} = \left[\phi_{\xi\zeta}^{J_{0}^{SE}e_{int}^{inc}J_{0}^{SE}}\right]_{\Xi^{\vec{J}_{0}^{SE}}\times\Xi^{\vec{J}_{0}^{SE}}} \tag{82.1}$$

$$\overline{\overline{\Phi}}^{J_{0}^{SE}e_{int}^{inc}M_{0}^{SE}} = \left[\phi_{\xi\zeta}^{J_{0}^{SE}e_{int}^{inc}M_{0}^{SE}}\right]_{\Xi^{\vec{J}_{0}^{SE}}\times\Xi^{\vec{M}_{0}^{SE}}} \tag{82.2}$$

$$\overline{\overline{\Phi}}^{J_{\cap}^{l}e_{int}^{inc}J_{0}^{SE}} = \left[\phi_{\xi\zeta}^{J_{\cap}^{l}e_{int}^{inc}J_{0}^{SE}}\right]_{\Xi^{\vec{J}_{\cap}^{l}}\times\Xi^{\vec{J}_{0}^{SE}}} \tag{82.3}$$

$$\overline{\overline{\Phi}}^{J_{\cap}^{l}e_{int}^{inc}M_{0}^{SE}} = \left[\phi_{\xi\zeta}^{J_{\cap}^{l}e_{int}^{inc}M_{0}^{SE}}\right]_{\Xi^{\vec{J}_{\cap}^{l}}\times\Xi^{\vec{M}_{0}^{SE}}} \tag{82.4}$$

$$\overline{\overline{\Phi}}^{J_{\cap}^{s}e_{int}^{inc}J_{0}^{SE}} = \left[\phi_{\xi\zeta}^{J_{\cap}^{s}e_{int}^{inc}J_{0}^{SE}}\right]_{\Xi^{\vec{J}_{\cap}^{s}}\times\Xi^{\vec{J}_{0}^{SE}}} \tag{82.5}$$

$$\overline{\overline{\Phi}}^{J_{\cap}^{s}e_{int}^{inc}M_{0}^{SE}} = \left[\phi_{\xi\zeta}^{J_{\cap}^{s}e_{int}^{inc}M_{0}^{SE}}\right]_{\Xi^{\vec{J}_{\cap}^{s}}\times\Xi^{\vec{M}_{0}^{SE}}} \tag{82.6}$$

and the submatrices in (77) are as follows

$$\overline{\overline{\Phi}}^{M_{0}^{SE}h_{int}^{inc}J_{0}^{SE}} = \left[\phi_{\xi\zeta}^{M_{0}^{SE}h_{int}^{inc}J_{0}^{SE}}\right]_{\Xi^{\vec{M}_{0}^{SE}}\times\Xi^{\vec{J}_{0}^{SE}}} \tag{83.1}$$

$$\overline{\overline{\Phi}}^{M_{0}^{SE}h_{int}^{inc}M_{0}^{SE}} = \left[\phi_{\xi\zeta}^{M_{0}^{SE}h_{int}^{inc}M_{0}^{SE}}\right]_{\Xi^{\vec{M}_{0}^{SE}}\times\Xi^{\vec{M}_{0}^{SE}}} \tag{83.2}$$

The elements in (78) are as follows

$$\phi_{\xi\zeta}^{J_{0}^{l}E_{met}^{sca}J_{0}^{l}} = (1/2)\left\langle \vec{b}_{\xi}^{\vec{J}_{0}^{l}} , -j\omega\mu_{0}\mathcal{L}_{0}\left(\vec{b}_{\zeta}^{\vec{J}_{0}^{l}}\right)\right\rangle_{L_{0}^{met}} \tag{84.1}$$

$$\phi_{\xi\zeta}^{J_{0}^{l}E_{met}^{sca}J_{0}^{s}} = (1/2)\left\langle \vec{b}_{\xi}^{\vec{J}_{0}^{l}} , -j\omega\mu_{0}\mathcal{L}_{0}\left(\vec{b}_{\zeta}^{\vec{J}_{0}^{s}}\right)\right\rangle_{L_{0}^{met}} \tag{84.2}$$

$$\phi_{\xi\zeta}^{J_{0}^{s}E_{met}^{sca}J_{0}^{l}} = (1/2)\left\langle \vec{b}_{\xi}^{\vec{J}_{0}^{s}} , -j\omega\mu_{0}\mathcal{L}_{0}\left(\vec{b}_{\zeta}^{\vec{J}_{0}^{l}}\right)\right\rangle_{S_{0}^{met}\cup\partial V_{0}^{met}} \tag{84.3}$$

$$\phi_{\xi\zeta}^{J_{0}^{s}E_{met}^{sca}J_{0}^{s}} = (1/2)\left\langle \vec{b}_{\xi}^{\vec{J}_{0}^{s}} , -j\omega\mu_{0}\mathcal{L}_{0}\left(\vec{b}_{\zeta}^{\vec{J}_{0}^{s}}\right)\right\rangle_{S_{0}^{met}\cup\partial V_{0}^{met}} \tag{84.4}$$

$$\phi_{\xi\zeta}^{J_{\cap}^{l}E_{met}^{sca}J_{0}^{l}} = (1/2)\left\langle \vec{b}_{\xi}^{\vec{J}_{\cap}^{l}} , -j\omega\mu_{0}\mathcal{L}_{0}\left(\vec{b}_{\zeta}^{\vec{J}_{0}^{l}}\right)\right\rangle_{L_{\cap}^{met}} \tag{84.5}$$

$$\phi_{\xi\zeta}^{J_{\cap}^{l}E_{met}^{sca}J_{0}^{s}} = (1/2)\left\langle \vec{b}_{\xi}^{\vec{J}_{\cap}^{l}} , -j\omega\mu_{0}\mathcal{L}_{0}\left(\vec{b}_{\zeta}^{\vec{J}_{0}^{s}}\right)\right\rangle_{L_{\cap}^{met}} \tag{84.6}$$

$$\phi_{\xi\zeta}^{J_{\cap}^{s}E_{met}^{sca}J_{0}^{l}} = (1/2)\left\langle \vec{b}_{\xi}^{\vec{J}_{\cap}^{s}} , -j\omega\mu_{0}\mathcal{L}_{0}\left(\vec{b}_{\zeta}^{\vec{J}_{0}^{l}}\right)\right\rangle_{S_{\cap}^{met}\cup\partial V_{\cap}^{met}} \tag{84.7}$$

$$\phi_{\xi\zeta}^{J_{\cap}^{s}E_{met}^{sca}J_{0}^{s}} = (1/2)\left\langle \vec{b}_{\xi}^{\vec{J}_{\cap}^{s}} , -j\omega\mu_{0}\mathcal{L}_{0}\left(\vec{b}_{\zeta}^{\vec{J}_{0}^{s}}\right)\right\rangle_{S_{\cap}^{met}\cup\partial V_{\cap}^{met}} \tag{84.8}$$

and the elements in (79) are as follows

$$\phi_{\xi\zeta}^{J_{0}^{l}E_{ext}^{sca}J_{0}^{SE}} = (1/2)\left\langle \vec{b}_{\xi}^{\vec{J}_{0}^{l}} , j\omega\mu_{0}\mathcal{L}_{0}\left(\vec{b}_{\zeta}^{\vec{J}_{0}^{SE}}\right)\right\rangle_{L_{0}^{met}} \tag{85.1}$$

$$\phi_{\xi\zeta}^{J_{0}^{l}E_{ext}^{sca}M_{0}^{SE}} = (1/2)\left\langle \vec{b}_{\xi}^{\vec{J}_{0}^{l}} , \mathcal{K}_{0}\left(\vec{b}_{\zeta}^{\vec{M}_{0}^{SE}}\right)\right\rangle_{L_{0}^{met}} \tag{85.2}$$

$$\phi_{\xi\zeta}^{J_{0}^{s}E_{ext}^{sca}J_{0}^{SE}} = (1/2)\left\langle \vec{b}_{\xi}^{\vec{J}_{0}^{s}} , j\omega\mu_{0}\mathcal{L}_{0}\left(\vec{b}_{\zeta}^{\vec{J}_{0}^{SE}}\right)\right\rangle_{S_{0}^{met}\cup\partial V_{0}^{met}} \tag{85.3}$$

$$\phi_{\xi\zeta}^{J_{0}^{s}E_{ext}^{sca}M_{0}^{SE}} = (1/2)\left\langle \vec{b}_{\xi}^{\vec{J}_{0}^{s}} , \mathcal{K}_{0}\left(\vec{b}_{\zeta}^{\vec{M}_{0}^{SE}}\right)\right\rangle_{S_{0}^{met}\cup\partial V_{0}^{met}} \tag{85.4}$$

$$\phi_{\xi\zeta}^{J_{\cap}^{l}E_{ext}^{sca}J_{0}^{SE}} = (1/2)\left\langle \vec{b}_{\xi}^{\vec{J}_{\cap}^{l}} , j\omega\mu_{0}\mathcal{L}_{0}\left(\vec{b}_{\zeta}^{\vec{J}_{0}^{SE}}\right)\right\rangle_{L_{\cap}^{met}} \tag{85.5}$$

$$\phi_{\xi\zeta}^{J_{\cap}^{l}E_{ext}^{sca}M_{0}^{SE}} = (1/2)\left\langle \vec{b}_{\xi}^{\vec{J}_{\cap}^{l}} , \mathcal{K}_{0}\left(\vec{b}_{\zeta}^{\vec{M}_{0}^{SE}}\right)\right\rangle_{L_{\cap}^{met}} \tag{85.6}$$

$$\phi_{\xi\zeta}^{J_{\cap}^{s}E_{ext}^{sca}J_{0}^{SE}} = (1/2)\left\langle \vec{b}_{\xi}^{\vec{J}_{\cap}^{s}} , j\omega\mu_{0}\mathcal{L}_{0}\left(\vec{b}_{\zeta}^{\vec{J}_{0}^{SE}}\right)\right\rangle_{S_{\cap}^{met}\cup\partial V_{\cap}^{met}} \tag{85.7}$$

$$\phi_{\xi\zeta}^{J_{\cap}^{s}E_{ext}^{sca}M_{0}^{SE}} = (1/2)\left\langle \vec{b}_{\xi}^{\vec{J}_{\cap}^{s}} , \mathcal{K}_{0}\left(\vec{b}_{\zeta}^{\vec{M}_{0}^{SE}}\right)\right\rangle_{S_{\cap}^{met}\cup\partial V_{\cap}^{met}} \tag{85.8}$$

and the elements $\phi_{\xi\zeta}^{J_{\cap}^{l}E_{met}^{sca}J_{0}^{l}}$, $\phi_{\xi\zeta}^{J_{\cap}^{l}E_{met}^{sca}J_{0}^{s}}$, $\phi_{\xi\zeta}^{J_{\cap}^{s}E_{met}^{sca}J_{0}^{l}}$, and $\phi_{\xi\zeta}^{J_{\cap}^{s}E_{met}^{sca}J_{0}^{s}}$ in (80) are given in the (84.5)-(84.8), and the other elements in (80) are as follows

$$\phi_{\xi\zeta}^{J_{0}^{SE}E_{met}^{sca}J_{0}^{l}} = (1/2)\left\langle \vec{b}_{\xi}^{\vec{J}_{0}^{SE}} , -j\omega\mu_{0}\mathcal{L}_{0}\left(\vec{b}_{\zeta}^{\vec{J}_{0}^{l}}\right)\right\rangle_{\partial V_{0}^{mat}} \tag{86.1}$$

$$\phi_{\xi\zeta}^{J_{0}^{SE}E_{met}^{sca}J_{0}^{s}} = (1/2)\left\langle \vec{b}_{\xi}^{\vec{J}_{0}^{SE}} , -j\omega\mu_{0}\mathcal{L}_{0}\left(\vec{b}_{\zeta}^{\vec{J}_{0}^{s}}\right)\right\rangle_{\partial V_{0}^{mat}} \tag{86.2}$$

and the elements in (81) are as follows

$$\phi_{\xi\zeta}^{M_{0}^{SE}H_{met}^{sca}J_{0}^{l}} = (1/2)\left\langle \vec{b}_{\xi}^{\vec{M}_{0}^{SE}} , \mathcal{K}_{0}\left(\vec{b}_{\zeta}^{\vec{J}_{0}^{l}}\right)\right\rangle_{\partial V_{0}^{mat}} \tag{87.1}$$

$$\phi_{\xi\zeta}^{M_{0}^{SE}H_{met}^{sca}J_{0}^{s}} = (1/2)\left\langle \vec{b}_{\xi}^{\vec{M}_{0}^{SE}} , \mathcal{K}_{0}\left(\vec{b}_{\zeta}^{\vec{J}_{0}^{s}}\right)\right\rangle_{\partial V_{0}^{mat}} \tag{87.2}$$

and the elements in (82) are as follows

$$\phi_{\xi\zeta}^{J_{0}^{SE}e_{int}^{inc}J_{0}^{SE}} = (1/2)\left\langle \vec{b}_{\xi}^{\vec{J}_{0}^{SE}} , -j\omega\mu_{0}\mathcal{L}_{0}\left(\vec{b}_{\zeta}^{\vec{J}_{0}^{SE}}\right)\right\rangle_{\partial V_{0}^{mat}} \tag{88.1}$$

$$\begin{aligned}\phi_{\xi\zeta}^{J_{0}^{SE}e_{int}^{inc}M_{0}^{SE}} &= (1/2)\left\langle \vec{b}_{\xi}^{\vec{J}_{0}^{SE}} , -\mathcal{K}_{0}\left(\vec{b}_{\zeta}^{\vec{M}_{0}^{SE}}\right)\right\rangle_{\partial V_{0}^{mat}} \\ &= (1/2)\left\langle \vec{b}_{\xi}^{\vec{J}_{0}^{SE}} , \frac{1}{2}\hat{n}_{\rightarrow mat}\times\vec{b}_{\zeta}^{\vec{M}_{0}^{SE}} - \text{P.V.}\,\mathcal{K}_{0}\left(\vec{b}_{\zeta}^{\vec{M}_{0}^{SE}}\right)\right\rangle_{\partial V_{0}^{mat}}\end{aligned} \tag{88.2}$$

$$\phi_{\xi\zeta}^{J_{\cap}^{l}e_{int}^{inc}J_{0}^{SE}} = (1/2)\left\langle \vec{b}_{\xi}^{\vec{J}_{\cap}^{l}} , -j\omega\mu_{0}\mathcal{L}_{0}\left(\vec{b}_{\zeta}^{\vec{J}_{0}^{SE}}\right)\right\rangle_{L_{\cap}^{met}} \tag{88.3}$$

$$\phi_{\xi\zeta}^{J_{\cap}^{l}e_{int}^{inc}M_{0}^{SE}} = (1/2)\left\langle \vec{b}_{\xi}^{\vec{J}_{\cap}^{l}} , -\mathcal{K}_{0}\left(\vec{b}_{\zeta}^{\vec{M}_{0}^{SE}}\right)\right\rangle_{L_{\cap}^{met}} \tag{88.4}$$

$$\phi_{\xi\zeta}^{J_{\cap}^{s}e_{int}^{inc}J_{0}^{SE}} = (1/2)\left\langle \vec{b}_{\xi}^{\vec{J}_{\cap}^{s}} , -j\omega\mu_{0}\mathcal{L}_{0}\left(\vec{b}_{\zeta}^{\vec{J}_{0}^{SE}}\right)\right\rangle_{S_{\cap}^{met}\cup\partial V_{\cap}^{met}} \tag{88.5}$$

$$\phi_{\xi\zeta}^{J_{\cap}^{s}e_{int}^{inc}M_{0}^{SE}} = (1/2)\left\langle \vec{b}_{\xi}^{\vec{J}_{\cap}^{s}} , -\mathcal{K}_{0}\left(\vec{b}_{\zeta}^{\vec{M}_{0}^{SE}}\right)\right\rangle_{S_{\cap}^{met}\cup\partial V_{\cap}^{met}} \tag{88.6}$$

and the elements in (83) are as follows

$$\begin{aligned}\phi_{\xi\zeta}^{M_{0}^{SE}h_{int}^{inc}J_{0}^{SE}} &= (1/2)\left\langle \vec{b}_{\xi}^{\vec{M}_{0}^{SE}} , \mathcal{K}_{0}\left(\vec{b}_{\zeta}^{\vec{J}_{0}^{SE}}\right)\right\rangle_{\partial V_{0}^{mat}} \\ &= (1/2)\left\langle \vec{b}_{\xi}^{\vec{M}_{0}^{SE}} , -\frac{1}{2}\hat{n}_{\rightarrow mat}\times\vec{b}_{\zeta}^{\vec{J}_{0}^{SE}} + \text{P.V.}\,\mathcal{K}_{0}\left(\vec{b}_{\zeta}^{\vec{J}_{0}^{SE}}\right)\right\rangle_{\partial V_{0}^{mat}}\end{aligned} \tag{89.1}$$

$$\phi_{\xi\zeta}^{M_0^{SE} h_{int}^{inc} M_0^{SE}} = (1/2)\left\langle \vec{b}_\xi^{\vec{M}_0^{SE}} \ , \ -j\omega\varepsilon_0 \mathcal{L}_0\left(\vec{b}_\zeta^{\vec{M}_0^{SE}}\right)\right\rangle_{\partial V_0^{mat}} \tag{89.2}$$

