A Note on Heat Multiplication Factor

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Abstract

Kelvin, one of the founders of thermodynamics, proposed an economical, thermodynamic method to heat houses. The method employs a combination of two Carnot heat engines. One engine runs in clockwise direction while the other runs in counterclockwise direction. This combination is claimed to provide much more heat into the house for a given amount of fuel used, compared to that obtained through burning that fuel inside the house. The ratio of the two heats, one obtained by Kelvin's method and the other obtained by the burning the fuel inside the house, is known as heat multiplication factor (HMF). This factor could theoretically be quite high (a typical calculation gives more than a factor of 6). We show in this note that Kelvin's method is fallacious - it is impossible to get any more heat by using Kelvin's method than the heat that could be obtained from combustion (burning) of the fuel.

Key Words: Heat multiplication factor, Gain factor, Thermal heating efficiency, Combination of Carnot cycles.

Introduction

In some parts of the world the ambient temperatures are very low, especially in winter. It becomes necessary to burn some fuel in a convenient place in the house to maintain it warm for comfortable living. It is an expensive affair. Conventionally, fuel is burnt inside the house to produce heat. Wm. Thomson (later Lord Kelvin) proposed an alternate method for heating houses that is claimed to be much more economical.

Lord Kelvin was one of the founders of the subject of thermodynamics. He formulated second law of thermodynamics. Absolute temperature scale is named after him as 'Kelvin scale of temperature' and the unit of temperature on this scale is kelvin (K). Based on his expertise in thermodynamics he proposed an economical method of heating buildings [1]. The method involves burning the fuel, not in the house as was done conventionally, but in a boiler maintaining it at a high temperature. A Carnot heat engine (CHE-1) is operated with the boiler as the high temperature heat source and the interiors of the house as the low temperature heat sink. A second Carnot heat engine (CHE-2) is operated in reverse direction so as to work as a heat pump (CHP). This CHP is operated with atmosphere as the low temperature heat source and the interiors of the house as the high temperature heat sink. The work delivered by CHE-1 is used to drive the CHP. The overall result of the process is that heat is absorbed from both the boiler and the atmosphere and delivered to the interiors of the house with no expenditure of work. We pay only for the fuel burnt in the boiler while we get some heat at no cost from the atmosphere, in Kelvin's method. The ratio of the heat obtained by Kelvin's method to that obtained by the combustion (burning) of the fuel is known as 'heat multiplication factor' (HMF) [2,3] or 'gain factor' denoted by letter G. It is also termed as 'thermal heating efficiency'. Janes is an ardent supporter of Kelvin's method [4]. He observes, while discussing this method, that the concept underlying this method completes the logical structure of classical thermodynamics and also offers nontrivial practical applications. Theoretical calculations show that the savings offered by this method could be substantial - as much as an order of magnitude higher. Thermodynamics books [2, 3, 5] include this topic. Students all over the world, especially in US, are asked to solve problems on this method in examinations [6]. Therefore, this method has taken roots in theory, though not in practice.

We show in this note that it is impossible to get, using Kelvin's method, any more heat into the house than the amount of heat that could be obtained by combustion of the fuel. In other words, we show that the free extra heat supposed to be obtained by Kelvin's method is a myth.

Kelvin's method

We discuss Kelvin's method for two different combinations of CHE's:

1. In this combination CHE-1 operates between adiabats S_1 and S_2, and isotherms T_1 and T_2 while CHE-2 operates between the same adiabats but with isotherms T_2 and T_3. $\Delta T_1 = (T_1 - T_2) = (T_2 - T_3) = \Delta T_2$. $\Delta S_1 = (S_2 - S_1) = \Delta S_2$, (see Fig. 2a)

2. In this combination CHE-1 operates between isotherms T_1 and T_2 while CHE-2 operates between isotherms T_2 and T'_3. $\Delta T_1 > (T_2 - T'_3) = \Delta T'_2$. $\Delta S_1 < (S_3 - S_2) = \Delta S_2$. $(0 < T_3 < T'_3 < T_2 < T_1)$, (see Fig. 2b).