Employing (63) and (64), the (69) can be rewritten as follows

$$\begin{aligned} P^{inp} &= \left(\bar{a}^{\{\vec{J}_0^l,\vec{J}_0^s,\vec{J}_0^{SE},\vec{J}_\cap^l,\vec{J}_\cap^s\}}\right)^H \cdot \bar{\bar{P}}^{inp}_{\{\vec{J}_0^l,\vec{J}_0^s,\vec{J}_0^{SE},\vec{J}_\cap^l,\vec{J}_\cap^s\}} \cdot \bar{a}^{\{\vec{J}_0^l,\vec{J}_0^s,\vec{J}_0^{SE},\vec{J}_\cap^l,\vec{J}_\cap^s\}} \\ &= \left(\bar{a}^{\{\vec{J}_0^l,\vec{J}_0^s,\vec{J}_\cap^l,\vec{J}_\cap^s,\vec{M}_0^{SE}\}}\right)^H \cdot \bar{\bar{P}}^{inp}_{\{\vec{J}_0^l,\vec{J}_0^s,\vec{J}_\cap^l,\vec{J}_\cap^s,\vec{M}_0^{SE}\}} \cdot \bar{a}^{\{\vec{J}_0^l,\vec{J}_0^s,\vec{J}_\cap^l,\vec{J}_\cap^s,\vec{M}_0^{SE}\}} \end{aligned} \tag{90}$$

here

$$\bar{\bar{P}}^{inp}_{\{\vec{J}_0^l,\vec{J}_0^s,\vec{J}_0^{SE},\vec{J}_\cap^l,\vec{J}_\cap^s\}} = \begin{bmatrix} \bar{\bar{I}} & 0 & 0 \\ 0 & \bar{\bar{I}} & 0 \\ 0 & 0 & \bar{\bar{I}} \\ 0 & 0 & \bar{\bar{T}}^{\{\vec{J}_0^{SE},\vec{J}_\cap^l,\vec{J}_\cap^s\}\to\vec{M}_0^{SE}}_{Magnetic\ Extinction} \end{bmatrix}^H \cdot \bar{\bar{P}}^{inp}_{\{\vec{J}_0^l,\vec{J}_0^s,\vec{J}_0^{SE},\vec{J}_\cap^l,\vec{J}_\cap^s,\vec{M}_0^{SE}\}} \cdot \begin{bmatrix} \bar{\bar{I}} & 0 & 0 \\ 0 & \bar{\bar{I}} & 0 \\ 0 & 0 & \bar{\bar{I}} \\ 0 & 0 & \bar{\bar{T}}^{\{\vec{J}_0^{SE},\vec{J}_\cap^l,\vec{J}_\cap^s\}\to\vec{M}_0^{SE}}_{Magnetic\ Extinction} \end{bmatrix} \tag{91}$$

$$\bar{\bar{P}}^{inp}_{\{\vec{J}_0^l,\vec{J}_0^s,\vec{J}_\cap^l,\vec{J}_\cap^s,\vec{M}_0^{SE}\}} = \begin{bmatrix} \bar{\bar{I}} & 0 & 0 \\ 0 & \bar{\bar{I}} & 0 \\ 0 & 0 & \bar{\bar{T}}^{\{\vec{J}_\cap^l,\vec{J}_\cap^s,\vec{M}_0^{SE}\}\to\vec{J}_0^{SE}}_{Electric\ Extinction} \\ 0 & 0 & \bar{\bar{I}} \end{bmatrix}^H \cdot \bar{\bar{P}}^{inp}_{\{\vec{J}_0^l,\vec{J}_0^s,\vec{J}_0^{SE},\vec{J}_\cap^l,\vec{J}_\cap^s,\vec{M}_0^{SE}\}} \cdot \begin{bmatrix} \bar{\bar{I}} & 0 & 0 \\ 0 & \bar{\bar{I}} & 0 \\ 0 & 0 & \bar{\bar{T}}^{\{\vec{J}_\cap^l,\vec{J}_\cap^s,\vec{M}_0^{SE}\}\to\vec{J}_0^{SE}}_{Electric\ Extinction} \\ 0 & 0 & \bar{\bar{I}} \end{bmatrix} \tag{92}$$

and

$$\bar{a}^{\{\vec{J}_0^l,\vec{J}_0^s,\vec{J}_0^{SE},\vec{J}_\cap^l,\vec{J}_\cap^s\}} = \begin{bmatrix} \bar{a}^{\vec{J}_0^l} \\ \bar{a}^{\vec{J}_0^s} \\ \bar{a}^{\vec{J}_0^{SE}} \\ \bar{a}^{\vec{J}_\cap^l} \\ \bar{a}^{\vec{J}_\cap^s} \end{bmatrix} = \begin{bmatrix} \bar{a}^{\vec{J}_0^l} \\ \bar{a}^{\vec{J}_0^s} \\ \bar{a}^{\{\vec{J}_0^{SE},\vec{J}_\cap^l,\vec{J}_\cap^s\}} \end{bmatrix} \tag{93}$$

$$\bar{a}^{\{\vec{J}_0^l,\vec{J}_0^s,\vec{J}_\cap^l,\vec{J}_\cap^s,\vec{M}_0^{SE}\}} = \begin{bmatrix} \bar{a}^{\vec{J}_0^l} \\ \bar{a}^{\vec{J}_0^s} \\ \bar{a}^{\vec{J}_\cap^l} \\ \bar{a}^{\vec{J}_\cap^s} \\ \bar{a}^{\vec{M}_0^{SE}} \end{bmatrix} = \begin{bmatrix} \bar{a}^{\vec{J}_0^l} \\ \bar{a}^{\vec{J}_0^s} \\ \bar{a}^{\{\vec{J}_\cap^l,\vec{J}_\cap^s,\vec{M}_0^{SE}\}} \end{bmatrix} \tag{94}$$

The matrices $\bar{\bar{I}}$ in (91)-(92) are the identity matrices with suitable orders.

### *D. To construct power-based CM sets in expansion vector space*

The matrix $\bar{\bar{P}}^{inp}_{BV}$ can be decomposed into its Hermitian parts as follows

$$\bar{\bar{P}}^{inp}_{BV} = \bar{\bar{P}}^{inp}_{BV;+} + j\,\bar{\bar{P}}^{inp}_{BV;-} \tag{95}$$

here $BV = \{\vec{J}_0^l,\vec{J}_0^s,\vec{J}_0^{SE},\vec{J}_\cap^l,\vec{J}_\cap^s\}$ or $\{\vec{J}_0^l,\vec{J}_0^s,\vec{J}_\cap^l,\vec{J}_\cap^s,\vec{M}_0^{SE}\}$, and [1]-[3], [7]-[8], [10]-[11]

$$\bar{\bar{P}}^{inp}_{BV;+} = \frac{1}{2}\left[\bar{\bar{P}}^{inp}_{BV} + \left(\bar{\bar{P}}^{inp}_{BV}\right)^H\right] \tag{96.1}$$

$$\bar{\bar{P}}^{inp}_{BV;-} = \frac{1}{2j}\left[\bar{\bar{P}}^{inp}_{BV} - \left(\bar{\bar{P}}^{inp}_{BV}\right)^H\right] \tag{96.2}$$

Obviously, both the $\bar{\bar{P}}^{inp}_{BV;+}$ and $\bar{\bar{P}}^{inp}_{BV;-}$ are Hermitian [12].

Based on the discussions in [10]-[11], it can be concluded that the matrix $\bar{\bar{P}}^{inp}_{BV;+}$ is positive definite or semi-definite. When the matrix $\bar{\bar{P}}^{inp}_{BV;+}$ is positive definite at frequency $f$, the Input-power-based Characteristic Mode (InpCM) set can be derived from solving the following generalized characteristic equation [13]

$$\bar{\bar{P}}^{inp}_{BV;-}(f)\cdot\bar{a}_\xi^{BV}(f) = \lambda_\xi(f)\,\bar{\bar{P}}^{inp}_{BV;+}(f)\cdot\bar{a}_\xi^{BV}(f) \tag{97}$$

for any $\xi = 1,2,\cdots,\Xi$. In (97), $\Xi = \Xi^{\vec{J}_0^l} + \Xi^{\vec{J}_0^s} + \Xi^{\vec{J}_0^{SE}} + \Xi^{\vec{J}_\cap^l} + \Xi^{\vec{J}_\cap^s}$, and

$$\bar{a}_\xi^{BV}(f) = \begin{bmatrix} \bar{a}_\xi^{\vec{J}_0^l}(f) \\ \bar{a}_\xi^{\vec{J}_0^s}(f) \\ \bar{a}_\xi^{\vec{J}_0^{SE}}(f) \\ \bar{a}_\xi^{\vec{J}_\cap^l}(f) \\ \bar{a}_\xi^{\vec{J}_\cap^s}(f) \end{bmatrix} \tag{98}$$

$$\bar{a}_\xi^{\vec{M}_0^{SE}}(f) = \bar{\bar{T}}^{\{\vec{J}_0^{SE},\vec{J}_\cap^l,\vec{J}_\cap^s\}\to\vec{M}_0^{SE}}_{Magnetic\ Extinction} \cdot \begin{bmatrix} \bar{a}_\xi^{\vec{J}_0^{SE}}(f) \\ \bar{a}_\xi^{\vec{J}_\cap^l}(f) \\ \bar{a}_\xi^{\vec{J}_\cap^s}(f) \end{bmatrix} \tag{99}$$

if $BV = \{\vec{J}_0^l,\vec{J}_0^s,\vec{J}_0^{SE},\vec{J}_\cap^l,\vec{J}_\cap^s\}$; $\Xi = \Xi^{\vec{J}_0^l} + \Xi^{\vec{J}_0^s} + \Xi^{\vec{J}_\cap^l} + \Xi^{\vec{J}_\cap^s} + \Xi^{\vec{M}_0^{SE}}$, and

$$\bar{a}_\xi^{BV}(f) = \begin{bmatrix} \bar{a}_\xi^{\vec{J}_0^l}(f) \\ \bar{a}_\xi^{\vec{J}_0^s}(f) \\ \bar{a}_\xi^{\vec{J}_\cap^l}(f) \\ \bar{a}_\xi^{\vec{J}_\cap^s}(f) \\ \bar{a}_\xi^{\vec{M}_0^{SE}}(f) \end{bmatrix} \tag{100}$$

$$\bar{a}_\xi^{\vec{J}_0^{SE}}(f) = \bar{\bar{T}}^{\{\vec{J}_\cap^l,\vec{J}_\cap^s,\vec{M}_0^{SE}\}\to\vec{J}_0^{SE}}_{Electric\ Extinction} \cdot \begin{bmatrix} \bar{a}_\xi^{\vec{J}_\cap^l}(f) \\ \bar{a}_\xi^{\vec{J}_\cap^s}(f) \\ \bar{a}_\xi^{\vec{M}_0^{SE}}(f) \end{bmatrix} \tag{101}$$

if $BV = \{\vec{J}_0^l,\vec{J}_0^s,\vec{J}_\cap^l,\vec{J}_\cap^s,\vec{M}_0^{SE}\}$. When the matrix $\bar{\bar{P}}^{inp}_{BV;+}$ is positive semi-definite at frequency $f_0$, the frequency $f_0$ can be determined by employing the modal impedance or admittance defined in [2] and by using frequency sweep technique depicted in [10]-[11]; after the frequency $f_0$ is determined, the characteristic values and vectors at $f_0$ can be obtained by using the following limitations [10]-[11]

$$\lambda_\xi(f_0) = \lim_{f\to f_0}\lambda_\xi(f) \tag{102}$$

$$\bar{a}_\xi^{BV}(f_0) = \lim_{f\to f_0}\bar{a}_\xi^{BV}(f) \tag{103}$$

$$\bar{a}_\xi^Y(f_0) \;=\; \lim_{f\to f_0} \bar{a}_\xi^Y(f) \tag{104}$$

here $Y = \vec{J}_0^l, \vec{J}_0^s, \vec{J}_0^{SE}, \vec{J}_\cap^l, \vec{J}_\cap^s, \vec{M}_0^{SE}$.

The characteristic currents $\{\vec{J}_{0;\xi}^l, \vec{J}_{\cap;\xi}^l\}$, $\{\vec{J}_{0;\xi}^s, \vec{J}_{\cap;\xi}^s\}$, and $\{\vec{J}_{0;\xi}^{SE}, \vec{M}_{0;\xi}^{SE}\}$ are as follows

$$\vec{J}_{0/\cap;\xi}^l(\vec{r}) \;=\; \bar{B}^{\vec{J}_{0/\cap}^l} \cdot \bar{a}_\xi^{\vec{J}_{0/\cap}^l} \quad,\quad \left(\vec{r} \in L_{0/\cap}^{met}\right) \tag{105}$$

$$\vec{J}_{0/\cap;\xi}^s(\vec{r}) \;=\; \bar{B}^{\vec{J}_{0/\cap}^s} \cdot \bar{a}_\xi^{\vec{J}_{0/\cap}^s} \quad,\quad \left(\vec{r} \in S_{0/\cap}^{met} \cup \partial V_{0/\cap}^{met}\right) \tag{106}$$

$$\vec{C}_{0;\xi}^{SE}(\vec{r}) \;=\; \bar{B}^{\vec{C}_0^{SE}} \cdot \bar{a}_\xi^{\vec{C}_0^{SE}} \quad,\quad \left(\vec{r} \in \partial V_0^{mat}\right) \tag{107}$$

here $C = J, M$, and $\vec{J}_\xi^{LE} = \vec{J}_{\cap;\xi}^l$, and $\vec{J}_{\cap;\xi}^{SE} = \vec{J}_{\cap;\xi}^s$, and

$$\vec{J}_\xi^l(\vec{r}) \;=\; \vec{J}_{0;\xi}^l(\vec{r}) + \vec{J}_{\cap;\xi}^l(\vec{r}) \quad,\quad \left(\vec{r} \in L^{met}\right) \tag{108}$$

$$\vec{J}_\xi^s(\vec{r}) \;=\; \vec{J}_{0;\xi}^s(\vec{r}) + \vec{J}_{\cap;\xi}^s(\vec{r}) \quad,\quad \left(\vec{r} \in S^{met} \cup \partial V^{met}\right) \tag{109}$$

$$\vec{J}_\xi^{SE}(\vec{r}) = \vec{J}_{0;\xi}^{SE}(\vec{r}) + \vec{J}_{\cap;\xi}^{SE}(\vec{r}) \quad,\quad \left(\vec{r} \in \partial V_{surf}^{mat}\right) \tag{110}$$

and then the characteristic fields are as follows

$$\begin{aligned}\vec{f}_{int;\xi}^{inc}(\vec{r}) &= \mathcal{f}_{int}^{inc}\left(\vec{J}_\xi^{LE},0\right) + \mathcal{f}_{int}^{inc}\left(\vec{J}_{\cap;\xi}^{SE},0\right) + \mathcal{f}_{int}^{inc}\left(\vec{J}_{0;\xi}^{SE},\vec{M}_{0;\xi}^{SE}\right)\\ &= \mathcal{f}_{int}^{inc}\left(\vec{J}_{\cap;\xi}^{l},0\right) + \mathcal{f}_{int}^{inc}\left(\vec{J}_{\cap;\xi}^{s},0\right) + \mathcal{f}_{int}^{inc}\left(\vec{J}_{0;\xi}^{SE},\vec{M}_{0;\xi}^{SE}\right) \quad,\quad \left(\vec{r} \in \operatorname{int} V^{mat}\right)\end{aligned} \tag{111}$$

$$\begin{aligned}\vec{F}_\xi^{tot}(\vec{r}) &= \mathcal{F}_{int}^{tot}\left(\vec{J}_\xi^{LE},0\right) + \mathcal{F}_{int}^{tot}\left(\vec{J}_{\cap;\xi}^{SE},0\right) + \mathcal{F}_{int}^{tot}\left(\vec{J}_{0;\xi}^{SE},\vec{M}_{0;\xi}^{SE}\right)\\ &= \mathcal{F}_{int}^{tot}\left(\vec{J}_{\cap;\xi}^{l},0\right) + \mathcal{F}_{int}^{tot}\left(\vec{J}_{\cap;\xi}^{s},0\right) + \mathcal{F}_{int}^{tot}\left(\vec{J}_{0;\xi}^{SE},\vec{M}_{0;\xi}^{SE}\right) \quad,\quad \left(\vec{r} \in \operatorname{int} V^{mat}\right)\end{aligned} \tag{112}$$

$$\begin{aligned}\vec{F}_{mat;\xi}^{sca}(\vec{r}) &= \begin{cases}\mathcal{F}_{ext}^{sca}\left(\vec{J}_\xi^{LE},0\right) + \mathcal{F}_{ext}^{sca}\left(\vec{J}_{\cap;\xi}^{SE},0\right) + \mathcal{F}_{ext}^{sca}\left(\vec{J}_{0;\xi}^{SE},\vec{M}_{0;\xi}^{SE}\right) &,\ \left(\vec{r} \in \operatorname{ext} V^{mat}\right)\\ \vec{F}_\xi^{tot}(\vec{r}) - \vec{f}_{int;\xi}^{inc}(\vec{r}) &,\ \left(\vec{r} \in \operatorname{int} V^{mat}\right)\end{cases}\\ &= \begin{cases}\mathcal{F}_{ext}^{sca}\left(\vec{J}_{\cap;\xi}^l,0\right) + \mathcal{F}_{ext}^{sca}\left(\vec{J}_{\cap;\xi}^{s},0\right) + \mathcal{F}_{ext}^{sca}\left(\vec{J}_{0;\xi}^{SE},\vec{M}_{0;\xi}^{SE}\right) &,\ \left(\vec{r} \in \operatorname{ext} V^{mat}\right)\\ \vec{F}_\xi^{tot}(\vec{r}) - \vec{f}_{int;\xi}^{inc}(\vec{r}) &,\ \left(\vec{r} \in \operatorname{int} V^{mat}\right)\end{cases}\end{aligned} \tag{113}$$

$$\begin{aligned}\vec{F}_{met;\xi}^{sca}(\vec{r}) &= \mathcal{F}_{met}^{sca}\left(\vec{J}_\xi^l\right) + \mathcal{F}_{met}^{sca}\left(\vec{J}_\xi^s\right)\\ &= \mathcal{F}_{met}^{sca}\left(\vec{J}_{0;\xi}^l\right) + \mathcal{F}_{met}^{sca}\left(\vec{J}_{\cap;\xi}^l\right) + \mathcal{F}_{met}^{sca}\left(\vec{J}_{0;\xi}^s\right) + \mathcal{F}_{met}^{sca}\left(\vec{J}_{\cap;\xi}^s\right) \ ,\ \left(\vec{r} \in \mathbb{R}^3 \setminus \partial D^{met}\right)\end{aligned} \tag{114}$$

here the domain $\partial D^{met}$ in (114) is the whole boundary of metal part, i.e., $\partial D^{met} = L^{met} \cup S^{met} \cup \partial V^{met}$. Then, the characteristic currents $\{\vec{J}_\xi^{vop}, \vec{M}_\xi^{vm}\}$ and the characteristic scattering and incident fields are as follows

$$\vec{J}_\xi^{vop}(\vec{r}) \;=\; j\omega\Delta\varepsilon_c \vec{E}_\xi^{tot}(\vec{r}) \quad,\quad \left(\vec{r} \in \operatorname{int} V^{mat}\right) \tag{115.1}$$

$$\vec{M}_\xi^{vm}(\vec{r}) \;=\; j\omega\Delta\mu \vec{H}_\xi^{tot}(\vec{r}) \quad,\quad \left(\vec{r} \in \operatorname{int} V^{mat}\right) \tag{115.2}$$

and

$$\vec{F}_\xi^{sca}(\vec{r}) \;=\; \vec{F}_{mat;\xi}^{sca}(\vec{r}) + \vec{F}_{met;\xi}^{sca}(\vec{r}) \ ,\ \left(\vec{r} \in \mathbb{R}^3 \setminus \partial D\right) \tag{116}$$

and

$$\vec{E}_{\zeta;\tan}^{inc}(\vec{r}) = -\lim_{\vec{r}'\to\vec{r}} \vec{E}_{\zeta;\tan}^{sca}(\vec{r}') \quad,\quad \left(\vec{r} \in \partial D^{met}\right) \tag{117.1}$$

$$\vec{F}_\xi^{inc}(\vec{r}) \;=\; \vec{f}_{int;\xi}^{inc}(\vec{r}) - \vec{F}_{met;\xi}^{sca}(\vec{r}) \ ,\ \left(\vec{r} \in \operatorname{int} V^{mat}\right) \tag{117.2}$$

here the $\Delta\varepsilon_c$ and $\Delta\mu$ in (115) are defined as $\Delta\varepsilon_c \triangleq \varepsilon_c - \varepsilon_0$, and $\Delta\mu \triangleq \mu - \mu_0$; the domain $\partial D$ in (116) is the union of $\partial D^{met}$ and $\partial V^{mat}$, i.e., $\partial D = \partial D^{met} \cup \partial V^{mat} = L^{met} \cup S^{met} \cup \partial V^{met} \cup \partial V^{mat}$; the subscript "$\tan$" in (117.1) represents the tangential component of field, and $\vec{r}' \in \mathbb{R}^3 \setminus \partial D$.

If the necessary orthogonalization is done for the degenerate modes, the characteristic currents and the characteristic fields satisfy the following input power orthogonality

$$\begin{aligned}P_\xi^{inp}\delta_{\xi\zeta} \;=\; &(1/2)\left\langle \vec{J}_\xi^l \oplus \vec{J}_\xi^s , \vec{E}_{\zeta;\tan}^{inc}\right\rangle_{L^{met}\cup S^{met}\cup\partial V^{met}}\\ &+(1/2)\left\langle \vec{J}_\xi^{vop}, \vec{E}_\zeta^{inc}\right\rangle_{V^{mat}} + (1/2)\left\langle \vec{H}_\xi^{inc}, \vec{M}_\zeta^{vm}\right\rangle_{V^{mat}}\end{aligned} \tag{118}$$

and the following output power orthogonality

$$\operatorname{Re}\{P_\xi^{out}\}\delta_{\xi\zeta} = \frac{1}{2}\oiint_{S_\infty}\left[\vec{E}_\zeta^{sca} \times \left(\vec{H}_\xi^{sca}\right)^*\right]\cdot d\vec{S} + \frac{1}{2}\left\langle \sigma\vec{E}_\xi^{tot}, \vec{E}_\zeta^{tot}\right\rangle_{V^{mat}} \tag{119.1}$$

$$\begin{aligned}\operatorname{Im}\{P_\xi^{out}\}\delta_{\xi\zeta} = 2\omega\Bigg\{&\left[\frac{1}{4}\left\langle \vec{H}_\xi^{sca}, \mu_0\vec{H}_\zeta^{sca}\right\rangle_{\mathbb{R}^3} - \frac{1}{4}\left\langle \varepsilon_0\vec{E}_\xi^{sca}, \vec{E}_\zeta^{sca}\right\rangle_{\mathbb{R}^3}\right]\\ &+\left[\frac{1}{4}\left\langle \vec{H}_\xi^{tot}, \Delta\mu\vec{H}_\zeta^{tot}\right\rangle_{V^{mat}} - \frac{1}{4}\left\langle \Delta\varepsilon\vec{E}_\xi^{tot}, \vec{E}_\zeta^{tot}\right\rangle_{V^{mat}}\right]\Bigg\}\end{aligned} \tag{119.2}$$

for any $\xi,\zeta = 1,2,\cdots,\Xi$, here the "$\delta_{\xi\zeta}$" is Kronecker delta symbol; the $\operatorname{Re}\{P_\xi^{out}\}$ and $\operatorname{Im}\{P_\xi^{out}\}$ are the real and imaginary parts of $P_\xi^{out}$. In addition, it is obvious that $P_\xi^{out} = P_\xi^{inp}$, i.e., the modal output power equals to the modal input power, because of the conservation law of energy [4].

The InpCM-based modal expansion formulation for metal-material combined object can be found in [1]-[2], and it is not repeated here. The other power-based CM sets, such as the Active power CM (ActCM) set, the Reactive power CM (ReactCM) set, and the Coupling power CM (CoupCM) set, can be constructed by using the methods given in [8] and [10]-[11], and they are not repeated here. The method to normalize various power-based CMs derived from LS-MM-EMP-CMT can be found in [2], and it is not repeated here.

## VI. Intrinsic Resonance and the Relevant Concepts

In this section, a power-based modal classification method [13] is simply retrospected, and then a series of new concepts *intrinsic resonance*, *intrinsic resonance equation*/*condition*, *intrinsic resonant mode*, *intrinsic resonance space*, and *intrinsic resonant CM set* are introduced.

### A. Power-based modal classification

If any operating state of scatterer is called as an operating mode (it is not restricted to the CM), the modes can be classified according to their modal powers.

**(a) According to modal active power:** Because the matrix $\bar{\bar{P}}_{BV;+}^{inp}$ is positive definite or semi-definite, then

$$\operatorname{Re}\{P^{inp}\} \;=\; \left(\bar{a}^{BV}\right)^H \cdot \bar{\bar{P}}_{BV;+}^{inp} \cdot \bar{a}^{BV} \;\geq\; 0 \tag{120}$$

for any operating state $\bar{a}^{BV}$. The modes corresponding to $\operatorname{Re}\{P^{inp}\} > 0$ are called as active modes, and the modes corresponding to $\mathrm{Re}\{P^{inp}\}=0$ are called as non-active modes.

**(b) According to modal radiated power [13]:** It can be proven that the modal radiated power $P^{rad}$ is non-negative for any mode, here the modal radiated power $P^{rad}$ is as follows

$$P^{rad} = \frac{1}{2}\oiint_{S_\infty}\left[\vec{E}^{sca}\times\left(\vec{H}^{sca}\right)^*\right]\cdot d\vec{S} \tag{121}$$

The modes corresponding to $P^{rad}>0$ are called as radiative modes, and the modes corresponding to $P^{rad}=0$ are called as non-radiative modes.

**(c) According to modal reactive power [13]:** The modal reactive power is as follows

$$\begin{aligned}\mathrm{Im}\left\{P^{inp}\right\} &= \left(\bar{a}^{BV}\right)^H\cdot\bar{\bar{P}}^{inp}_{BV;-}\cdot\bar{a}^{BV}\\ &= 2\omega\left\{\left[\frac{1}{4}\left\langle\vec{H}^{sca},\mu_0\vec{H}^{sca}\right\rangle_{\mathbb{R}^3}-\frac{1}{4}\left\langle\varepsilon_0\vec{E}^{sca},\vec{E}^{sca}\right\rangle_{\mathbb{R}^3}\right]\right.\\ &\quad\left.+\left[\frac{1}{4}\left\langle\vec{H}^{tot},\Delta\mu\vec{H}^{tot}\right\rangle_{V^{mat}}-\frac{1}{4}\left\langle\Delta\varepsilon\vec{E}^{tot},\vec{E}^{tot}\right\rangle_{V^{mat}}\right]\right\}\end{aligned} \tag{122}$$

The matrix $\bar{\bar{P}}^{inp}_{BV;-}$ is indefinite. The modes corresponding to $\mathrm{Im}\{P^{inp}\}<0$ are called as capacitive modes, and the modes corresponding to $\mathrm{Im}\{P^{inp}\}=0$ are called as resonant modes, and the modes corresponding to $\mathrm{Im}\{P^{inp}\}>0$ are called as inductive modes.

## B. *Intrinsic resonance and the relevant concepts*

Because the matrix $\bar{\bar{P}}^{inp}_{BV;+}$ is positive definite or semi-definite, $\left(\bar{a}^{BV}\right)^H\cdot\bar{\bar{P}}^{inp}_{BV;+}\cdot\bar{a}^{BV}=0$ if and only if $\bar{\bar{P}}^{inp}_{BV;+}\cdot\bar{a}^{BV}=0$ [12]. However, $\left(\bar{a}^{BV}\right)^H\cdot\bar{\bar{P}}^{inp}_{BV;-}\cdot\bar{a}^{BV}=0$ does not imply that $\bar{\bar{P}}^{inp}_{BV;-}\cdot\bar{a}^{BV}=0$, though $\bar{\bar{P}}^{inp}_{BV;-}\cdot\bar{a}^{BV}=0$ always implies that $\left(\bar{a}^{BV}\right)^H\cdot\bar{\bar{P}}^{inp}_{BV;-}\cdot\bar{a}^{BV}=0$, because the matrix $\bar{\bar{P}}^{inp}_{BV;-}$ is indefinite. It is equivalent to saying that

$$\begin{aligned}\bar{\bar{P}}^{inp}_{BV;+}\cdot\bar{a}^{BV}=0 \quad &\Leftrightarrow\quad \left(\bar{a}^{BV}\right)^H\cdot\bar{\bar{P}}^{inp}_{BV;+}\cdot\bar{a}^{BV}=0\\ &\Leftrightarrow\quad \text{Mode } \bar{a}^{BV} \text{ is non-active}\end{aligned} \tag{123}$$

and

$$\begin{aligned}\bar{\bar{P}}^{inp}_{BV;-}\cdot\bar{a}^{BV}=0 \quad &\underset{\nLeftarrow}{\Rightarrow}\quad \left(\bar{a}^{BV}\right)^H\cdot\bar{\bar{P}}^{inp}_{BV;-}\cdot\bar{a}^{BV}=0\\ &\Leftrightarrow\quad \text{Mode } \bar{a}^{BV} \text{ is resonant}\end{aligned} \tag{124}$$

In electromagnetic engineering society, the resonance is a very important concept, and the above (124) implies that the condition $\bar{\bar{P}}^{inp}_{BV;-}\cdot\bar{a}^{BV}=0$ is a stronger condition than $\left(\bar{a}^{BV}\right)^H\cdot\bar{\bar{P}}^{inp}_{BV;-}\cdot\bar{a}^{BV}=0$ to guarantee resonance. Based on this, the equation $\bar{\bar{P}}^{inp}_{BV;-}\cdot\bar{a}^{BV}=0$ can be called as intrinsic resonance equation/condition, if the equation $\left(\bar{a}^{BV}\right)^H\cdot\bar{\bar{P}}^{inp}_{BV;-}\cdot\bar{a}^{BV}=0$ is viewed as resonance equation/condition.

Then, the modes satisfying intrinsic resonance equation are called as intrinsic resonant modes. Obviously, all intrinsic resonant modes constitute a space, and this space is just the null space of matrix $\bar{\bar{P}}^{inp}_{BV;-}$, and it can be specifically called as intrinsic resonance space from the power-based point of view. However, it cannot be guaranteed that the set constituted by all resonant modes is a space.