A schematic diagram of Kelvin's method is depicted in Fig. 1. Kelvin's analysis goes along the following lines. CHE-1 absorbs Q_1 units of heat from the boiler - heat reservoir (HR) - at temperature T_1, delivers W units of work and rejects Q_{2c} (= $Q_1 - W$) units of heat into the house - HR at temperature T_2. CHE-2 operates in the reverse direction (anticlockwise direction) and acts as a Carnot heat pump (CHP). It absorbs Q_3 units of heat from the atmosphere - HR at temperature T_3 as well as the W units of work delivered by CHE-1, and rejects Q_{2p} (= $Q_3 + W$) units of heat into the house. Thus, for every combined cycle of operation of the two engines, we get $(Q_{2c} + Q_{2p}) = (Q_1 + Q_3)$ units of heat into the house in the place of Q_1 units that we get by directly burning the fuel inside the house. TS diagrams of the processes are shown in Fig. 2a and 2b.

Analysis of Cycle of Combination 1 (Fig. 2a)

Heat multiplication factor (HMF) is defined as the ratio of heat obtained by the thermodynamic method (Q_{2c} + Q_{2p}) to the heat obtained by combustion of the fuel, Q_1. All Qs and Ts are positive quantities.

$$HMF = \frac{(Q_{2c} + Q_{2p})}{Q_1} = \frac{(Q_1 + Q_3)}{Q_1} > 1 \qquad (Fig.2a) \tag{1}$$

$$HMF = \frac{(Q_{2c} + Q'_{2p})}{Q_1} = \frac{(Q_1 + Q'_3)}{Q_1} > 1 \qquad (Fig.2b) \tag{2}$$

Q's, T's and W are all positive quantities and are related as:

$$\frac{Q_1}{T_1} = \frac{Q_{2c}}{T_2} = \frac{Q_3}{T_3} = \Delta S_1, \quad \frac{Q'_{2p}}{T_2} = \frac{Q'_3}{T'_3} = \Delta S_2 = n\Delta S_1, \;\; n\ is\ a\ rational\ number \tag{3}$$

$$(Q_1 - Q_{2c}) = (Q_{2c} - Q_3) = (Q'_{2p} - Q'_3) = W \tag{4}$$

The cyclic processes in Fig. 2a (ΔS is the same for CHE-2 and CHE-1) must satisfy the following conditions:

$$\alpha Q_1 = \beta Q_{2c} = W, \qquad (T_1 - T_2) = (T_2 - T_3) \tag{5}$$

$$\alpha = \frac{W}{Q_1} = \frac{(Q_1 - Q_{2c})}{Q_1} = \frac{(T_1 - T_2)}{T_1} < 1 \tag{6}$$

$$\beta = \frac{W}{Q_{2c}} = \frac{(Q_{2c} - Q_3)}{Q_{2c}} = \frac{(T_2 - T_3)}{T_2} < 1 \tag{7}$$

$$\gamma = \frac{2W}{Q_1} = \frac{(Q_1 - Q_3)}{Q_1} = \frac{(T_1 - T_3)}{T_1} = 2\alpha < 1, \qquad \therefore \alpha < \frac{1}{2} \tag{8}$$

α, β and γ are the efficiencies of CHE-1, CHE-2 and CHE-3 respectively. From (6), (7) and (8) we get,

$$\alpha < \beta < \gamma < 1 \; and \; \frac{\alpha}{\beta} < 1 < \frac{\gamma}{\beta} = \frac{T_2(T_1 - T_3)}{T_1(T_2 - T_3)} = HMF \tag{9}$$

We note that CHE-3 operates between adiabats S_1 and S_2, and isotherms T_1 and T_3. It absorbs Q_1 units of heat from HR at T_1, delivers 2W units of work and rejects Q_3 units of heat to HR at T_3.

$$HMF = \frac{\gamma}{\beta} = \frac{\left(\frac{2W}{Q_1}\right)}{\left(\frac{W}{Q_2}\right)} = 2\frac{Q_2}{Q_1} = 2(1-\alpha) \geq 1, \qquad accordings \; as \;\; \alpha \leq 1/2 \tag{10}$$

Eq (10) shows that HMF =1, demands γ = β, α = 1/2 and $T_3 = 0$, which is impossible, since $T_3 > 0$. Surprisingly, we find from Eq (10), that higher the HMF, lower the efficiency of CHE-1! Such surprising results where devices with lower efficient components lead to higher efficient devices are consequences of the definition of efficiency of Carnot heat engine being less than one. We welcome inefficiency here!

W will not be enough to pump more heat than Q_3 through ΔT_2. Therefore, HMF cannot be greater than one; it can only be less than one. Hence, we conclude from this result that burning fuel inside house is the most efficient and the most economical way of heating a house.