Due to the object-oriented feature of EMP-CMT [8], [10]-[11], this paper introduces a new CM set, intrinsic resonant CM set, and it is defined as the basis of intrinsic resonance space. The intrinsic resonant CM set can be efficiently derived from solving the intrinsic resonance equation $\bar{\bar{P}}^{inp}_{BV;-}\cdot\bar{a}^{BV}=0$.

In addition, it is easy to prove that any intrinsic resonant mode $\bar{a}^{BV}_{res}$ (it is not necessarily the element of intrinsic resonant CM set) is orthogonal to any operating state $\bar{a}^{BV}$ of scatterer as follows

$$\begin{aligned}0 &= \left(\bar{a}^{BV}_{res}\right)^H\cdot\bar{\bar{P}}^{inp}_{BV;-}\cdot\bar{a}^{BV}\\ &= 2\omega\left\{\left[\frac{1}{4}\left\langle\vec{H}^{sca}_{res},\mu_0\vec{H}^{sca}\right\rangle_{\mathbb{R}^3}-\frac{1}{4}\left\langle\varepsilon_0\vec{E}^{sca}_{res},\vec{E}^{sca}\right\rangle_{\mathbb{R}^3}\right]\right.\\ &\quad\left.+\left[\frac{1}{4}\left\langle\vec{H}^{tot}_{res},\Delta\mu\vec{H}^{tot}\right\rangle_{V^{mat}}-\frac{1}{4}\left\langle\Delta\varepsilon\vec{E}^{tot}_{res},\vec{E}^{tot}\right\rangle_{V^{mat}}\right]\right\}\end{aligned} \tag{125.1}$$

$$\begin{aligned}0 &= \left(\bar{a}^{BV}\right)^H\cdot\bar{\bar{P}}^{inp}_{BV;-}\cdot\bar{a}^{BV}_{res}\\ &= 2\omega\left\{\left[\frac{1}{4}\left\langle\vec{H}^{sca},\mu_0\vec{H}^{sca}_{res}\right\rangle_{\mathbb{R}^3}-\frac{1}{4}\left\langle\varepsilon_0\vec{E}^{sca},\vec{E}^{sca}_{res}\right\rangle_{\mathbb{R}^3}\right]\right.\\ &\quad\left.+\left[\frac{1}{4}\left\langle\vec{H}^{tot},\Delta\mu\vec{H}^{tot}_{res}\right\rangle_{V^{mat}}-\frac{1}{4}\left\langle\Delta\varepsilon\vec{E}^{tot},\vec{E}^{tot}_{res}\right\rangle_{V^{mat}}\right]\right\}\end{aligned} \tag{125.2}$$

Then, $\left(\bar{a}^{BV}+\bar{a}^{BV}_{res}\right)^H\cdot\bar{\bar{P}}^{inp}_{BV;-}\cdot\left(\bar{a}^{BV}+\bar{a}^{BV}_{res}\right)=\left(\bar{a}^{BV}\right)^H\cdot\bar{\bar{P}}^{inp}_{BV;-}\cdot\bar{a}^{BV}$ for any operating state $\bar{a}^{BV}$, if the $\bar{a}^{BV}_{res}$ is an intrinsic resonant mode.

# VII. Some Typical Examples

In this section, the general formulations given in Sec. V are specialized to the special forms corresponding to some typical examples.

## A. *Scatterer is constructed by one piece of metal line and one piece of material body, and the line partially contacts with the body, but any part of the line is not submerged into the body*

In this subsection, the scatterer illustrated in Fig. 4 is considered. Its metal part only includes one piece of line, and the line partially contacts with the material body, but any part of

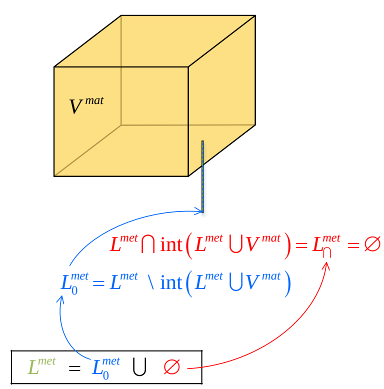


Fig. 4. A piece of metal line partially contacts with material body.

the line is not submerged into the material body. It is obvious that $L_{\cap}^{met}, S^{met}, \partial V^{met} = \varnothing$, so $\vec{J}_{\cap}^{l}, \vec{J}^{s} = 0$, and then the basic variables can be selected as $\{\vec{J}_0^l, \vec{J}_0^{SE}\}$ or equivalently selected as $\{\vec{J}_0^l, \vec{M}_0^{SE}\}$.

For this case, the input power operator (54) is specialized to the following (126)

$$
\begin{aligned}
&P^{inp}\\
&= -(1/2)\left\langle \vec{J}_0^l, \mathcal{E}_{met}^{sca}\left(\vec{J}_0^l\right)\right\rangle_{L_0^{met}} - (1/2)\left\langle \vec{J}_0^l, \mathcal{E}_{ext}^{sca}\left(\vec{J}_0^{SE}, \vec{M}_0^{SE}\right)\right\rangle_{L_0^{met}}\\
&\quad - \frac{\mu\,\Delta\varepsilon_c^*}{\mu_0\varepsilon_0 - \mu\varepsilon_c^*}\left[\frac{1}{2}\left\langle \vec{J}_0^{SE}, \mathcal{E}_{met}^{sca}\left(\vec{J}_0^l\right)\right\rangle_{\partial V_0^{mat}} + \frac{\mu_0}{\mu}\frac{1}{2}\left\langle \vec{M}_0^{SE}, \mathcal{H}_{met}^{sca}\left(\vec{J}_0^l\right)\right\rangle_{\partial V_0^{mat}}\right]\\
&\quad - \frac{\varepsilon_c\,\Delta\mu}{\varepsilon_0\mu_0 - \varepsilon_c\mu}\left[\frac{\varepsilon_0}{\varepsilon_c}\frac{1}{2}\left\langle \vec{J}_0^{SE}, \mathcal{E}_{met}^{sca}\left(\vec{J}_0^l\right)\right\rangle_{\partial V_0^{mat}}^* + \frac{1}{2}\left\langle \vec{M}_0^{SE}, \mathcal{H}_{met}^{sca}\left(\vec{J}_0^l\right)\right\rangle_{\partial V_0^{mat}}^*\right]\\
&\quad + \frac{\mu\,\Delta\varepsilon_c^*}{\mu_0\varepsilon_0 - \mu\varepsilon_c^*}\left[\frac{1}{2}\left\langle \vec{J}_0^{SE}, e_{int}^{inc}\left(\vec{J}_0^{SE}, \vec{M}_0^{SE}\right)\right\rangle_{\partial V_0^{mat}} + \frac{\mu_0}{\mu}\frac{1}{2}\left\langle \vec{M}_0^{SE}, h_{int}^{inc}\left(\vec{J}_0^{SE}, \vec{M}_0^{SE}\right)\right\rangle_{\partial V_0^{mat}}\right]\\
&\quad + \frac{\varepsilon_c\,\Delta\mu}{\varepsilon_0\mu_0 - \varepsilon_c\mu}\left[\frac{\varepsilon_0}{\varepsilon_c}\frac{1}{2}\left\langle \vec{J}_0^{SE}, e_{int}^{inc}\left(\vec{J}_0^{SE}, \vec{M}_0^{SE}\right)\right\rangle_{\partial V_0^{mat}}^* + \frac{1}{2}\left\langle \vec{M}_0^{SE}, h_{int}^{inc}\left(\vec{J}_0^{SE}, \vec{M}_0^{SE}\right)\right\rangle_{\partial V_0^{mat}}^*\right]
\end{aligned}
\tag{126}
$$

and the matrix form of (126) is as follows

$$
\begin{aligned}
P^{inp} &= \left(\bar{a}^{\{\vec{J}_0^l, \vec{J}_0^{SE}\}}\right)^H \cdot \overline{\overline{P}}{}^{inp}_{\{\vec{J}_0^l, \vec{J}_0^{SE}\}} \cdot \bar{a}^{\{\vec{J}_0^l, \vec{J}_0^{SE}\}}\\
&= \left(\bar{a}^{\{\vec{J}_0^l, \vec{M}_0^{SE}\}}\right)^H \cdot \overline{\overline{P}}{}^{inp}_{\{\vec{J}_0^l, \vec{M}_0^{SE}\}} \cdot \bar{a}^{\{\vec{J}_0^l, \vec{M}_0^{SE}\}}
\end{aligned}
\tag{127}
$$

here

$$
\overline{\overline{P}}{}^{inp}_{\{\vec{J}_0^l, \vec{J}_0^{SE}\}} = \begin{bmatrix} \overline{\overline{I}} & 0 \\ 0 & \overline{\overline{I}} \\ 0 & \overline{\overline{T}}{}^{\vec{J}_0^{SE} \to \vec{M}_0^{SE}}_{Magnetic\ Extinction} \end{bmatrix}^H \cdot \overline{\overline{P}}{}^{inp}_{\{\vec{J}_0^l, \vec{J}_0^{SE}, \vec{M}_0^{SE}\}} \cdot \begin{bmatrix} \overline{\overline{I}} & 0 \\ 0 & \overline{\overline{I}} \\ 0 & \overline{\overline{T}}{}^{\vec{J}_0^{SE} \to \vec{M}_0^{SE}}_{Magnetic\ Extinction} \end{bmatrix}
\tag{128}
$$

$$
\overline{\overline{P}}{}^{inp}_{\{\vec{J}_0^l, \vec{M}_0^{SE}\}} = \begin{bmatrix} \overline{\overline{I}} & 0 \\ 0 & \overline{\overline{T}}{}^{\vec{M}_0^{SE} \to \vec{J}_0^{SE}}_{Electric\ Extinction} \\ 0 & \overline{\overline{I}} \end{bmatrix}^H \cdot \overline{\overline{P}}{}^{inp}_{\{\vec{J}_0^l, \vec{J}_0^{SE}, \vec{M}_0^{SE}\}} \cdot \begin{bmatrix} \overline{\overline{I}} & 0 \\ 0 & \overline{\overline{T}}{}^{\vec{M}_0^{SE} \to \vec{J}_0^{SE}}_{Electric\ Extinction} \\ 0 & \overline{\overline{I}} \end{bmatrix}
\tag{129}
$$

and

$$
\bar{a}^{\{\vec{J}_0^l, \vec{J}_0^{SE}\}} = \begin{bmatrix} \bar{a}^{\vec{J}_0^l} \\ \bar{a}^{\vec{J}_0^{SE}} \end{bmatrix}
\tag{130}
$$

$$
\bar{a}^{\{\vec{J}_0^l, \vec{M}_0^{SE}\}} = \begin{bmatrix} \bar{a}^{\vec{J}_0^l} \\ \bar{a}^{\vec{M}_0^{SE}} \end{bmatrix}
\tag{131}
$$

In (128)-(129),

$$
\begin{aligned}
&\overline{\overline{P}}{}^{inp}_{\{\vec{J}_0^l, \vec{J}_0^{SE}, \vec{M}_0^{SE}\}}\\
&= -\overline{\overline{P}}{}^{\vec{J}_0^l \leftarrow \mathcal{E}_{met}^{sca}(\vec{J}_0^l)} - \overline{\overline{P}}{}^{\vec{J}_0^l \leftarrow \mathcal{E}_{ext}^{sca}(\vec{J}_0^{SE}, \vec{M}_0^{SE})}\\
&\quad - \frac{\mu\,\Delta\varepsilon_c^*}{\mu_0\varepsilon_0 - \mu\varepsilon_c^*}\left[\overline{\overline{P}}{}^{\vec{J}_0^{SE} \leftarrow \mathcal{E}_{met}^{sca}(\vec{J}_0^l)} + \frac{\mu_0}{\mu}\overline{\overline{P}}{}^{\vec{M}_0^{SE} \leftarrow \mathcal{H}_{met}^{sca}(\vec{J}_0^l)}\right]\\
&\quad - \frac{\varepsilon_c\,\Delta\mu}{\varepsilon_0\mu_0 - \varepsilon_c\mu}\left\{\frac{\varepsilon_0}{\varepsilon_c}\left[\overline{\overline{P}}{}^{\vec{J}_0^{SE} \leftarrow \mathcal{E}_{met}^{sca}(\vec{J}_0^l)}\right]^H + \left[\overline{\overline{P}}{}^{\vec{M}_0^{SE} \leftarrow \mathcal{H}_{met}^{sca}(\vec{J}_0^l)}\right]^H\right\}\\
&\quad + \frac{\mu\,\Delta\varepsilon_c^*}{\mu_0\varepsilon_0 - \mu\varepsilon_c^*}\left[\overline{\overline{P}}{}^{\vec{J}_0^{SE} \leftarrow e_{int}^{inc}(\vec{J}_0^{SE}, \vec{M}_0^{SE})} + \frac{\mu_0}{\mu}\overline{\overline{P}}{}^{\vec{M}_0^{SE} \leftarrow h_{int}^{inc}(\vec{J}_0^{SE}, \vec{M}_0^{SE})}\right]\\
&\quad + \frac{\varepsilon_c\,\Delta\mu}{\varepsilon_0\mu_0 - \varepsilon_c\mu}\left\{\frac{\varepsilon_0}{\varepsilon_c}\left[\overline{\overline{P}}{}^{\vec{J}_0^{SE} \leftarrow e_{int}^{inc}(\vec{J}_0^{SE}, \vec{M}_0^{SE})}\right]^H + \left[\overline{\overline{P}}{}^{\vec{M}_0^{SE} \leftarrow h_{int}^{inc}(\vec{J}_0^{SE}, \vec{M}_0^{SE})}\right]^H\right\}
\end{aligned}
\tag{132}
$$

and

$$
\overline{\overline{T}}{}^{\vec{J}_0^{SE} \to \vec{M}_0^{SE}}_{Magnetic\ Extinction} = \left(\overline{\overline{\Phi}}{}^{M_0^{SE} H_{int}^{tot} M_0^{SE}}\right)^{-1} \cdot \overline{\overline{\Phi}}{}^{M_0^{SE} H_{int}^{tot} J_0^{SE}}
\tag{133}
$$

$$
\overline{\overline{T}}{}^{\vec{M}_0^{SE} \to \vec{J}_0^{SE}}_{Electric\ Extinction} = \left(\overline{\overline{\Phi}}{}^{J_0^{SE} E_{int}^{tot} J_0^{SE}}\right)^{-1} \cdot \overline{\overline{\Phi}}{}^{J_0^{SE} E_{int}^{tot} M_0^{SE}}
\tag{134}
$$

The various matrices in (132) are as follows

$$
\overline{\overline{P}}{}^{\vec{J}_0^l \leftarrow \mathcal{E}_{met}^{sca}(\vec{J}_0^l)} = \begin{bmatrix} \overline{\overline{\Phi}}{}^{J_0^l E_{met}^{sca} J_0^l} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
\tag{135}
$$

$$
\overline{\overline{P}}{}^{\vec{J}_0^l \leftarrow \mathcal{E}_{ext}^{sca}(\vec{J}_0^{SE}, \vec{M}_0^{SE})} = \begin{bmatrix} 0 & \overline{\overline{\Phi}}{}^{J_0^l E_{ext}^{sca} J_0^{SE}} & \overline{\overline{\Phi}}{}^{J_0^l E_{ext}^{sca} M_0^{SE}} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
\tag{136}
$$

$$
\overline{\overline{P}}{}^{\vec{J}_0^{SE} \leftarrow \mathcal{E}_{met}^{sca}(\vec{J}_0^l)} = \begin{bmatrix} 0 & 0 & 0 \\ \overline{\overline{\Phi}}{}^{J_0^{SE} E_{met}^{sca} J_0^l} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
\tag{137}
$$

$$
\overline{\overline{P}}{}^{\vec{M}_0^{SE} \leftarrow \mathcal{H}_{met}^{sca}(\vec{J}_0^l)} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \overline{\overline{\Phi}}{}^{M_0^{SE} H_{met}^{sca} J_0^l} & 0 & 0 \end{bmatrix}
\tag{138}
$$

$$
\overline{\overline{P}}{}^{\vec{J}_0^{SE} \leftarrow e_{int}^{inc}(\vec{J}_0^{SE}, \vec{M}_0^{SE})} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \overline{\overline{\Phi}}{}^{J_0^{SE} e_{int}^{inc} J_0^{SE}} & \overline{\overline{\Phi}}{}^{J_0^{SE} e_{int}^{inc} M_0^{SE}} \\ 0 & 0 & 0 \end{bmatrix}
\tag{139}
$$

$$
\overline{\overline{P}}{}^{\vec{M}_0^{SE} \leftarrow h_{int}^{inc}(\vec{J}_0^{SE}, \vec{M}_0^{SE})} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \overline{\overline{\Phi}}{}^{M_0^{SE} h_{int}^{inc} J_0^{SE}} & \overline{\overline{\Phi}}{}^{M_0^{SE} h_{int}^{inc} M_0^{SE}} \end{bmatrix}
\tag{140}
$$

The procedures to construct the power-based CM sets corresponding to the structure in Fig. 4 are completely similar to the procedures given in Sec. V, so they will not be repeated here.

In fact, the structure in Fig. 4 has many applications in electromagnetic engineering society, such as the probe-fed Dielectric Resonator Antennas (DRAs) in which the probes partially contact with the DRAs [14].

In addition, the structure in Fig. 4 can be further specialized to the structures in Fig. 5, and the formulations corresponding to the structures in Fig. 5 are identical to the ones given in this subsection.

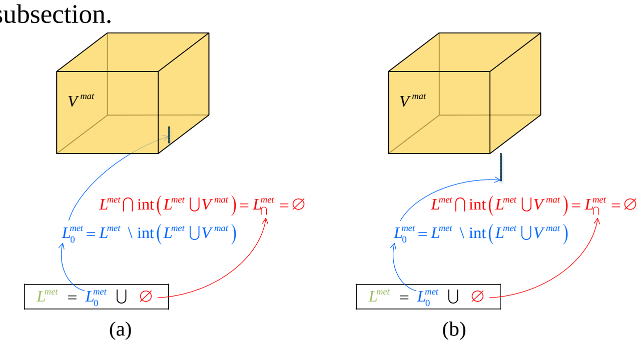



Fig. 5. (a) A piece of metal line completely contacts with material body; (b) a piece of metal line completely does not contact with material body.

### *B. Scatterer is constructed by one piece of metal line and one piece of material body, and the line is partially submerged into the body*

In this subsection, the scatterer illustrated in Fig. 6 is considered. Its metal part only includes one piece of line, and the line is partially submerged into the material body. It is obvious that $S^{met},\partial V^{met}=\varnothing$, so $\vec{J}^{s}=0$, and then the basic variables can be selected as $\{\vec{J}_0^{l},\vec{J}_0^{SE},\vec{J}_{\cap}^{l}\}$ or equivalently selected as $\{\vec{J}_0^{l},\vec{J}_{\cap}^{l},\vec{M}_0^{SE}\}$.

For this case, the input power operator (54) is specialized to the following (141)

$$
\begin{aligned}
&P^{inp}\\
&=-(1/2)\left\langle \vec{J}_0^{l}+\vec{J}_{\cap}^{l},\mathcal{E}_{met}^{sca}\left(\vec{J}_0^{l}\right)\right\rangle_{L_0^{met}\cup L_{\cap}^{met}}\\
&\quad-(1/2)\left\langle \vec{J}_0^{l}+\vec{J}_{\cap}^{l},\mathcal{E}_{ext}^{sca}\left(\vec{J}_0^{SE},\vec{M}_0^{SE}\right)\right\rangle_{L_0^{met}\cup L_{\cap}^{met}}\\
&\quad-\frac{\mu\,\Delta\varepsilon_c^{*}}{\mu_0\varepsilon_0-\mu\varepsilon_c^{*}}\left[\frac{1}{2}\left\langle \vec{J}_{\cap}^{l}\oplus\vec{J}_0^{SE},\mathcal{E}_{met}^{sca}\left(\vec{J}_0^{l}\right)\right\rangle_{\partial V_0^{mat}\cup L_{\cap}^{met}}+\frac{\mu_0}{\mu}\frac{1}{2}\left\langle \vec{M}_0^{SE},\mathcal{H}_{met}^{sca}\left(\vec{J}_0^{l}\right)\right\rangle_{\partial V_0^{mat}}\right]\\
&\quad-\frac{\varepsilon_c\,\Delta\mu}{\varepsilon_0\mu_0-\varepsilon_c\mu}\left[\frac{\varepsilon_0}{\varepsilon_c}\frac{1}{2}\left\langle \vec{J}_{\cap}^{l}\oplus\vec{J}_0^{SE},\mathcal{E}_{met}^{sca}\left(\vec{J}_0^{l}\right)\right\rangle_{\partial V_0^{mat}\cup L_{\cap}^{met}}^{*}+\frac{1}{2}\left\langle \vec{M}_0^{SE},\mathcal{H}_{met}^{sca}\left(\vec{J}_0^{l}\right)\right\rangle_{\partial V_0^{mat}}^{*}\right]\\
&\quad+\frac{\mu\,\Delta\varepsilon_c^{*}}{\mu_0\varepsilon_0-\mu\varepsilon_c^{*}}\left[\frac{1}{2}\left\langle \vec{J}_{\cap}^{l}\oplus\vec{J}_0^{SE},\mathcal{e}_{int}^{inc}\left(\vec{J}_0^{SE},\vec{M}_0^{SE}\right)\right\rangle_{\partial V_0^{mat}\cup L_{\cap}^{met}}+\frac{\mu_0}{\mu}\frac{1}{2}\left\langle \vec{M}_0^{SE},\mathcal{h}_{int}^{inc}\left(\vec{J}_0^{SE},\vec{M}_0^{SE}\right)\right\rangle_{\partial V_0^{mat}}\right]\\
&\quad+\frac{\varepsilon_c\,\Delta\mu}{\varepsilon_0\mu_0-\varepsilon_c\mu}\left[\frac{\varepsilon_0}{\varepsilon_c}\frac{1}{2}\left\langle \vec{J}_{\cap}^{l}\oplus\vec{J}_0^{SE},\mathcal{e}_{int}^{inc}\left(\vec{J}_0^{SE},\vec{M}_0^{SE}\right)\right\rangle_{\partial V_0^{mat}\cup L_{\cap}^{met}}^{*}+\frac{1}{2}\left\langle \vec{M}_0^{SE},\mathcal{h}_{int}^{inc}\left(\vec{J}_0^{SE},\vec{M}_0^{SE}\right)\right\rangle_{\partial V_0^{mat}}^{*}\right]
\end{aligned}
\tag{141}
$$

and the matrix form of (141) is as follows

$$
\begin{aligned}
P^{inp}&=\left(\bar{a}^{\{\vec{J}_0^{l},\vec{J}_0^{SE},\vec{J}_{\cap}^{l}\}}\right)^{H}\cdot\overline{\overline{P}}{}^{inp}_{\{\vec{J}_0^{l},\vec{J}_0^{SE},\vec{J}_{\cap}^{l}\}}\cdot\bar{a}^{\{\vec{J}_0^{l},\vec{J}_0^{SE},\vec{J}_{\cap}^{l}\}}\\
&=\left(\bar{a}^{\{\vec{J}_0^{l},\vec{J}_{\cap}^{l},\vec{M}_0^{SE}\}}\right)^{H}\cdot\overline{\overline{P}}{}^{inp}_{\{\vec{J}_0^{l},\vec{J}_{\cap}^{l},\vec{M}_0^{SE}\}}\cdot\bar{a}^{\{\vec{J}_0^{l},\vec{J}_{\cap}^{l},\vec{M}_0^{SE}\}}
\end{aligned}
\tag{142}
$$

here

$$
\overline{\overline{P}}{}^{inp}_{\{\vec{J}_0^{l},\vec{J}_0^{SE},\vec{J}_{\cap}^{l}\}}=\begin{bmatrix}\overline{\overline{I}}&0\\0&\overline{\overline{I}}\\0&\overline{\overline{T}}{}^{\{\vec{J}_0^{SE},\vec{J}_{\cap}^{l}\}\to\vec{M}_0^{SE}}_{Magnetic\ Extinction}\end{bmatrix}^{H}\cdot\overline{\overline{P}}{}^{inp}_{\{\vec{J}_0^{l},\vec{J}_0^{SE},\vec{J}_{\cap}^{l},\vec{M}_0^{SE}\}}\cdot\begin{bmatrix}\overline{\overline{I}}&0\\0&\overline{\overline{I}}\\0&\overline{\overline{T}}{}^{\{\vec{J}_0^{SE},\vec{J}_{\cap}^{l}\}\to\vec{M}_0^{SE}}_{Magnetic\ Extinction}\end{bmatrix}
\tag{143}
$$

$$
\overline{\overline{P}}{}^{inp}_{\{\vec{J}_0^{l},\vec{J}_{\cap}^{l},\vec{M}_0^{SE}\}}=\begin{bmatrix}\overline{\overline{I}}&0\\0&\overline{\overline{T}}{}^{\{\vec{J}_{\cap}^{l},\vec{M}_0^{SE}\}\to\vec{J}_0^{SE}}_{Electric\ Extinction}\\0&\overline{\overline{I}}\end{bmatrix}^{H}\cdot\overline{\overline{P}}{}^{inp}_{\{\vec{J}_0^{l},\vec{J}_0^{SE},\vec{J}_{\cap}^{l},\vec{M}_0^{SE}\}}\cdot\begin{bmatrix}\overline{\overline{I}}&0\\0&\overline{\overline{T}}{}^{\{\vec{J}_{\cap}^{l},\vec{M}_0^{SE}\}\to\vec{J}_0^{SE}}_{Electric\ Extinction}\\0&\overline{\overline{I}}\end{bmatrix}
\tag{144}
$$

and

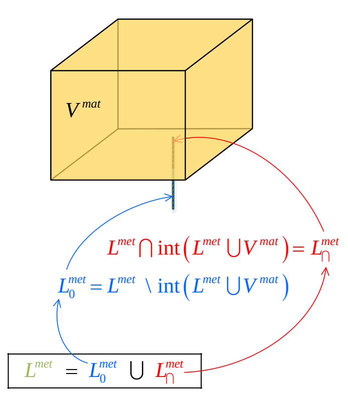



Fig. 6. A piece of metal line is partially submerged into material body.

$$
\bar{a}^{\{\vec{J}_0^{l},\vec{J}_0^{SE},\vec{J}_{\cap}^{l}\}}=\begin{bmatrix}\bar{a}^{\vec{J}_0^{l}}\\\bar{a}^{\vec{J}_0^{SE}}\\\bar{a}^{\vec{J}_{\cap}^{l}}\end{bmatrix}=\begin{bmatrix}\bar{a}^{\vec{J}_0^{l}}\\\begin{bmatrix}\bar{a}^{\vec{J}_0^{SE}}\\\bar{a}^{\vec{J}_{\cap}^{l}}\end{bmatrix}\end{bmatrix}
\tag{145}
$$

$$
\bar{a}^{\{\vec{J}_0^{l},\vec{J}_{\cap}^{l},\vec{M}_0^{SE}\}}=\begin{bmatrix}\bar{a}^{\vec{J}_0^{l}}\\\bar{a}^{\vec{J}_{\cap}^{l}}\\\bar{a}^{\vec{M}_0^{SE}}\end{bmatrix}=\begin{bmatrix}\bar{a}^{\vec{J}_0^{l}}\\\begin{bmatrix}\bar{a}^{\vec{J}_{\cap}^{l}}\\\bar{a}^{\vec{M}_0^{SE}}\end{bmatrix}\end{bmatrix}
\tag{146}
$$

In (143)-(144),

$$
\begin{aligned}
&\overline{\overline{P}}{}^{inp}_{\{\vec{J}_0^{l},\vec{J}_0^{SE},\vec{J}_{\cap}^{l},\vec{M}_0^{SE}\}}\\
&=-\overline{\overline{P}}{}^{\{\vec{J}_0^{l},\vec{J}_{\cap}^{l}\}\leftarrow\mathcal{E}_{met}^{sca}(\vec{J}_0^{l})}-\overline{\overline{P}}{}^{\{\vec{J}_0^{l},\vec{J}_{\cap}^{l}\}\leftarrow\mathcal{E}_{ext}^{sca}(\vec{J}_0^{SE},\vec{M}_0^{SE})}\\
&\quad-\frac{\mu\,\Delta\varepsilon_c^{*}}{\mu_0\varepsilon_0-\mu\varepsilon_c^{*}}\left[\overline{\overline{P}}{}^{\{\vec{J}_0^{SE},\vec{J}_{\cap}^{l}\}\leftarrow\mathcal{E}_{met}^{sca}(\vec{J}_0^{l})}+\frac{\mu_0}{\mu}\overline{\overline{P}}{}^{\vec{M}_0^{SE}\leftarrow\mathcal{H}_{met}^{sca}(\vec{J}_0^{l})}\right]\\
&\quad-\frac{\varepsilon_c\,\Delta\mu}{\varepsilon_0\mu_0-\varepsilon_c\mu}\left\{\frac{\varepsilon_0}{\varepsilon_c}\left[\overline{\overline{P}}{}^{\{\vec{J}_0^{SE},\vec{J}_{\cap}^{l}\}\leftarrow\mathcal{E}_{met}^{sca}(\vec{J}_0^{l})}\right]^{H}+\left[\overline{\overline{P}}{}^{\vec{M}_0^{SE}\leftarrow\mathcal{H}_{met}^{sca}(\vec{J}_0^{l})}\right]^{H}\right\}\\
&\quad+\frac{\mu\,\Delta\varepsilon_c^{*}}{\mu_0\varepsilon_0-\mu\varepsilon_c^{*}}\left[\overline{\overline{P}}{}^{\{\vec{J}_0^{SE},\vec{J}_{\cap}^{l}\}\leftarrow\mathcal{e}_{int}^{inc}(\vec{J}_0^{SE},\vec{M}_0^{SE})}+\frac{\mu_0}{\mu}\overline{\overline{P}}{}^{\vec{M}_0^{SE}\leftarrow\mathcal{h}_{int}^{inc}(\vec{J}_0^{SE},\vec{M}_0^{SE})}\right]\\
&\quad+\frac{\varepsilon_c\,\Delta\mu}{\varepsilon_0\mu_0-\varepsilon_c\mu}\left\{\frac{\varepsilon_0}{\varepsilon_c}\left[\overline{\overline{P}}{}^{\{\vec{J}_0^{SE},\vec{J}_{\cap}^{l}\}\leftarrow\mathcal{e}_{int}^{inc}(\vec{J}_0^{SE},\vec{M}_0^{SE})}\right]^{H}+\left[\overline{\overline{P}}{}^{\vec{M}_0^{SE}\leftarrow\mathcal{h}_{int}^{inc}(\vec{J}_0^{SE},\vec{M}_0^{SE})}\right]^{H}\right\}
\end{aligned}
\tag{147}
$$

and

$$
\overline{\overline{T}}{}^{\{\vec{J}_0^{SE},\vec{J}_{\cap}^{l}\}\to\vec{M}_0^{SE}}_{Magnetic\ Extinction}=\left(\overline{\overline{\Phi}}{}^{M_0^{SE}H_{int}^{tot}M_0^{SE}}\right)^{-1}\cdot\begin{bmatrix}\overline{\overline{\Phi}}{}^{M_0^{SE}H_{int}^{tot}J_0^{SE}}&\overline{\overline{\Phi}}{}^{M_0^{SE}H_{int}^{tot}J_{\cap}^{l}}\end{bmatrix}
\tag{148}
$$

$$
\overline{\overline{T}}{}^{\{\vec{J}_{\cap}^{l},\vec{M}_0^{SE}\}\to\vec{J}_0^{SE}}_{Electric\ Extinction}=\left(\overline{\overline{\Phi}}{}^{J_0^{SE}E_{int}^{tot}J_0^{SE}}\right)^{-1}\cdot\begin{bmatrix}\overline{\overline{\Phi}}{}^{J_0^{SE}E_{int}^{tot}J_{\cap}^{l}}&\overline{\overline{\Phi}}{}^{J_0^{SE}E_{int}^{tot}M_0^{SE}}\end{bmatrix}
\tag{149}
$$