Analysis of Cycle of Combination 2 (Fig. 2b)

The numerical examples that are given imply this type of combination. We show that this combination can do no better than what combination-1 could achieve.

Here, CHE-2' operates between adiabats S_1 and S_2, and isotherms T_2 and T'_3 ($> T_3$). It absorbs Q'_2 units of heat from HR at T_2, delivers W units of work and rejects Q'_3 units of heat to HR at T'_3. $\Delta S_1 < \Delta S_2$, $\Delta S_1 : \Delta S_2 = \Delta T_2 : \Delta T_1 = 1$: n. Efficiency β' is given by,

$$\beta' = \frac{W}{Q'_2} = \frac{(Q'_2 - Q'_3)}{Q'_2} = \frac{(T_2 - T'_3)}{T_2} < \frac{(T_2 - T_3)}{T_2} = \beta < 1 \tag{11}$$

If β' < α < β, then, it should clear that, what could not be achieved through a higher efficiency cyclic process (CHE-2) cannot be achieved through a lower efficiency cyclic process (CHE-2').

The equation for HMF takes the form,

$$HMF = \frac{T_2(T_1 - T'_3)}{T_1(T_2 - T'_3)} = \frac{T_2}{T_1}\left[\frac{(T_1 - T_2) + (T_2 - T'_3)}{(T_2 - T'_3)}\right] = \frac{T_2}{T_1}\left[1 + \frac{(T_1 - T_2)}{(T_2 - T'_3)}\right] \tag{12}$$

$$= \frac{T_2}{T_1}\left[1 + \frac{\Delta T_1}{\Delta T_2}\right] = \frac{T_2}{T_1}\left[1 + \frac{\Delta S_2}{\Delta S_1}\right] = \frac{T_2}{T_1}[1+n] = (1-\alpha)(1+n) \tag{13}$$

$$But, \quad HMF = 2(1-\alpha), \quad \therefore n = 1, \quad \therefore \; \Delta S_2 = \Delta S_1 \tag{14}$$

Therefore, the combination of cycles in Fig. 2b cannot achieve what cannot be achieved by the combination of cycles in Fig. 2a. HMF depends only on the efficiencies of the engines and has no regard for the relative values of the ΔS of the two Carnot cycles involved.

Thus the above analysis leads to the conclusion that Kelvin's process is impossible and heat multiplication by a thermodynamic cyclic process is a myth. Thermal heating efficiency can only be less than 1.

It is important to note that, when CHE-1 and CHE-2 are connected in series with HR at T_2 common to both, and when both are operated as heat engines (no heat pump), it is required that HR at T_2 suffers no change.

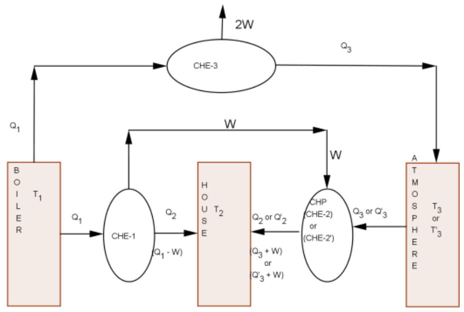

Fig.1 Diagram depicting Kelvin's thermodynamic process that brings about economy in heating houses

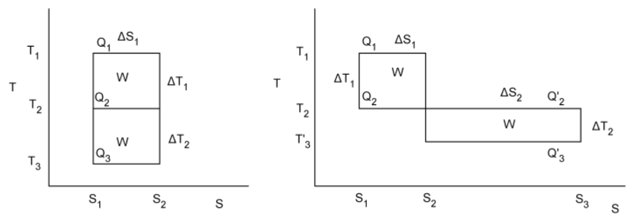

Fig. 2a.

Fig. 2b.

Fig. 2_a, 2_b TS diagrams depicting two different types of combination of two CHEs

Our result appeals to common sense. No process that satisfies conservation of energy can provide more heat for a given amount of fuel to a house than that obtained from burning that fuel inside the house.

Thermodynamics requires us to believe concepts such as: the efficiency of an ideal heat engine is less than one, the coefficient of performance, COP (efficiency) of an ideal heat pump/refrigerator is more than one, Heat multiplication is possible through thermodynamic processes, heat death of the universe and many others, which grate at commonsense.

The origin of these problems lies in the thermodynamic concept of heat. We believe the concept of heat needs a reinvestigation.

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