The various matrices in (147) are as follows

$$
\overline{\overline{P}}{}^{\{\vec{J}_0^{l},\vec{J}_{\cap}^{l}\}\leftarrow\mathcal{E}_{met}^{sca}(\vec{J}_0^{l})}=\begin{bmatrix}\overline{\overline{\Phi}}{}^{J_0^{l}E_{met}^{sca}J_0^{l}}&0&0&0\\0&0&0&0\\\overline{\overline{\Phi}}{}^{J_{\cap}^{l}E_{met}^{sca}J_0^{l}}&0&0&0\\0&0&0&0\end{bmatrix}
\tag{150}
$$

$$
\overline{\overline{P}}{}^{\{\vec{J}_0^{l},\vec{J}_{\cap}^{l}\}\leftarrow\mathcal{E}_{ext}^{sca}(\vec{J}_0^{SE},\vec{M}_0^{SE})}=\begin{bmatrix}0&\overline{\overline{\Phi}}{}^{J_0^{l}E_{ext}^{sca}J_0^{SE}}&0&\overline{\overline{\Phi}}{}^{J_0^{l}E_{ext}^{sca}M_0^{SE}}\\0&0&0&0\\0&\overline{\overline{\Phi}}{}^{J_{\cap}^{l}E_{ext}^{sca}J_0^{SE}}&0&\overline{\overline{\Phi}}{}^{J_{\cap}^{l}E_{ext}^{sca}M_0^{SE}}\\0&0&0&0\end{bmatrix}
\tag{151}
$$

$$
\overline{\overline{P}}{}^{\{\vec{J}_0^{SE},\vec{J}_{\cap}^{l}\}\leftarrow\mathcal{E}_{met}^{sca}(\vec{J}_0^{l})}=\begin{bmatrix}0&0&0&0\\\overline{\overline{\Phi}}{}^{J_0^{SE}E_{met}^{sca}J_0^{l}}&0&0&0\\\overline{\overline{\Phi}}{}^{J_{\cap}^{l}E_{met}^{sca}J_0^{l}}&0&0&0\\0&0&0&0\end{bmatrix}
\tag{152}
$$

$$
\overline{\overline{P}}{}^{\vec{M}_0^{SE}\leftarrow\mathcal{H}_{met}^{sca}(\vec{J}_0^{l})}=\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\\overline{\overline{\Phi}}{}^{M_0^{SE}H_{met}^{sca}J_0^{l}}&0&0&0\end{bmatrix}
\tag{153}
$$

$$
\overline{\overline{P}}{}^{\{\vec{J}_0^{SE},\vec{J}_{\cap}^{l}\}\leftarrow\mathcal{e}_{int}^{inc}(\vec{J}_0^{SE},\vec{M}_0^{SE})}=\begin{bmatrix}0&0&0&0\\0&\overline{\overline{\Phi}}{}^{J_0^{SE}e_{int}^{inc}J_0^{SE}}&0&\overline{\overline{\Phi}}{}^{J_0^{SE}e_{int}^{inc}M_0^{SE}}\\0&\overline{\overline{\Phi}}{}^{J_{\cap}^{l}e_{int}^{inc}J_0^{SE}}&0&\overline{\overline{\Phi}}{}^{J_{\cap}^{l}e_{int}^{inc}M_0^{SE}}\\0&0&0&0\end{bmatrix}
\tag{154}
$$

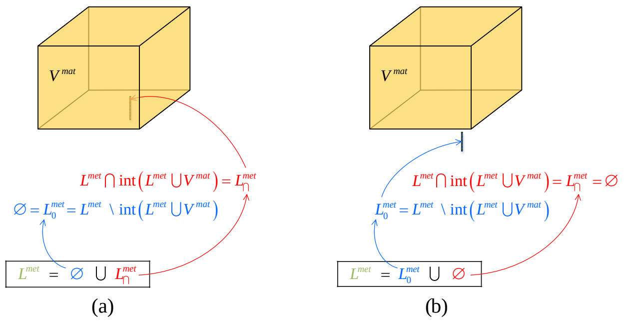



Fig. 7. (a) a piece of metal line is completely submerged into material body; (b) a piece of metal line completely does not contact with material body.

$$\bar{\bar{P}}^{\vec{M}_0^{SE}\leftarrow h_{int}^{inc}(\vec{J}_0^{SE},\vec{M}_0^{SE})} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \bar{\bar{\Phi}}^{M_0^{SE} h_{int}^{inc} J_0^{SE}} & 0 & \bar{\bar{\Phi}}^{M_0^{SE} h_{int}^{inc} M_0^{SE}} \end{bmatrix} \tag{155}$$

The procedures to construct the power-based CM sets corresponding to the structure in Fig. 6 are completely similar to the procedures given in Sec. V, so they will not be repeated here.

In fact, the structure in Fig. 6 has many applications in electromagnetic engineering society, such as the probe-fed DRAs in which the probes are partially submerged into the DRAs [14].

In addition, the structure in Fig. 6 can be further specialized to the structures in Fig. 7. The formulations corresponding to the structure in Fig. 7 (a) can be obtained by removing the terms corresponding to $\vec{J}_0^{\,l}$ from the formulations given in this subsection; the formulations corresponding to the structure in Fig. 7 (b) can be obtained by removing the terms corresponding to $\vec{J}_\cap^{\,l}$ from the formulations given in this subsection.

### *C. Scatterer is constructed by one piece of metal surface and one piece of material body, and the surface partially contacts with the body, but any part of the surface is not submerged into the body*

In this subsection, the scatterer illustrated in Fig. 8 is considered. Its metal part only includes one piece of surface, and the surface partially contacts with the material body, but any part of the surface is not submerged into the material body. It is obvious that $L^{met}, S_\cap^{met}, \partial V^{met} = \varnothing$, so $\vec{J}^{\,l}, \vec{J}_\cap^{\,s} = 0$, and then the

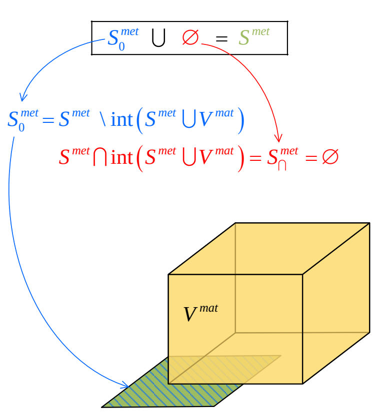



Fig. 8. A piece of metal surface partially contacts with material body.

basic variables can be selected as $\{\vec{J}_0^{\,s}, \vec{J}_0^{\,SE}\}$ or equivalently selected as $\{\vec{J}_0^{\,s}, \vec{M}_0^{\,SE}\}$.

For this case, the input power operator (54) is specialized to the following (156)

$$\begin{aligned} P^{inp} = & -(1/2)\left\langle \vec{J}_0^{\,s}, \mathcal{E}_{met}^{sca}\left(\vec{J}_0^{\,s}\right)\right\rangle_{S_0^{met}} - (1/2)\left\langle \vec{J}_0^{\,s}, \mathcal{E}_{ext}^{sca}\left(\vec{J}_0^{\,SE}, \vec{M}_0^{\,SE}\right)\right\rangle_{S_0^{met}} \\ & - \frac{\mu\,\Delta\varepsilon_c^*}{\mu_0\varepsilon_0 - \mu\varepsilon_c^*}\left[ \frac{1}{2}\left\langle \vec{J}_0^{\,SE}, \mathcal{E}_{met}^{sca}\left(\vec{J}_0^{\,s}\right)\right\rangle_{\partial V_0^{mat}} + \frac{\mu_0}{\mu}\frac{1}{2}\left\langle \vec{M}_0^{\,SE}, \mathcal{H}_{met}^{sca}\left(\vec{J}_0^{\,s}\right)\right\rangle_{\partial V_0^{mat}} \right] \\ & - \frac{\varepsilon_c\,\Delta\mu}{\varepsilon_0\mu_0 - \varepsilon_c\mu}\left[ \frac{\varepsilon_0}{\varepsilon_c}\frac{1}{2}\left\langle \vec{J}_0^{\,SE}, \mathcal{E}_{met}^{sca}\left(\vec{J}_0^{\,s}\right)\right\rangle_{\partial V_0^{mat}}^* + \frac{1}{2}\left\langle \vec{M}_0^{\,SE}, \mathcal{H}_{met}^{sca}\left(\vec{J}_0^{\,s}\right)\right\rangle_{\partial V_0^{mat}}^* \right] \\ & + \frac{\mu\,\Delta\varepsilon_c^*}{\mu_0\varepsilon_0 - \mu\varepsilon_c^*}\left[ \frac{1}{2}\left\langle \vec{J}_0^{\,SE}, e_{int}^{inc}\left(\vec{J}_0^{\,SE}, \vec{M}_0^{\,SE}\right)\right\rangle_{\partial V_0^{mat}} + \frac{\mu_0}{\mu}\frac{1}{2}\left\langle \vec{M}_0^{\,SE}, h_{int}^{inc}\left(\vec{J}_0^{\,SE}, \vec{M}_0^{\,SE}\right)\right\rangle_{\partial V_0^{mat}} \right] \\ & + \frac{\varepsilon_c\,\Delta\mu}{\varepsilon_0\mu_0 - \varepsilon_c\mu}\left[ \frac{\varepsilon_0}{\varepsilon_c}\frac{1}{2}\left\langle \vec{J}_0^{\,SE}, e_{int}^{inc}\left(\vec{J}_0^{\,SE}, \vec{M}_0^{\,SE}\right)\right\rangle_{\partial V_0^{mat}}^* + \frac{1}{2}\left\langle \vec{M}_0^{\,SE}, h_{int}^{inc}\left(\vec{J}_0^{\,SE}, \vec{M}_0^{\,SE}\right)\right\rangle_{\partial V_0^{mat}}^* \right] \end{aligned} \tag{156}$$

Its matrix form is as follows

$$\begin{aligned} P^{inp} & = \left(\bar{a}^{\{\vec{J}_0^s, \vec{J}_0^{SE}\}}\right)^H \cdot \bar{\bar{P}}^{inp}_{\{\vec{J}_0^s, \vec{J}_0^{SE}\}} \cdot \bar{a}^{\{\vec{J}_0^s, \vec{J}_0^{SE}\}} \\ & = \left(\bar{a}^{\{\vec{J}_0^s, \vec{M}_0^{SE}\}}\right)^H \cdot \bar{\bar{P}}^{inp}_{\{\vec{J}_0^s, \vec{M}_0^{SE}\}} \cdot \bar{a}^{\{\vec{J}_0^s, \vec{M}_0^{SE}\}} \end{aligned} \tag{157}$$

here

$$\bar{\bar{P}}^{inp}_{\{\vec{J}_0^s, \vec{J}_0^{SE}\}} = \begin{bmatrix} \bar{\bar{I}} & 0 \\ 0 & \bar{\bar{I}} \\ 0 & \bar{\bar{T}}^{\vec{J}_0^{SE}\rightarrow\vec{M}_0^{SE}}_{Magnetic\ Extinction} \end{bmatrix}^H \cdot \bar{\bar{P}}^{inp}_{\{\vec{J}_0^s, \vec{J}_0^{SE}, \vec{M}_0^{SE}\}} \cdot \begin{bmatrix} \bar{\bar{I}} & 0 \\ 0 & \bar{\bar{I}} \\ 0 & \bar{\bar{T}}^{\vec{J}_0^{SE}\rightarrow\vec{M}_0^{SE}}_{Magnetic\ Extinction} \end{bmatrix} \tag{158}$$

$$\bar{\bar{P}}^{inp}_{\{\vec{J}_0^s, \vec{M}_0^{SE}\}} = \begin{bmatrix} \bar{\bar{I}} & 0 \\ 0 & \bar{\bar{T}}^{\vec{M}_0^{SE}\rightarrow\vec{J}_0^{SE}}_{Electric\ Extinction} \\ 0 & \bar{\bar{I}} \end{bmatrix}^H \cdot \bar{\bar{P}}^{inp}_{\{\vec{J}_0^s, \vec{J}_0^{SE}, \vec{M}_0^{SE}\}} \cdot \begin{bmatrix} \bar{\bar{I}} & 0 \\ 0 & \bar{\bar{T}}^{\vec{M}_0^{SE}\rightarrow\vec{J}_0^{SE}}_{Electric\ Extinction} \\ 0 & \bar{\bar{I}} \end{bmatrix} \tag{159}$$

and

$$\bar{a}^{\{\vec{J}_0^s, \vec{J}_0^{SE}\}} = \begin{bmatrix} \bar{a}^{\vec{J}_0^s} \\ \bar{a}^{\vec{J}_0^{SE}} \end{bmatrix} \tag{160}$$

$$\bar{a}^{\{\vec{J}_0^s, \vec{M}_0^{SE}\}} = \begin{bmatrix} \bar{a}^{\vec{J}_0^s} \\ \bar{a}^{\vec{M}_0^{SE}} \end{bmatrix} \tag{161}$$

In (158)-(159), the matrices $\bar{\bar{T}}^{\vec{J}_0^{SE}\rightarrow\vec{M}_0^{SE}}_{Magnetic\ Extinction}$ and $\bar{\bar{T}}^{\vec{M}_0^{SE}\rightarrow\vec{J}_0^{SE}}_{Electric\ Extinction}$ can be obtained as (133) and (134), and

$$\begin{aligned} & \bar{\bar{P}}^{inp}_{\{\vec{J}_0^s, \vec{J}_0^{SE}, \vec{M}_0^{SE}\}} \\ = & -\bar{\bar{P}}^{\vec{J}_0^s\leftarrow\mathcal{E}_{met}^{sca}(\vec{J}_0^s)} - \bar{\bar{P}}^{\vec{J}_0^s\leftarrow\mathcal{E}_{ext}^{sca}(\vec{J}_0^{SE}, \vec{M}_0^{SE})} \\ & - \frac{\mu\,\Delta\varepsilon_c^*}{\mu_0\varepsilon_0 - \mu\varepsilon_c^*}\left[ \bar{\bar{P}}^{\vec{J}_0^{SE}\leftarrow\mathcal{E}_{met}^{sca}(\vec{J}_0^s)} + \frac{\mu_0}{\mu}\bar{\bar{P}}^{\vec{M}_0^{SE}\leftarrow\mathcal{H}_{met}^{sca}(\vec{J}_0^s)} \right] \\ & - \frac{\varepsilon_c\,\Delta\mu}{\varepsilon_0\mu_0 - \varepsilon_c\mu}\left\{ \frac{\varepsilon_0}{\varepsilon_c}\left[\bar{\bar{P}}^{\vec{J}_0^{SE}\leftarrow\mathcal{E}_{met}^{sca}(\vec{J}_0^s)}\right]^H + \left[\bar{\bar{P}}^{\vec{M}_0^{SE}\leftarrow\mathcal{H}_{met}^{sca}(\vec{J}_0^s)}\right]^H \right] \\ & + \frac{\mu\,\Delta\varepsilon_c^*}{\mu_0\varepsilon_0 - \mu\varepsilon_c^*}\left[ \bar{\bar{P}}^{\vec{J}_0^{SE}\leftarrow e_{int}^{inc}(\vec{J}_0^{SE}, \vec{M}_0^{SE})} + \frac{\mu_0}{\mu}\bar{\bar{P}}^{\vec{M}_0^{SE}\leftarrow h_{int}^{inc}(\vec{J}_0^{SE}, \vec{M}_0^{SE})} \right] \\ & + \frac{\varepsilon_c\,\Delta\mu}{\varepsilon_0\mu_0 - \varepsilon_c\mu}\left\{ \frac{\varepsilon_0}{\varepsilon_c}\left[\bar{\bar{P}}^{\vec{J}_0^{SE}\leftarrow e_{int}^{inc}(\vec{J}_0^{SE}, \vec{M}_0^{SE})}\right]^H + \left[\bar{\bar{P}}^{\vec{M}_0^{SE}\leftarrow h_{int}^{inc}(\vec{J}_0^{SE}, \vec{M}_0^{SE})}\right]^H \right\} \end{aligned} \tag{162}$$

The various matrices in (162) are as follows

$$\bar{\bar{P}}^{\vec{J}_0^s \leftarrow \mathcal{E}_{met}^{sca}(\vec{J}_0^s)} = \begin{bmatrix} \bar{\bar{\Phi}}^{J_0^s E_{met}^{sca} J_0^s} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{163}$$

$$\bar{\bar{P}}^{\vec{J}_0^s \leftarrow \mathcal{E}_{ext}^{sca}(\vec{J}_0^{SE},\vec{M}_0^{SE})} = \begin{bmatrix} 0 & \bar{\bar{\Phi}}^{J_0^s E_{ext}^{sca} J_0^{SE}} & \bar{\bar{\Phi}}^{J_0^s E_{ext}^{sca} M_0^{SE}} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{164}$$

$$\bar{\bar{P}}^{\vec{J}_0^{SE} \leftarrow \mathcal{E}_{met}^{sca}(\vec{J}_0^s)} = \begin{bmatrix} 0 & 0 & 0 \\ \bar{\bar{\Phi}}^{J_0^{SE} E_{met}^{sca} J_0^s} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{165}$$

$$\bar{\bar{P}}^{\vec{M}_0^{SE} \leftarrow \mathcal{H}_{met}^{sca}(\vec{J}_0^s)} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \bar{\bar{\Phi}}^{M_0^{SE} H_{met}^{sca} J_0^s} & 0 & 0 \end{bmatrix} \tag{166}$$

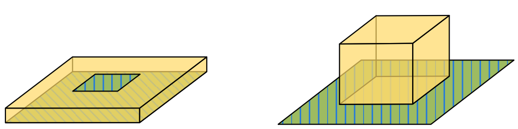
$$\bar{\bar{P}}^{\vec{J}_0^{SE} \leftarrow e_{int}^{inc}(\vec{J}_0^{SE},\vec{M}_0^{SE})} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \bar{\bar{\Phi}}^{J_0^{SE} e_{int}^{inc} J_0^{SE}} & \bar{\bar{\Phi}}^{J_0^{SE} e_{int}^{inc} M_0^{SE}} \\ 0 & 0 & 0 \end{bmatrix} \tag{167}$$

$$\bar{\bar{P}}^{\vec{M}_0^{SE} \leftarrow h_{int}^{inc}(\vec{J}_0^{SE},\vec{M}_0^{SE})} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \bar{\bar{\Phi}}^{M_0^{SE} h_{int}^{inc} J_0^{SE}} & \bar{\bar{\Phi}}^{M_0^{SE} h_{int}^{inc} M_0^{SE}} \end{bmatrix} \tag{168}$$

The procedures to construct the power-based CM sets corresponding to the structure in Fig. 8 are completely similar to the procedures given in Sec. V, so they will not be repeated here.

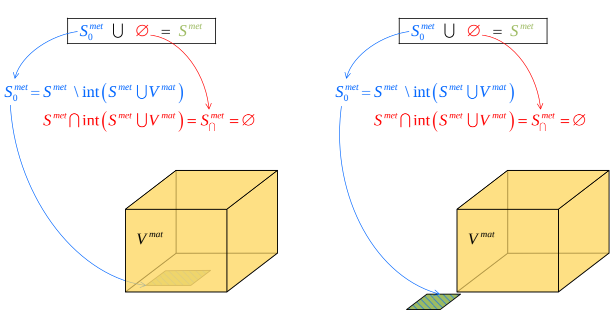
In fact, the structure in Fig. 8 has many applications in electromagnetic engineering society, such as the microstrip antennas [15] and the DRAs mounted on a metal plate [14], as illustrated in Fig. 9.

In addition, the structure in Fig. 8 can be further specialized to the structures in Fig. 10, and the formulations corresponding to the structures in Fig. 10 are identical to the ones given in this subsection.



(a) (b)

Fig. 9. (a) A rectangular microstrip antenna; (b) a rectangular DRA mounted on a metal palate.




(a) (b)

Fig. 10. (a) A piece of metal surface completely contacts with material body; (b) a piece of metal surface completely does not contact with material body.

## *D. Scatterer is constructed by one piece of metal surface and one piece of material body, and the surface is partially submerged into the body*

In this subsection, the scatterer illustrated in Fig. 11 is considered. Its metal part only includes one piece of surface, and the surface is partially submerged into the material body. It is obvious that $L^{met}, \partial V^{met} = \varnothing$, so $\vec{J}^l, \vec{J}_{\partial V_\cap^{met}}^{SE} = 0$, and then the basic variables can be selected as $\{\vec{J}_0^s, \vec{J}_0^{SE}, \vec{J}_\cap^s\}$ or equivalently selected as $\{\vec{J}_0^s, \vec{J}_\cap^s, \vec{M}_0^{SE}\}$, here $\vec{J}_\cap^s = \vec{J}_{open\ surf}^{SE}$.

For this case, the input power operator (54) is specialized to the following (169)

$$\begin{aligned} &P^{inp} \\ &= -(1/2)\left\langle \vec{J}_0^s + \vec{J}_\cap^s, \mathcal{E}_{met}^{sca}(\vec{J}_0^s)\right\rangle_{S_0^{met}\cup S_\cap^{met}} \\ &\quad -(1/2)\left\langle \vec{J}_0^s + \vec{J}_\cap^s, \mathcal{E}_{ext}^{sca}(\vec{J}_0^{SE}, \vec{M}_0^{SE})\right\rangle_{S_0^{met}\cup S_\cap^{met}} \\ &\quad -\frac{\mu\,\Delta\varepsilon_c^*}{\mu_0\varepsilon_0 - \mu\varepsilon_c^*}\left[\frac{1}{2}\left\langle \vec{J}_0^{SE} + \vec{J}_\cap^s, \mathcal{E}_{met}^{sca}(\vec{J}_0^s)\right\rangle_{\partial V_0^{mat}\cup S_\cap^{met}} + \frac{\mu_0}{\mu}\frac{1}{2}\left\langle \vec{M}_0^{SE}, \mathcal{H}_{met}^{sca}(\vec{J}_0^s)\right\rangle_{\partial V_0^{mat}}\right] \\ &\quad -\frac{\varepsilon_c\,\Delta\mu}{\varepsilon_0\mu_0 - \varepsilon_c\mu}\left[\frac{\varepsilon_0}{\varepsilon_c}\frac{1}{2}\left\langle \vec{J}_0^{SE} + \vec{J}_\cap^s, \mathcal{E}_{met}^{sca}(\vec{J}_0^s)\right\rangle_{\partial V_0^{mat}\cup S_\cap^{met}}^* + \frac{1}{2}\left\langle \vec{M}_0^{SE}, \mathcal{H}_{met}^{sca}(\vec{J}_0^s)\right\rangle_{\partial V_0^{mat}}^*\right] \\ &\quad +\frac{\mu\,\Delta\varepsilon_c^*}{\mu_0\varepsilon_0 - \mu\varepsilon_c^*}\left[\frac{1}{2}\left\langle \vec{J}_0^{SE} + \vec{J}_\cap^s, e_{int}^{inc}(\vec{J}_0^{SE}, \vec{M}_0^{SE})\right\rangle_{\partial V_0^{mat}\cup S_\cap^{met}} + \frac{\mu_0}{\mu}\frac{1}{2}\left\langle \vec{M}_0^{SE}, h_{int}^{inc}(\vec{J}_0^{SE}, \vec{M}_0^{SE})\right\rangle_{\partial V_0^{mat}}\right] \\ &\quad +\frac{\varepsilon_c\,\Delta\mu}{\varepsilon_0\mu_0 - \varepsilon_c\mu}\left[\frac{\varepsilon_0}{\varepsilon_c}\frac{1}{2}\left\langle \vec{J}_0^{SE} + \vec{J}_\cap^s, e_{int}^{inc}(\vec{J}_0^{SE}, \vec{M}_0^{SE})\right\rangle_{\partial V_0^{mat}\cup S_\cap^{met}}^* + \frac{1}{2}\left\langle \vec{M}_0^{SE}, h_{int}^{inc}(\vec{J}_0^{SE}, \vec{M}_0^{SE})\right\rangle_{\partial V_0^{mat}}^*\right] \end{aligned} \tag{169}$$

and the matrix form of (169) is as follows

$$\begin{aligned} P^{inp} &= \left(\bar{a}^{\{\vec{J}_0^s, \vec{J}_0^{SE}, \vec{J}_\cap^s\}}\right)^H \cdot \bar{\bar{P}}_{\{\vec{J}_0^s, \vec{J}_0^{SE}, \vec{J}_\cap^s\}}^{inp} \cdot \bar{a}^{\{\vec{J}_0^s, \vec{J}_0^{SE}, \vec{J}_\cap^s\}} \\ &= \left(\bar{a}^{\{\vec{J}_0^s, \vec{J}_\cap^s, \vec{M}_0^{SE}\}}\right)^H \cdot \bar{\bar{P}}_{\{\vec{J}_0^s, \vec{J}_\cap^s, \vec{M}_0^{SE}\}}^{inp} \cdot \bar{a}^{\{\vec{J}_0^s, \vec{J}_\cap^s, \vec{M}_0^{SE}\}} \end{aligned} \tag{170}$$

here

$$\bar{\bar{P}}_{\{\vec{J}_0^s, \vec{J}_0^{SE}, \vec{J}_\cap^s\}}^{inp} = \begin{bmatrix} \bar{\bar{I}} & 0 \\ 0 & \bar{\bar{I}} \\ 0 & \bar{\bar{T}}_{Magnetic\ Extinction}^{\{\vec{J}_0^{SE}, \vec{J}_\cap^s\}\rightarrow\vec{M}_0^{SE}} \end{bmatrix}^H \cdot \bar{\bar{P}}_{\{\vec{J}_0^s, \vec{J}_0^{SE}, \vec{J}_\cap^s, \vec{M}_0^{SE}\}}^{inp} \cdot \begin{bmatrix} \bar{\bar{I}} & 0 \\ 0 & \bar{\bar{I}} \\ 0 & \bar{\bar{T}}_{Magnetic\ Extinction}^{\{\vec{J}_0^{SE}, \vec{J}_\cap^s\}\rightarrow\vec{M}_0^{SE}} \end{bmatrix} \tag{171}$$

$$\bar{\bar{P}}_{\{\vec{J}_0^s, \vec{J}_\cap^s, \vec{M}_0^{SE}\}}^{inp} = \begin{bmatrix} \bar{\bar{I}} & 0 \\ 0 & \bar{\bar{T}}_{Electric\ Extinction}^{\{\vec{J}_\cap^s, \vec{M}_0^{SE}\}\rightarrow\vec{J}_0^{SE}} \\ 0 & \bar{\bar{I}} \end{bmatrix}^H \cdot \bar{\bar{P}}_{\{\vec{J}_0^s, \vec{J}_0^{SE}, \vec{J}_\cap^s, \vec{M}_0^{SE}\}}^{inp} \cdot \begin{bmatrix} \bar{\bar{I}} & 0 \\ 0 & \bar{\bar{T}}_{Electric\ Extinction}^{\{\vec{J}_\cap^s, \vec{M}_0^{SE}\}\rightarrow\vec{J}_0^{SE}} \\ 0 & \bar{\bar{I}} \end{bmatrix} \tag{172}$$

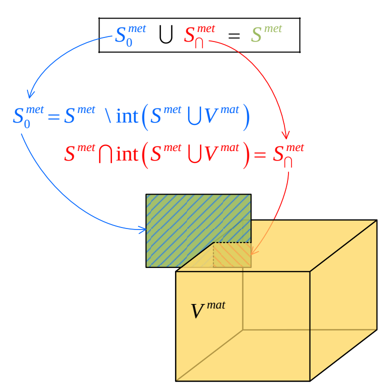



Fig. 11. A piece of metal surface is partially submerged into material body.

and

$$\bar{a}^{\{\vec{J}_0^s,\vec{J}_0^{SE},\vec{J}_\cap^s\}} = \begin{bmatrix} \bar{a}^{\vec{J}_0^s} \\ \bar{a}^{\vec{J}_0^{SE}} \\ \bar{a}^{\vec{J}_\cap^s} \end{bmatrix} = \begin{bmatrix} \bar{a}^{\vec{J}_0^s} \\ \begin{bmatrix} \bar{a}^{\vec{J}_0^{SE}} \\ \bar{a}^{\vec{J}_\cap^s} \end{bmatrix} \end{bmatrix} \tag{173}$$

$$\bar{a}^{\{\vec{J}_0^s,\vec{J}_\cap^s,\vec{M}_0^{SE}\}} = \begin{bmatrix} \bar{a}^{\vec{J}_0^s} \\ \bar{a}^{\vec{J}_\cap^s} \\ \bar{a}^{\vec{M}_0^{SE}} \end{bmatrix} = \begin{bmatrix} \bar{a}^{\vec{J}_0^s} \\ \begin{bmatrix} \bar{a}^{\vec{J}_\cap^s} \\ \bar{a}^{\vec{M}_0^{SE}} \end{bmatrix} \end{bmatrix} \tag{174}$$

In (171)-(172),

$$\begin{aligned} &\overline{\overline{P}}^{inp}_{\{\vec{J}_0^s,\vec{J}_0^{SE},\vec{J}_\cap^s,\vec{M}_0^{SE}\}} \\ &= -\overline{\overline{P}}^{\{\vec{J}_0^s,\vec{J}_\cap^s\}\leftarrow\mathcal{E}_{met}^{sca}(\vec{J}_0^s)} - \overline{\overline{P}}^{\{\vec{J}_0^s,\vec{J}_\cap^s\}\leftarrow\mathcal{E}_{ext}^{sca}(\vec{J}_0^{SE},\vec{M}_0^{SE})} \\ &\quad -\frac{\mu\,\Delta\varepsilon_c^*}{\mu_0\varepsilon_0-\mu\varepsilon_c^*}\left[\overline{\overline{P}}^{\{\vec{J}_0^{SE},\vec{J}_\cap^s\}\leftarrow\mathcal{E}_{met}^{sca}(\vec{J}_0^s)} + \frac{\mu_0}{\mu}\overline{\overline{P}}^{\vec{M}_0^{SE}\leftarrow\mathcal{H}_{met}^{sca}(\vec{J}_0^s)}\right] \\ &\quad -\frac{\varepsilon_c\,\Delta\mu}{\varepsilon_0\mu_0-\varepsilon_c\mu}\left\{\frac{\varepsilon_0}{\varepsilon_c}\left[\overline{\overline{P}}^{\{\vec{J}_0^{SE},\vec{J}_\cap^s\}\leftarrow\mathcal{E}_{met}^{sca}(\vec{J}_0^s)}\right]^H + \left[\overline{\overline{P}}^{\vec{M}_0^{SE}\leftarrow\mathcal{H}_{met}^{sca}(\vec{J}_0^s)}\right]^H\right\} \\ &\quad +\frac{\mu\,\Delta\varepsilon_c^*}{\mu_0\varepsilon_0-\mu\varepsilon_c^*}\left[\overline{\overline{P}}^{\{\vec{J}_0^{SE},\vec{J}_\cap^s\}\leftarrow e_{int}^{inc}(\vec{J}_0^{SE},\vec{M}_0^{SE})} + \frac{\mu_0}{\mu}\overline{\overline{P}}^{\vec{M}_0^{SE}\leftarrow h_{int}^{inc}(\vec{J}_0^{SE},\vec{M}_0^{SE})}\right] \\ &\quad +\frac{\varepsilon_c\,\Delta\mu}{\varepsilon_0\mu_0-\varepsilon_c\mu}\left\{\frac{\varepsilon_0}{\varepsilon_c}\left[\overline{\overline{P}}^{\{\vec{J}_0^{SE},\vec{J}_\cap^s\}\leftarrow e_{int}^{inc}(\vec{J}_0^{SE},\vec{M}_0^{SE})}\right]^H + \left[\overline{\overline{P}}^{\vec{M}_0^{SE}\leftarrow h_{int}^{inc}(\vec{J}_0^{SE},\vec{M}_0^{SE})}\right]^H\right\} \end{aligned} \tag{175}$$

and

$$\overline{\overline{T}}^{\{\vec{J}_0^{SE},\vec{J}_\cap^s\}\rightarrow\vec{M}_0^{SE}}_{Magnetic\ Extinction} = \left(\overline{\overline{\Phi}}^{M_0^{SE}H_{int}^{tot}M_0^{SE}}\right)^{-1}\cdot\begin{bmatrix}\overline{\overline{\Phi}}^{M_0^{SE}H_{int}^{tot}J_0^{SE}} & \overline{\overline{\Phi}}^{M_0^{SE}H_{int}^{tot}J_\cap^s}\end{bmatrix} \tag{176}$$

$$\overline{\overline{T}}^{\{\vec{J}_\cap^s,\vec{M}_0^{SE}\}\rightarrow\vec{J}_0^{SE}}_{Electric\ Extinction} = \left(\overline{\overline{\Phi}}^{J_0^{SE}E_{int}^{tot}J_0^{SE}}\right)^{-1}\cdot\begin{bmatrix}\overline{\overline{\Phi}}^{J_0^{SE}E_{int}^{tot}J_\cap^s} & \overline{\overline{\Phi}}^{J_0^{SE}E_{int}^{tot}M_0^{SE}}\end{bmatrix} \tag{177}$$

The various matrices in (175) are as follows

$$\overline{\overline{P}}^{\{\vec{J}_0^s,\vec{J}_\cap^s\}\leftarrow\mathcal{E}_{met}^{sca}(\vec{J}_0^s)} = \begin{bmatrix}\overline{\overline{\Phi}}^{J_0^sE_{met}^{sca}J_0^s} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \overline{\overline{\Phi}}^{J_\cap^sE_{met}^{sca}J_0^s} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0\end{bmatrix} \tag{178}$$

$$\overline{\overline{P}}^{\{\vec{J}_0^s,\vec{J}_\cap^s\}\leftarrow\mathcal{E}_{ext}^{sca}(\vec{J}_0^{SE},\vec{M}_0^{SE})} = \begin{bmatrix}0 & \overline{\overline{\Phi}}^{J_0^sE_{ext}^{sca}J_0^{SE}} & 0 & \overline{\overline{\Phi}}^{J_0^sE_{ext}^{sca}M_0^{SE}} \\ 0 & 0 & 0 & 0 \\ 0 & \overline{\overline{\Phi}}^{J_\cap^sE_{ext}^{sca}J_0^{SE}} & 0 & \overline{\overline{\Phi}}^{J_\cap^sE_{ext}^{sca}M_0^{SE}} \\ 0 & 0 & 0 & 0\end{bmatrix} \tag{179}$$

$$\overline{\overline{P}}^{\{\vec{J}_0^{SE},\vec{J}_\cap^s\}\leftarrow\mathcal{E}_{met}^{sca}(\vec{J}_0^s)} = \begin{bmatrix}0 & 0 & 0 & 0 \\ \overline{\overline{\Phi}}^{J_0^{SE}E_{met}^{sca}J_0^s} & 0 & 0 & 0 \\ \overline{\overline{\Phi}}^{J_\cap^sE_{met}^{sca}J_0^s} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0\end{bmatrix} \tag{180}$$

$$\overline{\overline{P}}^{\vec{M}_0^{SE}\leftarrow\mathcal{H}_{met}^{sca}(\vec{J}_0^s)} = \begin{bmatrix}0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \overline{\overline{\Phi}}^{M_0^{SE}H_{met}^{sca}J_0^s} & 0 & 0 & 0\end{bmatrix} \tag{181}$$

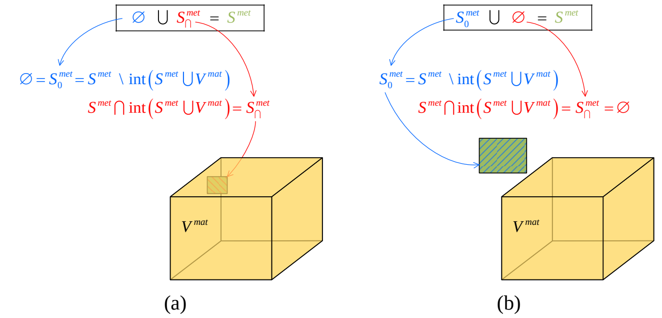



Fig. 12. (a) A piece of metal surface is completely submerged into material body; (b) a piece of metal surface completely does not contact with material body.

$$\overline{\overline{P}}^{\{\vec{J}_0^{SE},\vec{J}_\cap^s\}\leftarrow e_{int}^{inc}(\vec{J}_0^{SE},\vec{M}_0^{SE})} = \begin{bmatrix}0 & 0 & 0 & 0 \\ 0 & \overline{\overline{\Phi}}^{J_0^{SE}e_{int}^{inc}J_0^{SE}} & 0 & \overline{\overline{\Phi}}^{J_0^{SE}e_{int}^{inc}M_0^{SE}} \\ 0 & \overline{\overline{\Phi}}^{J_\cap^se_{int}^{inc}J_0^{SE}} & 0 & \overline{\overline{\Phi}}^{J_\cap^se_{int}^{inc}M_0^{SE}} \\ 0 & 0 & 0 & 0\end{bmatrix} \tag{182}$$

$$\overline{\overline{P}}^{\vec{M}_0^{SE}\leftarrow h_{int}^{inc}(\vec{J}_0^{SE},\vec{M}_0^{SE})} = \begin{bmatrix}0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \overline{\overline{\Phi}}^{M_0^{SE}h_{int}^{inc}J_0^{SE}} & 0 & \overline{\overline{\Phi}}^{M_0^{SE}h_{int}^{inc}M_0^{SE}}\end{bmatrix} \tag{183}$$

The procedures to construct the power-based CM sets corresponding to the structure in Fig. 11 are completely similar to the procedures given in Sec. V, so they will not be repeated here.

The structure in Fig. 11 can be further specialized to the structures in Fig. 12. The formulations corresponding to the structure in Fig. 12 (a) can be obtained by removing the terms corresponding to $\vec{J}_0^s$ from the formulations given in this subsection; the formulations corresponding to the structure in Fig. 12 (b) can be obtained by removing the terms corresponding to $\vec{J}_\cap^s$ from the formulations given in this subsection.

### *E. Scatterer is constructed by one piece of metal body and one piece of material body, and the metal body is partially "contacted with / submerged into" the material body*

In this subsection, the scatterer illustrated in Fig. 13 is considered. Its metal part only includes one piece of body, and the metal body can be viewed as being partially contacted with

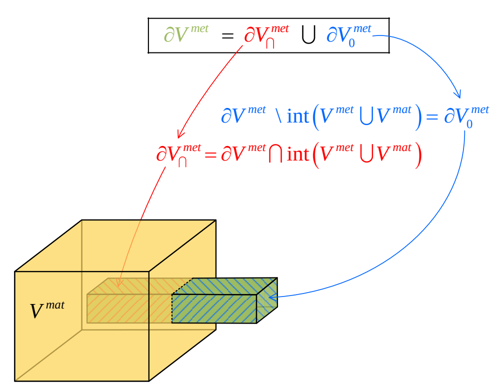



Fig. 13. A piece of metal body is partially contacted with a notched material body.

a notched material body, and also be viewed as being partially "submerged into" the material body. It is obvious that $L^{met}, S^{met} = \varnothing$, so $\vec{J}^{l}, \vec{J}^{SE}_{open\ surf} = 0$, and then the basic variables can be selected as $\{\vec{J}^{s}_{0}, \vec{J}^{SE}_{0}, \vec{J}^{s}_{\cap}\}$ or equivalently selected as $\{\vec{J}^{s}_{0}, \vec{J}^{s}_{\cap}, \vec{M}^{SE}_{0}\}$, here $\vec{J}^{s}_{\cap} = \vec{J}^{SE}_{\partial V^{met}_{\cap}}$.

For this case, the input power operator (54) is specialized to the following (184)

$$
\begin{aligned}
&P^{inp}\\
&= -(1/2)\left\langle \vec{J}^{s}_{0} + \vec{J}^{s}_{\cap}, \mathcal{E}^{sca}_{met}\left(\vec{J}^{s}_{0}\right)\right\rangle_{\partial V^{met}_{0} \cup \partial V^{met}_{\cap}}\\
&\quad -(1/2)\left\langle \vec{J}^{s}_{0} + \vec{J}^{s}_{\cap}, \mathcal{E}^{sca}_{ext}\left(\vec{J}^{SE}_{0}, \vec{M}^{SE}_{0}\right)\right\rangle_{\partial V^{met}_{0} \cup \partial V^{met}_{\cap}}\\
&\quad -\frac{\mu\,\Delta\varepsilon_c^*}{\mu_0\varepsilon_0 - \mu\varepsilon_c^*}\left[\frac{1}{2}\left\langle \vec{J}^{SE}_{0} + \vec{J}^{s}_{\cap}, \mathcal{E}^{sca}_{met}\left(\vec{J}^{s}_{0}\right)\right\rangle_{\partial V^{mat}_{0} \cup \partial V^{met}_{\cap}} + \frac{\mu_0}{\mu}\frac{1}{2}\left\langle \vec{M}^{SE}_{0}, \mathcal{H}^{sca}_{met}\left(\vec{J}^{s}_{0}\right)\right\rangle_{\partial V^{mat}_{0}}\right]\\
&\quad -\frac{\varepsilon_c\,\Delta\mu}{\varepsilon_0\mu_0 - \varepsilon_c\mu}\left[\frac{\varepsilon_0}{\varepsilon_c}\frac{1}{2}\left\langle \vec{J}^{SE}_{0} + \vec{J}^{s}_{\cap}, \mathcal{E}^{sca}_{met}\left(\vec{J}^{s}_{0}\right)\right\rangle^{*}_{\partial V^{mat}_{0} \cup \partial V^{met}_{\cap}} + \frac{1}{2}\left\langle \vec{M}^{SE}_{0}, \mathcal{H}^{sca}_{met}\left(\vec{J}^{s}_{0}\right)\right\rangle^{*}_{\partial V^{mat}_{0}}\right]\\
&\quad +\frac{\mu\,\Delta\varepsilon_c^*}{\mu_0\varepsilon_0 - \mu\varepsilon_c^*}\left[\frac{1}{2}\left\langle \vec{J}^{SE}_{0} + \vec{J}^{s}_{\cap}, \mathcal{e}^{inc}_{int}\left(\vec{J}^{SE}_{0}, \vec{M}^{SE}_{0}\right)\right\rangle_{\partial V^{mat}_{0} \cup \partial V^{met}_{\cap}} + \frac{\mu_0}{\mu}\frac{1}{2}\left\langle \vec{M}^{SE}_{0}, \mathcal{h}^{inc}_{int}\left(\vec{J}^{SE}_{0}, \vec{M}^{SE}_{0}\right)\right\rangle_{\partial V^{mat}_{0}}\right]\\
&\quad +\frac{\varepsilon_c\,\Delta\mu}{\varepsilon_0\mu_0 - \varepsilon_c\mu}\left[\frac{\varepsilon_0}{\varepsilon_c}\frac{1}{2}\left\langle \vec{J}^{SE}_{0} + \vec{J}^{s}_{\cap}, \mathcal{e}^{inc}_{int}\left(\vec{J}^{SE}_{0}, \vec{M}^{SE}_{0}\right)\right\rangle^{*}_{\partial V^{mat}_{0} \cup \partial V^{met}_{\cap}} + \frac{1}{2}\left\langle \vec{M}^{SE}_{0}, \mathcal{h}^{inc}_{int}\left(\vec{J}^{SE}_{0}, \vec{M}^{SE}_{0}\right)\right\rangle^{*}_{\partial V^{mat}_{0}}\right]
\end{aligned}
\tag{184}
$$

Its matrix expression is identical to the (170) in form.

The procedures to construct the power-based CM sets corresponding to the structure in Fig. 13 are completely similar to the procedures given in Sec. V, so they will not be repeated here.

In fact, the structure in Fig. 13 can be further specialized to the structures in Fig. 14. The formulations corresponding to the structure in Fig. 14 (a) can be obtained by removing the terms corresponding to $\vec{J}^{s}_{0}$ from the formulations corresponding to the structure in Fig. 13; the formulations corresponding to the structures in Fig. 14 (b) and (c) are identical to the formulations corresponding to the structure in Fig. 13; the formulations corresponding to the structure in Fig. 14 (d) can be obtained by removing the terms corresponding to $\vec{J}^{s}_{\cap}$ from the formulations corresponding to the structure in Fig. 13.

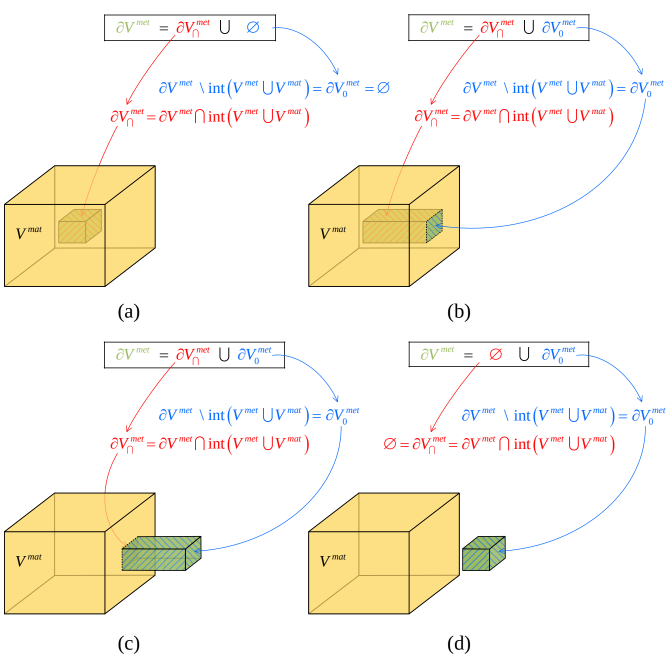



Fig. 14. (a) A piece of metal body is completely submerged into a material body; (b) a piece of metal body partially contacts with a notched material body; (c) a piece of metal body partially contacts with a material body; (d) a piece of metal body completely does not contact with a material body.

corresponding to the structure in Fig. 13.

## VIII. Conclusions

In this paper, a line-surface equivalent principle is established for the material body whose boundary includes some lines and open surfaces beside a closed surface. The traditional surface equivalent principle for the material body whose boundary is a closed surface can be viewed as the special case of the line-surface equivalent principle.

The applicable range of this Part II is larger than the previous Part I, for example, the formulations given in this Part II is not only suitable for the case that the metal lines are not submerged into material body, but also suitable for the case that some metal lines are completely or partially submerged into material body. The formulations corresponding to variable unification in expansion vector space are explicitly provided in this Part II, and the styles of these formulations are consistent with the variable unification formulations in the previously established Surface formulations of the EMP-CMT for Material bodies (Surf-Mat-EMP-CMP) [3], [7]-[8]. The number of arguments in the new input/output power operator provided in this Part II is less than the previous Part I, because the surface equivalent currents $\{\vec{J}^{SE}_{+}, \vec{M}^{SE}_{+}\}$ and $\{\vec{J}^{SE}_{-}, \vec{M}^{SE}_{-}\}$ are separately treated in Part I, but they are treated as a whole in this Part II. In addition, the input/output power operator used in Part I includes some volume integrals besides some line and surface integrals, whereas there does not exist any volume integral in the new input/output power operator provided in this Part II, so this Part II is a "real" LS-MM-EMP-CMT.

Due to the object-oriented feature of EMP-CMT, a new CM set, intrinsic resonant CM set, is introduced into the EMP-CMT family, and a series of new concepts related to intrinsic resonance are introduced.

The LS-MM-EMP-CMT provided in the previous Part I and this Part II has many valuable applications in electromagnetic engineering society, especially the antenna engineering community, for example, it can be efficiently utilized to analyze and design the antennas with a metal-material combined structure, such as the microstrip antennas, the probe-fed DRAs, and the aperture-fed DRAs mounted on a metal plate, etc.

## Acknowledgement

This work is dedicated to my mother.

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